

REVIEW OF THE MODEL DEVELOPMENT PROCESS and the Resulting Values for Wales

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CONSULTING SERVICES
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BACKGROUND

In June 2023 XXXXXX of the Valuation Office Agency (VOA) contacted the International Association of Assessing Officers (IAAO). As explained by XXXXX in his email, the VOA is an executive agency sponsored by His Majesty's Revenue and Customs. The VOA provides "British governments the valuations and property advice needed to support taxation and benefits." XXXXXX further explained the Welsh Government had commissioned the VOA to prepare for a proposed revaluation of all domestic properties in Wales.

As part of the assignment and undertaking of the VOA staff had spent the past few years building a bespoke Automated Valuation Model (AVM or model) to facilitate in the revaluation of all 1.5 million domestic properties in Wales. The VOA team responsible for the model development and refinement had previously contracted with IAAO seeking professional education courses to help provide a foundation of understanding regarding industry best practices on the topic of model building for mass appraisal.

XXXXXX initial email stated the project team had reached a point in the model development and performance testing that they felt it necessary to seek an independent third-party review of the model to ensure industry best practices were being followed and the indicated value outputs met IAAO's Technical Standards for performance. As the primary standard-setting organization in the field of mass appraisal and assessment administration, IAAO was in the unique position to be able to provide the requested performance audit via the Professional Consulting Services of IAAO (PCSIAAO).

An informational call was scheduled for VOA project leaders and PCSIAAO staff to meet one another and to share more about the project on June 30, 2023. XXXXXX, XXXXXX, and XXXXXX joined the call from the VOA project team and Shannon Hiss, Justin Eimers, and Ashley Lathrop joined the call on behalf of PCSIAAO. During the call the VOA project team provided some additional background information regarding the history of the current Welsh revaluation project as well as past revaluations in Wales.

Of note, the last successful revaluation of Wales was completed in 2005, with values based on the date April 1, 2003. The Welsh government tasked the VOA with a proposed revaluation as of April 1, 2023, with values in effect April 1, 2025. The VOA team further shared approximately 450,000 sales were included in the model development, they further elaborated of those sales they used approximately 3 years' worth of sales for houses, and 6 years' worth of sales for flats.

Following the additional calls and communication, the VOA team provided PCSIAAO with a high-level overview of initial model performance. The high-level analysis displayed the overall performance metrics for the entirety of domestic properties in Wales—the initial statistics indicated strong performance and reliability in values with a Median Sales Ratio of 1.004, Coefficient of Dispersion (COD) of 13.8, and a Price-Related Differential (PRD) of 1.032. Both the Median Ratio and COD fell well within acceptable performance measures as discussed within IAAO's Standard on Sales Ratio Studies. The PRD indicated some regressivity—meaning the lower value properties were valued at a higher rate relative to the higher value properties. However, given that these statistics relate to *all* domestic properties within Wales the indication of regressivity could be caused by any number of factors and warrants a more thorough analysis at the local level. Later in this report the statistical analysis and evaluation of the performance of the model is discussed at length.

The VOA published the Invitation to Tender *Professional Services - Independent Audit of Valuation Model (Welsh Revaluation)* on July 20, 2023, PCSIAAO responded to the tender on August 6, 2023. Following internal conversations and follow up discussions with PCSIAAO the VOA project team narrowed the scope of the proposed project. PCSIAAO submitted a revised response to the updated Invitation to Tender on August 22, 2023. The revised response and terms were accepted by the VOA and the contract to proceed was signed by both parties on August 31, 2023.

In addition to PCSIAAO staff, Joshua Myers was added to act as a subject matter expert to complete the statistical analysis and evaluation of the model performance. PCSIAAO staff, Mr. Myers, and VOA project team members coordinated and worked closely together throughout the remainder of the project. Several informational meetings were held throughout the months of September and October 2023 to discuss the history, development, and performance pieces of the bespoke Spatial Random Fields valuation models developed by VOA staff.

Due to strict data privacy laws within the United Kingdom, the project was paused through the months of November and December as the VOA and PCSIAAO worked through the many layers of requirements to ensure data security and privacy to allow for the necessary data transfer to allow the independent analysis of the data quality and model performance to be completed by PCSIAAO. PCSIAAO was able to successfully download the VOA data to the secure server on December 21, 2023. The project resumed in earnest in mid-January 2024, with weekly project check in calls scheduled every Thursday. During the weekly check-in calls questions about data and model performance were answered and additional background information was provided regarding the source of sales data information as well as the banding methodology and identification and model performance feedback to be utilized by VOA valuers as the revaluation project moves into the final phase of band selection and valuation of domestic properties.

We would like to thank all VOA staff for taking the time to meet for interviews throughout the course of this project—without their insights, explanations, and feedback this report would not be possible. The following report provides a high-level summary of both the sales data collection process and output presented to valuers during the valuation process. A significant portion of this report is spent analyzing the design and performance of the comparable sales valuation approach that is built upon Spatial Random Fields model. IAAO's Technical Standards will be referenced throughout this report. By necessity, the Technical Standards are broad in nature and assume ideal circumstances under which they will be applied by practitioners who are fully trained and experienced in mass appraisal. It is not unusual to find gaps between current operations within an office and IAAO's Technical Standards. The primary purpose of this report is to point out the gaps and offer suggestions for improvement. All recommendations are made with the understanding that their implementation is the responsibility of the client.

SALES DATA COLLECTION PRACTICES

Welsh Government requires all real property sold to have a Land Transaction Tax (WLTT) return completed and submitted to the Welsh Revenue Authority. Within the WLTT document buyers and sellers are required to complete and answer a series of questions regarding the property and the term of the sales transaction. The document seeks the following information:

- Name & contact information for both buyer and seller
 - Clarification of any relationship between the buyer and seller
- The date of the transaction and the sale price
- The location and description of the property
- The special considerations for the transaction (agricultural property, mineral rights conveyance, etc.)
- Property type (residential, non-residential, or mixed use)
- Improvements on the property (existing and/or any planned future development)
- Type of transaction (freehold, grant of a lease, assignment of lease, etc.)
- Interest transferred or created by the transaction
- Net Present Value yearly analysis if property is leased

In collecting all the above information, the Welsh Government has met both industry best practices as well as the expectations of sales data collection established in IAAO's Standard on Verification and Adjustment of Sales. As a matter of standard operations, the WLTT documents are automatically uploaded to the VOA's system, ensuring both an efficient transfer of sales information and timely notification to VOA staff of all sales transactions.

The VOA has further and extensive guidance for staff to utilize when reviewing and verifying sales. To compliment the data provided with the WLTT document, staff regularly compare internal ownership records to the Land Registry data to further ensure ownership and property records are current, correct, and complete. Further, the VOA also utilizes online third party resources and other internal resources such as internal mapping to help ensure staff record accurate information for all sales transactions.

Current software limitations prevent staff from being able to update records and verify sales transactions in real time. Creating a rather significant backlog of data for staff to process and work through whenever undertaking a revaluation project. However, VOA staff relayed to PCSIAAO that by the end of 2024 it is the goal of the agency to be able to update property records and verify sales in real time as the information is recorded. PCSIAAO strongly recommends the VOA vigorously pursue this objective. Doing so will not only make the next revaluation easier, but also ensure that the recorded data is accurate, thorough, and complete for all properties.

