

A Gaussian Markov Random Field model for the valuation of domestic properties in Wales, UK, using INLA

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Abstract

Location is a key factor in the valuation of residential properties, but it can be a challenge to capture its impact well. This paper considers a Gaussian Markov Random Field (GMRF) approach to modelling property values that enables the location element of the model to be considered as a smooth process and allows a more flexible correlation structure than alternatives. The model was built to value approximately 1.5 million properties in Wales, UK, for a revaluation planned by the Welsh Government. Due to the large scale of the model a Bayesian technique, known as the Integrated Nested Laplace Approximation (INLA), was employed to make the model computationally feasible. This method allowed spatial dependence to be accounted for and improved model performance over a mixed model with nested geographical variables.

Introduction

Council tax is a property tax in England and Wales which is based on the open market value of a domestic property at a set Antecedent Valuation Date (AVD). The AVD in Wales is currently April 1, 2003, whilst in England properties have not been re-valued since the introduction of council tax in 1993 (using an AVD of April 1, 1991). Council tax is a banded system, where properties within a range of values are placed into the same band (see Valuation Office (2024) for more information). The Valuation Office, part of His Majesty's Revenue and Customs, is responsible for banding domestic properties for council tax purposes. Local Authorities are responsible for billing properties and receive the tax revenue raised.

The Welsh Government, as part of their aim to make Council Tax more fair, announced a revaluation of Welsh Council Tax Bands (See Welsh Government (2021) and Welsh Government (2023) for more details on this). Initial plans were for this to be implemented on April 1, 2025 so the VO was commissioned to undertake a revaluation to place properties into new bands based on an AVD of 1st April 2023. Following a public consultation, the Welsh Government decided to introduce the changes more slowly and have a revaluation in 2028.

The VO's aim was to develop an automated valuation model (AVM) which could be used to assist valuation, with the model providing initial valuations. Targeted valuer reviews were then conducted, with greater emphasis in areas where the model performed less well or where estimates were considered close to a band margin. The valuations would be verified or amended by a valuer.

The AVM is built using sales evidence, property attributes and location data. It comprises an initial regression model and a comparables model. The regression model gives an initial point estimate for each property. The comparables model selects the five most comparable properties (based on their attributes, proximity to the subject and date of sales information) and uses these to adjust the initial estimates and to calculate levels of reliability for the final, comparable adjusted estimates. The focus of this paper is on the initial regression model.

There are about 1.5 million residential properties in Wales in a diverse range of settings, including rural, urban, coastal, and industrial areas. The adopted modelling approach therefore needs to be capable of handling a large dataset while dealing suitably with the inherent spatial relationships present in residential property values. This paper demonstrates using a Gaussian Markov Random Field (GMRF) (Rue and Held 2005) approach, using a Bayesian approximation (INLA, Rue et al. (2009)), and compares performance against a mixed effects model with a hierarchy of geographic areas included as random effects.

Data

Sales Data

The VOA's primary source of sales data in Wales is the Welsh Land Transaction Tax (WLTT) data; this is supplemented with HM Land Registry Price Paid data. For more information about these data sources, see Welsh Government (2024) and HM Land Registry (2014).

An AVM typically values properties at a fixed point in time. For the Council Tax revaluation in Wales, the model was developed to estimate the values as of 1 April 2023. To achieve this, the sales data used to build the model were adjusted using the Office for National Statistics (ONS) House Price Index (Office for National Statistics 2023).

For model building and testing, 90% of the available sales data were split into three groups to be used in 3-fold cross-validation, with 10% of data being retained for final model validation. In the final model applied to the full list of properties in Wales, all valid sales were used to build the model.

Property Attribute Data

Property attribute data come from VO's operational database: this is a comprehensive source collected over a number of years. For building the model, property attributes present at the time of the sale were used. For model prediction the latest property attributes were used.

In addition to the property attributes held centrally by VO, plot size data from Land Registry (HMLR) and Ordnance Survey (OS) have been used.

Location Data

The location data used in the final AVM consist of the Billing Authority (a broad administrative area responsible for the collection of Council Tax) associated with each property and the geographical coordinates of the properties. Coordinate data are available for the vast majority (approximately 99.7%) of properties, with postcode centroid coordinates being used as a proxy for making predictions on properties with no available coordinates.

Earlier models used the UK's census geography areas which are a set of nested geographical regions. The Office for National Statistics (ONS) produces various levels of geography after each census. A summary of the methodology can be found in Wyszomierski et al. (2023). These are based more on demographics but do take some housing variables into account and provide ready-made geographies. These are split into 3 tiers:

- Output Areas (OAs): 40 – 250 households
- Lower layer Super Output Areas (LSOAs): 400 – 1,200 households; typically 4-5 OAs.
- Middle layer Super Output Areas (MSOAs) – 2,000 – 6,000 households; typically 4-5 LSOAs.

Testing against market areas previously defined by valuers showed that these premade geographies performed as well.

Data improvement

To improve the accuracy and usability of the model, VO staff undertook data improvement activity:

- Where required, property data were brought up-to-date and data gaps were filled.
- As sales evidence underpins the revaluation, outliers were reviewed by valuers, enabling corrections to be made to property attribute data and sales not representative of open-market value to be removed.

Exploratory Data Analysis

Introduction

Before beginning any model development, a phase of exploratory data analysis was carried out. This analysis was used to inform the development of an initial base model.

