



Modelling heat emitters within the Home Energy Model

A technical explanation of the methodology

Acknowledgements

This methodology has been developed for the Department for Energy Security & Net Zero by a number of organisations and individuals, including Sustenic, Qidos, Scene Connect, City Science, Hoare Lea, Oxford Brookes University, University of Bath, 10-x, Building Research Establishment (BRE), AECOM, Kiwa Ltd., Loughborough University Enterprises Limited, Chris Martin and John Tebbit.

Quality assurance has been undertaken by a consortium led by Etude, including Levitt Bernstein, People Powered Retrofit, University of Strathclyde's Energy Systems Research Unit, Julie Godefroy Sustainability, and UCL.

Document reference: HEM-TP-16

Document version: v2.0

Issue date: January 2026

Home Energy Model version: HEM 1.0



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Background to the Home Energy Model

What is the Home Energy Model?

The [Home Energy Model \(HEM\)](#) is a calculation methodology designed to assess the energy performance of homes, which will replace the government's [Standard Assessment Procedure \(SAP\)](#).

Where can I find more information?

This document is part of a wider package of material relating to the Home Energy Model.

Home Energy Model technical documentation (e.g. this document)

What: This document is one of a suite of [technical documents](#), which explain the calculation methodology in detail. New documents will be added, and the content amended, when necessary to ensure documentation is sufficiently comprehensive. This will usually, but not always, occur alongside the release of a new version of HEM.

Audience: The technical documentation will be of interest to those who want to understand the detail of how the Home Energy Model works and how different technologies are treated.

The Home Energy Model consultation and government response

What: The [Home Energy Model consultation](#) introduces the overhaul to the SAP methodology and sought views on the approach taken by the new Home Energy Model. The [Home Energy Model consultation](#) summarises the feedback to the consultation and the actions taken subsequently in development, ahead of the initial release of HEM.

Audience: The Home Energy Model consultation will be of interest to those seeking a general introduction to HEM and its role in government policy on domestic energy performance.

The Home Energy Model reference code

What: The full Python source code for the Home Energy Model core engine has been published as a [Git repository](#). Note the reference code for official HEM wrappers is published separately.

Audience: The reference code will be of interest to those who want to understand how the model has been implemented in code, and those wishing to fully clarify their

understanding of the new methodology. It will also be of interest to any potential contributors to the Home Energy Model or those wishing to use it within their own projects.

Related content

This paper sets out the methodology for modelling heat emitters within the core Home Energy Model. Other relevant [papers on the core Home Energy Model](#) include:

- HEM-TP-04 Space heating and cooling demand
- HEM-TP-12 Heat pump methodology
- HEM-TP-14 Boiler methodology
- HEM-TP-15 Heat batteries
- HEM-TP-17 Controls

For further information on relevant assumptions made within the FHS assessment wrapper, please see [HEMFHS-TP-02 FHS space heating and cooling demand assumptions](#).

To understand how this methodology has been implemented in computer code, please see:

`src/hem_core/heating_systems/emitters.py`

Methodology

This document provides a description of how wet heat emitters (radiators, wet underfloor heating, fan coil emitters) are modelled in the Home Energy Model (HEM).

Wet emitters are heated by the circulation of hot water, supplied by a heat source such as a heat pump or boiler, through each emitter via a pipework circuit. Emitter heat output is determined by the emitter characteristics and the temperature difference between the emitters and the surrounding air. A fraction of the heat emitted is assumed to be convective, with the remainder radiative. This convective fraction is taken from the input file and applied in HEM's core heat balance equations.

The responsiveness of radiators and underfloor heating (UFH) depends on their thermal mass - their rate of warm-up and cool-down is modelled in HEM.

Fan coils operate in a slightly different way to radiators and UFH but share some elements of the methodology. The main difference is that their output is looked up directly from manufacturer data, rather than being physically modelled.

The space heating demand is calculated for each zone in the order they appear in the input file¹ (see HEM-TP-04 Space heating and cooling demand). This becomes an input to the emitter calculation for that zone, which calculates the energy required from the heat source (e.g. heat pump) given the emitter properties (e.g. thermal mass). For radiators and UFH, the final emitter temperature and output is then calculated, which is limited by the maximum emitter temperature and the maximum heat output of the heat source. The final emitter temperature is then stored and becomes the initial emitter temperature in the next timestep. The emitter output is returned to the space heating calculation so that the resultant temperature of the zone can be calculated. For fan coils, which are assumed to have negligible thermal mass, the maximum output is looked up from manufacturer's performance data.