On the matter of timeliness of sales verification and data entry IAAO's *Standard on the Verification and Adjustment of Sales* states (emphasis added) on page 13:

*"The effectiveness of sales validation is partly a function of when it is performed. Contacting parties to the sale shortly after the sale occurred can help improve response rates and accuracy of responses, as parties are more likely to recall circumstances and details of the sale ... **In general, sales should be validated within three months of occurrence.**"*

The VOA's extensive and comprehensive protocols for reviewing and validating sales transactions is commendable. Processes for reviewing useful and non-useful sales and the protocols for documentation by staff as the sales are reviewed allows for extensive analysis of the sales and allows VOA management to conduct routine quality assurance checks on the sales verification and validation work done by staff. Routine quality assurance on the sales verification processes ensures that all the data going into the valuation model is complete and accurate, which in turns leads to more reliable estimates of value produced by the valuation model.

GUIDANCE TO VALUERS ON BANDING

To facilitate property banding, the VOA has developed several bespoke Power BI dashboards and tools to help valuers efficiently work through the banding process. Patch and Batch overview allows valuers to see all sales available. Valuers are able to work a batch by selecting baseline typical property to act as the representative sample for the properties that fall within a specific band. Based on the sales within the batch and the selected parameters for the bands the valuer is then able to review the properties within the batch. As properties are reviewed, particularly those on the fringes of the bands, valuers are able to select whether the current identified band is correct—and if it is not, they are able to provide feedback as to why they feel it is not correct.

That feedback is kept in the system for both those who quality control (QC) the banding work and relaying back to the modeling team to ensure any trends or important factors about the model performance can be onboarded and necessary adjustments made. The QC process developed by the VOA has both a separate Power BI dashboard and extensive documentation for protocols and workflow processes. Those conducting the QC can read the valuer feedback and review overall band performance within specific market areas to ensure both consistency and completeness in the work done by valuers. Once the QC has been completed, staff can add additional comments demonstrating the QC review has been completed and any findings from that QC review.

Another powerful, and recently developed, tool is the "Stand Back and Look" Power BI dashboard that allows QC to be conducted by allowing VOA staff to review color coded property bands as overlaid on a map. An example would be a group of terraced homes all identified in band A2, while one unit is in band B. The color-coded map would help valuers and those completing QC to quickly identify this property as an "outlier" and work to confirm whether the banding for that unit needs to be modified. This tool will prove to be particularly valuable in both the banding process, as well as during banding disputes with property owners. Staff will be able to quickly confirm whether the property in dispute has been treated equitably and fairly when compared

to similar units in its immediate area.

An important consideration with the banding and valuation process is that the tools developed by the VOA staff for this revaluation are truly bespoke to the project. Staff are creating protocols and best practices as the process is developed and the project is further refined. It will be very valuable to conduct a final QC of all the work done by staff throughout the project. Most specifically, it will be important to review the initially selected bands as compared to the final bands selected for properties after the dust settles from all disputes raised by property owners.

The “Stand Back and Look” tool (or something similar) will allow valuers and modelers to review performance and seek to identify patterns of potential deficiencies in performance. Additionally, valuers and modelers should have an opportunity to gather and discuss what worked, what didn’t work, and what each side had wished the other had done throughout this revaluation. Allowing time for constructive feedback will allow for growth and, hopefully, more efficient, fair, and equitable revaluations in the future.

ANALYSIS OF MODEL DEVELOPMENT PRACTICES

An important part of this project, before embarking on the sales ratio study, was to examine the model development practices of VOA, including their statistical code, relevant processes, and goals. PCSIAAO had multiple meetings with VOA staff modelers and analysts. During these meetings, VOA staff walked through each relevant model development script, answered questions about the process of model specification and calibration, and explained the logic behind certain process-related decisions.

PCSIAAO presents the following key findings with respect to the model development practices:

Statistical Computing

- The bulk of the work was done in R, an open-source language for statistical computing. R is arguably one of the most powerful and flexible tools that is available.
- R scripts were documented, well organized, and well-maintained over time.
- These scripts also ran without significant errors and were effective at producing the desired output to explore the data, specify and calibrate models, and to produce the desired output.
- R scripts contain all the key elements necessary to conduct the model development process.

Model Progression

- VOA displayed a strong desire to improve their valuation model development practices.
- VOA showed an admirable willingness to use outside consulting services, such as Highland Statistics, to seek improvements to their original mixed model structure.
- Adequate time and resources were allocated for model development and model review was made a priority.
- A version of the comparable sales model based on the mixed model was compared to the version of the comparable sales model based on the SRF model to see the degree to which the addition of the comparable sales stage to the mixed model accounted for the spatial variation. This was a valuable comparison to make.

Spatial Random Fields (SRF) Model

- The SRF model is based on a bayesian approach and includes a set of fixed global model coefficients along with the spatial component of the model, with the spatial component of the model being driven by a set of model parameters.
- SRF parameters were determined using a reasonable process based on the nature of the variogram and the required computational time.
- Competing parameter sets and specifications for the SRF model were compared using the WAIC and DIC, a good practice.

Staffing

- Modelers and analysts had the requisite statistical skill, resourcefulness, and appraisal knowledge to develop these models.
- Staff members conducted themselves with the highest degree of professionalism.
- Staff members have a firm grasp over their respective roles.
- There is a strong team mentality present within the group assigned tasks in model development.

Reliability Scoring

- A reliability score is assigned to each property according to a detailed algorithm which seeks to identify which comparable sales model values are most in need of a manual review. Such a manual review could improve upon the accuracy of the comparable sales model estimates.
- The reliability score is conceptually sound and the development process is generally well reasoned.
- There is a clear association between the reliability score and the accuracy of the comparable sales model.

Design

- VOA has a well-designed model development process.
- There are two versions of the houses model and the flats model, a full and a reduced model.
 - The full version of the houses model includes both plot size and a parking variable value, but the reduced version does not. The reduced version is only applicable for houses without plot size or a parking variable value present.
 - The full version of the flats model includes a parking variable value, but the reduced version does not. The reduced version is only applicable for flats without a parking variable value. This was a good design decision because these variables could be used to form a better predicted value for those properties that have values present.

- The models are developed and compared using a training set (60% of the sample) and a test set (30% of the sample), which work on an iterative basis via cross-validation to perform model validation. A holdout sample (10% of the sample) is used to form a basis for comparing competing models and is not included in the cross-validation process. Stratified random sampling is used to allocate sales to a given partition. These are good model development practices.
- Model documentation is done comprehensively and superlatively via statistical output, tables, and plots using tools such as Rmarkdown and Power BI.
- The comparable sales process is well designed overall. Comparable sale selection is also done well and according to a good set of weighting criteria.

Data

- The VOA staff rightly focuses significant time and energy on data quality assurance.
- The houses model ultimately used sales between April 1, 2020 and June 30, 2023 (three years and three months of sales) and the flats model ultimately used sales between April 1, 2017 and June 30, 2023 (six years and three months of sales). These ranges are reasonable and it was within good practices to include more years of sales for the flats model.
- The VOA staff used a helpful process of data imputation for properties missing certain data elements, such as the parking variable. This is a good way to populate missing data, move more properties over to the full version of the model, and increase overall model accuracy.