The analysis considered the data available, its reliability and its completeness. For potential explanatory variables, their relationships with both property value and other possible explanatory variables were examined. This helped in the following ways:

- identifying the key predictors of property value
- identifying potential transformations of variables
- avoiding multicollinearity between explanatory variables

Details of the variables used in the models can be found in Appendix A

Key Findings

The analysis led to the following key model features:

- Variables identified as the most important for inclusion in the base model:
 - Dwelling Area – size of the property in square metres
 - Dwelling Type – the type of property; e.g. detached, terraced
 - Location variables (BA, MSOA, LSOA)

The (HPI adjusted) sale price was transformed with a natural logarithm transformation, resulting in a multiplicative model.

A log transformation was also used for dwelling area, resulting in an ‘elastic’ model where the rate of increase in price will change as the dwelling area increases. This reflected the expectation of valuer colleagues.

The number of rooms and bedrooms were excluded as explanatory variables due to multicollinearity with the dwelling area variable.

Mixed Effects Model

Introduction

In the early stages of model development, a base model was developed; this included the key explanatory variables identified during the exploratory data analysis and the nested census geography variables. The hierarchical nature of these geographical variables led to the choice of a mixed effects model structure; mixed effects models allow correlation between observations in the same geographical regions to be modelled through inclusion of random intercepts and slopes. Mixed effects models, also commonly known as multi-level or hierarchical models, are commonly used for geographical modelling to account for differences between geographical regions. See Gelman and Hill (2007), for example, for further details of this approach.

One big advantage of using mixed effects models is that, for geographic areas with small sample sizes, the model is able to pool information from higher-level regions in the hierarchy.

Model Specification

After developing the mixed effects base model, further analysis was carried out to improve the model by including additional explanatory variables. Separate models were built for houses and bungalows (referred to as houses) and for flats and maisonettes (referred to as flats). The structures of the final mixed models are given in Table 1.

Table 1: Model structure for the houses and flats models

	Houses	Flats
Fixed Effects	log(dwelling area), dwelling type, age, subsidized flag, parking, log(plot size), Billing Authority	log(dwelling area), dwelling type, age, subsidized flag, parking, bathroom count
Interactions	log(dwelling area) * dwelling type, age * dwelling type, Billing Authority * dwelling type	log(dwelling area) * dwelling type
Random Intercept	MSOA, LSOA (nested variables)	MSOA, LSOA (nested variables)
Random Slope	log(dwelling area)	log(dwelling area)

In addition to these models for houses and flats, reduced versions of the houses and flats models were also built. These models exclude certain variables which were sometimes missing and were applied to a minority of properties. The results presented in this paper are for the full models.

Results

Results presented are based on applying the model to the 10% validation dataset which was not used to build the model.

Table 2 shows the percentages of predictions falling within 10% and 25% of the adjusted sales price and table 3 shows the high-level standard ratio studies metrics used to assess how well the model is performing (See IAAO (2013) for more details on these standards). At this level, the mixed model appears to be performing quite well, with a combined COD of 13.2 for both houses and flats models and with around 88% of predictions being within 25% of the true value. The houses model performs better than the flats model, with the flats model showing signs of vertical inequity. Associated confidence intervals for the metrics in Table 3 are given in Appendix B.

Table 2: Percentages of predictions from the mixed model falling within 10% and 25% of the adjusted sales price

	Predictions within 10%	Predictions within 25%
All	51%	88%
Houses	52%	88%
Flats	46%	83%

Table 3: Ratio studies metrics for the mixed model

	COD	COV	PRD	PRB	Median Ratio
All	13.2	0.20	1.031	-0.051	0.992
Houses	12.9	0.19	1.029	-0.052	0.991
Flats	15.0	0.22	1.043	-0.080	0.999

Figures 1 and 2 show variograms of the residuals from the houses and flats mixed models respectively. A variogram of residuals shows how similar the residuals are for locations at different distances apart: if locations that are closer together have more similar residuals than those further apart, then the residuals are spatially autocorrelated. For both houses and flats models, the variograms show the presence of spatial autocorrelation in the residuals up to a distance of around 400m. This means that the model’s assumption of independent residuals is not valid but also means that the model has not been able to fully capture the spatial patterns dependence in the data. This is addressed further in the next section.