Emitters with the same performance characteristics within a zone can be combined in the input file and treated as a single larger emitter. Where there is more than one type of emitter within a zone (e.g. radiators with different performance characteristics, or a mixture of radiators and UFH), these must be entered as separate emitter types in the input file. However, only one type of fan coil can be assigned per zone and fan coils cannot currently be combined with radiators or UFH within the same zone because the modelling approaches are incompatible.

Each zone is currently treated as though independent from the other(s) – this may be revisited in future, but for the time being this means no consideration is given to hydraulic balancing.

¹ In practice this means the first zone is prioritised because its heat demand is met before later zones. This becomes relevant when there is insufficient heating capacity.

Allowance for the pre-heating of emitters or use of setback temperatures is dealt with by the controls module, where specified, and is therefore accounted for in the space heating demand for the timestep that is input to the emitter calculation. See HEM-TP-17 Controls.

1. Flow and return temperatures and maximum emitter temperature

1.1 Flow temperature

The emitter module requires the Eco-design control class (I to VIII)² to be specified and the flow and return temperature depend on the class of the controller used:

- For controllers without weather compensation (classes I, IV, V and VIII), the flow temperature is assumed to be fixed at the design flow temperature from the input file.
- For weather compensating controllers (classes II, III, VI and VII) the flow temperature is dependent on the outside temperature. The minimum flow temperature is also required (the maximum, or design, flow temperature is entered regardless) along with the outside air temperatures at which the maximum and minimum flow temperatures will be set. Linear interpolation is used to calculate the flow temperature based on the outdoor temperature.

1.2 Return temperature

Since the return temperature (of water leaving the emitter) depends on the heat output from the emitter and the heat output depends on the mean emitter temperature (which depends on the return temperature), an iterative solver is used to determine the return temperature and corresponding heat output that can be achieved for a given flow temperature.

To perform this iteration, an initial estimate is made of the return temperature (see below), which allows the power output of the emitter to be calculated. This power output is then used to recalculate the return temperature using equation (1). This is repeated until the power output does not change between iterations, ensuring a thermal balance is reached.

$$\Delta T = \frac{P}{\dot{p}} / (c_w \times \rho) \quad (1)$$

where:

²These classes are fully defined on page 16 of this document from the Official Journal of the European Union - [https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52014XC0703\(01\)](https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52014XC0703(01)). In brief they are: Class I - On/off Room Thermostat. Class II - Weather compensator control, for use with modulating heaters. Class III - Weather compensator control, for use with on/off output heaters. Class IV - TPI10 room thermostat, for use with on/off output heaters. Class V - Modulating room thermostat, for use with modulating heaters. Class VI - Weather compensator and room sensor, for use with modulating heaters. Class VII - Weather compensator and room sensor, for use with on/off output heaters. Class VIII – Multi-sensor room temperature control, for use with modulating heaters.

ΔT is the difference between the flow and return temperatures in Kelvin.

P is the power output of the emitter(s) in kW.

\dot{p} is the flow rate in m³/s.

c_w is the specific heat capacity of water in kJ/kgK

ρ is the density of water in kg/m³.

If there is recirculated water fed back into the flow via a bypass system³, the blended flow temperature is calculated as a weighted average of the flow and return temperatures and used in place of the flow temperature from the heat generator to determine the emitter temperature and its heat output. In this case, the emitter flow temperature will therefore be lower than the flow temperature from the heat source.

There is a difference in approach depending on whether the water flow rate is fixed or variable. Whether the system has a fixed or variable flow rate circulation pump is an input to the model. In the case of a fixed flow rate, the difference between the flow and return temperature across the emitter will vary according to the flow temperature and its power output. In the case of variable flow rate, it is assumed the flow varies to achieve the target temperature difference across the emitter (from the input file) within the bounds of its minimum and maximum flow rate.

Fixed flow rates: The return temperature in °C is initially estimated to be 6/7ths of the flow temperature in °C, prior to being refined by iteration. For flow temperatures above 70°C this is capped at a 60°C return temperature. The 6/7ths rule-of-thumb originated from an old MCS heat pump sizing guide⁴.

