HMG's Partial Equilibrium Trade Model (PE-TRAde or PETRA)

1. Introduction

His Majesty's Government (HMG) is pursuing a number of changes to trade policy.

It is vital that HMG makes full use of all available analytical tools to both inform policy decisions before the event (so-called *ex ante* analysis) and assess their impacts during and after (*ex post* analysis). These tools include (but are not limited to) a suite of trade models each with different strengths and areas of focus and based across different departments. This fits with the recommendations of the Trade Modelling Review's Expert Panel[1] and is essential for robust and comprehensive economic analysis in line with the principles of the Government Economic Service.

The DBT (2025) working paper[2] details how HMG uses a range partial equilibrium models[3] by presenting results from running simulations of a notional trade agreement between two countries. It uses these to show how these models' function, what they can deliver, what are the key parameters that drive their results and how sensitive the results are to changes in these parameters.

One such model is the UK Partial Equilibrium Trade (or PETRA) model. A Partial Equilibrium (PE) trade model is essentially a set of equations based on economic theory that incorporates a range of factors which influence the prices and sales of products, whether imported or produced domestically. In this case, PETRA estimates the economic impact of changes in barriers to trade, such as tariff rates and non-tariff measures. It estimates the changes to a range of variables from their initial values, including equilibrium prices, domestic production, trade and welfare.

PE models tend to be used in trade policy analysis to assess the potential impact of new policies such as agreeing a Free Trade Agreement (FTA) between two countries, although they can also be used to analyse the impact of past policy changes. They provide a means of testing how impacts might vary depending on the nature of the policy changes and on which changes might be more significant than others. For an FTA, they can estimate the impact on the countries directly involved in the FTA and on third countries.

Like all partial equilibrium models, PETRA focuses on the direct impact of a policy change on a particular sector. It does not incorporate general equilibrium effects that might result from policy changes, for example from a reallocation of resources or changes in capital allocation, relative wages or employment. This makes it easier to see the potential 'first order' causes and effects from the policy changes being modelled. Its simpler structure means it is less computationally complex and data intensive than general equilibrium models and provides greater scope for sectoral disaggregation.

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[1] Trade modelling review expert panel: report (Jan 2022) https://www.gov.uk/government/publications/trade-modelling-review-expert-panel-report-and-recommendations/trade-modelling-review-expert-panel-report

[2] Working paper illustrating the use of HMG's partial equilibrium trade models (July 2025) https://www.gov.uk/government/publications/partial-equilibrium-trade-models-modelling-paper

[3] The document for another PE model can be found at https://www.gov.uk/government/publications/partial-equilibrium-d-pe-d-model-modelling-paper

PE models simulate possible impacts resulting from a policy change; they are not a forecast, but rather are intended to guide their users to the potential direction of movement and order of magnitude of possible changes as well as how sensitive these might be to variations in the policy changes.

PETRA is 'static', which means it simulates the change from an initial equilibrium period, based on historical data, to a new equilibrium once all the impacts of the policy change that are being modelled have worked their way through the sectors in the model. It does not predict the path of how the economy will move to its new equilibrium. Nor does it consider how other factors such as demographics or productivity may change over time.

How long a PE model's sectors may take to reach their new equilibrium is likely to vary depending on the nature of the policy change and the characteristics of the sectors being modelled. It is probable that a new equilibrium may be reached in a modelling sense faster in a PE than in a Computable General Equilibrium (CGE) model, such as the GTAP model used by the Department to simulate the potential impact of FTAs, as labour and capital markets might be expected to adjust more slowly than goods markets and therefore to take more time to reach their new equilibrium and unlike CGE models PE models do not assume that labour or capital markets adjust.

PETRA is intended to complement the results from other HMG models, especially the CGE model, by being able to simulate potential impacts at a more disaggregated product level. Because it requires less data than CGE models, it can be run for more detailed sectors. The standard PETRA dataset has around 120 manufacturing and agricultural food sectors but does not include any services, minerals or plants and animals. If more detailed data is available, it can be broken down further, as has been done for certain food sectors in the Arm+ version of the model (see section 2.2).

Even though PETRA offers more disaggregated analysis than CGE models, its 100+ sectors are still relatively broad and can contain a wide variety of products with different characteristics and preferences. Therefore, caution should be taken before assuming that the results for each sector necessarily apply to all products. Instead results need to be interpreted bearing in mind the following differences between products contained within the sector. For example, motor vehicles (ISIC 2910) contains all types of vehicles from passenger cars through buses to commercial vehicles such as vans and lorries. Different countries may have very different patterns of demand for these or indeed for varieties within these categories such as for small cars compared to SUVs within the vehicles sector.

Moreover, tariff and non-tariff barriers may vary considerably across the products within a sector, especially for food products. There is thus a risk of aggregation bias where the average barriers for a sector may not be appropriate for all products within the sector, especially if there are significant peak barriers for some products.

The PE framework does not consider how changes in wages, employment, or investment in one sector affect other sectors.

Nor does the standard (Armington) version[4] of the model, consider how sectors might be affected by changes in the costs of their inputs.

[4] The imperfect competition version with intermediates version of PETRA is a partial exception, as it does incorporate part of the impact of a shock to a sector's suppliers.

2. Versions of the Model

Several versions of PETRA have been developed. The different versions of the model have significantly different structures, reflecting different theories found in the academic literature about how trade works, but all seek to estimate the price of a product which equates demand and supply. The different versions must be run independently. However, the same scenarios and data can be simulated with the different versions of the model to see the impact that using different versions of the model has on the results.