Figure 1: Variogram of the residuals from the houses mixed model

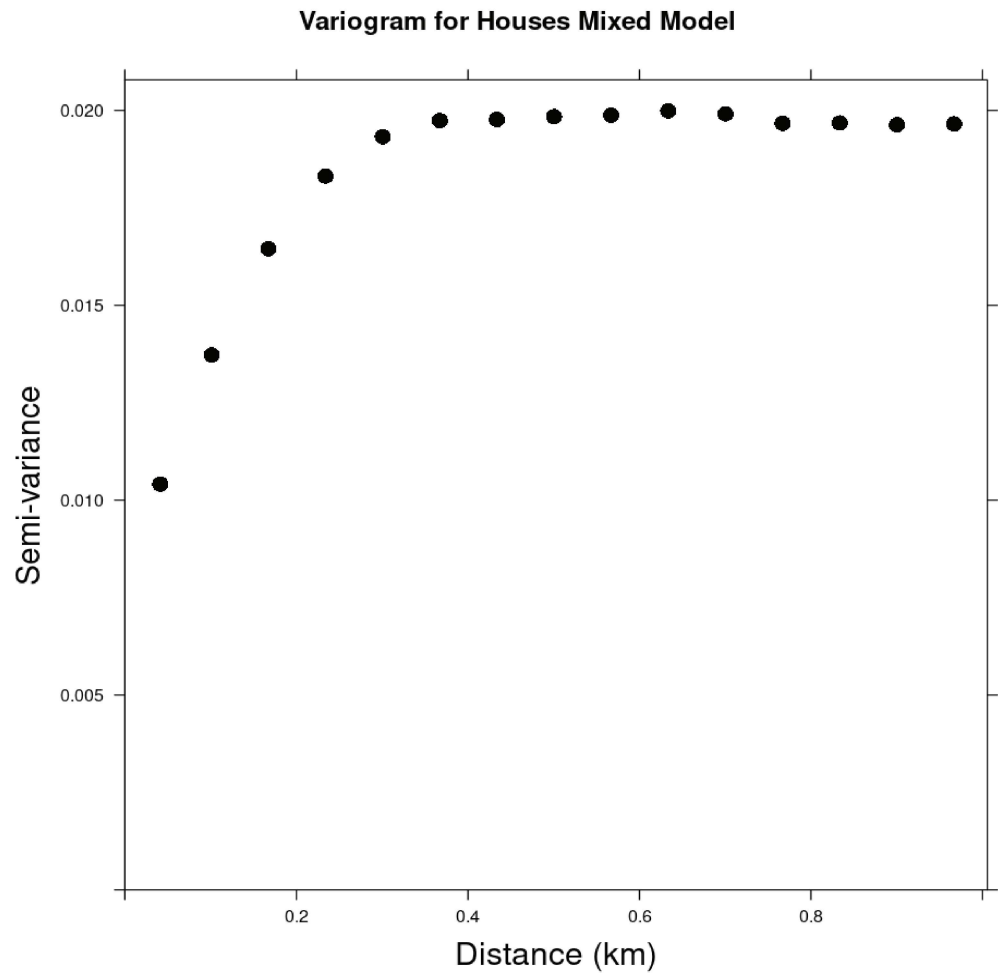
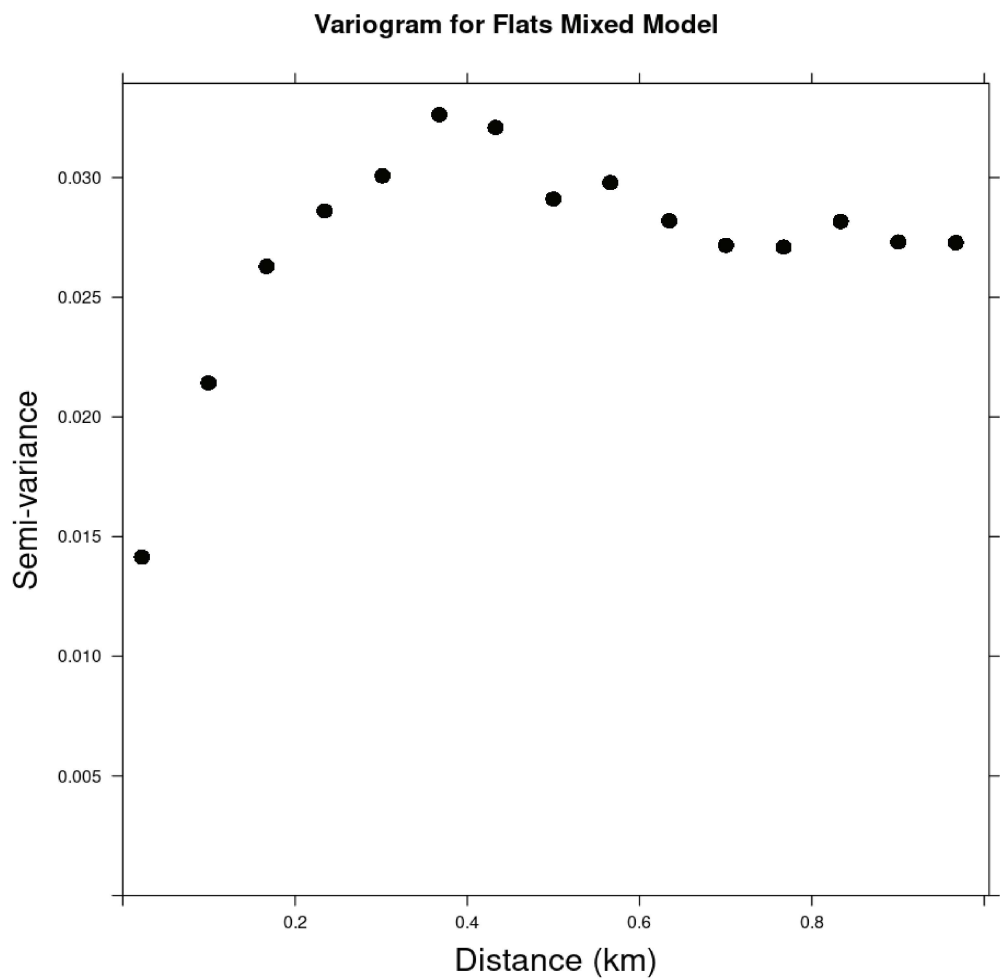


Figure 2: Variogram of the residuals from the flats mixed model



Gaussian Markov Random Field Model Introduction

The presence of spatial autocorrelation in the residuals of the mixed effects model demonstrated the need for an alternative modelling approach. The mixed model, with random effects based on the nested geographical structure, was not able to account for the spatial correlation in the data.

Arcaya et al. (2012) and Wolf et al. (2021) discuss the differences between spatial dependence and “platial” dependence. Platial dependence occurs when observations within the same geographical area are considered related, whereas with spatial dependence observations can be related whether or not they are in the same region. Wolf et al. (2021) comment on the limitation of a hierarchical model for modelling spatial dependence: the mixed effects approach does not account for the specific locations and is therefore not able to model spatial dependence but rather can model platial dependence.

Values of nearby properties are related regardless of whether or not they are in the same geographical region, so an alternative modelling approach was required. To overcome this issue, the values of properties can be considered to vary smoothly over Wales, rather than by fixed geographical regions. Gaussian random fields (GRFs) can be used to model such continuous spatial processes (see, for example, Abrahamsen (1997)). Details of the approach used are given in the following section.