Variable flow rates: The return temperature in this case is initially assumed to be equal to the flow temperature minus the emitter design temperature difference, which comes from the input file. If the required flow rate is within the flow rate range specified in the input file, then the initial return temperature assumption is valid, and no further iteration is required to calculate the return temperature. If the required flow rate is outside the flow rate range specified in the input file, then the minimum or maximum value is used and the return temperature is calculated using the iteration process described above.

³ A bypass is a plumbing/control feature that feeds a proportion of the 'return' water (leaving the emitters) back to mix with the 'flow' (going to the emitters) without it going through the heat source. This may be necessary in some systems to limit the emitter temperature to acceptable levels (e.g. in the case of UFH) or to maintain a suitable temperature difference across the heat generator.

⁴ Unfortunately this document is no longer available and the 6/7ths estimate is not referred to in newer MCS guidance here: <https://mcs-certified.com/wp-content/uploads/2020/07/Heat-Pump-Guide.pdf>. However, as this is only used as an initial guess, so the value is not critical.

1.3 Maximum emitter temperature

The maximum achievable temperature of the emitter is calculated as the average of the final flow temperature (the blended temperature in the case of a system with bypass) and return temperature from the iterative calculation.

2. Energy input from heat source

HEM's core heat balance module determines how much heat is required for each zone. This is the energy output required from the heat emitters. In turn this heat must be provided by the heat generator. This section describes how the amount of heat required from the heat generator is calculated.

If the space heating demand on the emitters is zero, then the emitters will be cooling down or at steady state, and no heat will be provided to the emitter by the heat generator. In this case, the calculation skips the following steps and goes straight to section 3. Emitter temperature and output achieved

2.1 Emitter temperature required to meet space heating demand

The energy required to heat the zone, E_{heat_zone} , is converted into a power output demand by dividing by the timestep. The calculated power output demand is therefore the power needed to achieve the set point within the current timestep.

In the case of fan coils, as they are assumed to have negligible thermal mass, the emitter temperature is simply the average of the flow and return temperature. The energy input from the heating source is therefore equal to the energy required by the zone, unless limited by the maximum power output of the fan coil at that emitter temperature (see section 3.2 Fan coils).

In the case of radiators, the following equation (from the 2020 ASHRAE Handbook⁵, modified to sum the output from multiple emitters) gives the power output for a given emitter temperature:

$$P_{heat_zone} = \sum_i c_i (T_E(t) - T_{rm}(t - 1))^{n_i} \quad (2)$$

where:

P_{heat_zone} is power output required to heat the zone, in kW.

$T_E(t)$ is the mean emitter temperature in timestep t .

$T_{rm}(t - 1)$ is the final internal air temperature achieved in the room/zone in the previous timestep; this is used to avoid a circularity in the calculation.

⁵ 2020 ASHRAE Handbook, HVAC Systems and Equipment, Chapter 36, 2. Ratings of Heat-distributing units, Corrections for Nonstandard Conditions, Equation (1), page 644.

c_i and n_i are constants from the characteristic equation of emitters derived from BS EN 442 tests⁶, for emitter i . The constant, c_i , may be entered on a “per meter length” basis, alongside the length of the emitter, and HEM will calculate the resulting c_i required for equation (2).

The characteristics of UFH differ from those of radiators. However, the code approximates wet UFH systems such that they can be treated in the same way as radiators, also using equation (2). Specifically, c is calculated from heat output per m² of floor area and the thermal mass is calculated from the equivalent thermal mass per m² of floor area in kJ/m²K. These values are inputs, pre-calculated according to BEAMA guidance. The exponent n is set equal to 1.

Due to the emitter temperature/power circularity mentioned in section 1.2 Return temperature, the calculation iterates on equation (2) until it finds the emitter temperature required to provide the required power output for the zone. Note that equation (2) sums over all radiator / UFH types in the zone to give a single emitter temperature that provides the correct level of heat output. As noted earlier, fan coils cannot currently be combined with radiators/UFH within a zone.