TABLE 1. SRF Model Specifications

SRF Model	Full or Reduced	Specification
Houses	Full	log_size_std + dwelling_type_subgroup + log_size_std:dwelling_type_subgroup + age_code_grouped + dwelling_type_subgroup:age_code_grouped + subsidised + parking_code_group + log_plotsize_std + billing_auth_id + billing_auth_id:dwelling_type_subgroup + spatial component
Houses	Reduced	log_size_std + dwelling_type_subgroup + log_size_std:dwelling_type_subgroup + age_code_grouped + dwelling_type_subgroup:age_code_grouped + subsidised + billing_auth_id + billing_auth_id:dwelling_type_subgroup + spatial component
Flats	Full	log_size_std + dwelling_type_subgroup + log_size_std:dwelling_type_subgroup + age_code_grouped + subsidised + parking_code_group + bathroom_count_group + spatial component
Flats	Reduced	log_size_std + dwelling_type_subgroup + log_size_std:dwelling_type_subgroup + age_code_grouped + subsidised + bathroom_count_group + spatial component

Table 1 lists the specifications of the spatial random fields (SRF) models used by the VOA in the proposed Welsh revaluation. Overall, the model development practices employed by the VOA are generally well constructed. Nevertheless, the following changes are recommended to help improve different aspects of the model development process:

- Comments and descriptions in the code should be kept more current so the code is immediately more readable by someone previously unfamiliar with it.
- Sales should be allocated to a given partition only once and these partitions should be stored or otherwise kept constant throughout the model development process. PCSIAAO observed a minor issue related to this matter during the Welsh revaluation project.

The VOA should strengthen its computing environment, add more resources, and work on optimizing the current codebase to be able to test finer mesh parameters in the SRF model that would produce finer meshes. This recommendation is based on the fact that:

- Some spatial autocorrelation was still present within approximately 200 meters on the variogram from the houses SRF model. The VOA staff reported testing finer meshes in an effort to fully remove that spatial autocorrelation would be computationally impractical based on the current level of available resources.
- Consideration should be given to consolidating certain variables, or other variable management strategies, during model specification in order to create sensible model adjustments in cases where coefficients are similar or, because of small sample size, have non-sensical coefficients. This would be important when explaining these models to valuers internally.
- In model scripts, model quality assurance measures should include confidence intervals (or an equivalent test) for statistics that are computed to evaluate model performance with respect to best practice standards and internal office performance expectations.
- Once the raw values come out of the comparable sales model, they are used as the basis for the categorization of each property into one of the value bands where each property in the band pays tax on the same value. This tax policy presents an interesting consideration to the valuation team.
 - Model development is often a cost/benefit analysis. More complicated and time-consuming modeling approaches should be adopted with consideration as to whether the extra effort and complication afford the valuation office with additional benefits like improved model precision or explainability.
 - In this case, if the resulting values are compressed into a series of value bands, an important consideration for VOA should be the degree to which values are categorized into the correct band. Ultimately, under such a banded tax system, the valuation exercise is also, to a certain degree, a categorization problem. In addition to comparing the COD and the other statistics put forth in this report, it then becomes helpful to see how much a difference one model makes over another with respect to this classification problem.
 - It is recommended that VOA consider this question as one factor in deciding between models in the future. If exact value bands are not yet available, an approximate or estimated bands could be used.
- The reliability scoring uses an inaccuracy measure to assist in evaluating its efficacy and this inaccuracy measure is calculated based on an assumption that the measure of central tendency is equal to 1, which is not always the case. The median sales ratio should be used in the formula instead.
- VOA should examine developing reliability scores separately by dwelling type subgroup. While adding some complexity, this recommendation may result in less ad hoc scaling parameter estimation and a reduction in the iterative nature of the tuning process. This may also result in a better and more useful set of reliability scores.

PURPOSE OF RATIO STUDY

The purpose of a sales ratio study is to evaluate assessment performance (IAAO 2013, p.7). A sales ratio is the estimated value of a property divided by its sale price. For example, a sales ratio of 40% (or 0.40) means that the assessed is set to 40% of that of the sale price. The estimated value is sometimes called the predicted price, the appraised value, or the assessed value; this report may use these terms synonymously and interchangeably. Sale prices, which are valid for analysis and otherwise recorded appropriately, are the “most objective estimates of market value” and are used as the basis for evaluating assessment performance in a sales ratio study (IAAO 2013, p.7). A set of well-collected, properly validated sales is a critically important part of a sales ratio study. Ratio studies analyze a set of ratios to determine the degree to which groups of assessed values accurately reflect market value. The IAAO *Standard on Ratio Studies* states the following on page 7:

“There are two major aspects of appraisal accuracy: level and uniformity. Appraisal level refers to the overall ratio of appraised values to market values. Level measurements provide information about the degree to which goals or certain legal requirements are met. Uniformity refers to the degree to which properties are appraised at equal percentages of market value.”

Appraisal estimates of market value are typically not used in a ratio study instead of sale prices unless validated sales are largely unavailable; even then, appraisal estimates of market value must be calculated independently with no knowledge of the equalization rate.

All value estimates, whether produced by VOA or by an external appraiser, are inherently subject to a degree of statistical error depending on several factors such as the accuracy of available property data and the skill of the appraiser or model developer. The *IAAO Standard on Ratio Studies* sets professional standards for assessment level and uniformity that recognize there is some degree of imperfection in assessed values. Sales ratio studies can answer the question of whether a set of assessed value estimates meets the acceptable standards as promulgated by the IAAO with respect to assessment level and uniformity.

A population is the full set of properties defined by a set of criteria, and a sample is a subset of properties which is drawn from a given population. For the purposes of a sales ratio study, a sales sample is drawn from the population of all properties by the fact that the properties in the sample sold during the relevant time period and meets all other data constraints and filters. A foundational principle of ratio studies is that conclusions can be made about the assessment performance for the population of properties using the sales sample (IAAO 2013, pp.7-8), so long as the sales sample has been appropriately collected, is free from statistically and practically significant selective reappraisal, and is sufficiently representative of the population (IAAO 2013, p.11).

KEY USES OF RATIO STUDIES

The IAAO *Standard on Ratio Studies* lists the key uses of ratio studies as follows (Page 7):

- measurement and evaluation of the level and uniformity of mass appraisal models
- internal quality assurance and identification of appraisal priorities
- determination of whether administrative or statutory standards have been met
- determination of time trends
- adjustment of appraised values between reappraisals

SEVEN STEPS IN A RATIO STUDY

Ratio studies generally involve the seven basic steps listed below. (IAAO 2013, p. 8)

1. Define the Purpose, Scope, and Objectives

Every well-constructed ratio study has an intended purpose, which is broadly defined according to end goal or key questions being posed to the researcher. The scope and objectives of the ratio study are then defined accordingly.

2. Design

The design of the ratio study is the methodologies by which the purpose, scope, and objectives of the ratio study are investigated. The design includes the choice of analyses, statistical tests, and means of presenting the results.