Overview of approach

The high-level model structure is similar to the mixed model structure, with fixed effects terms for the property attributes and a random effects part for the spatial element. The difference is in how the spatial component is parametrized to model a continuous process.

Due to the scale of the model, modelling the Gaussian random field directly would not be feasible. A Gaussian Markov random field (GMRF) model, an approximation to the Gaussian random field, results in a sparse precision matrix (the inverse of the covariance matrix) and makes the model feasible (see Simpson et al. (2012)).

To make this kind of model possible computationally, a Bayesian technique known as the Integrated Nested Laplace Approximation (INLA) is used. INLA was developed by Rue et al. (2009) and uses a stochastic partial differential equation (SPDE) approach to estimate the spatial autocorrelation (See Lindgren et al. (2011) for more details). The R-INLA package (R-INLA Project 2025) makes it relatively easy to fit these models in R.

The SPDE approach used by INLA enables approximation of continuously indexed models with a Matérn covariance structure. The Matérn covariance function for two points separated by a distance, d , is given by the formula:

$$C_v(d) = \sigma^2 \frac{2^{1-v}}{\Gamma(v)} \left(\sqrt{2v} \frac{d}{\rho} \right)^v K_v \left(\sqrt{2v} \frac{d}{\rho} \right)$$

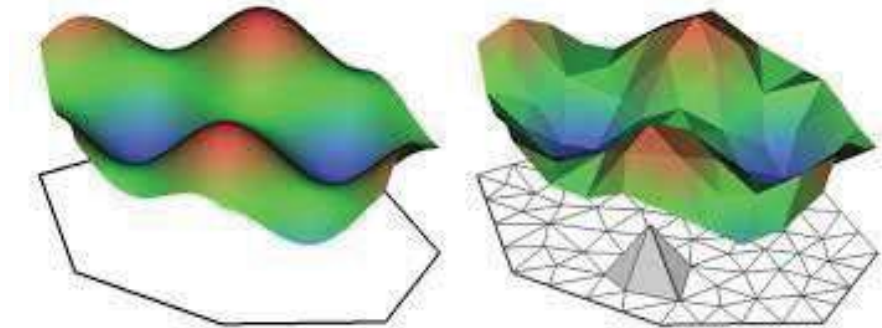
where

- $\Gamma(\cdot)$ is the gamma function;
- $K_v(\cdot)$ is the modified Bessel function of the second kind;
- σ^2 is the marginal variance of the spatial field;
- ρ controls the range of the spatial process;
- ν controls the smoothness of the spatial process.

The smoothness parameter, ν , is difficult to estimate so is usually fixed; in the models presented in this paper, the default value in the R-INLA package ($\nu = 1$) is used. This leaves the variance and range parameters to be estimated.

INLA requires the spatial process to be approximated by defining a triangular mesh (see figure 3). A finer mesh leads to a better approximation to the smooth surface but is computationally more intensive. There is therefore a trade-off between capturing lower-level spatial patterns and the computational time required to run the models.

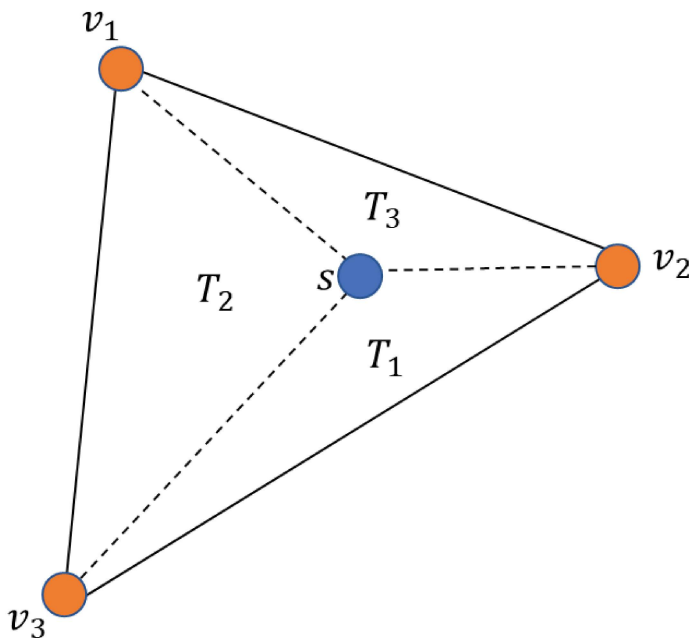
Figure 3: Illustration of the continuous field and its approximation with triangular mesh (figure taken from <https://ourcodingclub.github.io/>, licensed under CC BY-SA 4.0)



The model estimates the value of the spatial process at each triangle vertex in the mesh. The value of the spatial process at any point in the spatial field can then be estimated from the values at the three vertices of the triangle within which the new location lies.

Figure 4 illustrates one triangle of the mesh with vertices v_1 , v_2 , and v_3 .

Figure 4: Illustration of one triangle from the triangular mesh



For point s , which lies inside the triangle, the spatial component $Z(s)$ would be calculated as:

$$Z(s) \approx \frac{A_1}{A} Z_1 + \frac{A_2}{A} Z_2 + \frac{A_3}{A} Z_3,$$

where

- A_1 , A_2 , and A_3 are the areas of the smaller triangles T_1 , T_2 , and T_3
- A is the total area of the large triangle (i.e. $A = A_1 + A_2 + A_3$)
- Z_1 , Z_2 , and Z_3 are the spatial components at vertices v_1 , v_2 , and v_3

The modelling approach used does have some limitations: primarily, the Matérn correlation function is stationary and isotopic. This means that the statistical properties of the spatial process are assumed not to vary by location (stationarity), and that the correlation

depends only on the distance between locations and not the direction (isotropy). As stated by Bakka et al. (2018), these assumptions are rarely entirely true but are made to simplify the construction of valid covariance functions. Bakka et al. (2018) do consider more complex models to address these limitations, but these have not been explored here.