If the emitter temperature at the start of the timestep is already higher than the required temperature, the emitters are allowed to cool (i.e. with no heat input from the heat source) to below the required temperature in order that the cumulative energy output during this period is the same as it would have been had the emitter started the timestep at the required temperature and maintained this temperature. The time taken to complete this phase is calculated by iteration, and the emitter temperature from the final iteration is recorded to be used later – see section 2.3 Energy input required to reach maximum emitter temperature. Note that it is possible for this cool-down period to last for the entire timestep.

2.2 Energy input to emitters required to meet space heating demand

The energy provided to the emitters by the heat source must include the energy needed to warm the emitters up to the required temperature when accounting for their thermal mass, $E_{heat_emitter}$, as well as the energy needed to increase/maintain the temperature of the zone, E_{heat_zone} .

Equation (3) gives the energy required to warm the emitters:

$$E_{heat_emitter} = K_E \times (T_E(t) - T_E(t - 1)) \quad (3)$$

where:

$T_E(t)$ is the emitter temperature in timestep t .

$T_E(t - 1)$ is the emitter temperature at the end of the previous timestep.

K_E is the thermal mass of the emitters.

⁶ Note that c is referred to as k_m in BS EN 442, but the equations are in different units. Therefore, k_m must be divided by 1,000 to give c , so that equation (2) gives a result in kW rather than W. Please see Heat Emitter Input Estimator for more details on how to calculate the c and n values.

The thermal mass may be calculated by HEM based on the specific thermal mass, where the units and calculation depend on the emitter type. For radiators, the thermal mass can be entered on a “per meter length” basis alongside the length of the emitter and for underfloor heating on a “per m²” basis alongside the floor area. In each case, HEM calculates the thermal mass to be used in equation (3). Entering the required emitter temperature into equation (3) gives the energy needed to warm the emitters to the required temperature. Note that if the emitter temperature is already above the required temperature, then the result of equation (3) will be negative.

Equation (4) gives the total energy required to meet the space heating demand:

$$E_{heat_required} = E_{heat_zone} + E_{heat_emitter} \quad (4)$$

However, this calculation of the energy input required is not yet limited by the maximum emitter temperature (see next section).

2.3 Energy input required to reach maximum emitter temperature

The energy input provided by the heat source must be limited so that the emitter temperature does not exceed its maximum or provide more energy than the zone requires. This is achieved by dividing the timestep into three periods:

1. The period during which emitters that begin the timestep at a higher temperature than required temperature are cooling down (see section 2.1 Emitter temperature required to meet space heating demand).
2. The period during which the emitters are warming up to the maximum temperature or cooling down to the maximum temperature. Where the emitter starts the timestep below the maximum emitter temperature, it is assumed to be heated at the maximum rate the heat generator can supply. Where the emitter starts the timestep above the maximum temperature⁷ it is assumed that the heat source output will be zero until the maximum emitter temperature is reached.
3. The period during which the emitters maintain the maximum temperature.

The power input from the heat source during the first (cool-down) period is zero, allowing the emitters to cool.

During the second (warm-up) period the power input from the heat source is the maximum power output of the heat source (calculated by the relevant heat source module, e.g., heat pump module). Note that this will be reduced if the heat source has already allocated some output to higher-priority services⁸.

⁷ For example if the maximum temperature has been reduced compared to the previous timestep due to weather compensation adjusting flow and return temperatures.

⁸ For example, if half of its maximum energy output for the timestep is required to provide water heating, only the remaining half of its power is available for space heating.

Then, the time required to reach the maximum emitter temperature is calculated by iteratively solving the following differential equation (derivation in [Annex A](#)) using the heat source power input figure determined above as the power input, P_{input} :

$$\frac{d\Delta T}{dt} = \frac{P_{input} - \sum_i c_i \times (\Delta T)^{n_i}}{K_E} \quad (5)$$

where ΔT is the difference between the emitter temperature and the room temperature.

It is possible for the cool-down and/or warm-up periods to last for the entire timestep, in which case there is no power input associated with the third (fixed temperature) period. If the emitters do reach the maximum temperature during the timestep, then the power input required to maintain this temperature during the warm-up period is calculated using equation (2).

Over the full timestep, the energy input needed to heat the emitters to the maximum temperature is therefore the sum of the energy input for the three periods described above. This is calculated by multiplying the length of each period by the power input in that period.

2.4 Energy input from heat source

The total energy input required from the heat source is therefore the lower of:

- Energy input required to meet space heating demand.
- Energy input required to reach maximum emitter temperature.