3. Stratification

Stratification is the process of dividing the sale properties in the ratio study into two or more groups called stratum and then running the ratio study in each stratum as well as for the overall set of properties. Stratification can be a useful tool to provide a more detailed picture of assessment performance (IAAO 2013, p.9). In ratio studies, a stratification framework should be derived according to several factors, including the goal of the sales ratio study, the availability of appropriate variables to use as a basis for stratification, and the number of sales in each proposed stratum.

4. Collection and Preparation of Market Data

It is important to accurately collect appropriate market data, such as sales, to use in a ratio study and to prepare it for analysis. This includes the sales verification (sales validation) process where information about the sale and the sale property are verified and a decision is made on whether a given sale is valid for analysis. It also includes the preparation of all sales data in an appropriate format. Part of this process in a sales ratio study is defining an appropriate sale date range, where all verified sales within the sale date range are candidates for inclusion in the ratio study.

5. Matching of Appraisal and Market Data

Once market data has been collected and prepared in a usable format, it must be appropriately matched with appraisal data. This forms the basis for the ratio analysis because the ratios analyzed in the study are simply the assessed (or appraised) value divided by the proxy for market value. In the case of a sales ratio study, this proxy for market value is the validated sale price. Properties that are fundamentally different between the sale date and the appraisal data are not matched appropriately and should be removed from the study.

6. Statistical Analysis

The statistical analysis generates the key statistics evaluated in the ratio study. These analyses are defined in the ratio study design according to the purpose, scope, and objectives of the study. The researcher has the power to choose the most appropriate set of statistical analyses based on their experience, knowledge of relevant standards, and statistical knowledge.

7. Evaluation and Use of Result

The key statistics generated from the statistical analysis must be interpreted and evaluated, with the end goal of fashioning them into a usable set of results.

STATISTICAL BACKGROUND

Calculating a Sales Ratio

A sales ratio study is an analysis of sales ratios. A sales ratio is calculated by taking an appraised (or assessed) value and dividing it by the sale price of a valid sale transaction. Consider the following example:

- An office building has an assessed value of \$1,000,000 as of the assessment date.
- The same office building has a verified and validated sale price of \$4,000,000 during sale date range included in the study. Assume that no time adjustment to the sale price is appropriate in this case.
- The sales ratio is equal to $\$1,000,000 / \$4,000,000 = 0.25$ or 25%.
- This means that the assessed value is set to 25% of the sale price in this instance.

All sales ratios included in the study are aggregated for stratification and statistical analysis. The sales ratios for multiple parcel sales are calculated in the same way except that the assessed values for all of the parcels in the sale are added together to form an aggregated assessed value for the sale.

Outlier Handling

Outliers have unusual values that are far from the center of the distribution and could occur for a variety of reasons. According to the IAAO *Standard on Ratio Studies* on page 13:

“The validity of ratio study statistics used to make inferences about population parameters could be compromised by the presence of outliers that distort the statistics computed from the sample.”

Therefore, it is necessary that outliers be dealt with appropriately in a sales ratio study. It is often best practice to first investigate outliers in case they are a result of data that can be corrected, such as an inaccurately recorded sale price. If outliers are unable to be corrected, they are then candidates for removal using an accepted statistical procedure in accordance with the size removal restrictions outlined in Appendix B of the *Standard on Ratio Studies*. Sales ratio outliers are sales ratios with unusually small or large values which could distort certain measures like the Coefficient of Dispersion (COD).

Confidence Intervals

The primary concern of ratio studies is to make conclusions about the population of properties based on a sample. Because not all properties sell in a given period of time, all properties that do sell make up a sales sample of the population of properties. Point estimate statistics calculated from a sales sample inherently contain sampling error, defined as the type of error resulting solely from the sampling process (IAAO 2013, p.43). For example, if 100 samples are drawn from a given population of properties, then 100-point estimate statistics will be calculated. The difference between the 100-point estimate statistics is explained by sampling error.

Confidence intervals account for sampling error and thus serve as a measure of the precision for the calculated point estimate statistic as an estimate of the unknown population parameter with a given degree of confidence. For example, if the point estimate of the median sales ratio is 20% and the 90% confidence interval for the unknown population median sales ratio is 19% to 21%, then the best estimate of the unknown population median ratio is 20% and that it can be said with 90% confidence that the population median sales ratio is in the range of 19% to 21%.

Statistical Hypothesis Testing

Statistical hypothesis testing is used to make conclusions about a population based on a sample. Confidence intervals can also be used to conduct statistical hypothesis testing. In fact, conclusions about the population of properties, including non-compliance with IAAO Standards, cannot be made without using statistical hypothesis testing to account for sampling error (IAAO 2013, p.15). If the confidence interval overlaps a standard range, then that statistic is said to have met the standard, regardless of the value of the statistic's point estimate (IAAO 2013, pp.34-35). A variety of statistical tests can be employed depending on the design of the ratio study.

Sale Price Time Trend Analysis

Sale price time trends measure sale price fluctuations over a given date range. There are a variety of methods available to measure sale price trends and certain methods are preferred in certain contexts. Time adjustments can be derived from sale price time trend models as a multiplicative factor to adjust each sale to the estimated market value as of a given date.

Assessment Level Statistics

Ratio studies typically include measures of assessment level and assessment uniformity. The assessment level is a measure of central tendency for the distribution of sales ratios. This is sometimes referred to as the equalization rate. Depending on the purpose for which the study is being made, different measures of assessment level may be used such as the median ratio or the weighted mean ratio. The IAAO *Standard on Ratio Studies* states that the mean ratio is generally not the preferred measure of the appraisal level (IAAO 2013, p. 28).

- The median sales ratio is the middle value of the sales ratios sorted in increasing order, if the number of sales ratios is odd, or the average of the two middle sales ratios, if the number of sales ratios is even. The median is resistant to the effect of sales ratio outliers (IAAO 2013, p.13). According to the *IAAO Standard on Ratio Studies*, “the median is the generally preferred measure of central tendency for direct equalization, monitoring of appraisal performance, or evaluation of the need for a reappraisal” (IAAO 2013, p.27).
- The weighted mean sales ratio is the mean ratio weighted by sale price. This is sometimes called the aggregate ratio and can be calculated by dividing the sum of the assessed values by the sum of the sale prices. Larger sale prices will have a greater effect on the value of the weighted mean than smaller sale prices. For example, a \$5,000,000 sale will have ten times the weight of a \$500,000 sale in the calculation of the weighted mean. Unlike the median, the weighted mean is not resistant to either sales ratio outliers or large value outliers. Large sale price value outliers can distort the weighted mean (IAAO 2013, p.28). In medium to large-sized samples, wide confidence intervals for the weighted mean may be indicative of the effect that only a few larger-valued sales have on the value of this statistic.

Measures of Appraisal Level

Assessment uniformity is a measure of the variability of the ratio distribution. Ratio studies are usually concerned with a general measure of overall variability, like the Coefficient of Dispersion (COD), and a measure of variability that specifically measures vertical inequity, like the Coefficient of Price-Related Bias (PRB).