Application of approach

Key Considerations

In applying this modelling approach, several things are considered and set up carefully to produce a suitable model:

- The mesh: a finer mesh will result in better predictions but will require more computing power and take longer to run.
- Priors: INLA is a Bayesian technique which means prior distributions must be specified for the model parameters.

Additionally, after fitting different models, the most suitable model needs to be chosen. The approach to these aspects of the model application is detailed below.

Model Choice

Several Bayesian model criteria are available from the `inla()` function (see Held et al. (2010) and Gelman et al.(2014) for more information).

The Deviance Information Criterion (DIC) (Spiegelhalter et al. 2002) and the Widely Applicable Information Criterion (WAIC) (Watanabe 2013) measure the quality of a model by factoring in both the model fit and the model complexity, penalizing more complex models. These criteria were used in determining the fineness of the mesh and in choosing the fixed effects included in the final model.

Mesh Set-up

In general, the choice of mesh is a trade-off between capturing lower-level features of the spatial process (by using a mesh with small triangles) and computing time required to run the models. There are other considerations to be made when setting up the mesh; full details can be found in Lindgren and Rue (2015), for example.

To avoid building a mesh with too many small triangles in locations with clustered input data, there is a cutoff parameter in the R-INLA mesh function. Additionally, the edge lengths of the triangles should be less than the range of the spatial process so there is a max. edge parameter that sets the maximum allowable triangle edge length.

The settings used in the models are given in table 4. A finer mesh was used in the flats model than for the houses model as the DIC and WAIC showed sufficient improvement for flats but not for houses.

Table 4: Mesh parameters for houses and flats models

Parameter	Houses Model	Flats Model
cutoff	120m	100m
max.edge	600m	500m

Prior Distributions

The parameters of the Matérn Covariance Function must be given prior distributions. These are chosen to be fairly uninformative so that a strong belief about the values of these parameters is not imposed. The same priors were used for both the houses and flats models.

Penalized Complexity (PC) priors (see Simpson et al., 2017) can easily be defined using probability statements about the parameters. The priors were set up as follows:\

- The range (ρ) – this is the range of the spatial process (measured in km). For the prior, a value at which it was unlikely for the range to be below was chosen and the prior was set with the following probability statement:

$$\text{probability } (\rho < 1\text{km}) = 0.05$$

- The scale parameter (σ): This was chosen based on the value of σ resulting from an ‘intercept only model’ (a model with no spatial effect or covariates). It is unlikely for σ to be larger than that of the intercept only model so the prior distribution is specified as

$$\text{probability } (\sigma > \sigma_{\text{intercept only}}) = 0.05.$$

The Fitted Model

In terms of the fixed effects, the structure of the final model follows the structure of the mixed model given in table 4. The random effects part of the mixed model is replaced by the spatial field. This means that in addition to the spatial random effect, the model contains a Billing Authority (BA) fixed effect and an interaction between the BA and dwelling type. Including the BA effect allows higher level geographical differences to be captured while allowing the spatial field to capture lower-level spatial dependence; this improved the model’s performance.

Figure 5 illustrates the combined BA and residual spatial effect across Wales: because of the interaction between BA and dwelling type, this is only for the houses model’s representative dwelling type category (semi-detached houses). Note that the effects have been back-transformed so show multiplicative adjustments for the price. The majority of the adjustments are seen to be downward adjustments, and this is due to the choice of Cardiff BA as the representative BA: Cardiff BA includes the capital city and typically has higher property values than other parts of Wales.

The impact of including a BA fixed effect in addition to the spatial random effect is clearly visible with approximate BA boundaries being visible in figure 5. Where sales data are sparse the combined spatial effect will be closer to the BA effect, but when there are sufficient sales data then there can be more variability in the combined spatial effect.