2.1 Armington

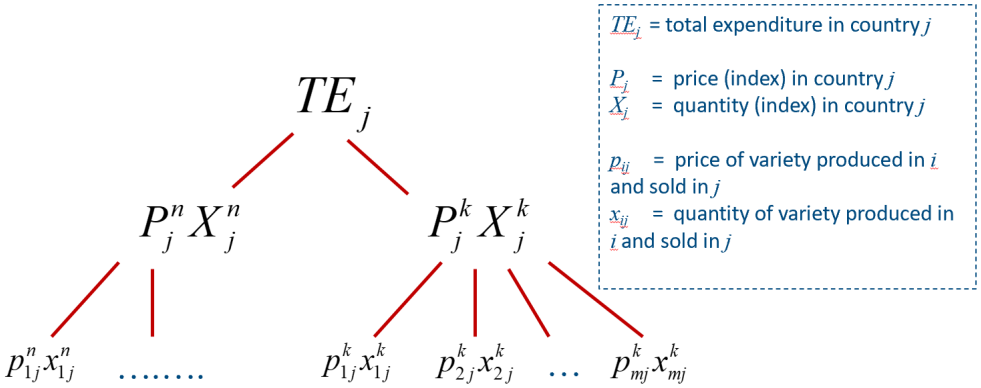
The simplest, or Armington version, is based around the concept that each country supplies a different variety of a product. Equilibrium for each sector is estimated by equating supply with demand in each market for each product.

Goods are differentiated by the country in which they are produced. Markets in each country are competitive, with the number of varieties of each product equal to the number of countries included in the simulation. Firms are price takers and price equals marginal cost.

The elasticity of demand and hence the slope of the demand curve is derived from a two stage Constant Elasticity of Substitution (CES) process: in the first stage overall demand for the product is determined by the overall elasticity of demand, whilst in the second stage demand is split between different varieties of the product according to the Armington elasticity of substitution.

This enables the model to capture consumers preference for variety. Effectively it is assumed that people like and consume different versions of a product from different sources.

Figure 1. Structure of Demand in the Armington model



This chart shows the structure of demand in the Armington model as described.

Historical data is used to estimate consumers underlying preferences for the different varieties of a product, on the assumption that this represents the base equilibrium position, reflecting historical relative prices and consumer preferences. This process is described in section 3.1. below. Then when a policy change is applied that alters relative prices, a new simulated equilibrium position is calculated based on these new prices and existing consumer preferences.

The supply curve in each market is assumed to be upward sloping as marginal cost rises as output rises, implying a decreasing return to scale in each market. Supply is assumed to be

elastic, hence the supply curve is relatively flat. More details on all the elasticities in the model can be found in Annex 2.

The equilibrium level of prices and output in each market are solved independently, so changes in one market do not directly affect any other markets.

2.2 Arm+ Extension

A number of features have been added to the basic Armington model, to make the Arm+ extension. These were developed from work by the Department for Environment, Food and Rural Affairs (Defra) to address some common issues when modelling agricultural trade. However, these features are not exclusive to agricultural products and can be applied to other sectors. The features are:

a) Small Shares Extension (SSE):

One limitation with models like PETRA is that they can struggle to simulate the potential impact on sectors where there is no or minimal historical trade as a result of prohibitive barriers or missing data (which is often recorded as a zero trade flow). PETRA works in percentage terms. So, even if the barriers are removed and PETRA suggests very large percentage increases in trade, these are unlikely to translate into more than small changes in the absolute level of trade[5], which can be below what might be regarded as realistic outcomes.

The SSE deals with this issue by replacing historical trade levels in such cases with proxied levels of trade that are estimated to represent what trade might have been if the prohibitive barriers were not in place. These proxies are based on the importing country's total consumption and the exporting country's exports share of global consumption. The model then estimates the impacts of the policy change using this higher 'representative' level of imports. As the model is now simulating a policy change to a higher baseline of trade, the same changes in tariffs and NTMs should generate larger changes in absolute trade flows which may provide a more realistic reflection of the expected impact of removing high barriers to trade.

b) Supply Redirection Extension (SRE):

In the Armington version of PETRA, all markets are modelled independently of each other. A UK producer's exports to Country A are estimated on the basis of Country A's market conditions. There is no facility for these exports to be sent to other markets, either the domestic market or third countries, if demand changes in Country A. Especially in agri-food trade, econometric evidence suggests this is not an accurate representation of producer behaviour. If UK exports face a new tariff in Country A, then exports to that country will fall, but supply will be redirected to the domestic market and the Rest of the World (as well as production potentially falling). This type of result is supported by historical econometric analysis of agri-food trade.

The SRE deals with this issue by linking markets, so that supply to one market can be influenced by changes in supply to other markets. It means that producers can react and

[5] A big percentage increase of a small number is still a relatively small number. For example, if the current value of a trade flow is only £1m and the model predicts an increase of 500%, this would result in a relatively low new absolute trade flow of £6m.

redirect some output to third countries if a policy change leads to a significant change in their market share in one country.

c) More detailed products:

A couple of important agri-food sectors are modelled at a more detailed commodity level than in the standard PETRA dataset. These are:

- ISIC 1010 Meats sector is instead modelled as: cattle meat, sheep meat, pig meat and poultry meat,
- ISIC 1050 Dairy sector is instead modelled as: butter and cheese.

