**Bilateral Monopoly Revisited:**
**Price Formation, Efficiency and Countervailing Powers**

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Abstract. In this paper, I revisit and synthesize the rich literature on price formation in bilateral monopoly. I show how traditional flat-rate price posting (e.g. price setting and price taking) is akin to Nash bargaining over wholesale price with subsequent 'right-to-manage', while two-part tariffs are akin to bilaterally efficient Nash bargaining over both wholesale price and quantity. Outcomes under the former protocol nest price posting and the cases of pure monopoly and pure monopsony. Outcomes under the latter protocol nest all-or-nothing offers, the Walrasian outcome under two-sided price taking and trace out the contract curve. With lopsided bargaining power, outcomes under right-to-manage can lead to socially superior outcomes to those that are bilaterally efficient, but may also lead to socially inferior over production. Last, effects of bargaining power on markups, markdowns and cost pass-through are characterized.

JEL Classification: L13, L42, D42, C78, J42.

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## 1. Introduction

In recent years, there has been a resurgent interest in understanding and characterizing market power in supply chains, including issues of buyer power in labor and intermediate goods markets, processes of price formation and their effects on markups and markdowns[1].

Trade between firms is of central importance to economic outcomes, especially in a world with increasingly complex and long supply chains, sometimes including large numbers of firms. Recent empirical work includes Decarolis and Rovigatti (2019) in the market for online advertising, Avignon and Guigue (2022) on the French dairy market, Hahn (2023) on German

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‡All figures in this paper were generated in *Mathematica*. The codes are available upon request.

1See e.g. the Economic Research Strategy of the UK's Competition and Markets Authority (2023) and Shastitko et al. (2018) on competition policy issues in bilateral monopoly. See also Syverson (2024) for a nice overview of recent research on markups and markdowns.

car manufacturing and Molina (2024) on the French bottled water industry. Lee et al. (2021) provide an overview of structural empirical work on contracting in vertical markets.

Three fundamental questions arise when considering trade between firms. First, does the exercise of market power in vertical industry structures lead to inefficiencies that are different from those without the vertical structure, e.g. when a monopolist sells directly to final consumers rather than to another firm? Since the work of Spengler (1950), it has been known that vertical structures and supply chains may create inefficiencies that depend delicately on the contractual terms that govern their trade. Second, does the distribution of market power between firms in bilateral monopoly matter for efficiency and final outcomes? Although this question was first raised explicitly in the work of Galbraith (1952) in the context of countervailing powers, the answer was already implicitly answered by the early literature on bilateral monopoly, e.g. in the work of Bowley (1924, 1928), which noted that allocating the price posting power to one firm or the other would give rise to widely different production decisions, with potentially different welfare properties. Last, what is the appropriate way to model price formation in trade between firms? To solve for equilibrium, the modeler must make some choices about the institution that determines the contractual terms in agreements between firms. This amounts to making assumptions about who chooses the price and who chooses the output (or both, as the case may be). The problem is that this choice is not innocuous, as it amounts to making direct assumptions about who has the power to make such decisions, which are highly payoff relevant.

Early precursors to the theory of bilateral monopoly include Cournot (1838), Menger (1871) and a number of other illustrious economists. Excellent surveys of the very early treatments are found in Machlup and Taber (1960) and Ståhl (1978). More formal analyses, focused on the models treated in the current work, kicked off with contributions by Bowley (1924, 1928), Wicksell (1925), Fellner (1947), Morgan (1949) and Farouker (1957). Dobbs and Hill (1993) are unusual in extending the analysis of bilateral monopoly to settings with uncertain demand and asymmetric information.

A central theme in the early literature was that of indeterminacy, of intermediate goods prices, of quantities and sometimes of both. As Bowley (1924, 1928) pointed out, for any given wholesale price determined either through bilateral agreement or imposed by third parties, the two firms may disagree about the quantity to be produced. For the upstream seller will want to produce according to its supply function (which equalizes the wholesale price to its marginal costs), while the buyer will want to produce according to its demand function (which equalizes the wholesale price to its marginal revenue product).[2] Yet both firms cannot simultaneously control output and in either case, the chosen quantity will generally not maximize the joint profits of the vertical structure. Similarly, even if firms do agree on an output level, e.g. that

[2]In recent work, Avignon et al. (2024) consider a non-Walrasian model of bilateral monopoly in which trade is dictated by the short side of the market. Each party signals desired supply and demand for a given wholesale price and a rationing rule (rather than market clearing) then determines the quantity traded. This type of solution concept formally belongs to the class of 'rationed equilibria' considered by Benassy (1989). Demirer and Rubens (2024) consider a similar restriction on wholesale prices, but solves the model simultaneously rather than sequentially, using the Nash-in-Nash approach.

which is bilaterally efficient, they may still disagree on the wholesale price, for this is what determines the distribution of rents between the firms.[3] While these issues were not resolved in subsequent research, the issue was circumvented by imposing specific assumptions on the bargaining protocol, such as assuming that one firm posts prices and the other firm acts as a price taker. This approach has become standard in the literature on vertical oligopoly, where it is commonplace to assume that firms are price setters downstream but price takers upstream. Examples include Wicksell (1925), Spengler (1950), Greenhut and Ohta (1979), Salinger (1988), DeGraba (1990), Abiru (1998), Ishikawa and Spencer (1999), Mukherhjee and Mukherjee (2003), Ghosh and Morita (2007b), Peitz and Reisinger (2014), Nagurney (2006, 2022), Ghosh et al. (2022) and Walsh (2020). This approach effectively extends the price formation process commonly adopted by firms facing final consumers, to settings in which firms sell to other firms. As noted by Machlup and Taber (1960), this amounts to assuming a very specific allocation of bargaining power between firms, often done for convenience rather than in order to reflect any specific economic reality in the industry under consideration.

Specific competition policy issues also spring from the adoption of this process of price formation. Spengler (1950) first articulated how flat-rate pricing in supply chains could lead to double marginalization and inefficient production, an issue that has remained a staple in the policy debate to the present day (see Rey and Verge, 2005, and Kwoka and Slade, 2020).

Given the very specific nature of price posting models, it is natural to consider settings in which both parties have some influence over the terms of trade. Galbraith (1952) introduced the notion of countervailing powers, namely the idea that strong downstream firms could use their buyer power to rein in the seller power of powerful upstream firms, potentially passing on some of their gains to final consumers. The natural language for such questions is that of bargaining and although the early literature on bilateral monopoly was explicitly couched in terms of negotiation and bargaining power, it was not until after the formulation of the Nash bargaining solution (Nash, 1950) that the bargaining process was explicitly brought to bear on the analysis of price formation between firms.[4] Although the Nash bargaining solution was originally formulated in terms of final payoffs, the literature on bilateral monopoly has considered the application of this solution concept to determining either all or only a subset of the relevant variables. Under *complete* bargaining, the firms maximize their joint profits over all choice variables, namely both total output and a wholesale price to be paid by the buyer to the seller. Under *partial* bargaining, the firms negotiate over the wholesale price only, and one of the firms can subsequently choose how much to transact at that negotiated price. Complete bargaining was considered by McDonald and Solow (1981) and Manning (1987) in labor contexts and by Björnerstedt and Stennek (2007), Ghosh and Morita (2007a) and Collard-Wexler et al. (2019) in industrial organization contexts. Partial bargaining over wholesale prices (or wages) was considered by McDonald and Solow (1981), Manning (1987), Dobson (1997) and Layard et al. (1991) in labor settings and by Horn and Wolinsky (1988),

[3] For a modern texbook discussion of these issues, see Scherer and Ross (1990).

[4] See Foldes (1964) for an alternative bargaining procedure in bilateral monopoly based on differential rates of time preference.

Mukherjee (2008), Naylor (2002) and others in industrial organization models. Alviarez et al. (2023) and others make use of similar machinery to analyze applications to international supply chains. Last, Lachowska et al. (2022) review the labor literature on wage posting versus wage negotiation.

The presence of double marginalization is intimately connected to the use of linear, flat-rate contracts and it is well known that non-linear contracts are one way to eliminate the inefficiencies associated with multiple firms exercising market power in a non-coordinated fashion. Two-part tariffs allow the firms to separate the question of production (and thus the determination of the level of the joint profits of the vertical structure) from that of division of rents between firms. With linear contracts, the wholesale price serves the dual purpose of determining both the level of surplus and the distribution of rents and in doing so, fails to maximize joint profits. It turns out that there is a similar phenomenon when moving between partial bargaining and complete bargaining. The former bargaining protocol generalizes price posting, while the latter generalizes bilaterally efficient contracting through a cost-plus contract that effectively endogenizes the lump-sum component of the two-part tariff.

Price setting by an upstream firm and by a downstream firm, respectively, may appear to represent two extreme cases in a unified model in which price is negotiated between firms, but this is not the case. Price setting by one or the other firm is in fact a special case of two different models. In negotiated price models, it is true that the price setting component is replaced by a bargaining process that returns some compromise price, but the subsequent choice of output is still made by one of the firms. The outcome with price setting by the upstream firm is the special case where it has all the bargaining power at the price negotiation stage but the downstream firm has all the power at the quantity setting stage, while the outcome with price setting by the downstream firm is the special case in which it has all the bargaining power at the price negotiation stage but the upstream firm has all the power at the quantity setting stage. Price posting models and models of partial bargaining over wholesale prices (but not over final quantities) thus have in common that one of the firms has the ability to react to the previously determined price as it wishes. This highlights the importance of the bargaining protocol for final outcomes and as such, choosing one protocol over another should be justified by the modeler.

The remainder of the paper is structured as follows. In Section 2, I set out the model and derive a number of preliminary results. In Section 3, I analyze some important benchmark solutions that play a role in the subsequent analysis of bargaining. In Section 4, I analyze outcomes under price posting. In Section 5, I analyze outcomes under generalized Nash bargaining. In Section 6, I consider markups and markdowns and characterize how these depend on the bargaining protocol and on the relative bargaining power of the two firms. Section 7 contains the discussion.

## 2. MODEL AND PRELIMINARIES

Consider a supply chain with one upstream firm $U$ and one downstream firm $D$. The downstream firm buys its inputs from the upstream firm and in turn sells its product to final

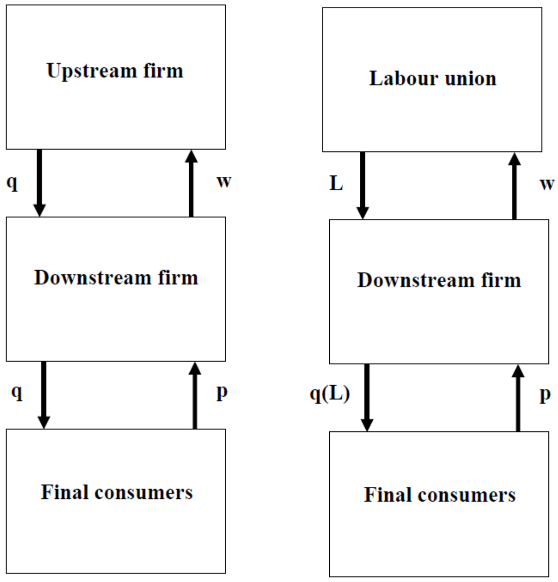



Figure 1: Supply chain with one upstream and one downstream firm.

consumers. In the production of each unit of final output, one unit of intermediate input is needed. This implies that the downstream firm's marginal revenue product of inputs equals its marginal revenue from final sales. This setup is illustrated in Figure 1. The figure also illustrates the typical setup in labour economics models of wage bargaining between a firm and a labor union. Final market demand is given by the inverse demand function

$$p(q) = a - bq \tag{1}$$

with $a, b > 0$. The upstream firm produces the intermediate good at total cost

$$C(q) = cq + dq^2 \tag{2}$$

with $c, d \geq 0$ and $a > c$. The downstream firm can turn the intermediate good into the final good at no cost, but must pay the upstream firm a wholesale price $w$ per unit of the intermediate good. The firms always have the option to remain inactive and thus will produce only when it yields non-negative profits. The analysis extends in a straightforward manner to settings in which the downstream firm has additional (decreasing returns to scale) production costs and the qualitative features also extend to more general specifications of the cost and demand functions.