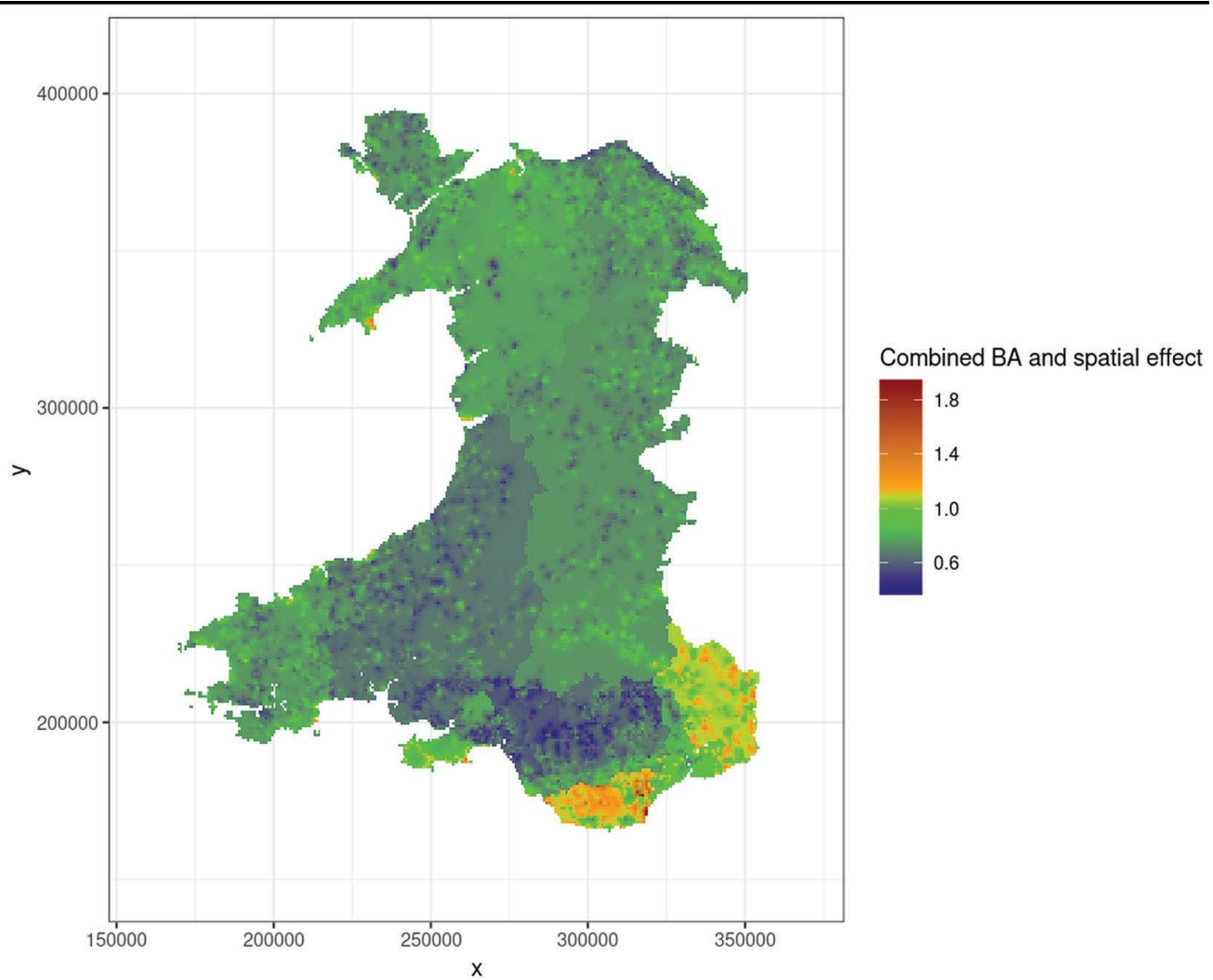


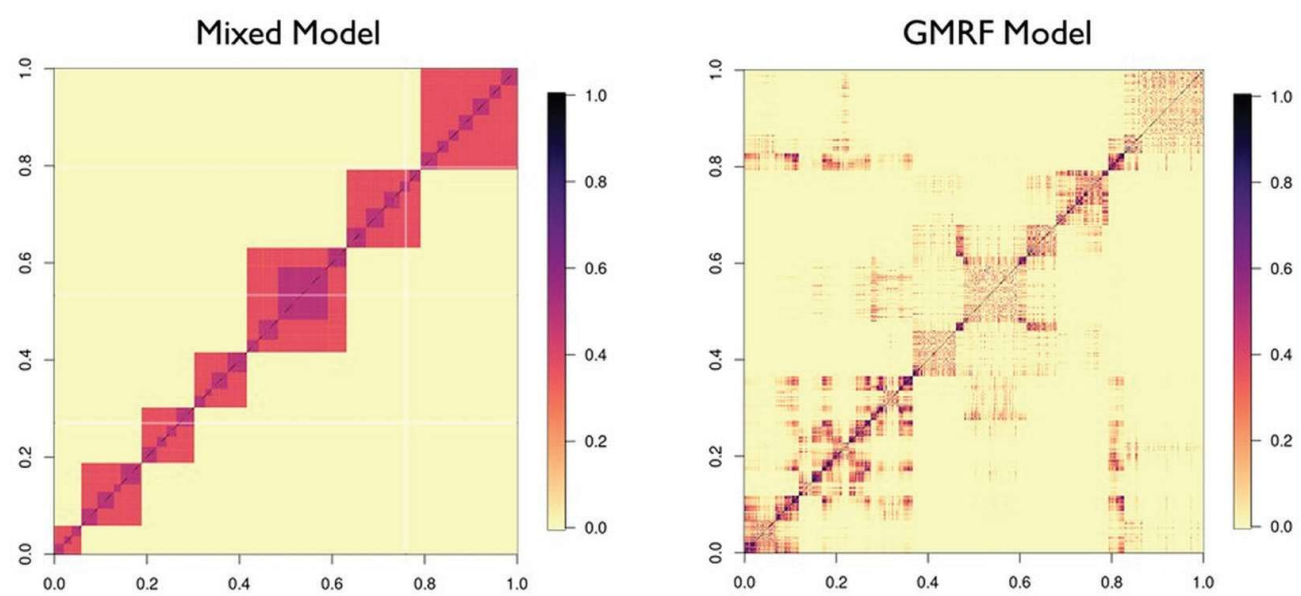
Figure 5: Combined BA and residual spatial effect across Wales

Model Comparisons

Correlation Structure

The mixed model allows correlation between observations within the same geographical regions to be modelled but will assume independence between property values in two different geographical regions. This can be seen by a comparison of the correlation structures from the two houses models for a sub-region of Wales given in figure 6. The mixed model correlation matrix is rigid, only allowing correlation between values in the same geographical regions (shown in the block-like structure). The implied GMRF field correlation matrix, however, shows that correlation is possible between values of properties in different geographical regions, while the values of properties within the same region may not necessarily be correlated. This demonstrates the benefit of using such a model to account for spatial dependence and was also highlighted by Sarafian et al. (2019) who compare the performance of mixed and GMRF models for predicting air pollution levels.

Figure 6: Correlation matrix structures for the mixed and GMRF houses models, for a sub-region of Wales



Spatial Autocorrelation

Figures 7 and 8 show the variograms for the residuals of the houses and flats models respectively; variograms for both the mixed model and GMRF model are given to aid comparison.