2.3 Imperfect Competition without intermediates

The Armington version of PETRA has many of the features of perfect competition, such as many buyers and sellers and firms having no effective market power. Whilst this may be appropriate for some sectors, such as those with homogeneous products or dominated by commodities, it is not representative of many manufacturing sectors. For them, the Imperfect Competition extension of PETRA may provide a more useful guide.

In the Imperfect Competition version of the model, whilst there are still many buyers and sellers consumer preferences for different products are assumed to be sufficiently strong that firms have some market power. The equilibrium output and prices are those that enable a representative firm to maximise their profits by equating when their marginal cost of producing a product matches the marginal revenue they receive from selling the product.

The elasticity of demand for a variety in a country is a weighted average of the demand and Armington elasticities reflecting the variety's market share in that country. The higher its market share, the more demand elasticity reflects overall elasticity of demand, the less it is the more it reflects the Armington elasticity of the product.

Supply is now driven by the representative firm's marginal cost function, including the price of intermediate inputs. This cost function incorporates:

- The representative firm's mark-up,
- Possibility of economies or diseconomies of scale,
- Changes in the costs of the representative firm's inputs (see intermediates below).

In this version of PETRA, equilibrium price and output levels are now determined across all markets jointly, as there are potential multi-market linkages. For example, if there are returns to scale then a change in the representative firm's output in one market will affect its marginal costs and hence competitiveness in other markets.

Perfect Competition is a special case of the IC version of the model, which uses the structure of the IC model (equilibrium determined where representative firm's MC = MR), but removes the assumption that firms have market power, so again price = MC = MR and there are no returns to scale.

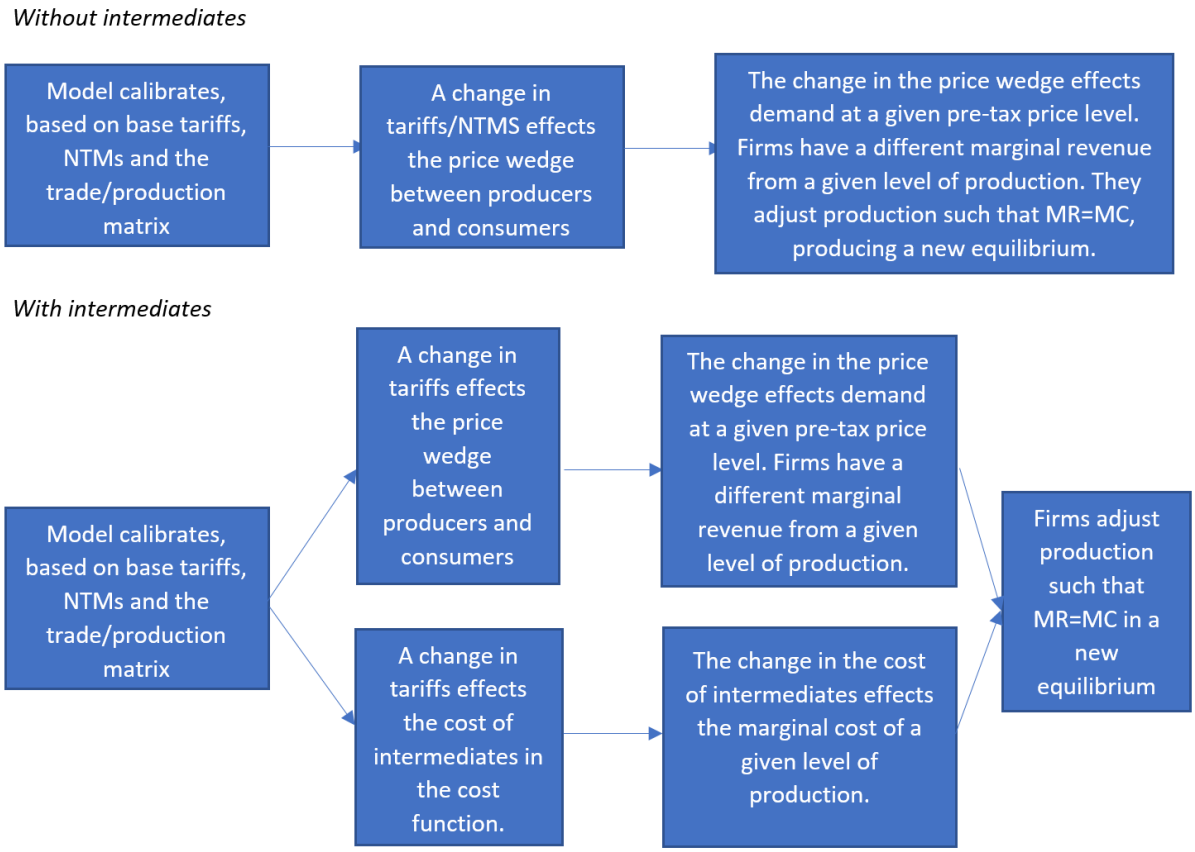
2.4 Intermediates option

The Imperfect Competition version of PETRA still solves for each sector independently, ignoring changes in the prices of other products. But for some sectors supply chains and the price of the intermediate inputs they use are important. For them, the intermediates extension of PETRA was developed.

The intermediates extension incorporates the impact of changes in the prices of intermediate inputs when calculating the representative firm's marginal costs and hence equilibrium price. However, this is not a general equilibrium model, so whilst the price and quantity of final products will adjust to reflect changes in the relative price of intermediate inputs, the weight assigned to the price of each intermediate input used to calculate a firm's marginal cost will not change. For example, if a policy change leads to a fall in the UK price of steel imported from the US this will feed through into the marginal costs of representative firms that use American steel, but if US steel historically accounted for 0.2% of their inputs then it is assumed that it will still account for 0.2% of their inputs. This is a simplification (as whilst it might hold in the short run if the representative firms are tied by contracts to their existing suppliers, it is unlikely to persist in the long term when supply contracts can be renegotiated) but was adopted to avoid computational issues as the model could otherwise struggle to find a unique equilibrium.