Note that the total energy input is not allowed to cause the emitter temperature to exceed its maximum limit. Therefore, the heat source cannot continue increasing the emitter temperature, even if the demand is still not satisfied, once the emitter has reached its temperature limit.

The energy input required from the heat source is passed to the relevant heat source module (e.g. the heat pump module) along with the flow and return temperatures (which may affect the efficiency of the heat source). The heat source module then calculates the energy that can be provided to the emitters (which is returned to the emitter module) and the associated fuel consumption required (which is stored for later). This takes into account the reduced time available from the heat source due to any initial cool down period by reducing the power available from the heat source in proportion to the fraction of the timestep taken up by the cool-down period.

3. Emitter temperature and output achieved

3.1 Radiators and underfloor heating

If the emitter temperature required to meet the space heating demand is higher than the maximum temperature, and if the calculations described above determine that the maximum emitter temperature could be reached within the timestep, then the final emitter temperature achieved is taken to be the maximum emitter temperature.

Otherwise, the temperature of the emitters at the end of the timestep is calculated by iteratively solving equation (5) using the power input actually provided by the heat source. If there was no space heating demand, then the power input is zero.

The calculation of emitter temperature is based on the following assumptions:

- During any period when the emitters are cooling down, the heat source provides no heat.
- During any period when the emitters are warming up, the heat source output is front-loaded (but subsequent to completion of any cool down period) and delivers the maximum output until the emitters have reached the required temperature. The time taken to reach this state is recorded.
- Once the required temperature is reached, the power from the heat source is reduced to the level needed to maintain this temperature for the remainder of the timestep.

If higher priority services (such as domestic hot water) are required during the timestep, this is accounted for by reducing the power available for the current service proportionately over the whole timestep. This is a simplification because it ignores the specific timing of the events, which could in practice cause intermittent operation or delayed start for the current event. In practice, the error that this introduces into the calculation is small. Note that this power reduction interacts multiplicatively with the reduction in service time caused by the cool-down period, whereby this fraction of the timestep is not available for heat output. For example, if the cool-down period lasts for half of the timestep, and half of the capacity of the heat source has been used for higher-priority services, then only a quarter of the full capacity of the heat source will be available for the current service.

If the final emitter temperature calculated is less than the internal air temperature (taken from the previous timestep to avoid circularity) then the final emitter temperature is reset to the air temperature. In this situation equation (6), below, is used to calculate the energy absorbed by the emitter during the cooling process from the initial emitter temperature down to the final temperature (internal air temperature), giving a negative emitter output (representing heat absorbed from the room into the thermal mass), ensuring the conservation of energy.

Once the final emitter temperature has been obtained, the energy output by the emitters can be calculated by rearranging equation (4) to give the equation below:

$$E_{heat_zone} = E_{heat_required} - E_{heat_emitter} \quad (6)$$

where the energy used to warm up emitters is calculated by entering the final emitter temperature into equation (5).

The energy output by the emitters is then returned to the space heating calculation so that the final zone temperature can be calculated.

3.2 Fan coils

The treatment of fan coil outputs is much simpler than the treatment of radiators and UFH because fan coils are assumed to have negligible thermal mass and their output is based entirely on interpolation of test data.

For fan coil emitters, the maximum output is determined from;

- the temperature difference between room temperature and the average of the flow and return temperatures of the emitter, and
- interpolation of test data using this temperature difference.

The actual output is the minimum of either a) the power output required or b) the maximum output the fan coil can achieve. The fan power is interpolated from test data based on the heat output required and the flow/return temperature⁹.

4 Heat losses

Thermal energy losses from space heating distribution pipework to the dwelling are calculated explicitly and accounted for in the internal gains and required emitter output. For radiators and wet underfloor systems, an additional emitter is modelled to represent the properties of the uninsulated distribution pipework, while it is assumed that no losses are incurred from insulated pipework. For fan coil emitters, thermal energy losses from both insulated and uninsulated pipework are calculated (based on the steady state conditions including the flow temperature and internal air temperature) and subtracted from the required output of the fan coil emitters while being included in the required output of the heating technology.

5 Unmet demand

If the maximum amount of heat the emitters can provide in a given timestep is less than is required by the zone and there is no backup heating system, the shortfall is recorded and reported as unmet demand in the output file. See HEM-TP-04 for details.