Coefficient of Dispersion (COD)

The Coefficient of Dispersion (COD) is a general measure of assessment precision. It is equal to the average absolute deviation of the sales ratios from the median sales ratio divided by the median sales ratio and is calculated as follows: (IAAO 2013, p. 13)

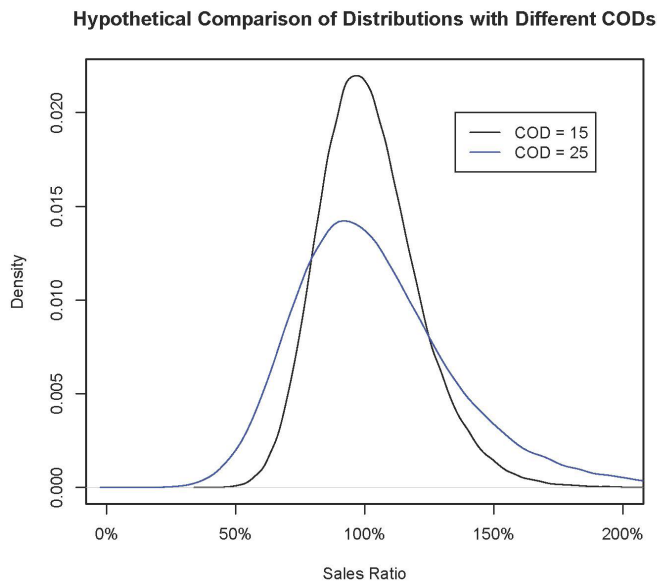
1. Calculate the median sales ratio
2. Subtract the median sales ratio from each ratio
3. Take the absolute value of the calculated differences
4. Sum the absolute differences

5. Divide by the number of ratios to obtain the average absolute deviation
6. Divide by the median sales ratio
7. Multiply by 100

The COD is based on the median and is therefore not resistant to the effect of ratio outliers. The COD provides a general measure of how tightly the sales ratios are distributed around the median sales ratio. The lower the COD, the more tightly the sales ratios are distributed around the median sales ratio. Conversely, the higher the COD, the more spread out the sales ratios are around the median sales ratio.

Figure 1 provides a useful visualization of the COD where two hypothetical sales ratio distributions with different CODs are superimposed. The density lines show how the sales ratios fall in a distribution around the median of the sales ratios. The COD is a measure of this variability. Notice how the sales ratio distribution with a COD of 15 is more tightly distributed around the median sales ratio and the sales ratio distribution with a COD of 25 is more spread out. This means that the lower the COD, the more tightly distributed the sales ratios are around the median sales ratio and so the more accurate the assessed values are.

FIGURE 1. A Hypothetical Comparison of Two CODs: 15 and 25



The *IAAO Standard on Ratio Studies* has promulgated standard ranges for the COD based on the type of property. There are two different tables of acceptable standard ranges (found on p.17 and p. 34 of the Standard, respectively) and both are shown in **Figure 2** & **Figure 3** for context. Demonstrating the predictive error inherent in the assessment process, the lower end of each acceptable range is 5% and this is thought to be the best COD actually achievable in most circumstances assuming the assessment process has been conducted properly (IAAO 2013, p.19).

FIGURE 2. IAAO Standard Ranges for the COD by Type of Property

Type of property—General	Type of property—Specific	COD Range**
Single-family residential (including residential condominiums)	Newer or more homogeneous areas	5.0 to 10.0
Single-family residential	Older or more heterogeneous areas	5.0 to 15.0
Other residential	Rural, seasonal, recreational, manufactured housing, 2-4 unit family housing	5.0 to 20.0
Income-producing properties	Larger areas represented by large samples	5.0 to 15.0
Income-producing properties	Smaller areas represented by smaller samples	5.0 to 20.0
Vacant land		5.0 to 25.0
Other real and personal property		Varies with local conditions

These types of property are provided for guidance only and may not represent jurisdictional requirements.

* Appraisal level for each type of property shown should be between 0.90 and 1.10, unless stricter local standards are required.

PRD's for each type of property should be between 0.98 and 1.03 to demonstrate vertical equity.

PRD standards are not absolute and may be less meaningful when samples are small or when wide variation in prices exist. In such cases, statistical tests of vertical equity hypotheses should be substituted (see table 1-2).

** CODs lower than 5.0 may indicate sales chasing or non-representative samples.

FIGURE 3. IAAO Standard Ranges for the COD by Type of Property

General Property Class	Jurisdiction Size/Profile/Market Activity	COD Range
Residential improved (single family dwellings, condominiums, manuf. housing, 2-4 family units)	Very large jurisdictions/densely populated/newer properties/active markets	5.0 to 10.0
	Large to mid-sized jurisdictions/older & newer properties/less active markets	5.0 to 15.0
	Rural or small jurisdictions/older properties/depressed market areas	5.0 to 20.0
Income-producing properties (commercial, industrial, apartments)	Very large jurisdictions/densely populated/newer properties/active markets	5.0 to 15.0
	Large to mid-sized jurisdictions/older & newer properties/less active markets	5.0 to 20.0
	Rural or small jurisdictions/older properties/depressed market areas	5.0 to 25.0
Residential vacant land	Very large jurisdictions/rapid development/active markets	5.0 to 15.0
	Large to mid-sized jurisdictions/slower development/less active markets	5.0 to 20.0
	Rural or small jurisdictions/little development/depressed markets	5.0 to 25.0
Other (non-agricultural) vacant land	Very large jurisdictions/rapid development/active markets	5.0 to 20.0
	Large to mid-sized jurisdictions/slower development/less active markets	5.0 to 25.0
	Rural or small jurisdictions/little development/depressed markets	5.0 to 30.0

These types of property are provided for general guidance only and may not represent jurisdictional requirements.

* The COD performance recommendations are based upon representative and adequate sample sizes, with outliers trimmed and a 95% level of confidence.

* Appraisal level recommendation for each type of property shown should be between 0.90 and 1.10.

* PRD's for each type of property should be between 0.98 and 1.03 to demonstrate vertical equity. However, PRD standards are not absolute and may be less meaningful when samples are small or when wide variation in prices exist. In such cases, statistical tests of vertical equity hypotheses should be substituted.

* Alternatively, assessing officials can rely on the PRB, which is less sensitive to atypical prices and ratios. PRB coefficients should generally fall between -.05 and .05. PRBs that are statistically significant and less than -0.10 or greater than 0.10 indicate unacceptable vertical inequities.

* CODs lower than 5.0 may indicate sales chasing or non-representative samples.