Using this option also makes it more difficult to predict the potential impact of policy changes as there can now be two factors driving changes in the equilibrium. Firstly, there is the effect that the change the relative prices (or price wedge due to tariffs and NTMs) has on demand and secondly there is also now the impact through the supply side through changing the price of intermediates in the representative firm's cost function, as illustrated in the box on the following page.

The impact of using this option varies across sectors. It has a greater impact on sectors that make greater use of imported inputs, especially when those inputs face significant barriers to trade.



This diagram visualises the processes of the model with and without intermediates as described in the text.

2.5 Int+ Extension

The Armington and Imperfect competition versions of PETRA focus on changes in trade policy and trade costs. The Int+ version was developed to add domestic policies that could shift relative prices in the model.

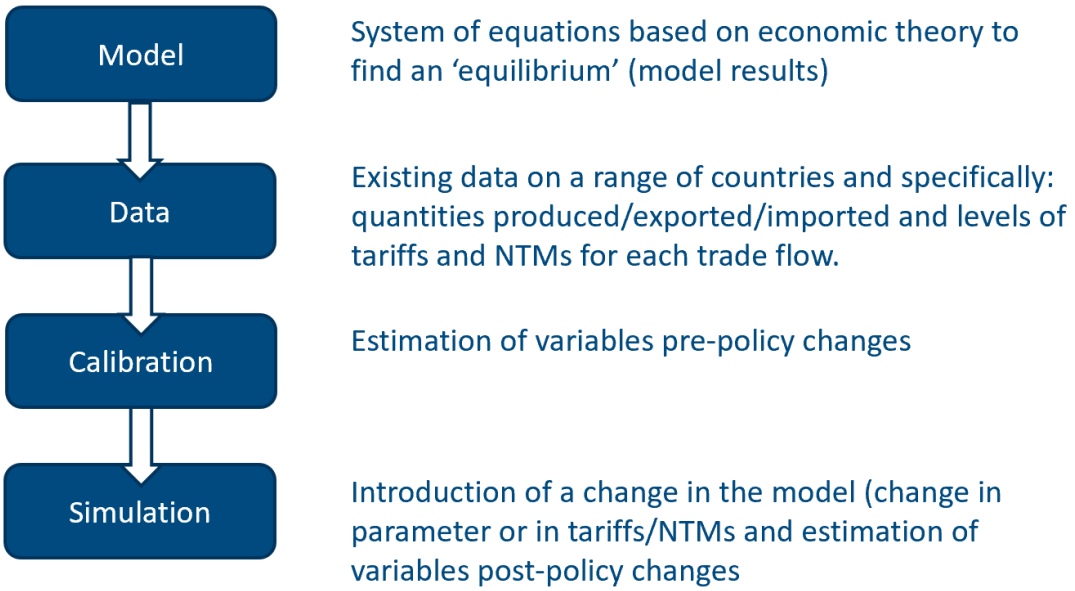
This version of PETRA includes domestic policy measures which change the variable costs of production in a producing country wherever the goods are sold. It is assumed that they can be assigned an ad valorem equivalent (AVE) value that captures the impact they have on relative prices. This option can be used to analyse policies that decrease (such as subsidies) or increase the variable costs of production in a producing country.

This version of PETRA also allows for cross-sectoral as well as cross-supplier substitution of intermediate inputs. For example, if the price of US steel imported into the UK falls then not only will there be substitution to US steel from that produced by other countries but there could also be substitution to use steel rather than other materials. It is assumed that the elasticity of substitution between different varieties of the same intermediate product is much higher than the elasticity of substitution between different intermediates, as it is easier for metal-using sectors to substitute one countries steel for another's than to substitute other materials such as aluminium or plastics for steel.

3. Running the Model

PETRA can simulate potential changes in output, prices and trade for over 100 sectors[6] for a range of countries (currently up to sixteen) from a policy change that changes the relative price of products in at least one country.

3.1. Calculation Process



This diagram visualises the calculation procedure of the model as described in the text.

The process starts by defining a set of equations, based on economic theory, that determine how equilibrium is reached. As outlined in the previous section, these equations vary depending on the version of the model used.

The next stage is to gather sufficient data to enable the model to run. This is critical as PETRA's outputs are dependent on the quality of its inputs. The data required are outlined later in the Data section.

When the model is run, its first stage is to calibrate its equations using historical data. This establishes the value of key parameters such as consumer tastes, their preference for the different varieties of the product and the production function (level of technological frontier) for each product in the model's equations.

It then estimates simulation values by applying the policy changes being modelled and then re-solving its equations using the changed values of tariffs and NTMs and the calibrated parameters.

3.2. Modifying the Baseline

PETRA's base dataset uses 2017 data. Given the changes that have occurred in the UK's trading relationship as of 2017, it can be useful to consider an alternative baseline using

[6] The basic dataset includes uses ISIC4 sectors covering manufacturing and foods from 1010 (processing and preserving of meats) to 3290 (other manufacturing not elsewhere classified) for which data is available. The agricultural extension of the model includes some more disaggregation food sectors for example meats is split between cattle meat, sheep meat, pig meat and poultry and other meats.

more recent tariff and NTM rates, such as the UK Global Tariff (UKGT), when assessing the potential impact of policy changes such as a new FTA.