For ease of reference and to simplify later exposition, I introduce the following notation:

$$\begin{aligned} AR(q) &= a - bq && (3)\\ MR(q) &= a - 2bq && (4)\\ MMR(q) &= a - 4bq && (5) \end{aligned}$$

The first two of these functions are simply the downstream firm's average and marginal revenue curves respectively, the former coinciding with the inverse demand function of final consumers. In some settings, the marginal revenue function $MR(q)$ will dictate the downstream firm's demand for the upstream firm's inputs when the latter sets the wholesale price, and hence we will need to make use of the marginal revenue from the perspective of the upstream firm. This is what the curve labeled $MMR(q)$ is, and it bears the same relation to the $MR(q)$ curve as the $MR(q)$ curve does to the $AR(q)$ curve (it is not simply a derivative of it).

Similarly, let

$$AC(q) = c + dq \tag{6}$$

$$MC(q) = c + 2dq \tag{7}$$

$$MMC(q) = c + 4dq \tag{8}$$

The first two functions are the upstream firm's average and marginal cost curves, respectively. In some settings, we will need to make use of the marginal cost function as perceived by the downstream firm, when it itself sets the wholesale price and takes into account that its input demand influences the marginal cost at which the input can be obtained from the upstream firm. The curve labeled $MMC(q)$ is that function, and it bears the same relation to the $MC(q)$ curve as the $MC(q)$ curve does to the $AC(q)$ curve (and again, is not simply a derivative of it).

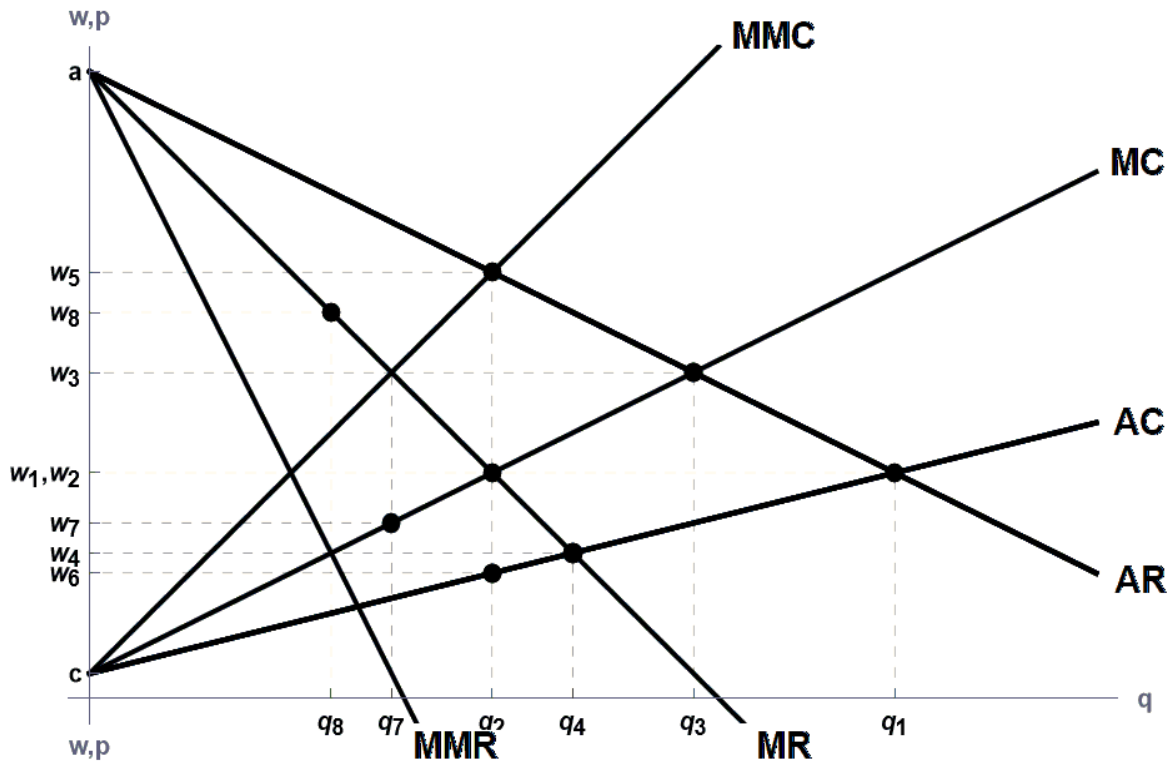
Last, rearranging the $MR(q)$ and $MC(q)$ curves, respectively, we get the functions

$$D(w) = \frac{a-w}{2b}, \quad S(w) = \frac{w-c}{2d} \tag{9}$$

The former function is the downstream firm's demand for the intermediate product, when it takes the wholesale price $w$ as given. The latter function is the upstream firm's supply of the intermediate good, when it takes the wholesale price $w$ as given. The curves are illustrated in Figure 2, which also labels key intersection points corresponding to solutions of different benchmark cases derived in the following sections.

Note that for a firm to have market power, it must face a firm who is earning some rents that can be extracted. This means that for the upstream firm to have power in the intermediate goods market, the downstream firm must have market power in the final goods market. This is the case when the latter faces a downward sloping inverse demand function (so $b > 0$). Similarly, for the downstream firm to have any power in the intermediate goods market, the upstream firm must have decreasing returns to scale (so $d > 0$), leading to an upward-sloping supply function.

**2.1. Profits and iso-profit curves.** For some generic agreement $(q, w)$ with a lump-sum transfer $A \geq 0$ from the downstream firm to the upstream firm, the profits of the two firms are



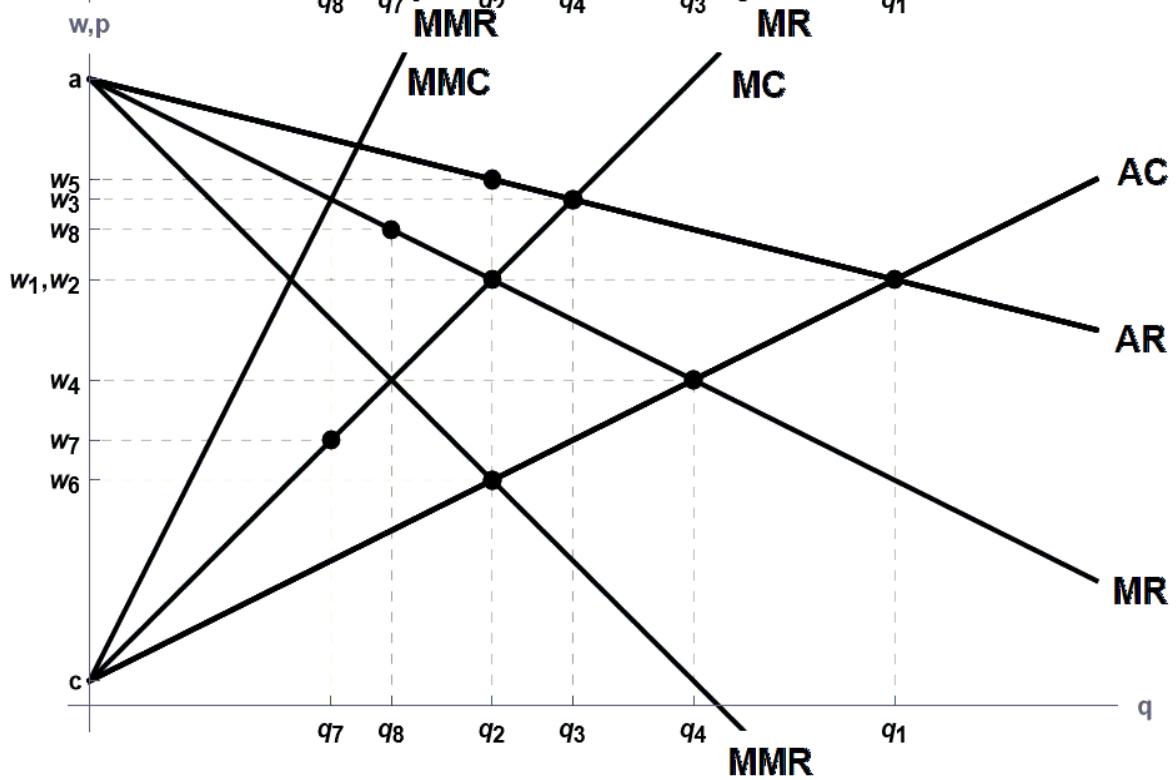




Figure 2: Cost and revenue curves and key intersection points. Upper graph shows case with $b > d$; lower graph shows case with $d > b$.

given by

$$\pi^U(w,q) = (w-c-dq)q + A \quad (10)$$
$$\pi^D(w,q) = (a-bq-w)q - A \quad (11)$$

The iso-profit curves in $(q,w)$-space have slopes

$$\frac{\Delta w}{\Delta q}|_{\Delta\pi^U(w,q)=0} = \frac{-(w-c-2dq)}{q} \quad (12)$$
$$\frac{\Delta w}{\Delta q}|_{\Delta\pi^D(w,q)=0} = \frac{a-w-2bq}{q} \quad (13)$$

From this it follows that the former slope is zero for $w = MC(q)$, while the latter slope is zero for $w = MR(q)$. In other words, the slope of the upstream firm's iso-profit curve is zero along its supply function $S(w)$, while the slope of the downstream firm's iso-profit curve is zero along its demand function $D(w)$. For a given output $q$, the upstream firm's profits are increasing in the wholesale price $w$, while the downstream firm's profits are decreasing. Some iso-profit curves are illustrated in Figure 3.

## 3. Some useful benchmarks

A central theme throughout this paper will be the extent to which different protocols for price formation induce efficient outcomes. For later comparison, we start by characterizing different procedures that lead to a series of different types of outcomes. First, I characterize the first-best outcome. Next, I consider the zero-profit outcome in which both firms break even. I then consider the outcome under two-sided price taking behavior, in which neither firm acts as a price setter. I then consider the two scenarios in which a firm finds itself with market power in one sector but either buys from or sells to a perfectly competitive sector (which are denoted as *pure monopoly* and *pure monopsony*, respectively). Last, I consider settings in which one firm is so dominant that it may impose a take-it-or-leave-it offer on the other party that covers all aspects of the transaction. These benchmarks will all play a role in subsequent analysis in which the parties engage in bilateral Nash bargaining over some (or all) aspects of the agreement. In what follows, the second-order conditions for optimality are satisfied, yet omitted.

**3.1. First best outcome.** For some output $q$, aggregate welfare in this economy is given by

$$W = (a-c-bq/2-dq)\,q \quad (14)$$

The first-order condition can be written as

$$AR(q) = a - bq = c + 2dq = MC(q) \quad (15)$$

and the optimal output and resulting price is

$$q^* = \frac{a-c}{b+2d} \tag{16}$$

$$p^* = \frac{2ad+bc}{b+2d} \tag{17}$$

Note that because the downstream firm does not have any costs over and above what it pays the upstream firm for its inputs, the social optimum is achieved when the supply of the upstream firm (which reflects the actual productions costs) equals final consumer demand.

**3.2. The zero-profit outcome.** If the upstream and the downstream sectors are perfectly competitive, then an upstream seller receives only its average costs, while the downstream buyer is charged its average revenue for the input. Equalizing average costs and average revenues yields

$$AC(q) = c + dq = a - bq = AR(q) \tag{18}$$

The perfectly zero-profit output is then

$$q_1 = \frac{a-c}{b+d} \tag{19}$$

In turn, retail and wholesale prices are

$$p_1 = w_1 = AC(q_1) = AR(q_1) = \frac{ad+bc}{b+d} \tag{20}$$

In this setting, $\pi_1^U = \pi_1^D = 0$. This solution is illustrated in Figure 2. Note that with only one firm upstream and one firm downstream, the zero-profit outcomes generally does not obtain (except for specific parameterizations). Last, note also that this outcome is the one that maximises consumer surplus, subject to both firms breaking even, yet it is not the social optimum.