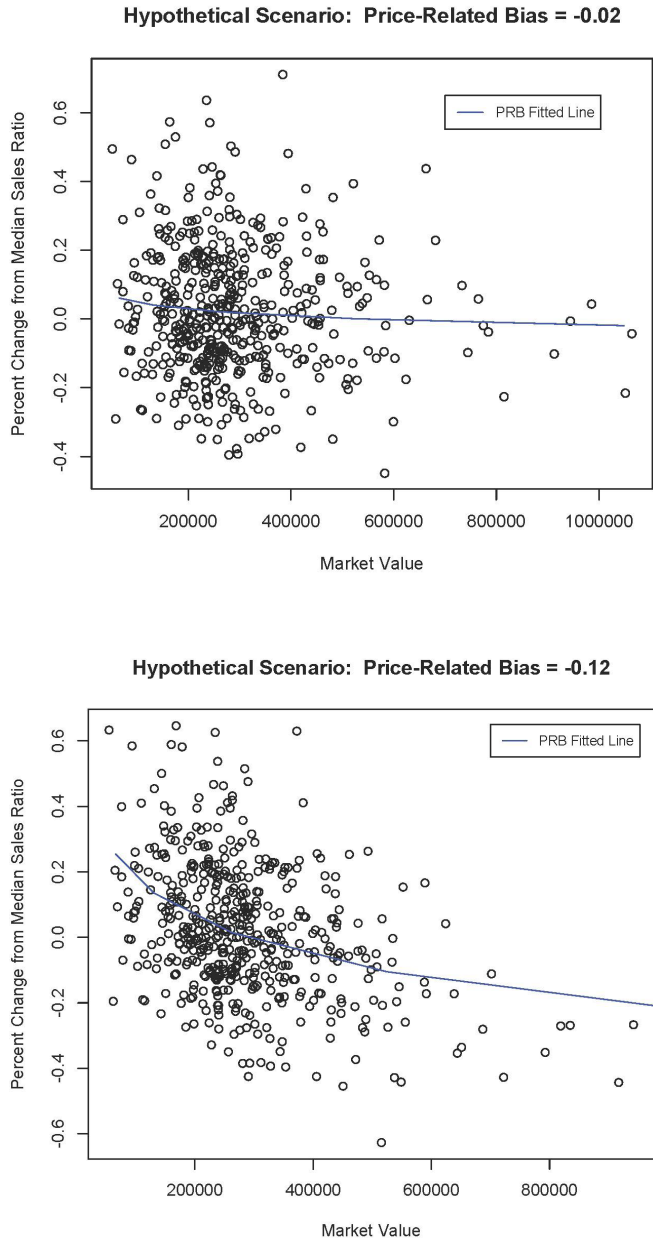
Coefficient of Price-Related Bias (PRB)

The Coefficient of Price-Related Bias (PRB) is a measure of market-value vertical inequity and is based on regressing the percent difference between the sales ratios and the median sales ratio versus the log base 2 of a market value proxy. Market-value vertical inequity is when properties with different market values have different levels of assessment. The PRB gives an indication of whether the vertical inequity, if any, is generally in a regressive (favoring higher-valued properties) or progressive (favoring lower-valued properties) direction by its sign. If the PRB is negative, the vertical inequity is regressive; if the PRB is positive, the vertical inequity is progressive.

The PRB is generally statistically superior to the Price-Related Differential (PRD), an alternative statistic used to measure vertical inequity, chiefly because it is less biased toward regressivity. The PRD is based on the weighted mean and therefore suffers from problems when there are large value outliers; the PRB also suffers from this same problem but to a lesser extent. The *IAAO Standard on Ratio Studies* says, "The coefficient of price-related bias (PRB) provides a more meaningful and easily interpreted index of price-related bias than the PRD" (IAAO 2013, p.14). Therefore, the PRB is used in this analysis rather than the PRD. The PRB can be interpreted as the percentage by which sales ratios rise (if the PRB is positive) or fall (if the PRB is negative) as the appraised values double (e.g. a PRB coefficient of 0.04 means that as appraised values double the sales ratios increase by 4%, indicating progressive vertical inequity).

The *IAAO Standard on Ratio Studies* states that the PRB standard range is -0.10 to 0.10 and that values outside of this range constitute unacceptable vertical inequities (IAAO 2013, p.36). IAAO also gives general guidance that values of the PRB should fall within the range of -0.05 to 0.05 (IAAO 2013, p.36).

Figure 4 provides a useful visualization of the PRB where two hypothetical sales ratio distributions with different PRBs are shown side by side. Here, the percent change between the sales ratios and the median sales ratio are plotted against the market value proxy of the property. This is the typical PRB set-up, except that the market value is expressed in a monetary-scale instead of the log-scale for ease of understanding. The two PRBs show varying degrees of regressive bias, with one being mildly regressive but within acceptable limits (PRB = -0.02) and the other being more regressive and outside acceptable limits (PRB = -0.12). The PRB fitted line is displayed for both cases. The PRB fitted line is almost flat in the case where the PRB = -0.02 but is noticeably steeper and more regressive in the case where the PRB = -0.12. As the PRB becomes increasing regressive, note that lower-valued properties are being appraised at a greater and greater percentage of their market value than higher-valued properties.

FIGURE 4. A Hypothetical Comparison of Two PRBs: -0.02 and -0.12

Selective Reappraisal

Selective reappraisal occurs when sold and un-sold parcels are appraised differently. When practically significant and statistically significant selective reappraisal is present, inferences about the population of properties made from the sales ratio study sample will not be accurate (IAAO 2013, p.59). In order to properly design a test for selective reappraisal, it is important to first identify a sale date range that could have been subject to selective reappraisal, if present.

CURRENT RATIO STUDY AND RELATED STATISTICAL ANALYSIS

This section describes the current project in terms of the seven basic steps of creating a sales ratio study that were described previously.

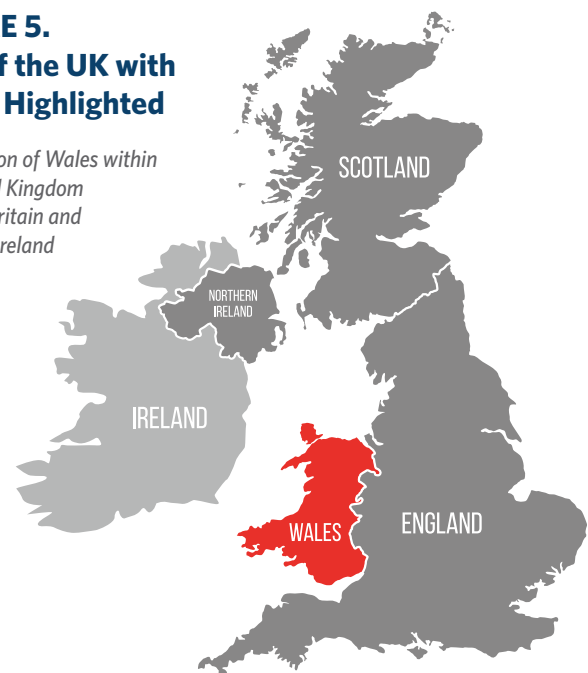
STEP 1: Define the Purpose, Scope, and Objectives

The Valuation Office Agency (VOA) produced a set of model estimates with an effective date April 1, 2023, to use in the general revaluation of Wales. This project's purpose is to conduct an independent sales ratio study and other associated statistical analyses to investigate the model development process and the quality of the final values.

This sales ratio study is designed and oriented around this purpose and was conducted using estimates from three different models that were explored by the VOA in the model development process: a mixed model, a spatial random fields model, and a comparable sales model. The set of final values were derived by the comparable sales model using a spatial random fields model value to make adjustments between the subject properties and their comparable sales.