Whilst the 'new baseline' tariff and NTM rates could be combined with the historical trade and production data in the base dataset this would lead to an inconsistency in the calibration process. This is because the calibration, which sets the value of the taste and supply parameters in the model, uses the baseline tariffs and NTMs. In this case the 'new baseline' tariff and NTMs would not be consistent with the historic trade and production data which reflect the actual tariffs and NTMs that applied in 2017.

Instead, in such circumstances the model is run twice. In the first simulation, the 'new baseline' is the simulation scenario. And in the second simulation, the 'new baseline' plus the policy change to be modelled is the simulation scenario. Then the marginal impact of the policy change being modelled is the difference between the results of the two simulations.

Whilst the Armington and Imperfect Competition versions use the standard two stage approach outlined above, other versions of the model require additional calculation steps.

3.3. Arm+ SSE extension

The SSE runs within the standard model steps. But there is an additional stage in which the model checks to see whether the SSE should be applied to a sector. It is activated if a sector meets all of the following criteria:

a) Large policy change: There is a sufficiently large, planned policy change (change in tariffs and NTMs), i.e. one that would mean that there was no longer a prohibitive barrier to trade in place.

b) Minimum Consumption Threshold: The importing country consumes a meaningful amount of the product, whether domestically produced or imported from other countries. So, SSE it is not used if a country does not consume a product.

c) Maximum Import Threshold: The exporter currently must not export significant amount of this product to the importer – as otherwise the current barrier wouldn't be effectively prohibiting trade.

d) Global Export Share Threshold: The exporter's historic market share in the importer's market must be less the exporter's share of the global market, which is a check that the barrier in this particular market is prohibitive compared to other markets.

Once the model has established that a trade flow meets the criteria for using the SSE, it creates the new 'representative' level of imports. This calculation is based on:

3.1 The importing country's total consumption,
3.2 The exporting country's exports as a share of global consumption.[7]

This new 'representative' level of imports is fed into the model through a change to the preference parameter. The model is then run and solved as normal.

3.4. Arm+: Supply Redirection Extension

The Supply Redirection adjustment can be applied as an extra step to the model by users as part of their chosen scenario. When it is used, the model is first solved as standard. Then

[7] Except for the UK. For the UK, the UK's non-EU exports as a share of global consumption (minus EU consumption) are used instead. This is because the UK has extremely close links historical with the EU. So, including EU exports in the UK's exports may overstate its competitiveness with the non-EU Rest of the World.

changes to the chosen producer's supply in one market can be distributed onto their supply curves to their other markets in proportion to the size of these other markets. The model is then re-solved, and these results are used as the new equilibrium.

The following example illustrates how this works in practice.

Scenario: The modelled policy change is a large tariff is introduced onto County A's exports to Country B, but there are no other changes in tariffs or NTMs. Historically, Country A's supply (excluding its exports to country B) is split: with 80% going to the domestic market, 20% to the Rest of the World.

The first run of the model might find that Country A's exports to Country B could fall by £100m. Using the splits above, the SRE extension would add £80m to Country A's domestic supply curve and £20m to the Country A's Rest of World supply curve.

The model is then re-run using these revised Country A supply curves. This impacts the overall results. Without the SRE, the only effect on Country A's exports would be a reduction in exports to Country B. With the extension, exports to Country B will fall, but exports to the Rest of the World, and supply to the domestic market would both increase.

3.5. Intermediates in Imperfect Competition model

To incorporate the changes in costs due to changes in the barriers facing imported intermediate inputs, an extra loop is introduced into the calculation of the simulation equilibrium, making it a two-stage process. In the first stage the model re-estimates the relative price of intermediate inputs based on the policy change. It its second stage it feeds these relative price changes into the cost function of the representative firm to estimate its new cost function which is then used to estimate the new simulated equilibrium.

In this second stage the representative firm uses the same weights for its use of intermediates when calculating its marginal costs as it used in the first stage of the run. These weights are fixed, based on World Input Output Data (WIOD) data. The representative firm's average intermediate marginal cost is therefore calculated as the weighted average of the post-policy change price of each intermediate product from each country.

The model solves to ensure equilibrium in final goods markets, based on the direct effect of the policy change to tariffs and NTMs in that sector and the impact of a first-round effect from changes in tariffs and NTMs on intermediate goods used by the sector.

4. Interpreting Results

PETRA generates the following results: change in prices, production, trade and welfare (consumer and producer surplus and tariff revenue) for all countries included in the simulation. Whilst the model is driven by the percentage change in relative prices which it uses to estimate the percentage changes in these variables, these results are also converted into changes in absolute values, in base year (i.e. 2017) prices. In addition, trade values are broken down between all the partner countries modelled.

PETRA simulates results for all countries and sectors that are included in the scenario that is modelled. It thus includes estimates of the impact on third countries from a policy change such as an UK FTA.

The impact of a policy change typically depends on:

- Magnitude of historical relationship,
- Size of the policy change,

- Elasticities (responsiveness of demand and supply to the relative price changes).

When there are few relative price changes it is relatively simple to see how the model works. If a policy change leads to a fall in the relative price of one variety of a product, then this will:

- Lead to a fall in the average price of that product in the market (as this is the trade weighted average of the prices of all varieties of the product).
- Due to this fall in the price consumption will rise, leading to an increase in the quantity produced.
- Lead to a shift in demand away from varieties whose relative prices have risen to the variety whose relative prices have fallen.