**3.3. Two-sided price taking.** Next, I consider the setting in which both firms act as price takers. When each firm acts as a price taker, then the equilibrium output is determined by the intersection of the demand and supply curves $D(w)$ and $S(w)$, i.e. where

$$MR(q) = a - 2bq = c + 2dq = MC(q) \tag{21}$$

This leads to output

$$q_2 = \frac{a-c}{2(b+d)} \tag{22}$$

The associated prices and profits are

$$w_2 = \frac{ad+bc}{b+d} \tag{23}$$
$$p_2 = \frac{ab+2ad+bc}{2(b+d)} \tag{24}$$
$$\pi_2^U = \frac{d}{b+d}\frac{(a-c)^2}{4(b+d)} \tag{25}$$
$$\pi_2^D = \frac{b}{b+d}\frac{(a-c)^2}{4(b+d)} \tag{26}$$
$$\pi_2 \equiv \pi_2^U + \pi_2^D = \frac{(a-c)^2}{4(b+d)} \tag{27}$$

This solution is illustrated in Figure 2.

Note that the outcome $(q_2, w_2)$ under two-sided price taking behavior has the Walrasian property that given the wholesale price $w_2$, each firm is maximizing its profits by supplying and demanding output $q_2$, respectively. For that reason, this benchmark will be referred to as the Walrasian equilibrium, but it should be noted that this refers to a Walrasian equilibrium in the *factor market*, not in the *product market*, as these do not generally obtain at the same time expect under special circumstances.

In the Walrasian outcome, the downstream and upstream firms split total industry profits $\pi_2$ according to shares $b/(b+d)$ and $d/(b+d)$ respectively, reflecting the relative elasticities of costs and final demand. These weights will play an important role under bargaining, and I will refer to them as the *Walrasian weights*.

Last, note that because the Walrasian equilibrium is at the intersection of the offer curves of the two firms (i.e. the demand and supply curves, respectively), it can be implemented in Nash equilibrium by considering a simultaneous-move game in which the firms each submit commitments $(q, w)$. This is in the spirit of the work by Binmore (1987).

**3.4. Pure monopoly and pure monopsony.** To isolate the effect of monopoly or monopsony power, we consider the cases of so-called pure monopsony and pure monopoly, respectively. These are the cases considered by Robinson (1933, pp. 52 and 220). Farouker (1957) argues that these solutions are appropriate when one of the firms can impose terms but has incomplete knowledge of the demand or supply function of the other firm, as the case may be. A dominant upstream firm will choose a point on its supply curve that ensures that the downstream firm receives the market rate of return, i.e. its average revenue product (the retail price). A dominant downstream firm will in turn choose a point on its demand curve that ensures that the upstream firm receives the market rate of return, i.e. its average costs.

**Pure monopsony.** Suppose that the firm sells its output to a perfectly competitive output sector, where

$$p = AR(q) = a - bq \tag{28}$$

and acts as a monopsonist on the input market. Assuming that the firm ignores its influence on the input price $p$ and takes it as given, the monopsonist maximizes its profits

$$\pi_3 = pq - (c + dq)q \tag{29}$$

The first-order condition yields

$$p - c - 2dq = 0 \tag{30}$$

which upon substitution of the price $p$ and rearrangement yields

$$MC(q) = c + 2dq = a - bq = AR(q) \tag{31}$$

The solution is then

$$\begin{align} q_3 &= \frac{a-c}{b+2d} \tag{32} \\ w_3 &= \frac{2ad+bc}{b+2d} \tag{33} \\ p_3 &= \frac{2ad+bc}{b+2d} \tag{34} \\ \pi_3^U &= d\left(\frac{a-c}{b+2d}\right)^2 \tag{35} \\ \pi_3^D &= 0 \tag{36} \end{align}$$

This solution is illustrated in Figure 2. Note that because the outcome under pure monopsony is where marginal costs of inputs equal the marginal revenue of final outputs, this is in fact the socially optimal outcome.

**Pure monopoly.** Suppose that the firm buys its inputs from a perfectly competitive input sector, where

$$w = AC(q) = c + dq \tag{37}$$

and acts as monopolist on the output market. Assuming that the firm ignores its influence on the input price $w$ and takes it as given, the monopolist maximizes its profits

$$\pi_4 = (a - bq)q - wq \tag{38}$$

The first-order condition gives

$$a - 2bq - w = 0 \tag{39}$$

which upon substitution of wholesale price $w$ and rearrangement yields

$$MR(q) = a - 2bq = c + dq = AC(q) \tag{40}$$

The solution is then

$$\begin{aligned}
q_4 &= \frac{a-c}{2b+d} && (41)\\
w_4 &= \frac{ad+2bc}{2b+d} && (42)\\
p_4 &= \frac{ab+ad+bc}{2b+d} && (43)\\
\pi_4^U &= 0 && (44)\\
\pi_4^D &= b\left(\frac{a-c}{2b+d}\right)^2 && (45)
\end{aligned}$$

This solution is illustrated in Figure 2.

**3.5. Take-it-or-leave-it offers.** Consider a setting in which one or the other firm has complete control of all decisions and can present the other firm with a take-it-or-leave-it offer $(q, w)$. We distinguish between the two cases in which the offer is made by the downstream and the upstream firm, respectively. The only constraint on the offer is that it must respect the weaker firm's participation constraint, i.e. yield non-negative profits. Such all-or-nothing offers were also discussed by Fellner (1947). He noted that relative to firms that must respond to this type of offer, price taking firms who are free to demand or supply as much as they wish at a quoted price retain a non-trivial influence over final outcomes. This contrasts to the case with constant returns to scale technology, in which a price-taking firm is kept to its participation constraint

**Upstream firm makes offer.** Suppose that the seller makes an offer to the buyer, consisting of a pair $(q, w)$. For such an offer to be accepted, the buyer must break even and so we need

$$\pi^D = (a - bq - w)q \geq 0 \qquad (46)$$

which is equivalent to requiring that

$$w \leq a - bq = AR(q) \qquad (47)$$

so that the buyer pays no more than its average revenue product. Keeping the buyer to its participation constraint, we can write the seller's profits as

$$\pi^U = (a-c)q - (b+d)q^2 \qquad (48)$$

The first-order condition for optimality is

$$a - c - 2(b+d)q = 0 \qquad (49)$$

We therefore have that in this scenario,

$$\begin{aligned} q_5 &= \frac{a-c}{2(b+d)} && (50) \\ w_5 &= \frac{ab+2ad+bc}{2(b+d)} && (51) \\ p_5 &= \frac{ab+2ad+bc}{2(b+d)} && (52) \\ \pi_5^U &= \frac{(a-c)^2}{4(b+d)} && (53) \\ \pi_5^D &= 0 && (54) \end{aligned}$$

Note that this output is identical to the bilaterally efficient output achieved via full integration (or equivalently, when both firms behave as price takers). This solution is illustrated in Figure 2.

**Downstream firm makes offer.** Suppose that the buyer makes an offer to the seller, consisting of a pair $(q, w)$. For such an offer to be accepted, the seller must break even and so we need

$$\pi^U = (w - c - dq)q \geq 0 \tag{55}$$

This is equivalent to requiring that

$$w \geq c + dq = AC(q) \tag{56}$$

so that the seller covers its average costs. Keeping the seller to its participation constraint, we can write the buyer's profits as

$$\pi^D = (a-c)q - (b+d)q^2 \tag{57}$$

The first-order condition for optimality is

$$a - c - 2(b+d)q = 0 \tag{58}$$

This problem is identical to that of the downstream firm when it makes an offer to the seller and so the chosen output is the same. This yields the solution

$$\begin{aligned} q_6 &= \frac{a-c}{2(b+d)} && (59) \\ w_6 &= \frac{ad+2bc+cd}{2(b+d)} && (60) \\ p_6 &= \frac{ab+2ad+bc}{2(b+d)} && (61) \\ \pi_6^U &= 0 && (62) \\ \pi_6^D &= \frac{(a-c)^2}{4(b+d)} && (63) \end{aligned}$$

This solution is illustrated in Figure 2.

## 4. POSTED PRICES

We now consider the setting in which the price is wholly determined by one party, while the other party takes this price as given and chooses how much to supply or demand at that price, as the case may be. In other words, this setting corresponds to the commonplace assumption that one of the parties acts as the price taker, while the other firm is the price setter. This approach was introduced by Wicksell (1925) and Bowley (1928), but see also Ståhl (1978) for a critical discussion. As noted by Machlup and Taber (1960), this approach extends the standard price-taking assumption from consumer theory and employs it to model trade between firms. The analysis of double marginalization by Spengler (1950) relies explicitly on such price taking by downstream firms. Blair et al. (1989) strongly criticized price posting based solutions as they assume that one of the two firms behaves competitively, although it is alone on its side of the transaction.

Under posted prices, even the price taker influences the price in equilibrium, for the price setter takes into account the quantity reaction to the posted price.

**4.1. Downstream firm sets price.** Consider the scenario in which the downstream firm sets the wholesale price $w$, the upstream firm takes this price as given and then chooses how much to supply. In this scenario, the downstream firm is a price setter both in the final goods market and in the market for the intermediate good. The downstream firm therefore chooses its output knowing how it will influence the retail price $p$ by moving along the inverse demand for the final good and the wholesale price $w$, since it will influence the supply $S(w)$ of the intermediate good. Substituting the upstream firm's supply function into the downstream firm's profit function, the latter chooses output to maximize profits

$$\pi^D = (a-c)q - (b+2d)\,q^2 \tag{64}$$

The first-order condition is

$$a - c - 2(b+2d)q = 0 \tag{65}$$

which reduces to

$$MR(q) = a - 2bq = c + 4dq = MMC(q) \tag{66}$$

The equilibrium output is then

$$q_7 = \frac{a-c}{2(b+2d)} \tag{67}$$

and the associated prices and profits are given by

$$\begin{aligned} w_7 &= \frac{ad+bc+cd}{b+2d} && (68)\\ p_7 &= \frac{ab+bc+4ad}{2(b+2d)} && (69)\\ \pi_7^U &= d\left(\frac{a-c}{2(b+2d)}\right)^2 && (70)\\ \pi_7^D &= \frac{(a-c)^2}{4(b+2d)} && (71) \end{aligned}$$

This solution is illustrated in Figure 2. Note that while the equilibrium output $q_7$ is determined by the intersection of the $MR(q)$ and $MMC(q)$ curves, the corresponding wholesale price $w_7$ is read on the upstream firm's supply function $S(w)$.

**4.2. Upstream firm sets price.** Consider the scenario in which the upstream firm sets wholesale price $w$, the downstream firm takes this price as given and then chooses how much to demand from the upstream firm at this price and then sell to final consumers. Formally, this is equivalent to the upstream firm choosing the wholesale price $w$ along the demand function $D(w)$ with a view to maximize its profits. Upon substitution of the downstream firm's demand function into the upstream firm's profit function, the latter's objective is to maximize

$$\pi^U = (a-c)q - (2b+d)q^2 \tag{72}$$

The first-order condition is

$$a - c - 2(2b+d)q = 0 \tag{73}$$

which reduces to

$$MC(q) = c + 2dq = a - 4bq = MMR(q)$$

This yields equilibrium output

$$q_8 = \frac{(a-c)}{2(2b+d)} \tag{74}$$

and associated prices and profits

$$\begin{aligned} w_8 &= \frac{ab+bc+ad}{2b+d} && (75)\\ p_8 &= \frac{3ab+2ad+bc}{2(2b+d)} && (76)\\ \pi_8^U &= \frac{(a-c)^2}{4(2b+d)} && (77)\\ \pi_8^D &= b\left(\frac{a-c}{2(2b+d)}\right)^2 && (78) \end{aligned}$$

This solution is illustrated in Figure 2. Note that while the equilibrium output $q_8$ is determined by the intersection between the $MC(q)$ and $MMR(q)$ curves, the corresponding wholesale price

$w_8$ is read on the downstream firm's demand function $D(w)$.

**4.3. Comparing across scenarios.** A long-standing question in competition policy is whether buyer power by downstream firms in the intermediate goods market works to reduce the distortions created by seller power by upstream firms. Whether this is the case turns out to depend on parameter values. Direct comparison of the different output levels yields the following ranking across scenarios:

**Proposition 1.**

$$\text{When } b > d : q_1 > q_3 > q_4 > q_2 = q_5 = q_6 > q_7 > q_8 \tag{79}$$

$$\text{When } b = d : q_1 > q_3 = q_4 > q_2 = q_5 = q_6 > q_7 = q_8 \tag{80}$$

$$\text{When } b < d : q_1 > q_4 > q_3 > q_2 = q_5 = q_6 > q_8 > q_7 \tag{81}$$

In other words, buyer power may, but need not, induce more efficient output levels. The reason is that both a powerful seller and a powerful buyer have different (but related) incentives to reduce output below the bilaterally efficient level. A monopolist seller that faces a downward-sloping demand curve on the output market will take into account that by increasing output, it will reduce what it earns on all inframarginal units. Similarly, a monopsonist that faces an upward-sloping supply curve on the input market will take into account that by increasing output (and hence input demand), it will increase what it pays for all inframarginal input units. In either case, a firm with market power will have an incentive to withhold output and produce below the socially efficient level.[5] The withholding incentive of the downstream firm is stronger than that of the upstream firm exactly when the price elasticity of demand for the intermediary good is higher than the price elasticity of supply. This is exactly the case when $b < d$.