FIGURE 5.
Map of the UK with Wales Highlighted

The location of Wales within the United Kingdom of Great Britain and Northern Ireland



STEP 2: Design

Recall that the design of the ratio study is the set of methodologies by which the purpose, scope, and objectives of the ratio study are investigated. This design includes the choice of analyses, statistical tests, and means of presenting the results. This study is performed according to the precepts laid out in the IAAO *Standard on Ratio Studies* (2013). However, this section describes key elements of the design of this sales ratio study that are worth noting.

Outlier Handling

Sales ratio outliers were trimmed according to the “1.5 x IQR procedure” with a log transformation of the sales ratios used to better achieve distributional symmetry, which is recommended in Appendix B of the *IAAO Standard on Ratio Studies* (IAAO 2013).¹ In this study, sales ratio outliers are trimmed by class in the primary stratification scheme.

Statistical Hypothesis Testing

All conclusions made in this sales ratio study are made using statistical hypothesis testing so that inferences about the population of all commercial properties can be made. All confidence intervals used in this analysis are 90% two-sided intervals, and all statistical hypothesis tests are stated informally and are done at the $\alpha = 5\%$ level. The appropriate 90% confidence interval bound yields the correct comparison value for a one-sided statistical test because the bounds of a 90% confidence interval are equivalent to the bounds of an upper and lower 95% one-sided confidence interval (IAAO 2013, p.18). Confidence intervals for the median sales ratio, the Coefficient of Dispersion, and the Coefficient of Price-Related Bias are calculated using a resampling procedure called bootstrapping.

A specific statistical test used in this sales ratio study is the Chi-Squared Test which, in the context of a contingency table analysis investigating sales sample representativeness, can detect whether counts between groups are distributed differently for two different samples. Permutation tests are used to detect whether two median sales ratios or coefficient of dispersions are different. Permutation tests are used because no parametric test for these measures exists.

Sale Price Time Trend Analysis

Sale price time trend models are fit in this sales ratio study using the sale price per square foot method, where the sale price divided by the dwelling size is regressed against some function of the month of sale. In this study, these sale price time trends are investigated separately for each dwelling type group and compared to the HPI time trend adopted by VOA. This was done in order to determine whether VOA was right to use the HPI time trend rather than develop their own custom trends in house. It is important in the model development process that the sale prices receive the necessary adjustment, if any, to reflect market value as of the effective date of valuation. The sale price per square foot method was used rather than inverse sales ratio trend method because a recent and reliable set of historical value estimates did not exist when VOA made the decision to use the HPI time trends.

In developing custom time trends, this analysis removes outliers through a process of fitting an initial approximate trend model with the outliers included, using this initial trend model to calculate an initial time-adjusted sale price through which outliers are identified, then a final trend model is calculated without those outliers and used to calculate the final set of time-adjusted sale prices. This is done to eliminate outliers appropriately where the sale price per square foot statistically varies with time. Outliers are removed using the aforementioned “1.5 x IQR procedure” with the log correction.

Assessment Level Statistics

The median is the most appropriate measure of assessment level to use in this case because it is the best measure for monitoring appraisal performance. The standard used for the assessment level is 90% to 110% (IAAO 2013, pp. 34-35) of the estimate of fair market value. The *IAAO Standard on Ratio Studies*

¹ This procedure takes the natural log of the sales ratios, finds the interquartile range (IQR) of the resulting logged ratios, multiplies that IQR by 1.5, and then adds that quantity to the third quartile to achieve the upper trimming bound and then subtracts that same quantity to the first quartile to achieve the lower trimming bound. Sales ratios below the lower trimming bound or above the upper trimming bound are then eliminated (or trimmed) from the study.

recommends this standard range for several reasons, including potential inflation or deflation during reappraisal cycles that extend beyond one year, the potential lack of available resources, and other limiting conditions “that may constrain the degree of accuracy that is possible and cost-effective within an assessment jurisdiction” (IAAO 2013, pp.18,33-34). If an estimate of the assessment level is not found to be statistically significantly outside of this range, then it will be concluded that this estimate of the assessment level falls in an acceptable range around the 100% target level.

Assessment Uniformity Statistics

The IAAO standard range used for the COD in this report is 5% to 15% for all strata, because Wales is a very large jurisdiction with an active market and a certain degree of heterogeneity (IAAO 2013, p.34).

Selective Reappraisal

This study does not explicitly test for selective reappraisal because it uses a holdout sample which by definition could not have been selectively reappraised.

STEP 3: Stratification

In this sales ratio study, sales are stratified in a variety of ways for the determination of time trends, for an evaluation of sales sample representativeness, and for the calculation of the sales ratio statistics. Sales are stratified by dwelling type, dwelling type subgroup, dwelling type group, middle layer super output area, dwelling size range, age range, floor count or floor level code, bedroom count range, bathroom count range, subsidized status, or certain combinations of two of these variables.

Sample Representativeness

Sample representativeness means that the properties in the sales sample used for the sales ratio study are sufficiently representative of the properties in the population. A specific test for sample representativeness is not prescribed in the IAAO *Standard on Ratio Studies*, but the Standard clearly advises that sample representativeness be investigated (IAAO 2013, p.11).

In this report, sample representativeness is evaluated by testing whether the distribution in the sales sample is different than that of the population of properties with respect to the combination of dwelling type subgroup and dwelling area range. This is done via a Chi-Squared Test using a contingency table of sample and population parcel counts. There is not one absolute standard for what constitutes being sufficiently representative. When the Chi-Squared Test shows a statistically significant difference between the distribution of the sales sample counts and the population counts across the defined strata, that is taken as evidence that the sales sample is not sufficiently representative of the population and remedial action is taken by weighting the sales sample in the calculation of the sales ratio statistics to better reflect the distribution of properties in the population.

It must be noted that when sample and population sizes are large, as in this case, slight differences between the sales sample and population counts across the various strata usually result in a statistically significant difference even if the differences between the two groups are typically relatively slight. The full population dataset was not available for this study, but this test was conducted based on counts submitted by the VOA.

The Chi-Squared Test shows that there is enough statistical evidence to conclude that the sales sample is distributed differently than the population of properties with respect to dwelling size range for each dwelling type subgroup (p-value < 0.0001 for each dwelling type subgroup). Therefore, sales ratio study statistics

should be calculated using weighting to achieve results that are more representative of the population of properties. In this case, however, the differences between the distribution of the counts across the various strata were typically fairly small.

STEP 4: Collection and Preparation of Market Data

Sales, valuation, and property-level data were first requested via a formal data request submitted to the VOA in October 2023. Data was then delivered to the IAAO project team in January 2024 that met all specifications. All sales delivered to the VOA were already marked as valid for inclusion in the sales ratio study by the VOA. Then, reasonable data filters were applied to remove valid sales that represented likely data errors or validation errors.

The sales data was partitioned into a training sample, a test sample, and a holdout sample. The holdout sample comprises 10% of the sample and the holdout sample values are truly predicted on an out-of-sample basis from each of the three models. Therefore, this sales ratio study is based on the holdout sample sales alone, unless otherwise noted.