The net impact on the producers of different varieties in a market that reduces its barriers will depend on how much they gain from the increase in the overall size of the market compared to how much they are affected from the change in their relative competitiveness. Except for producers who face lower barriers who will gain from both the increase in the size of the market and the improvement in their relative competitiveness.

When there are several changes in relative prices, for example if modelling policy changes affecting trade with more than one country, then although there are the same drivers of change, estimating how a producer of one variety will fare is more complex, as it will depend on the change in their relative price and the average price from all the policy changes.

The magnitude of the historical relationship is relevant because the larger a country's historical share of a market the more it will be affected by the change in relative prices. Similarly, if an FTA is agreed between two countries involving similar levels of ambition but one country accounts for a much greater share of the other's exports, then it is the country whose trade will be more significantly affected that would show the greater absolute changes.

Whilst the main impacts of the policy change will be on the countries directly involved, other countries are also affected by the same factors, but the impact will generally be limited unless they account for a large share of the markets being affected by the policy changes or see a very large change in their relative price competitiveness. Impacts on consumers and producers tend to be inversely related. Whilst lower prices can be negative for producers they will increase consumer surplus, as will any increase in consumption.

4.1. Differences between Armington & Imperfect Competition outputs

The Armington and Imperfect Competition versions of the model can generate quite a mercantilist narrative for producers. With these versions of the model, a policy change that only reduces a county's barriers to imports (such as unilateral liberalisation) tends to be negative for that country's producers as there will be increased competition from the lower barriers to imports into their domestic market. It is possible that domestic producers may gain if domestic consumption is boosted sufficiently by the lower prices that there is scope for domestic supply to rise as well as imports, but this is unlikely to occur unless the effect of the income elasticity is greater than the Armington elasticity.

The impact on total welfare will depend on how the reduction in government revenue from lower tariffs, and likely decline in producer surplus, compare to the gains in consumer welfare from lower prices and increased consumption.

The impact of a policy change that results in a lowering of barriers by both FTA partners on producers will depend on how gains from increased exports to the FTA partner offset the greater competition in the domestic market.

In the Imperfect Competition version of the model the results are more complicated as with all markets being solved jointly; a change in one market will affect others. And if returns to scale are present, this version of the model can project greater impacts from liberalisation than the Armington version, as firms that gain increased sales will see a reduction in their costs, improving their competitiveness, enabling them to gain further market share. And the further fall in price will increase demand, increasing the consumers gain.

The Imperfect Competition with Intermediates introduces further factors that can influence the impact on producers, notably that reductions in their costs as a result of changes in the price of their intermediate inputs can make them more competitive in both their home and overseas markets, enabling them to gain market share. So, with this version of model even unilateral liberalisation could be beneficial to domestic producers if their gains from their lower costs plus the likely increase in domestic consumption outweighs the impact of greater competition from imports.

When interpreting PE results, it is important to remember that they do not include any general equilibrium effects, and as such represent a partial story as they do not reflect possible changes in wages, employment, capital allocation, etc that will affect consumers and producers. Nor, with the exception of the Arm+ version of PETRA, do they incorporate any supply redirection.

5. Sensitivity Analysis

Modelling exercises are inherently uncertain and present a stylised representation of the trading relationship in order to gauge the broad range of possible results from a policy change. So, in addition to modelling central scenarios, it can be helpful to conduct sensitivity analysis by varying: the core parameters (such as elasticities) within the model, policy changes (such as changes in tariffs and NTMs) and even some of the model's structural assumptions (as the SSE and SRE do) and seeing how these affect the model's results. This especially applies to key parameters, such as the Armington elasticity of substitution or to any parameters based on assumptions. Examining the range of values, the model generates using such sensitivity testing can provide a more comprehensive picture of the potential impacts than relying on one central scenario as well as providing a guide to the potential variability of the impacts. The econometrically estimated Armington elasticities include high and low values which provide a 95% confidence interval around the central estimated elasticity.

6. Choosing which Version to Use

Which version of PETRA to use for a simulation will depend on the characteristics of the sectors being modelled (for example the Armington versions may be more appropriate for commodities or homogeneous products) and the information that is being sought (for example if concerned about the need for small shares adjustment or supply redirection might use Arm+ version; if need to examine potential supply chain effects then might use the Imperfect Competition with Intermediates or Int+ versions).

Annex 1

Principle Equations - examples of some key equations

Supply and demand equations used in Armington model

Supply and demand depend on the relative prices but also on a range of parameters:

$$\begin{cases} Supply & x_{ij} = k_{ij} p_{pij}^{\beta} \\ Demand & x_{ij} = a_{ij} b_j p_{ij}^{-\sigma} P_j^{\sigma-\mu} \end{cases}$$

$$where \begin{cases} k_{ij} is\ the\ constant\ parameter\ of\ supply \\ a_{ij}\ is\ the\ taste\ parameter\ of\ demand \\ b_j\ is\ the\ income\ parameter\ of\ demand \\ \beta\ is\ the\ supply\ elasticity\ of\ substitution \\ \mu\ is\ the\ aggregated\ (income)\ demand\ elasticity \\ \sigma\ is\ the\ Armington\ elasticity\ \text{of substitution}\ (between\ \text{varieties}) \\ P_{pij}\ is\ Producer\ price\ (price\ less\ tariff\ \&NTM)\ of\ i\ in\ j \\ p_{ij} is\ the\ price\ of\ variety\ produced\ in\ i\ and\ sold\ in\ j \\ P_j\ is\ the\ consumers'\ price\ index\ in\ j \end{cases}$$

Supply parameters:

- k_{ij} is the constant parameter of supply (e.g. captures capacity, productivity, technology, skills).
- β is the supply elasticity of substitution – it captures how supply is sensitive to prices. This determines the slope of the supply curve.