The ranking shows that when either or both firms earn more than the market rate of return, then the output produced in equilibrium is socially too low. Price posting allows the responder to add a margin above its costs, which creates a distortion of output. Yet when one firm is so dominant that it can dictate terms to the other firm completely, then the responder has no such ability and thus the outcome is bilaterally efficient.

Note that under price setting by the upstream firm, the downstream firm still retains a significant amount of market power due to its ability to choose the output at that posted price. This is evidenced by its ability to set a positive markup. In contrast, when it receives a take-it-or-leave-it offer from the upstream firm, it is kept to its participation constraint, earning no profits at all. Fouraker (1957) was the first to note that a truly dominant firm would present its counterpart with a take-it-or-leave it offer, rather than allow it to respond with a quantity of its own choosing (i.e. along its demand or supply function, as the case may be).

According to Blair et al. (1989), there is at least a dozen textbook treatments of the bilateral monopoly problem that postulate that a negotiated solution will fall somewhere in

[5]We also note that because of the linearity of the system, we have $q_1 = 2q_2$, $q_3 = 2q_7$ and $q_4 = 2q_8$.

the "range" $(q_7, w_7) \rightarrow (q_8, w_8)$ between the two price-posting outcomes, presumably arrived at as a compromise between the two firms' "preferred" solutions. This is somewhat odd, for any movement between these points involves both a change in wholesale price and quantity. As we will confirm later, Nash bargaining over the terms of trade will typically not lead to any solutions in this range, but may deliver these points as extreme solutions to completely different bargaining procedures. The notion that bargaining will lead to some compromise between the monopoly outcome and the monopsony outcome is therefore not well-founded. Morgan (1949) argues that away from cases of extreme dominance (in which the outcomes will be the take-it-or-leave-it ones), agreement will be on the locus connecting the points $(q_7, w_7) \rightarrow (q_2, w_2) \rightarrow (q_8, w_8)$. Again, this solution involves the comparison of price-quantity combinations on the upstream firm's supply function with combinations on the downstream firm's demand function. Yet it is not clear why either firm would want to cede the right to set output to the other firm, even if a wholesale price could be agreed upon.

For completeness, note that

$$\lim_{d \to 0} |q_1 - q_3| = \lim_{d \to 0} |q_2 - q_4| = 0 \tag{82}$$

This means that as the production technology approaches constant returns to scale, the pure monopsony outcome approaches the zero-profit outcome, while the pure monopoly outcome approaches the bilaterally efficient outcome under two-sided price taking.

**4.4. Elimination of double marginalization.** Because the output under two-sided price taking is guided by the true costs and benefits of the upstream firm and the downstream firm and neither charge a markup, this outcome also maximises joint profits $\pi = \pi^U + \pi^D$. Indeed, when firms can credibly make all-or-nothing offers, they have no incentive to distort output away from the bilaterally efficient level.

As seen above, price posting, whether by the upstream firm or the downstream firm, leads to inefficiently low production. The reason (first articulated by Spengler, 1950) is a familiar one, namely that the two firms choose output levels to maximize each their own profits and in doing so, will not ensure that the marginal cost of production equals the marginal revenue product. Two standard ways of avoiding this inefficiency is outright integration (which leads to the same outcome as that under two-sided price taking behavior) or to use non-linear contracts. A two-part tariff that will induce the bilaterally efficient output is a *cost-plus* contract in which the downstream firm pays the upstream firm

$$T(q) = A + (c + dq)q \tag{83}$$

This transfer ensures that the downstream firm demands and sells output such that joint profits

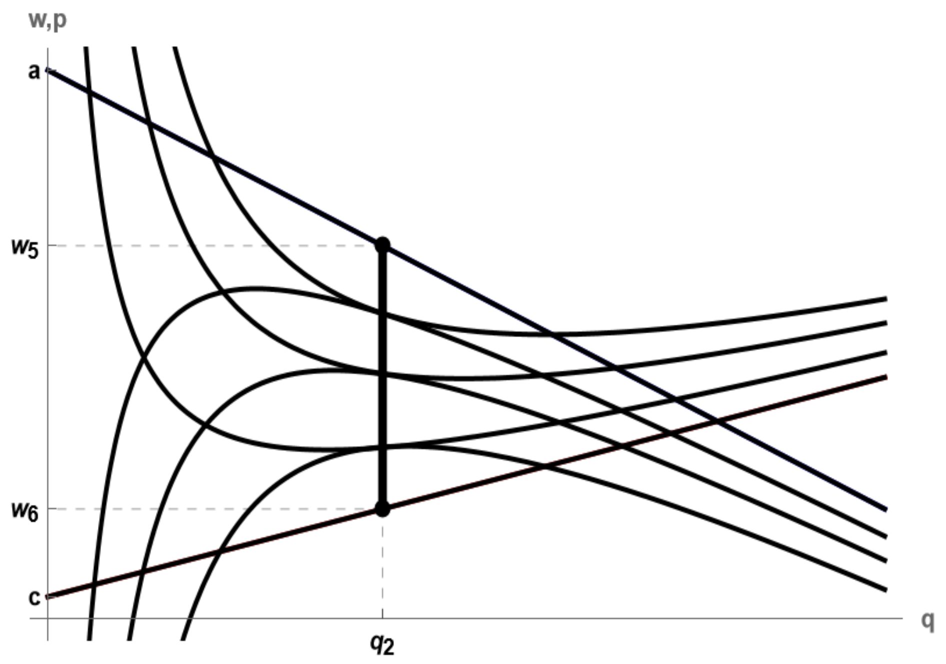



Figure 3: Contract curve in $(q, w)$-space. Along the contract curve, the iso-profit curves are at points of tangency.

are maximized. To see this, note that its profits are

$$\begin{aligned}
\hat{\pi}^D &= pq - T(q) && (84)\\
&= (a - c - (b + d)q)q - A && (85)\\
&= \pi^U + \pi^D - A && (86)
\end{aligned}$$

These profits correspond to the joint profits of the vertical structure, less the lump-sum payment $A$, which regulates the rents earned by the two parties. Note that in the special case of constant returns to scale, where $d = 0$, this contract stipulates that the inputs are priced at marginal cost $c$. Figure 3 illustrates the contract curve of Pareto efficient outcomes, i.e. the agreements $(q, w)$ that are bilaterally efficient. Denote the set of those agreements by $\mathcal{C}_e$. As can be seen from the figure, along the contract curve the iso-profit curves of the two firms are at points of tangency, showing that there are no reallocations that can make either firm better off without leaving the other firm worse off. This is also easily seen from expressions (12)-(13), which confirm that

$$\frac{\Delta w}{\Delta q}|_{\Delta\pi^U=0} = \frac{\Delta w}{\Delta q}|_{\Delta\pi^D=0} \tag{87}$$

whenever output is $q = q_2$, irrespective of the wholesale price $w$.

## 5. Price formation via Nash bargaining

While the early literature on bilateral monopoly explicitly appealed to notions of bargaining and bargaining power, it did not have at its disposal a microfounded or axiomatized bargaining framework like Nash bargaining. Subsequent work has directly embraced the generalized Nash bargaining solution to different aspects of the terms of trade. We will consider two such protocols that appear in the literature, which I term *complete* and *partial* Nash bargaining, respectively.

**5.1. Complete bargaining.** Under complete bargaining, the two firms negotiate directly over pairs $(q, w)$. For any agreement $(q, w)$, the profits of the two firms are

$$\pi^U(q, w) = (w - c - dq)q \tag{88}$$
$$\pi^D(q, w) = (a - bq - w)q \tag{89}$$

The agreement reached through generalized Nash bargaining is the solution to the problem

$$\max_{(q,w)} ((a - bq - w)q)^\gamma ((w - c - dq)q)^{1-\gamma} \tag{90}$$

where $\gamma \in [0, 1]$ denotes the bargaining power of the downstream firm. Solving the two first-order conditions yields the solution

$$q_e(\gamma) = \frac{a - c}{2(b + d)} = q_2 \tag{91}$$
$$w_e(\gamma) = \frac{ad + bc + [\gamma c + (1 - \gamma)a](b + d)}{2(b + d)} \tag{92}$$
$$p_e(\gamma) = \frac{ab + 2ad + bc}{2(b + d)} \tag{93}$$
$$\pi_e^U(\gamma) = \frac{(1 - \gamma)(a - c)^2}{4(b + d)} \tag{94}$$
$$\pi_e^D(\gamma) = \frac{\gamma(a - c)^2}{4(b + d)} \tag{95}$$

It is easily verified that $w_e(\gamma) = (1-\gamma)AR(q_2)+\gamma AC(q_2)$. The solution is illustrated in Figure 4 in $(\pi^D, \pi^U)$-space. As the bargaining power $\gamma$ of the downstream firm increases from zero to one, the solution moves along the bargaining set from the point $(\pi_5^D, \pi_5^U)$ to the point $(\pi_6^D, \pi_6^U)$.

Note that the output is always at the bilaterally efficient level $q_2$, but that the wholesale price varies with the bargaining power so that it ranges between the average cost of the seller and the average revenue product of the buyer. That is,

$$\lim_{\gamma \to 1} w_e(\gamma) = w_6 \tag{96}$$
$$\lim_{\gamma \to 0} w_e(\gamma) = w_5 \tag{97}$$

and so the wholesale price traces the range $[AC(q_2), AR(q_2)]$ as the bargaining power shifts between firms. This also means that the efficient bargaining outcome contains the two take-it-or-leave-it offers as special cases, namely when $\gamma = 0$ or $\gamma = 1$, respectively. Define the

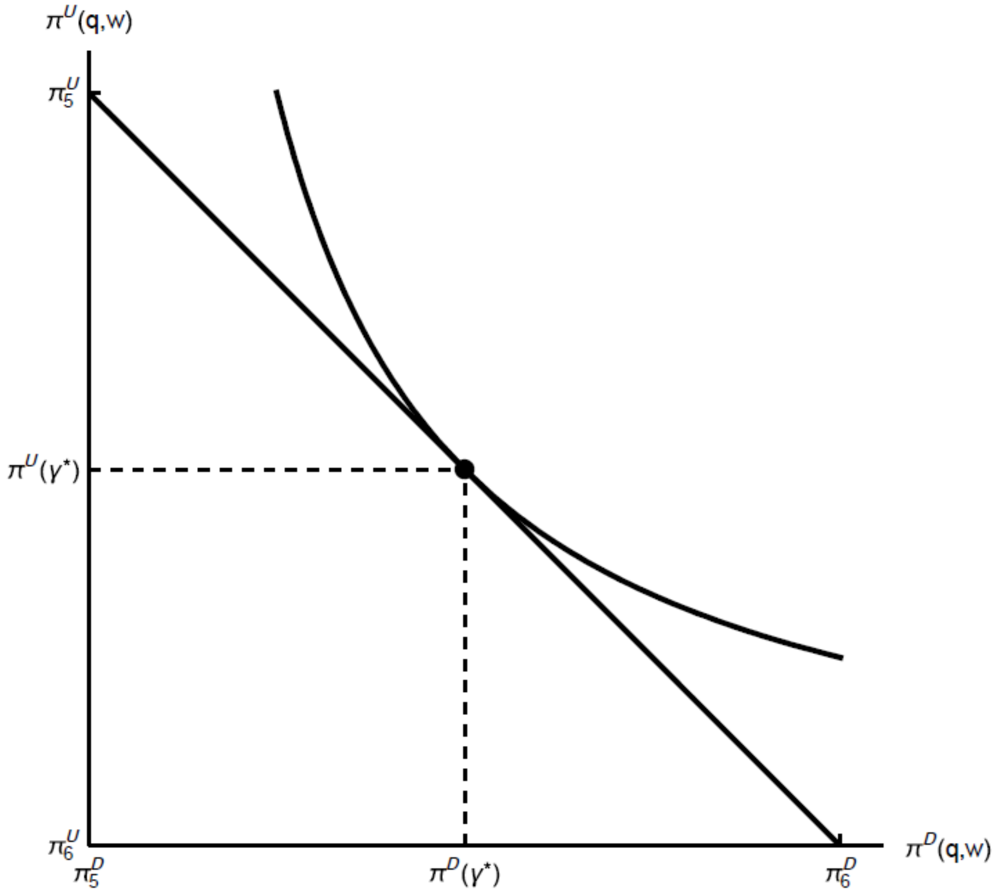



Figure 4: The Nash bargaining solution under complete bargaining.

contract curve under complete bargaining as

$$\mathcal{C}_e \equiv \{(q,w) \in \mathbb{R}^2_+ : q_e(\gamma) = q_2 \text{ and } w \in [w_6, w_5]\} \tag{98}$$

Here subscript $e$ stands for (bilaterally) efficient bargaining. The contract curve $\mathcal{C}_e$ is illustrated in Figure 5.