For houses, the variogram indicates that using GMRF model has reduced the range of spatial autocorrelation in the residuals slightly, with the curve flattening at around 300m compared with around 400m for the mixed model. For flats, however, using the GMRF model appears to have removed the spatial autocorrelation in the residuals.

Figure 7: Variograms of the residuals for the mixed and GMRF houses models
Variograms for GMRF and Mixed Models for Houses

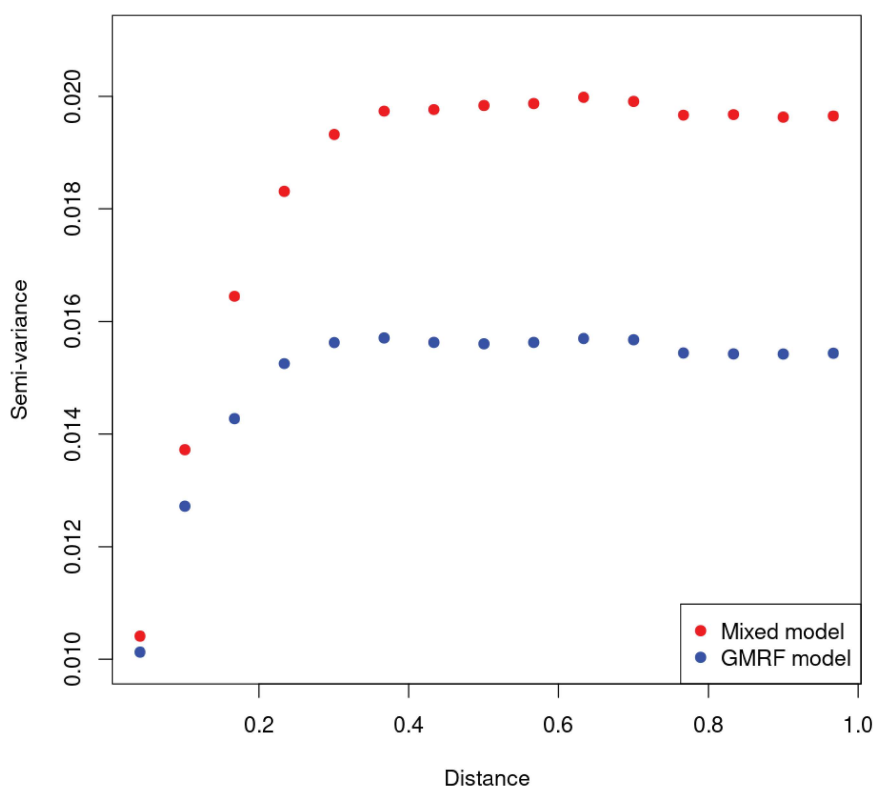
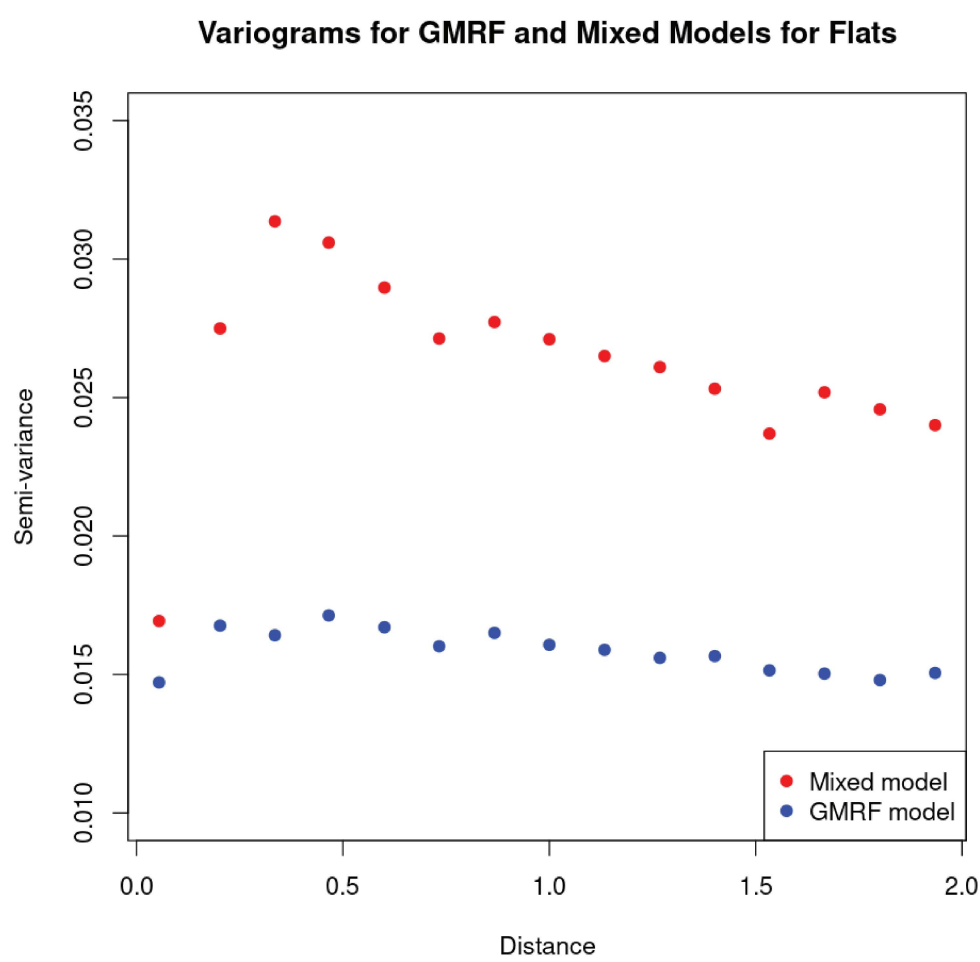


Figure 8: Variograms of the residuals for the mixed and GMRF flats models



Ratio Studies

Table 5 shows the percentages of predictions falling within 10% and 25% of the adjusted sales price and table 6 shows the high-level standard ratio studies metrics used to assess how well the model is performing. Both tables also show the values for the mixed model for comparison.