Demand parameters:

- a_{ij} is the taste parameter capturing the consumers in j preference for the variety produced in i.
- b_j is the income parameter.
- The demand has two types of elasticity of substitution (capturing how demand is sensitive to prices and determining the slope of the demand curve):
    - μ the 'aggregated' demand elasticity – captures how consumers will substitute between different goods as prices change (i.e. total demand for cars in the UK increases as the price of cars in the UK falls).
    - σ the 'Armington' demand elasticity – captures how consumers will substitute between different varieties as their relative prices change (e.g. consuming good A coming from country A compared to consuming good A coming from country B).

The parameters a_{ij}, b_j & k_{ij} are estimated when the model is calibrated.

Marginal Revenue and Marginal Cost Equations in the Imperfect Competition version of the model

In imperfect competition, consumers can be charged a price that is greater than the representative firm's marginal cost (i.e. $P_j > MC$). We therefore need to define the marginal revenue and marginal cost to determine the quantity produced and its price (the MR=MC equilibrium condition).

$$\begin{cases} Marginal\ cost & MC = (1 - inte)AVC \\ Marginal\ revenue & MR = \dfrac{P_j}{1 + tt}\left(1 + \dfrac{1}{\varepsilon_d}\right) \end{cases}$$

$$where \begin{cases} inte \text{ is the internal economies of scale} \\ AVC \text{ is the average variable cost} \\ P_j \text{ is the consumers'price index in j} \\ tt \text{ is the tariff + NTM (at the sectoral level)} \\ \varepsilon_d \text{ is the demand elasticity of substitution} \end{cases}$$

The demand elasticity function is defined such that:

$$\varepsilon_d = -(1 - fsh_2) * sig - fsh\_2 * na$$

$$where \begin{cases} fsh\_2 \text{ is the variety's share of a country's expenditure} \\ sig \text{ is the Armington elasticity of substitution} \\ na \text{ is the aggregated demand elasticity} \end{cases}$$

The demand elasticity function is a weighted average of the Armington elasticity (sig) and the aggregated demand elasticity (na) weighted by the variety's share of a country's expenditure (or the market share of the representative firm from country i of expenditure in country j).

The Average variable cost function is defined as:

$$avc = \left[\sum_k supsh * \left(p_{\ int}(1 + tar_{int})\right)^{1-gam}\right]^{\frac{1}{1-gam}}$$

$$where \begin{cases} supsh \text{ is the share of intermediates from each supplier} \\ gam \text{ is the elasticity of substitution between intermediates from different sources} \\ p_{\ int} \text{is the price of intermediates} \\ tar_{int} \text{ is the tariff + NTM on intermediates} \end{cases}$$

Note this is a simplified version of the MC equations. An option exists to incorporate primary as well as intermediate inputs.

The Int+ version of the model has a more complex MC equation to allow for substitution between intermediates.

Annex 2: Data and Key Parameters

Like most models, PETRA is heavily dependent on the scope and quality of its data. It uses the following data:

Production:

Based on ISIC4 (rev 4) data for 2017 extracted from the OECD SDBS database. When this data is not available a variety of alternatives are used including: drawing on national datasets, using UNIDOs INDSTAT database, apportioning values from more aggregated ISIC sectors (ISIC2), uprating earlier years SDBS data by growth in exports and inferring production values from export intensity ratios in Input:Output databases. When necessary, adjustments are made to ensure internal consistency. Values are converted into sterling using IMF average year exchange rates.

Bilateral Trade flows:

Are based on HS6 (rev2012) data for 2017 extracted from the UN Comtrade Data (using the World Bank's WITS database). Uses the average of both countries flows, i.e. UK exports to the US are the average of UK exports to the US and US imports from the UK. If one bilateral flow is missing, uses the available flow. If 2017 data is missing uses latest available years data. HS6 mapped to ISIC4 sectors using concordance derived by HMG. If necessary, adjustments are made to world trade totals to ensure internal consistency. Values are converted into sterling using IMF average year exchange rates

Tariffs:

Are the Ad valorem equivalents (AVEs) effectively applied tariff rates in 2017, or the latest available year if 2017 data was not available. They are based on HS6 (rev2012) data drawn from WITS database. Specific, mixed and other non-AVE tariffs are converted into tariffs using the UNCTAD method provided by WITS. HS6 mapped to ISIC4 sectors as trade weighted averages using concordance derived by HMG. MACMAPS occasionally used as alternative source when WITS data not available.

NTMs:

It is assumed that NTMs can be expressed as AVEs which have a similar effect as tariffs (but without any revenue going to government), i.e. raise the effective price of imported products. Recognised simplification, required to enable the model to function. NTM AVEs are taken from estimates for each sector supplied by UKTPO, based on estimates made by Cadot & Gourdon in 2016 for 21 HS sections. Have two values for each sector, one representing the estimated AVE between countries which have a trade agreement, the other higher value representing the average NTM between two countries that don't have a trade agreement.

World Input Output (WIOD) data:

Only used when the Intermediates option in the Imperfect Competition version of the model is run. Provides the share of a product's intermediate costs that consists of inputs of each sector from each country in the model.