The outcome under efficient bargaining essentially implements a cost-plus contract with transfer

$$T_\gamma(q) = \pi_e^U(\gamma) + C(q) \tag{99}$$

$$= \frac{(1-\gamma)(a-c)^2}{4(b+d)} + (c+dq)q \tag{100}$$

from the downstream firm to the upstream firm. To see this, it suffices to note that

$$\hat{\pi}^D(\gamma) = pq - T_\gamma(q) \tag{101}$$

$$= (a-c-(b+d)q)q - \frac{(1-\gamma)(a-c)^2}{4(b+d)} \tag{102}$$

$$= \pi^U + \pi^D - \frac{(1-\gamma)(a-c)^2}{4(b+d)} \tag{103}$$

Under complete bargaining, because output does not depend on the relative bargaining power of the two firms, neither do the retail price, consumer surplus or total profits.

Last, direct inspection confirms that

$$w_e(\gamma) \leq w_2 \Leftrightarrow \gamma \geq \frac{b}{b+d} \equiv \gamma^* \tag{104}$$

This means that relative to the benchmark of two-sided price taking behavior, higher bargaining power for a firm changes the wholesale price in that firm's favour. The critical value of the bargaining weight $\gamma^*$ will play an important role in the comparison between complete and partial bargaining. It is a measure of the relative sensitivity of costs and revenues to an increase in output, i.e. of the relative magnitudes of the parameters $b$ and $d$.

Blair and Kaserman (1987) propose a pricing rule designed to ensure that industry profits are maximized while giving a share $\gamma$ of the joint profits to the downstream firm. Specifically, the pricing rule is the wholesale price $w$ that solves the problem

$$\pi^D = (a-w-bq)q = \gamma(a-c-(b+d)q)q = \gamma(\pi^U + \pi^D) \tag{105}$$

Solving this equality yields wholesale price

$$w^* = (a-bq) - \gamma(a-c-q(b+d)) \tag{106}$$

Upon substitution of $q = q_2$, it is easily verified that $w^* = w_e(\gamma)$, i.e. that their pricing rule exactly coincides with the wholesale price under complete bargaining.[6]

**5.2. Partial bargaining.** Under complete bargaining, the two parties negotiate over pairs $(q, w)$. Suppose in contrast that bargaining is only over the wholesale price $w$ and that one of the parties is subsequently assigned the power to choose an output level $q$. In this case, we need to distinguish between two cases, namely the case in which the buyer chooses output and the case in which the seller chooses output.

In this bargaining protocol, the Pareto criterion of the Nash bargaining solution applies for a *given* output $q$ and then one of the two firms is allocated the decision to choose this output as it sees fit. Manning (1987) considers a more general sequential bargaining protocol in which the parties first negotiate over one variable and then separately over the other variable. Milliou et al. (2009) go one step further and allow firms to negotiate over both the contract *form* and the contract *terms*.

**Upstream firm sets output.** When the upstream firm sets the output, it will choose a point on its supply curve $S(w)$. In this case, the profits of the two firms are given by

$$\pi_u^U(w) \equiv \pi^U(w, S(w)) = \frac{(w-c)^2}{4d} \tag{107}$$

$$\pi_u^D(w) \equiv \pi^D(w, S(w)) = \frac{(w-c)(2ad+bc-w(b+2d))}{4d^2} \tag{108}$$

where subscript $u$ denotes that the quantity is chosen by the upstream firm. The agreement reached through generalized Nash bargaining is the solution to the problem

$$\max_{w \geq 0} \left(\frac{(w-c)(2ad+bc-w(b+2d))}{4d^2}\right)^{\gamma} \left(\frac{(w-c)^2}{4d}\right)^{1-\gamma} \tag{109}$$

The solution to this problem is

$$q_u(\gamma) = \frac{(2-\gamma)(a-c)}{2(b+2d)} \tag{110}$$

$$w_u(\gamma) = \frac{bc+2ad-\gamma d(a-c)}{b+2d} \tag{111}$$

$$p_u(\gamma) = \frac{2a(b+2d)-b(2-\gamma)(a-c)}{2(b+2d)} \tag{112}$$

$$\pi_u^U(\gamma) = \frac{(a-c)^2 d(\gamma-2)^2}{4(b+2d)^2} \tag{113}$$

$$\pi_u^D(\gamma) = \frac{(a-c)^2\gamma(2-\gamma)}{4(b+2d)} \tag{114}$$

Define the contract curve under this bargaining protocol as

$$\mathcal{C}_u \equiv \{(q, w) \in \mathbb{R}_+^2 : q = q_u(\gamma) \text{ and } w = w_u(\gamma)\} \tag{115}$$

[6] Blair and Kaserman (1987) state that *"the franchisor would set the wholesale price equal to the cost of production plus the agreed upon share, $\alpha$, times the optimal integrated monopoly markup over all costs"*.

Here subscript $u$ denotes that output is chosen by the upstream firm. The contract curve $\mathcal{C}_u$ is illustrated in Figure 5. When the quantity is chosen by the upstream firm, the generalized Nash bargaining solution traces out the portion of the supply curve $S(w)$ which is both (i) Pareto efficient and (ii) individually rational for the downstream firm. Condition (i) means that $q \geq q_8$, while condition (ii) means that $q \leq q_3$. To see this, note that

$$\frac{\partial \pi_u^U(w)}{\partial q} = a - c - 2(2b+d)q \leq 0 \Leftrightarrow q \geq q_8 \tag{116}$$

$$\frac{\partial \pi_u^D(w)}{\partial q} = 2bq > 0 \tag{117}$$

This means that when the downstream firm is a price taker, it will prefer to produce as much output as possible. Although the upstream firm will set output $q_8$ under price posting, the downstream firm is better off moving down along its demand curve till it reaches the output $q_4$, the highest level consistent with the upstream firm's participation constraint. It should be emphasized that points on the demand curve $D(w)$ for which $q \in [0, q_8]$ are not Pareto efficient, for in this range, output can be increased while increasing the profits of *both* firms.

It is worth pointing out that increasing the bargaining power of the firm that chooses output has the effect of increasing output. It is also easily verified that

$$q_u(\gamma) \geq q_2 \Leftrightarrow \gamma \leq \gamma^* \tag{118}$$

This means that if the firm that has the right to manage also has significant bargaining power, then it will set a (bilaterally) inefficiently high output, i.e. even higher than the output that it would set under a take-it-or-leave-it offer or under bilaterally efficient bargaining. Yet this is socially desirable, as it brings output closer to (but never higher than) the socially optimal output $q_3$.

It is easy to verify that

$$q_u(\gamma) = \frac{(2-\gamma)(a-c)}{2(b+2d)} = (1-\gamma)q_3 + \gamma q_7 \tag{119}$$

and therefore that key benchmark solutions are obtained as special cases under this bargaining protocol:

$$\lim_{\gamma \to 1} q_u(\gamma) = q_7 \tag{120}$$

$$\lim_{\gamma \to 0} q_u(\gamma) = q_3 \tag{121}$$

$$\lim_{\gamma \to 1} w_u(\gamma) = w_7 \tag{122}$$

$$\lim_{\gamma \to 0} w_u(\gamma) = w_3 \tag{123}$$

**Downstream firm sets output.** When the downstream firm sets the output, it will choose a level on its demand curve $D(w)$. In this case, the profits of the two firms are given by

$$\pi_d^U(w) \equiv \pi^U(w, D(w)) = \frac{(a-w)(w(2b+d)-ad-2bc)}{4b^2} \tag{124}$$

$$\pi_d^D(w) \equiv \pi^D(w, D(w)) = \frac{(a-w)^2}{4b} \tag{125}$$

where subscript $d$ denotes that the quantity is chosen by the downstream firm. The agreement reached through generalized Nash bargaining is the solution to the problem

$$\max_{w \geq 0} \left(\frac{(a-w)^2}{4b}\right)^{\gamma} \left(\frac{(a-w)(w(2b+d)-ad-2bc)}{4b^2}\right)^{1-\gamma} \tag{126}$$

The solution to this problem is

$$q_d(\gamma) = \frac{(1+\gamma)(a-c)}{2(2b+d)} \tag{127}$$

$$w_d(\gamma) = \frac{ab+bc+ad-\gamma b(a-c)}{2b+d} \tag{128}$$

$$p_d(\gamma) = \frac{2a(2b+d)-b(1+\gamma)(a-c)}{2(2b+d)} \tag{129}$$

$$\pi_d^U(\gamma) = \frac{(a-c)^2(1-\gamma)(1+\gamma)}{4(2b+d)} \tag{130}$$

$$\pi_d^D(\gamma) = \frac{(a-c)^2 b(1+\gamma)}{4(2b+d)^2} \tag{131}$$

Define the contract curve under this bargaining protocol as

$$\mathcal{C}_d \equiv \{(q,w) \in \mathbb{R}_+^2 : q = q_d(\gamma) \text{ and } w = w_d(\gamma)\}$$

The contract curve $\mathcal{C}_d$ is illustrated in Figure 5. When the quantity is chosen by the downstream firm, the generalized Nash bargaining solution traces out the portion of the demand curve $D(w)$ which is both (i) Pareto efficient and (ii) individually rational for the upstream firm. Condition (i) means that $q \geq q_7$, while condition (ii) means that $q \leq q_4$. To see this, note that

$$\frac{\partial \pi_d^U(w)}{\partial q} = 2dq > 0 \tag{132}$$

$$\frac{\partial \pi_d^D(w)}{\partial q} = a-c-2(b+2d)q \leq 0 \Leftrightarrow q \geq q_7 \tag{133}$$

This means that when the upstream firm is a price taker, it prefers to produce as much as possible. That is, although the downstream firm will set output $q_7$ under price posting, the upstream firm would prefer the highest possible output consistent with the downstream firm's participation constraint, moving up along the supply curve till the output $q_4$ is reached. It should be emphasized that points on the supply curve $S(w)$ for which $q \in [0, q_7]$ are not Pareto efficient, for in this range, output can be increased while increasing the profits of *both* firms.

Again, we have that

$$q_d(\gamma) \geq q_2 \Leftrightarrow \gamma \geq \gamma^* \tag{134}$$

Note that as $\gamma$ increases $\gamma^*$, output approaches the pure monopoly level $q_4$. This leads to an unambiguous decrease in industry profits, but the effects on overall social welfare are ambiguous, because the ranking of the (socially optimal) monopsony level $q_3$ and the pure monopoly level $q_4$ depends on the elasticities of costs and final demand.

It is easy to verify that

$$q_d(\gamma) = \frac{(1+\gamma)(a-c)}{2(2b+d)} = \gamma q_4 + (1-\gamma)q_8 \tag{135}$$

and therefore that key benchmark solutions are obtained as special cases under this bargaining protocol:

$$\lim_{\gamma \to 0} q_d(\gamma) = q_8 \tag{136}$$
$$\lim_{\gamma \to 1} q_d(\gamma) = q_4 \tag{137}$$
$$\lim_{\gamma \to 0} w_d(\gamma) = w_8 \tag{138}$$
$$\lim_{\gamma \to 1} w_d(\gamma) = w_4 \tag{139}$$

**5.3. Comparing across protocols.** The bargaining sets $\mathcal{C}_e$, $\mathcal{C}_u$ and $\mathcal{C}_d$ under the three protocols are compared in Figure 5 in $(q, w)$-space, which also illustrates that all the benchmark cases $(q_i, w_i)$, $i = 2, ..., 8$ considered earlier obtain as special cases of one of the three protocols as the bargaining weight $\gamma$ is varied. In the bargaining literature, it is more common to illustrate the solution in $(\pi^D, \pi^U)$-space as this also elucidates and compares which payoffs are feasible under the three different protocols.