Table 5: Percentages of predictions falling within 10% and 25% of the adjusted sales price, for both the mixed model and GMRF model

	Model	Predictions within 10%	Predictions within 25%
All	MM	51%	88%
	GMRF	54%	90%
Houses	MM	52%	88%
	GMRF	54%	90%
Flats	MM	46%	83%
	GMRF	55%	88%

Table 6: Ratio studies metrics for both the mixed model and GMRF model

	Model	COD	COV	PRD	PRB	Median Ratio
All	MM	13.2	0.20	1.031	-0.051	0.992
	GMRF	12.2	0.18	1.027	-0.044	0.992
Houses	MM	12.9	0.19	1.029	-0.052	0.991
	GMRF	12.1	0.19	1.026	-0.047	0.991
Flats	MM	15.0	0.22	1.043	-0.080	0.999
	GMRF	12.5	0.18	1.033	-0.064	1.001

Overall, the GMRF model outperforms the mixed model across all metrics. The improvement for the flats model is particularly notable, bringing the model performance much closer to that of the houses model.

Discussion

This paper has considered the use of a Gaussian Markov Random Field to model residential property values in Wales, UK, as a smoothly varying process.

Location is a key factor in the valuation of a residential property. This model formulation has enabled greater flexibility in terms of the correlation structure compared to alternatives such as the mixed effects model, thus enabling spatial dependence to be captured more fully. This has resulted in reduced spatial autocorrelation in the residuals and has improved model performance. The improvement was particularly notable for the flats model, where the variogram indicated no spatial autocorrelation in the residuals.

The inclusion of the BA level fixed effect and the BA – dwelling type interaction, in addition to the spatial field, improved model performance. This means that in areas where sales data are scarce the spatial component is based more heavily on the BA level adjustment.

The presence of low-range autocorrelation in the residuals of the houses model could be investigated further in the future; the inclusion of additional explanatory variables or a finer mesh, while being a lot more computationally intensive, could resolve this. Additionally, extensions to the modelling approach, such as those mentioned in Bakka et al. (2018) to address the limitations of stationarity and isotropy in the correlation structure, could be considered.

While being a more mathematically complex model than some alternatives, it is intuitive in its broad approach: a property’s value is based on its attributes and is then adjusted for its location, where the location adjustments vary smoothly over the geographical space.

The approach is easy to implement, even for large datasets, through the R-INLA package. Other techniques such as Geographically Weighted Regression (GWR), kriging and other non-Bayesian techniques can be a lot more time consuming and may not be feasible for larger datasets. Additionally, the property valuations are more explainable than those from black box approaches such as XGBoost and this is an important consideration when valuing properties for taxation purposes

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Appendix A

Table of descriptions for model variables where a variable shows as being grouped in parentheses, a grouping of the original variable, identified during the analysis process, was used.

Variable	Description
Dwelling type (grouped)	For example; a flat, a terraced house or a detached house.
Dwelling area	The size of the property in m ² . Properties are measured in line with VO's code of measuring practice – guidance on how different properties are measured is available on GOV.UK .
Bathroom count (grouped)	A room is only classed as a bathroom if it has at least 3 fittings; for example, toilet, basin and shower. This means a toilet with just a basin would not be included.
Age code (grouped)	A code indicating the time period within which the property was built.
Parking code (grouped)	A code indicating the type of parking at a property. For example, a garage or open parking space.
Plot Size	The area of the curtilage of the property in m ² .
Subsidised	Variable indicating if the property has been purpose-built as social housing by a Local Authority, Housing Association or other public body.
Billing Authority	The Billing Authority in which a property is located
MSOA and LSOA	Office for National Statistics (ONS) Census geographical regions (LSOAs are nested within MSOAs and MSOAs are nested within BAs)

Appendix B

IAAO Ratio Studies metrics with associated confidence intervals

	COD	COV	PRD	PRB	Median Ratio
All - Mixed	13.2 (13.0, 13.5)	0.20 (0.18, 0.23)	1.031 (1.028, 1.034)	-0.051 (-0.057, -0.045)	0.992 (0.989, 0.995)
All - GMRF	12.2 (12.0, 12.5)	0.18 (0.17, 0.22)	1.027 (1.025, 1.029)	-0.044 (-0.050, -0.039)	0.992 (0.989, 0.996)
Houses - Mixed	12.9 (12.7, 13.3)	0.19 (0.17, 0.23)	1.029 (1.027, 1.032)	-0.052 (-0.060, -0.045)	0.991 (0.988, 0.994)
Houses - GMRF	12.1 (11.9, 12.4)	0.19 (0.16, 0.22)	1.026 (1.023, 1.028)	-0.047 (-0.054, -0.040)	0.991 (0.988, 0.994)
Flats - Mixed	15.0 (14.2, 15.7)	0.22 (0.20, 0.24)	1.043 (1.038, 1.049)	-0.080 (-0.097, -0.063)	0.999 (0.993, 1.008)
Flats - GMRF	12.5 (12.0, 13.1)	0.18 (0.17, 0.20)	1.033 (1.028, 1.038)	-0.064 (-0.078, -0.050)	1.001 (0.992, 1.009)