Number of Representative firms in a sector:

Only used in the Imperfect Competition version of the model. This parameter captures the concentration of each sector. Intuitively sectors with high number of representative firms are likely to be considered more competitive than sectors with low number of representative firms. Estimates of N, by sector, were supplied by UKTPO. They vary across sectors but are assumed to be the same across countries. Based on estimate of the number of

"equivalent sized" enterprises that exist in a SIC sector. Sensitivity testing suggests that varying N has a limited impact on results.

Price elasticities of demand:

Based on estimates supplied by UKTPO that were derived from Kee-Nicita-Olarreaga (2009) and Ghodsi-Gruebler-Stehrer (2016) estimates. These demand elasticities vary by sector but not by country.

Armington elasticities:

Two sets of elasticities have been used with PETRA.

The first is a set of estimates supplied by UKTPO. This uses two standard values across all countries, one of 6 for sectors with homogeneous products and one of 3 for sectors with heterogeneous (differentiated) products. Model does not incorporate home bias; the same Armington elasticity is applied between domestically produced and imported goods as is to imports from different countries.

The second set were estimated by DBT econometrically using the method developed by Fontagne et.al. (2022).[8]

The following gravity model was run to estimate the tariff elasticities at the product level.

$$X_{ijk,t} = \exp\left[\theta_{ik,t} + \theta_{jk,t} + \beta_k \ln\left(1 + \tau_{ijk,t}\right) + \gamma_k \ln\left(d_{ij}\right) + \zeta_k Z_{ij}\right] \times \epsilon_{ijk,t} \qquad \forall k \in K$$

The tariff elasticity is $\beta_k = -\delta_k$ in the usual CES framework, with δ_k being the elasticity of substitution between varieties of a given HS6 product exported by different countries. The above equation includes importer-time ($\theta_{jk,t}$) and exporter-time ($\theta_{ik,t}$) fixed effects. (Z_{ij}) represents a set of control variables, which includes dummies for (i) a common colony, (ii) a common border, and (iii) a common language; $\tau_{ijk,t}$ is the ad valorem tariff on the CIF[9] price of good k. The MFN tariff imposed by j is captured by the importer-time fixed effect. To address heteroscedasticity[10] in the error term, a non-linear Poisson Pseudo Maximum Likelihood – PPML – as the baseline estimator was estimated.

In the original Fontagne paper Armington elasticities were estimated at HS6 level. As elasticities are expected to vary depending on the granularity at which they are estimated and to minimise the risk of aggregation bias, we estimated our elasticities using the ISIC4 level data used in our standard PETRA dataset.

Unrealistic and extreme values were managed in a variety of ways. Products that generated unrealistic values (such as positive elasticities) were discarded from the input sheet and the model recalibrated. Remaining extreme values were replaced by the median values for their ISIC2 divisions. A min/max cap of 1 and 9 were imposed on any remaining extreme values.

Some caveats:

- There was a lack of high-quality tariff data for some sectors which may have produced biased results for those sectors.
- Results may be skewed in one direction when aggregating from HS6 to ISIC4 level and mask trade behaviours across sub-sectors.

[8] Fontagne et. al. (2022), https://www.cepii.fr/cepii/en/bdd_modele/bdd_modele_item.asp?id=35

[9] Cost, Insurance and Freight

[10] The variance of the errors is non-constant across observations.

Elasticities may vary across countries. In particular, if trade for some sectors is influenced by intra-regional standards, elasticity for a country within a trading block may not be representative of that of countries outside that block.

Price elasticities of supply:

Estimates supplied by UKTPO. Use two standard values for all sectors: 6 for domestically supplied products and 15 for imported products.

Annex 3: History of Model Development

The PETRA model was developed for HMG by InterAnalysis following their winning of a tender that was launched by DIT 30 May 2018, “Tender for Multi-Market Partial Equilibrium Model for the Department for International Trade”, Tender Reference: DN341902.

The Armington model and a dataset to run it was delivered by the end of 2018; the Imperfect Competition version of the model and an extended dataset were delivered in 2019, although further refinements were carried out in 2020.

HMG has since carried out further development of the model (notably the ARM+ extension) as well as making further expansions to the dataset.

The Int+ version of the model was developed in 2024.

InterAnalysis[11] have a long experience of working on trade and PE models, using models of both imperfect and perfect competition since the mid-1980s. This has included:

- Developing a sophisticated partial equilibrium model under imperfect competition with economies of scale used in the EU’s Cecchini Report.
- Using PE models in analysis for the Competition and Markets Authority.
- Developing the TAPES (Trade Analysis using Partial Equilibrium Simulations) model, variants of which have been used in a range of contexts from the impact of an EPA between the EU and Cariforum to the impact on third countries of TTIP and in EU SIAs. More recently, it has been used by UKPTO to evaluate the possible impacts of Brexit.

[11] See: https://tradesift.com/. InterAnalysis was a business spun out from the University of Sussex that drew on trade experts based at Sussex University. Although it ceased to operate in 2023, its members continue to be active in the fields of trade and trade modelling including at the UKTPO (UK Trade Policy Observatory)

References:

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Some examples of use of TAPES model:

Potential effects of the proposed transatlantic trade and investment partnership on selected developing countries, see: https://www.gov.uk/research-for-development-outputs/potential-effects-of-the-proposed-transatlantic-trade-and-investment-partnership-on-selected-developing-countries

Trade SIA for the negotiation of a PCA between the EU and China, see: https://trade.ec.europa.eu/doclib/docs/2010/june/tradoc_146246.pdf