In the upper panel of Figure 6, the bargaining sets under complete and partial bargaining are compared. The figure illustrates that the possible outcomes in $(\pi^D, \pi^U)$-space under partial bargaining are everywhere below those under complete bargaining, except when the bargaining weights are $(\gamma^*, 1-\gamma^*)$, in which case they exactly coincide. It is worth mentioning that starting at either of the points of contact $(\pi_d^U(\gamma^*), \pi_d^U(\gamma^*))$ and $(\pi_u^U(\gamma^*), \pi_u^U(\gamma^*))$, a movement in either direction along the bargaining sets (corresponding to a change in the bargaining weights), necessarily involves an increase in the profits of one firm, but a decrease in the profits of the other. The upward-sloping segment on the left-hand side figure, from the origin to point $(\pi_8^U, \pi_8^D)$, corresponds to the segment of the downstream firm's demand function along which $q \in [0, q_8]$. Similarly, the upward-sloping segment on the right-hand side figure, from the origin to point $(\pi_7^U, \pi_7^D)$, corresponds to the segment of the upstream firm's supply function along which $q \in [0, q_7]$.

In the lower panel of Figure 6, the Nash bargaining solutions under partial bargaining are illustrated, for a few different bargaining weights. Consider first the left-hand side figure, which

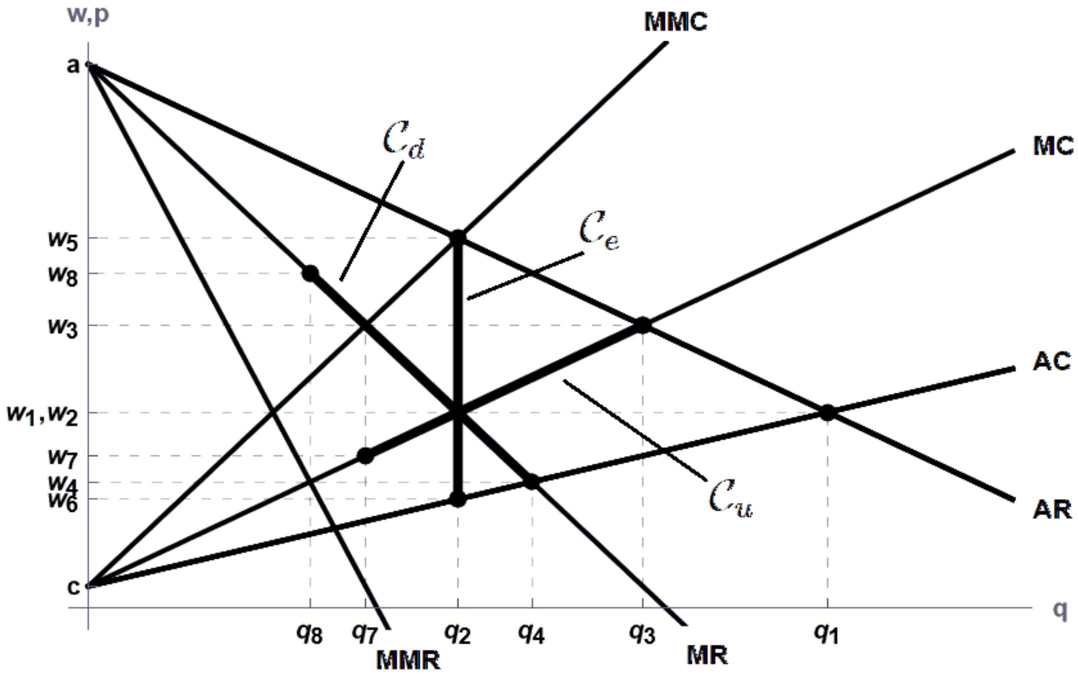



Figure 5: Contract curves in $(q, w)$-space under complete and partial Nash bargaining compared.

corresponds to the case in which movement is along the downstream firm's demand function. When the upstream firm has all the bargaining power, the solution yields equilibrium profits $(\pi_8^U, \pi_8^D)$, i.e. the same solution as under price setting by the upstream firm. Note that this in fact yields the highest possible profit for the upstream firm, subject to choosing points along $D(w)$. As the downstream firm's bargaining power increases, the solution moves rightward along the bargaining set, ending up in point $(\pi_4^U, \pi_4^D)$, i.e. the pure monopoly solution, in which the upstream firm earns no profits and those of the downstream firm are at their highest. Similarly, the right-hand side figure corresponds to the case in which movement is along the upstream firm's supply function. When the downstream firm has all the bargaining power, the outcome is point $(\pi_7^U, \pi_7^D)$, i.e. the same solution as under price setting by the downstream firm. This yields the highest possible profits for the downstream firm, subject to being on a point along $S(w)$. As the bargaining power of the upstream firm increases, the solution moves leftward along the bargaining set, ending in point $(\pi_3^U, \pi_3^D)$, i.e. the pure monopsony solution, in which the downstream firm earns no profits and those of the upstream firm are at their highest. Last, when the bargaining weight of the downstream firm is exactly at the magical level $\gamma^*$, we have

$$\pi_d^U(\gamma^*) = \pi_u^U(\gamma^*) = \pi_e^U(\gamma^*) \quad (140)$$

$$\pi_d^D(\gamma^*) = \pi_u^D(\gamma^*) = \pi_e^D(\gamma^*) \quad (141)$$

and the solutions under all three bargaining protocols coincide.

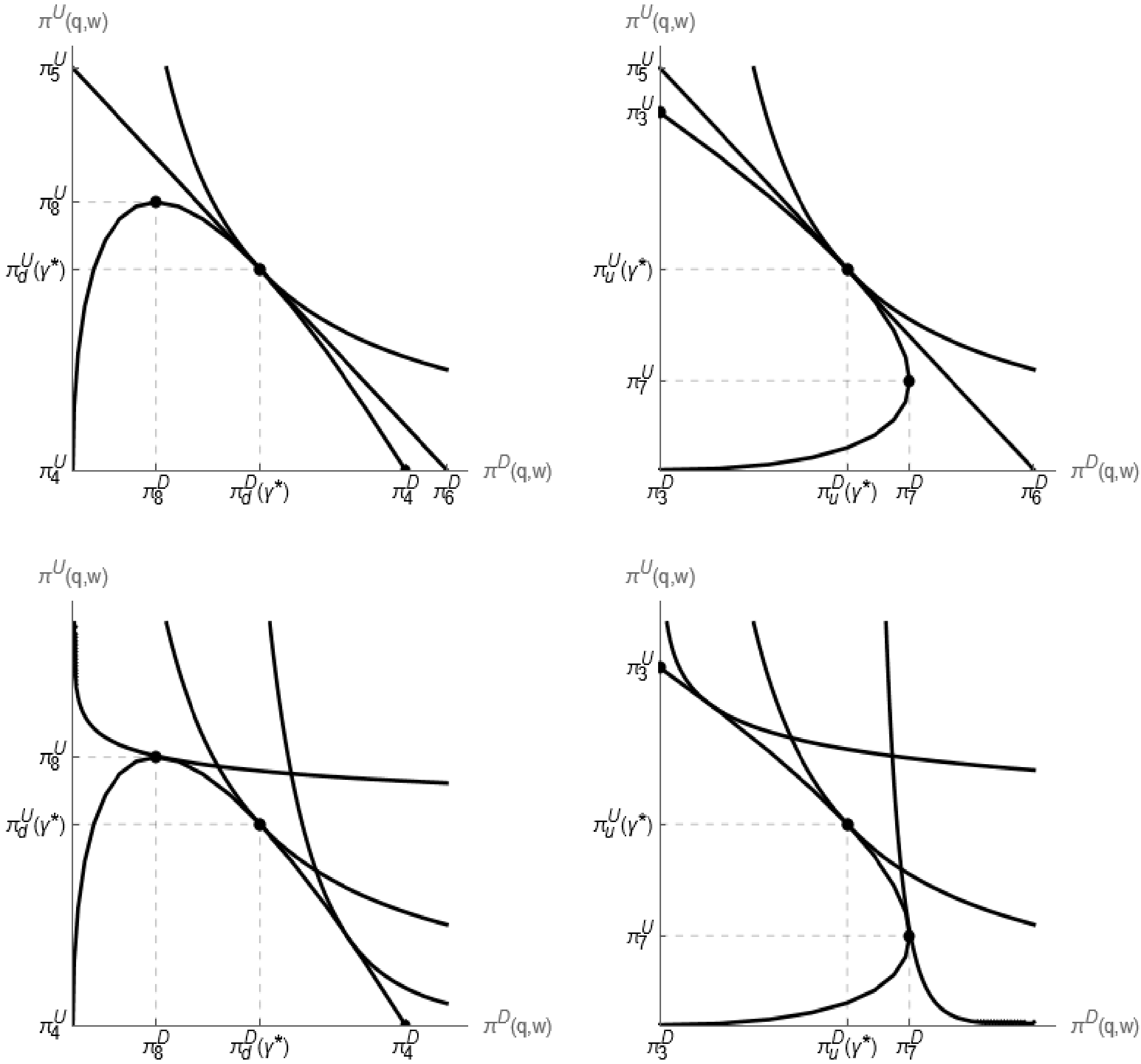



Figure 6: Bargaining sets and bargaining solutions under complete and partial bargaining. Upper panel: bargaining sets compared; lower panel: bargaining solutions compared. Left column: downstream firm sets output; right column: upstream firm sets output.

In the present analysis, we have considered the consequences of different bargaining protocols, taking as given that both firms are tied to (or have committed to) a particular procedure for negotiation. Yet it is interesting to consider conditions under which either firm would prefer one protocol over the other. Fouraker (1957) suggested such a comparison between price posting and movement along the contract curve.[7] It is straightforward to confirm that

$$\pi_e^U(\gamma) \geq \pi_u^U(\gamma) \Leftrightarrow \gamma \leq \gamma^* \tag{142}$$
$$\pi_e^D(\gamma) \geq \pi_u^D(\gamma) \Leftrightarrow \gamma \geq \gamma^* \tag{143}$$
$$\pi_e^U(\gamma) \geq \pi_d^U(\gamma) \Leftrightarrow \gamma \leq \gamma^* \tag{144}$$
$$\pi_e^D(\gamma) \geq \pi_d^D(\gamma) \Leftrightarrow \gamma \geq \gamma^* \tag{145}$$

From the comparative statics outlined above, it is straightforward to compare the two firms' profits across bargaining protocols. It is immediately clear that in the special case $\gamma = \gamma^*$, the two firms are in fact indifferent because the three protocols implement the same agreement $(q_2, w_2)$. Consider then an increase in the bargaining power $\gamma$. Depending on which firm has the ability to choose output, an increase in $\gamma$ will move the agreement from $(q_2, w_2)$ to some other agreement $(q', w')$ along either $S(w)$ or $D(w)$.

Note that because $\pi_6^U = \pi_4^U = 0$ (in both cases, the output lies on the $AC(q)$ curve), we have

$$\lim_{\gamma \to 1} \pi_e^U(\gamma) - \pi_d^U(\gamma) = 0 \tag{146}$$

Similarly, because $\pi_5^D = \pi_3^D = 0$ (in both cases, the output lies on the $AR(q)$ curve), we have

$$\lim_{\gamma \to 0} \pi_e^D(\gamma) - \pi_u^D(\gamma) = 0 \tag{147}$$

These limits are confirmed in Figure 7, which illustrates and compares the two firms' profits under the different bargaining protocols.