6 Circulation pump and auxiliary energy

The energy used by the circulation pump providing flow through the heat source is calculated in the relevant heat source module. Similarly, any additional pumps (e.g. secondary circulation where a buffer tank is used) and the electrical power consumed by the heat source, are also dealt with in the relevant heat source module. See papers relating to each heat source for details:

⁹ For a given flow temperature, the fan speed/power is varied to give the required heat output. The heat output and fan power data from this is looked-up is included in the input file.

- HEM-TP-12 Heat pump methodology
- HEM-TP-14 Boiler methodology
- HEM-TP-15 Heat batteries

Future development

The emitter calculation is currently done on a per-zone basis, treating the emitters in each zone as independent from each other when in reality they may be on the same circuit and therefore not separately controlled. This is an area for potential future development.

The model allows for different values of constants from the characteristic equation of emitters to be entered within a zone. This means that different types of emitters can be modelled in combination, however this does not extend to fan coil heaters in combination with other types. Allowing these as part of a mixed-emitter system is an area for potential future development.

Pump power is currently handled in the individual heat source modules (see section 6 *Circulation pump and auxiliary energy*), but in future these calculations could potentially be consolidated into the emitter module. Pump power is currently assumed to be constant for the required duration of heat output based on a value provided as part of the input file. The calculation of variable flow rates offers the potential to better estimate the pump power and auxiliary energy consumed within a timestep in future.

Fan assisted radiators are not yet included. These may be added in future.

More generally, the methodology could in future adopt some features (or larger sections) from EN 15316-2:2017 and EN 15316-3:2017 that are not already included, for example:

- System losses to outside spaces
- Calculation of necessary pipework lengths based on dimensions
- Calculation of required pump power, taking account of pressure loss
- Specific treatment for radiant panels
- Hydraulic balancing

Annex A – Derivation of differential equation for emitter temperature

A differential equation for change rate of emitter temperature was derived, to be solved iteratively to calculate the emitter temperatures.

Heat balance equation for emitters (conservation of energy):

$$\frac{K_E \times (T_E(t) - T_E(t-1))}{timestep} = P_{input} - P_{output} \quad (7)$$

where:

$T_E(t)$ is mean emitter temperature at the current timestep.

$T_E(t - 1)$ is mean emitter temperature at the previous timestep.

K_E is thermal mass of emitters

P_{input} is power input in Watts

P_{output} is power output in Watts

Power output from emitter (equation from 2020 ASHRAE Handbook p644, equation 1):

$$P_{output} (kW) = c \times (T_E(t) - T_{rm})^n \quad (8)$$

where:

T_{rm} is the air temperature in the room/zone.

c and n are constants from the characteristic equation of emitters (e.g., derived from BS EN 442 tests)

Substituting power output equation into heat balance equation gives:

$$\frac{K_E \times (T_E(t) - T_E(t-1))}{timestep} = P_{input} - c \times (T_E(t) - T_{rm})^n \quad (9)$$

Rearranging gives:

$$\frac{T_E(t) - T_E(t-1)}{timestep} = \frac{P_{input} - c \times (T_E(t) - T_{rm})^n}{K_E} \quad (10)$$

which gives the differential equation as timestep goes to zero:

$$\frac{dT_E}{dt} = \frac{P_{input} - c \times (T_E - T_{rm})^n}{K_E} \quad (11)$$

If T_{rm} is assumed to be constant over the time period, then the rate of change of T_E is the same as the rate of change of ΔT , where:

$$\Delta T = T_E - T_{rm} \quad (12)$$

Therefore, the differential equation can be simplified to an expression in terms of ΔT :

$$\frac{d\Delta T}{dt} = \frac{P_{input} - c \times (\Delta T)^n}{K_E} \quad (13)$$

Equations (8) to (13) are for individual emitters. To allow for multiple emitters per zone, the power output term becomes a sum over each emitter, i :

$$P_{heat_zone} = \sum_i c_i (T_E(t) - T_{rm}(t - 1))^{n_i} \quad (14)$$

$$\frac{d\Delta T}{dt} = \frac{P_{input} - \sum_i c_i \times (\Delta T)^{n_i}}{K_E} \quad (15)$$