**Random proposals.** As noted by Muthoo (1999), bargaining parties are usually better off when they act as proposers than when they respond to proposals and therefore assigning the proposer role randomly to one of the parties can be viewed as a short-cut for a more complicated bargaining procedure. In such models, the probability with which a party becomes the proposer plays the role of the bargaining strength. Chemla (2003) and Nocke and Rey (2018) consider one-shot procedures in which the proposer gets to make a take-it-or-leave-it offer to the responder. Under such proposals, output is always at the bilaterally efficient level and the ex-ante expected profits of the parties are simply the average payoffs $\gamma(\pi^U + \pi^D)$ and $(1-\gamma)(\pi^U + \pi^D)$ for the upstream and the downstream firm, respectively. In this case, the outcome under random proposer assignment is both outcome equivalent and payoff equivalent to the outcome under complete bargaining and the solution $(q_e(\gamma), w_e(\gamma), p_e(\gamma))$ is replicated ex ante.

[7] Mukherjee and Mukherjee (2024) also compare the welfare properties of alternative price setting procedures.

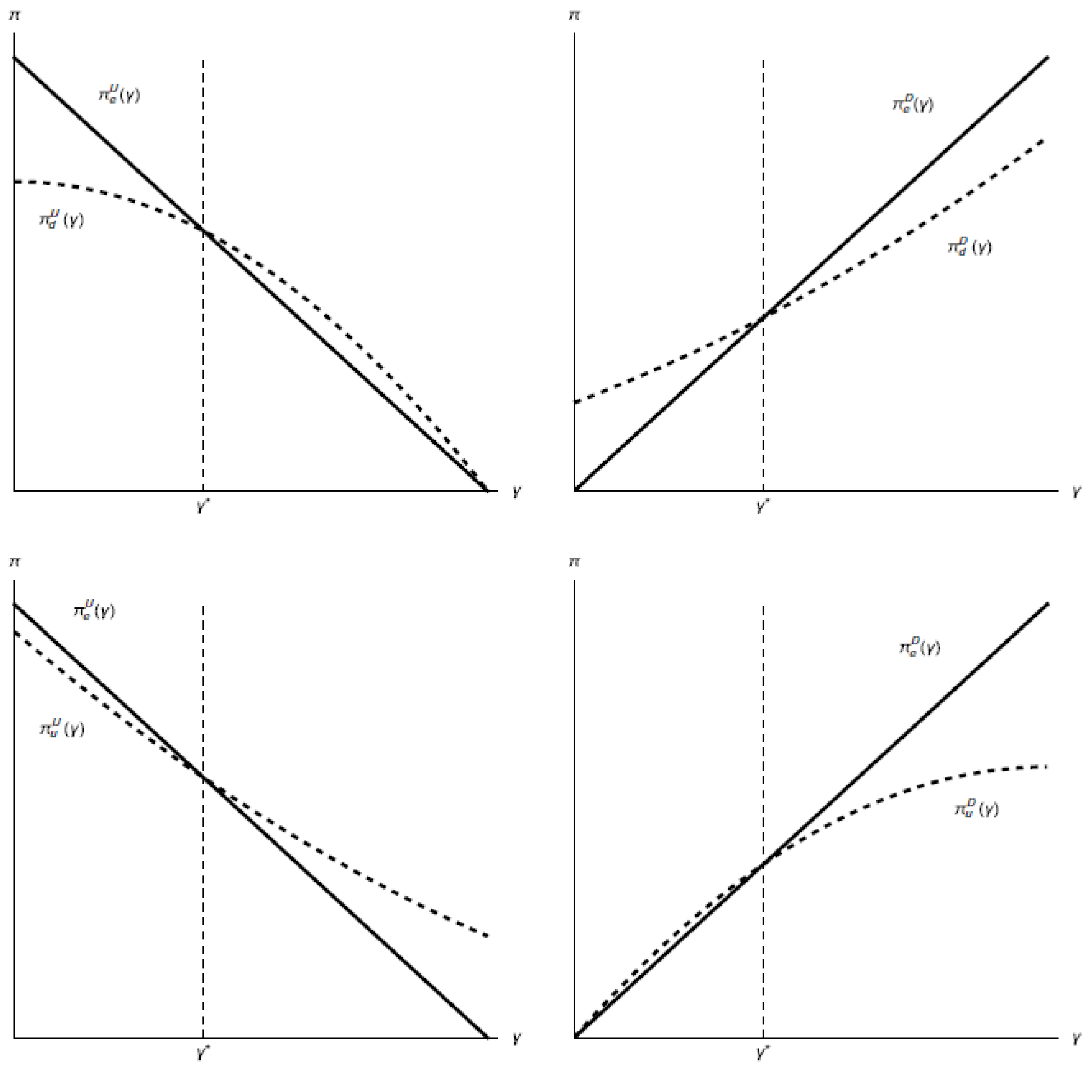



Figure 7: Profits under complete and partial bargaining. Upper panel: downstream firm sets output; lower panel: upstream firm sets output. Left column: upstream firm's profits; right column: downstream firm's profits.

As an alternative to this procedure, one can also consider a setting in which one of the firms is randomly assigned the role of proposing a price and then one of the firms is assigned the role of choosing output at that price. Under such a procedure, the outcome would be ex post equivalent to what would be chosen under one of the two price posting models. Ex ante, the solution would be

$$q_r(\gamma) = \gamma q_8 + (1-\gamma) q_7 \quad (148)$$
$$w_r(\gamma) = \gamma w_8 + (1-\gamma) w_7 \quad (149)$$
$$p_r(\gamma) = \gamma p_8 + (1-\gamma) p_7 \quad (150)$$

## 6. Bargaining Strength versus Competitive Pressure

A firm's ability to influence outcomes when dealing with other firms is succinctly captured in the notion of bargaining power, but bargaining power is shorthand for other features of the environment, such as the degree of direct or indirect competition that the firm faces for its services. It is therefore of interest to explicitly relate the outcomes under bargaining with those under bilateral oligopoly when one or the other side becomes increasingly competitive. In the appendix, I solve the model in which the upstream sector consists of $M \geq 1$ symmetric firms who sell a homogeneous intermediate good to the downstream sector, which consists of $N \geq 1$ symmetric firms producing a homogeneous final good. Production costs and final demand are as in the case of bilateral monopoly, which obtains as the special case with $M = N = 1$. As in the case of bilateral monopoly, the model can be solved with either price setting by upstream firms or price setting by downstream firms. Aggregate output under these two settings are

$$Q_d(M,N) = \frac{MN(a-c)}{2dN + b(M+1)(N+1)} \quad (151)$$
$$Q_u(M,N) = \frac{MN(a-c)}{(Mb+2d)(N+1)} \quad (152)$$

where subscript $d$ denotes that the output is set by downstream firms, while subscript $u$ denotes that output is set by upstream firms. Taking limits shows that

$$\lim_{M\to\infty} Q_d(M,1) = \lim_{M\to\infty} Q_u(M,1) = \frac{a-c}{2b} = \lim_{d\to 0} q_2 \quad (153)$$
$$\lim_{N\to\infty} Q_d(1,N) = \frac{a-c}{2(b+d)} = q_2 \quad (154)$$
$$\lim_{N\to\infty} Q_u(1,N) = \frac{a-c}{b+2d} = q_3 \quad (155)$$

Furthermore, it can be shown that

$$\lim_{M,N\to\infty} Q_d(M,N) = \lim_{M,N\to\infty} Q_u(M,N) = \frac{a-c}{b} = \lim_{d\to 0} q_3 = \lim_{d\to 0} q_1 \quad (156)$$

To understand these results, it is useful to consider the effects of competition upstream and downstream separately, as these are qualitatively different. First, consider the case with a single downstream firm and consider an increase in the number of upstream firms $M$. Whether prices are set upstream or downstream, as the upstream sector grows, each firm produces a vanishing share of aggregate output. This has two effects. First, the markup charged by upstream firms becomes negligible, thereby eliminating double marginalisation. In addition, because each firm produces a vanishing amount, production technology becomes approximately constant returns to scale. The upshot of these observations is that output approaches the bilaterally efficient level under constant returns to scale (which is also seen by the lack of the parameter $d$ in the output levels). Next, consider the case with a single upstream firm and consider an increase in the number of downstream firms $N$. Suppose that prices are set by the upstream sector. In this case, the downstream firms are standard Cournot competitors that exert the usual market power in competing for final consumers. As their number increases, their derived input demand function approaches the demand function for final goods and so the one upstream firm effectively faces the demand function of a vertically integrated bilateral monopolist. As a consequence, the equilibrium output approaches the bilaterally efficient Walrasian output. Now suppose that prices are set by the downstream sector and consider an increase in the number of downstream competitors. As their number increases, they each have less and less influence on the retail price downstream and on the wholesale price upstream and therefore their aggregate demand function for the intermediate good approaches the demand function for the final good. This means that the upstream firm, which in this scenario takes into account its influence on costs but not their influence on the retail price (because it is a price taker in the intermediate goods market, by assumption), is in the exact same position as the pure monopsonist. In consequence, the equilibrium output approaches the first best level. Last, when the number of firms upstream and downstream becomes very large, the outcome approaches the point at which average revenue equals average costs, yielding the zero profit outcome. which is in this case also the social optimum.

## 7. Markups, markdowns and pass-through

There has been a surge in interest in buyer and seller power in labor and in intermediate goods markets and in the measure of such power, including firm markups and markdowns. In this section, I briefly consider the effects that bargaining has on firm markups and markdowns, defined as follows:

$$\mu_U \equiv w - MC(q) \tag{157}$$

$$\mu_D \equiv p - w \tag{158}$$

$$\delta_D \equiv MR(q) - w \tag{159}$$

As usual, the markups are a measure of the upstream and downstream firm's seller power in the intermediate and final goods markets, respectively, while the markdown is a measure of the downstream firm's buyer power in the intermediate goods market. As the markups and markdowns will depend on the bargaining protocol, we will consider these in turn.

**7.1. Complete bargaining.** Under complete bargaining, the markups and markdowns are given by

$$\mu_U = \frac{(a-c)\left[(1-\gamma)b-\gamma d\right]}{2(b+d)} \tag{160}$$

$$\mu_D = \frac{(a-c)\gamma}{2} \tag{161}$$

$$\delta_D = \frac{-(a-c)\left[(1-\gamma)b-\gamma d\right]}{2(b+d)} \tag{162}$$

Note that in this case, output is fixed and thus there is no quantity effect. Also, it is worth noting that $\delta_D = -\mu_U$ and that $\mu_U \geq 0$ when $\gamma \leq \gamma^*$. This follows straightforwardly from inequality (104), the definitions of $\mu_U$ and $\delta_D$ and the fact that $MR(q_2) = MC(q_2)$. This means that if the upstream firm charges a positive markup (which happens when it is sufficiently strong), then the downstream firm necessarily has a negative markdown (which happens when it is sufficiently weak). While this may seem to imply that either one or the other firm is earning negative profits, this is not the case as both earn non-negative profits as long as $w \in [AC(q_2), AR(q_2)]$, which holds under complete bargaining. While one of the two firms earns a negative profit on the marginal unit it sells, this loss is more than compensated by what it earns on the inframarginal units.

**7.2. Partial bargaining: upstream firm sets output.** Under partial bargaining with output set by the upstream firm, the markups and markdown are

$$\mu_U = 0 \tag{163}$$

$$\mu_D = \frac{(a-c)\gamma}{2} \tag{164}$$

$$\delta_D = \frac{-(a-c)\left[(1-\gamma)b-\gamma d\right]}{b+2d} \tag{165}$$

While the markdown of the downstream firm is the same as under complete bargaining and can thus be of either sign, depending on bargaining strengths, its markup is non-negative. In turn, the upstream firm's markup is in this case zero. The reason is that when the upstream firm sets output, it chooses points along the $MC(q)$ curve, which also determines the magnitude of the wholesale price $w$.

**7.3. Partial bargaining: downstream firm sets output.** Under partial bargaining with output set by the downstream firm, the markups and markdown are

$$\mu_U = \frac{(a-c)\left[(1-\gamma)b - \gamma d\right]}{2b+d} \quad (166)$$

$$\mu_D = \frac{(a-c)b(1+\gamma)}{2(2b+d)} \quad (167)$$

$$\delta_D = 0 \quad (168)$$

While the markup of the upstream firm is the same as under complete bargaining and can thus be of either sign, depending on bargaining strengths, the downstream firm's markup is strictly positive. In turn, the downstream firm's markdown is in this case zero. The reason is that when the downstream firm sets output, it chooses points along the $MR(q)$ curve, which also determines the magnitude of the wholesale price $w$.

**7.4. Comparing across protocols.** The comparative statics with respect to bargaining weight $\gamma$ are summarised in the following table:

| | $q$ | $w$ | $p$ | $\mu_U$ | $\mu_D$ | $\delta_D$ |
|---|---|---|---|---|---|---|
| Complete bargaining over $(q, w)$ | 0 | $-$ | 0 | $-$ | $+$ | $+$ |
| Partial bargaining: upstream firm sets $q$ | $-$ | $-$ | $+$ | 0 | $+$ | $+$ |
| Partial bargaining: downstream firm sets $q$ | $+$ | $-$ | $-$ | $-$ | $+$ | 0 |

As the bargaining power of the downstream firm $\gamma$ increases, the equilibrium wholesale price $w$ unambiguously decreases. Yet the effect on output is ambiguous and depends on the bargaining protocol. Under complete bargaining, there is no quantity response, and just a vertical downward movement along the contract curve. When the downstream firm chooses output, the decrease in wholesale price is accompanied by an increase in output, corresponding to a rightward move down along the downstream firm's demand function. When the upstream firm chooses output, the decrease in wholesale price is accompanied by a decrease in output, corresponding to a leftward move along the upstream firm's supply function. Of course, this does not hurt the downstream firm overall, for the effect of reduced supply is in this case factored in by the generalized Nash bargaining solution, which is constrained to lie on the supply curve. When the bargaining power of the downstream firm increases, its markup always increases, while its markdown increases as long as it is not the firm to choose output. In turn, the markup of the upstream firm decreases, unless it is the firm choosing the output.

**Cost pass-through.** Since price formation depends on the bargaining power of the two firms in the supply chain, it is interesting to study how this influences the degree to which movements in costs and market conditions are transmitted through to final consumers. It is straightforward to confirm that

$$\frac{\partial p_u(\gamma)}{\partial c} = \frac{b(2-\gamma)}{2(b+2d)} > 0 \tag{169}$$

$$\frac{\partial^2 p_u(\gamma)}{\partial c \partial \gamma} = \frac{-b}{2(b+2d)} < 0 \tag{170}$$

$$\frac{\partial p_d(\gamma)}{\partial c} = \frac{b(1+\gamma)}{2(2b+d)} > 0 \tag{171}$$

$$\frac{\partial^2 p_d(\gamma)}{\partial c \partial \gamma} = \frac{b}{2(2b+d)} > 0 \tag{172}$$

Note that while an increase in the cost parameter $c$ unambiguously increases the retail price $p$ under either partial bargaining protocol, the effect of an increase in the bargaining power $\gamma$ on the extent of pass-through depends on whether the upstream or the downstream form sets the output once wholesale prices have been negotiated.

## 8. Discussion

The economic analysis of how firms compete for business from costumers, oligopoly theory, is a well-developed field and forms the core of industrial organization. Yet traditional oligopoly theory mostly sidesteps issues of trade between firms along the supply chain from raw inputs to final goods. The analysis of price formation along supply chains is not only important in its own right, but also central to understanding the level of final prices and their impact on overall social welfare. In this paper, I have drawn on the literature on bilateral monopoly and Nash bargaining to unify the analysis of how trade between vertically linked firms is conducted. I show that partial Nash bargaining over wholesale prices generalizes models of price posting with linear tariffs, while complete Nash bargaining over wholesale prices and output generalizes contracts with two-part tariffs. I show that while the outcomes under complete bargaining are bilaterally efficient, the outcomes under partial bargaining may either increase or decrease social welfare, depending on which firm chooses output and the bargaining strength of the two parties.

For completeness, it should be noted that there are alternative ways of considering contracting between the bilateral monopolists. Some authors consider settings in which firms, rather than setting prices sequentially as is the case under price posting, instead move simultaneously. Because of the one-to-one technology, quantities of the intermediary good and the final good must be equal in equilibrium. If firms set unconditional prices, an equilibrium may not exist. One way around this is to assume that the firms commit to markups, rather than to prices. In the Appendix, I solve the model under this alternative assumption.

## 9. Appendix: competition in markups

In this appendix, I solve the model under the assumption that the firms compete in markups, as described in Choi (1991), Young (1991) and Irmen (1997). Define the downstream firm's *markup* over the wholesale price as

$$m \equiv p - w \tag{173}$$

We can then rewrite the two firms' profits as

$$\pi^U = \left(w - c - d\left(\frac{a-w-m}{b}\right)\right)\frac{a-w-m}{b} \tag{174}$$

$$\pi^D = m\left(\frac{a-w-m}{b}\right) \tag{175}$$

Assume that the firms simultaneously set $w$ and $m$ to maximize their profits, respectively. Note the implication of writing the problems of the two firms in this way. First, the downstream firm essentially commits to a specific *reaction* to the upstream firm's wholesale price $w$, rather than to a retail price $p$. In addition, the upstream firm takes this reaction as given when making its decision. The key difference to the usual sequential solution procedure is that the two firms choose these reactions simultaneously. In a sense, this solution concept is related to the conjectural variations equilibrium, in which firms make their decisions simultaneously, but taking as given some conjectured reaction by the other firm.

The first-order conditions for profit maximization are then

$$a(b+2d) - b(2w+m-c) - 2d(m+w) = 0 \tag{176}$$

$$a - 2m - w = 0 \tag{177}$$

This yields the solution

$$q_9 = \frac{bc + 2a(b+d)}{3b+2d} \tag{178}$$

$$w_9 = \frac{ab + 2bc + 2ad}{3b+2d} \tag{179}$$

$$m_9 = \frac{ab - bc}{3b+2d} \tag{180}$$

$$p_9 = \frac{a-c}{3b+2d} \tag{181}$$

$$\pi_9^U = \frac{(a-c)^2(b+d)}{(3b+2d)^2} \tag{182}$$

$$\pi_9^D = \frac{b(a-c)^2}{(3b+2d)^2} \tag{183}$$

## 10. Appendix: bilateral oligopoly

Consider a model of vertical oligopoly in which an upstream sector consisting of $M \geq 1$ symmetric firms produce a homogeneous intermediate good at cost

$$C(q) = cq + dq^2 \tag{184}$$

with $c, d \geq 0$. A downstream sector of $N \geq 1$ symmetric firms turn each unit of the intermediate good into a homogeneous final good at zero marginal cost, which is then sold to final consumers.

The demand for the final good is characterised by the inverse demand function

$$p = a - bQ \tag{185}$$

with $a, b > 0$ and $a > c$, where $Q \equiv \sum_{i=1}^{N} q_i$ and $q_i$ is the output of firm $i$. Denote by $Q_{-i}$ the aggregate output of all firms except $i$.

**10.1. Two-sided price taking.** As a useful benchmark, we start by considering the case in which both sides of the market take the wholesale price $w$ as given and decide how many units to supply and demand, respectively. The equilibrium price and output pair $(w^*, Q^*)$ are then determined through market clearing.

To derive the aggregate demand, consider the problem of a typical downstream firm $i$, given by

$$\max_{q_i} \pi_i = \max_{q_i} (a - bq_i - bQ_{-i} - w)q_i \tag{186}$$

Rearranging the first-order condition[8] yields the best response

$$q_i = \frac{a - w - bQ_{-i}}{2b} \tag{187}$$

In symmetric equilibrium, $q_i = q$ and $Q_{-i} = Q - q_i = (N-1)q$, which yields individual and aggregate outputs on the market for the final product

$$q_i = \frac{a - w}{b(N+1)} = D_i(w) \tag{188}$$

$$Q = \frac{N}{N+1}\frac{a-w}{b} = \sum_{i=1}^{N} D_i(w) = D(w) \tag{189}$$

Because of the one-to-one technology, this function also constitutes the downstream sector's aggregate demand for the intermediate good.

To characterise aggregate supply, assume that upstream firms take the wholesale prices $w$ as given. They then solve the problem

$$\max_{q_j} (w - c - dq_j)q_j \tag{190}$$

The first-order condition[9] is given by

$$w - c - 2dq_j = 0 \tag{191}$$

[8]The second-order condition is satisfied since $-2b \leq 0$.
[9]The second-order condition is satisfied since $-2d \leq 0$.

which leads to individual and aggregate supply

$$q_j = \frac{w-c}{2d} = S_j(w) \tag{192}$$

$$Q = M\left(\frac{w-c}{2d}\right) = \sum_{j=1}^{M} S_j(w) = S(w) \tag{193}$$

Solving the market clearing condition $D(w) = S(w)$ yields the Walrasian equilibrium

$$w^* = \frac{M(N+1)bc + 2Nad}{M(N+1)b + 2dN} \tag{194}$$

$$Q^* = \frac{MN(a-c)}{M(N+1)b + 2dN} \tag{195}$$

**10.2. Upstream firms price setters, downstream firms price takers.** Assume that the downstream firms compete in quantities for final consumers but act as *price takers upstream*, i.e. they behave as if they cannot influence the wholesale price $w$ that results from quantity setting behaviour by the upstream sector. In this setting, the upstream firms choose equilibrium outputs along the downstream sector's aggregate demand function. To this end, note that the equation for downstream demand $D(w)$ can be rearranged to yield

$$w = a - \frac{b(N+1)}{N}Q \tag{196}$$

This is the demand function facing the upstream sector, i.e. it is the output of the intermediate output demanded in equilibrium by the downstream sector, for a given wholesale price $w$.

Given this, we can turn to the problem facing a typical firm in the upstream sector. It is given by

$$\max_{q_j} \pi_j = \max_{q_j} \left(\frac{aN - b(N+1)(q_j + Q_{-j})}{N} - c - dq_j\right) q_j \tag{197}$$

The first-order condition[10] is

$$a - c - b\left(\frac{N+1}{N}\right) Q_{-j} - 2\left(\frac{b(N+1)}{N} + d\right) q_j = 0 \tag{198}$$

which leads to the best response function

$$q_j = \frac{a - c - b\left(\frac{N+1}{N}\right) Q_{-j}}{2\left(\frac{b(N+1)+Nd}{N}\right)} \tag{199}$$

[10]The second order condition is satisfied as $-2\left(\frac{b(N+1)+dN}{N}\right) < 0$.

Letting $q_j = q$ and $Q_{-j} = (M-1)q$, we get the equilibrium output

$$q_j = \frac{M(a-c)}{2dN + b(M+1)(N+1)} \tag{200}$$

$$Q = \frac{MN(a-c)}{2dN + b(M+1)(N+1)} \tag{201}$$

**10.3. Upstream firms price takers, downstream firms price setters.** In this setting, the downstream sector's equilibrium demand occurs along the supply function of the upstream sector. The upstream sector's aggregate supply function $S(w)$ can be rewritten as

$$w = c + \left(\frac{2d}{M}\right) Q \tag{202}$$

Given this relationship, we can now write the maximization problem of the downstream firms, who take into account how their production decisions influence both downstream retail prices and upstream wholesale prices:

$$\max_{q_i} \left(a - c - \left(\frac{2d + Mb}{M}\right)(q_i + Q_{-i})\right) \tag{203}$$

The first-order condition[11] is

$$a - c - \left(\frac{Mb + 2d}{M}\right) Q_{-i} - 2\left(\frac{Mb + 2d}{M}\right) q_i = 0 \tag{204}$$

The best response function is then

$$q_i = \frac{a - c - \left(\frac{Mb+2d}{M}\right) Q_{-i}}{2\left(\frac{Mb+2d}{M}\right)} \tag{205}$$

Letting $q_i = q$ and $Q_{-i} = (N-1)q$, we get the equilibrium output

$$q_i = \frac{M(a-c)}{(Mb+2d)(N+1)} \tag{206}$$

$$Q = \frac{MN(a-c)}{(Mb+2d)(N+1)} \tag{207}$$

**10.4. Comparison of two settings.** Straightforward comparison shows that

$$Q_u \geq Q_d \Leftrightarrow (N+1)b \geq 2d \tag{208}$$

This generalizes the condition we found under bilateral monopoly. Also, it shows that what determines the direction is a comparison between a scaled version of the slope of the demand curve $b$ and the slope of the supply function $2d$. Notably, the condition is independent of the number of upstream firms $M$.

[11] The second-order condition is satisfied since $-2\left(\frac{Mb+2d}{M}\right) \leq 0$.

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