Data Filters

Data filters are applied in order to produce a sales sample that is appropriate for analysis. Properties that meet the following criteria are excluded from the study:

- 1.** A zero-valued sale price
- 2.** A zero-valued predicted price for the mixed model, the spatial random fields model, or the comparable sales model
- 3.** A missing dwelling type, dwelling type subgroup, or a dwelling type group
- 4.** A missing output area, lower layer super output area, middle layer super output area, or billing authority.
- 5.** A property with a dwelling area greater than 1000 square meters.

These data filters did not remove any sales from the study. This resulted in a sample size of 13,846 sales in the holdout sample to be used for analysis.

Due to the sheer size of the dataset and the number of stratification schemes being employed, the sample counts before and after outlier trimming are not reported nor are the summary statistics for the sales sample.

STEP 5: Matching of appraisal and market data

The matching of appraisal and market data was handled through a comparison of the data at the time of sale to the data as of the effective valuation date. Sales were eliminated from the sample unless the following variables matched between the two samples:

- Dwelling Area
- Output Area
- Lower Layer Super Output Area
- Middle Layer Super Output Area

- Billing Authority
- Dwelling Type
- Dwelling Type Subgroup
- Dwelling Type Group
- Mainroom Count
- Bathroom Count
- Geo Plot Size
- Age Code

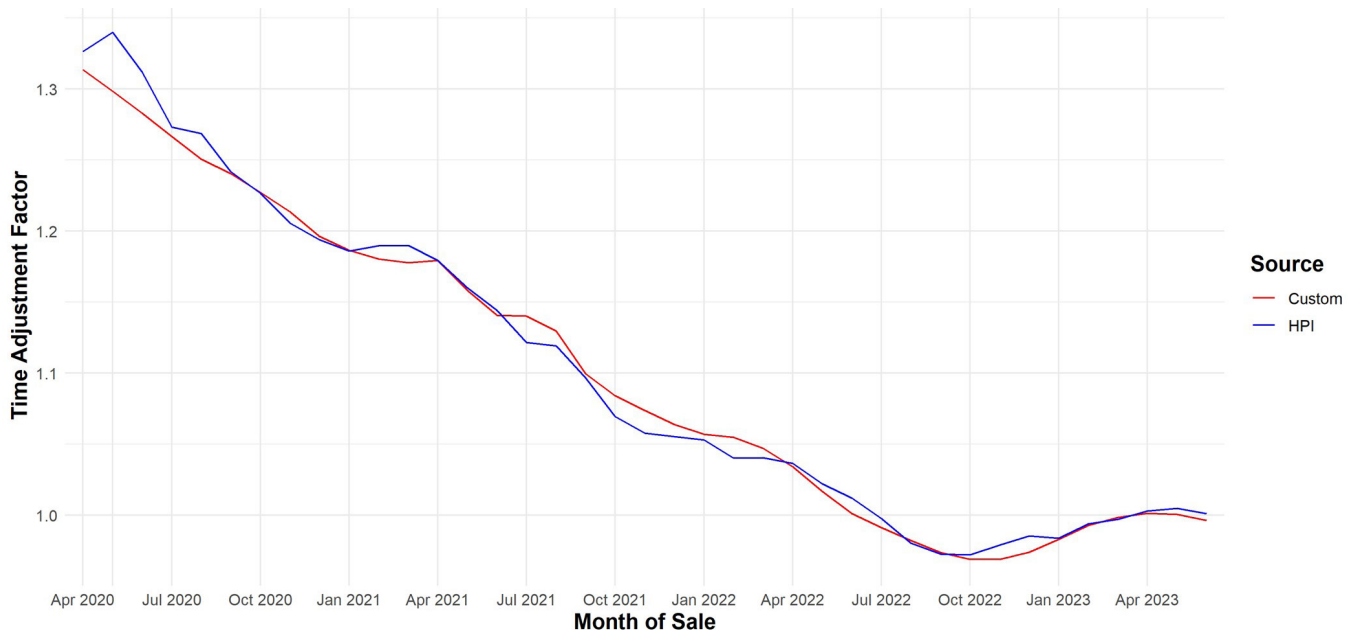
STEP 6: Statistical Analysis

The statistical analysis in this ratio study is broken down into sections according to the flow of logic and the priorities identified in the study's purpose, scope, and objectives. Key findings are given on an on-going basis along with the results but are also listed again at the end of the section.

Evaluation of the Use of the HPI

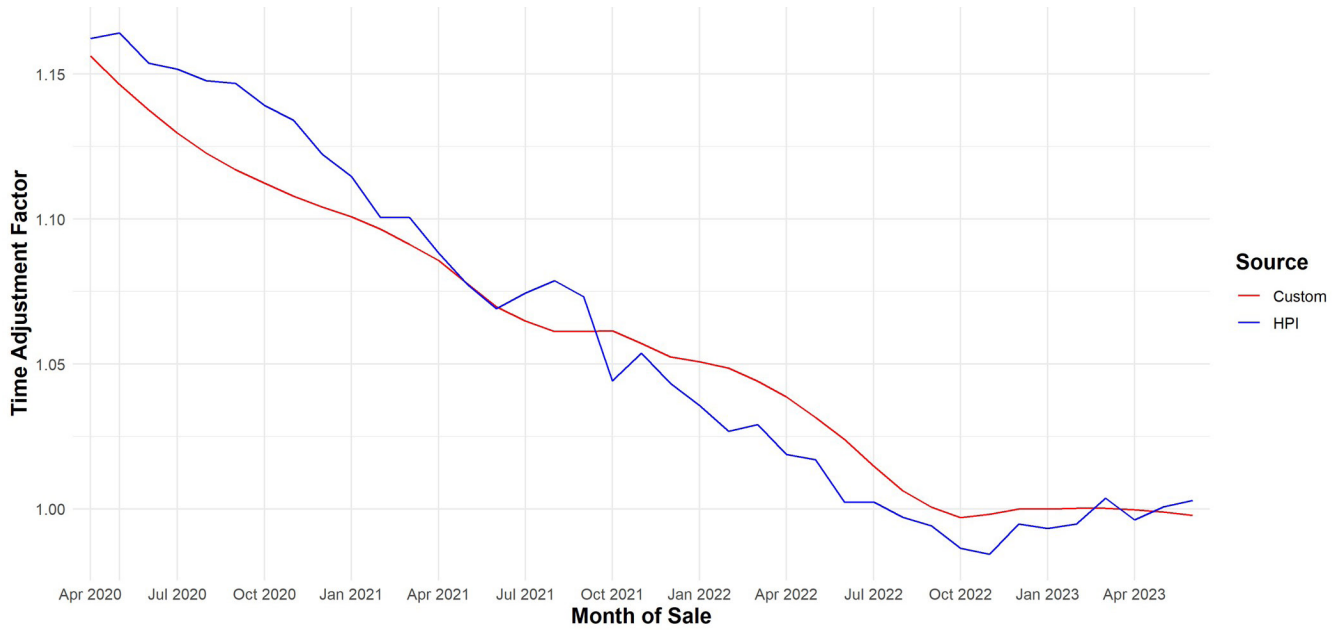
The VOA adopted the HPI time trends for use in their models and ratio studies. **Figures 6 - 9** illustrate the degree to which HPI trends mirror custom-developed trends.

FIGURE 6. Comparison of Time Trends for Detached Properties



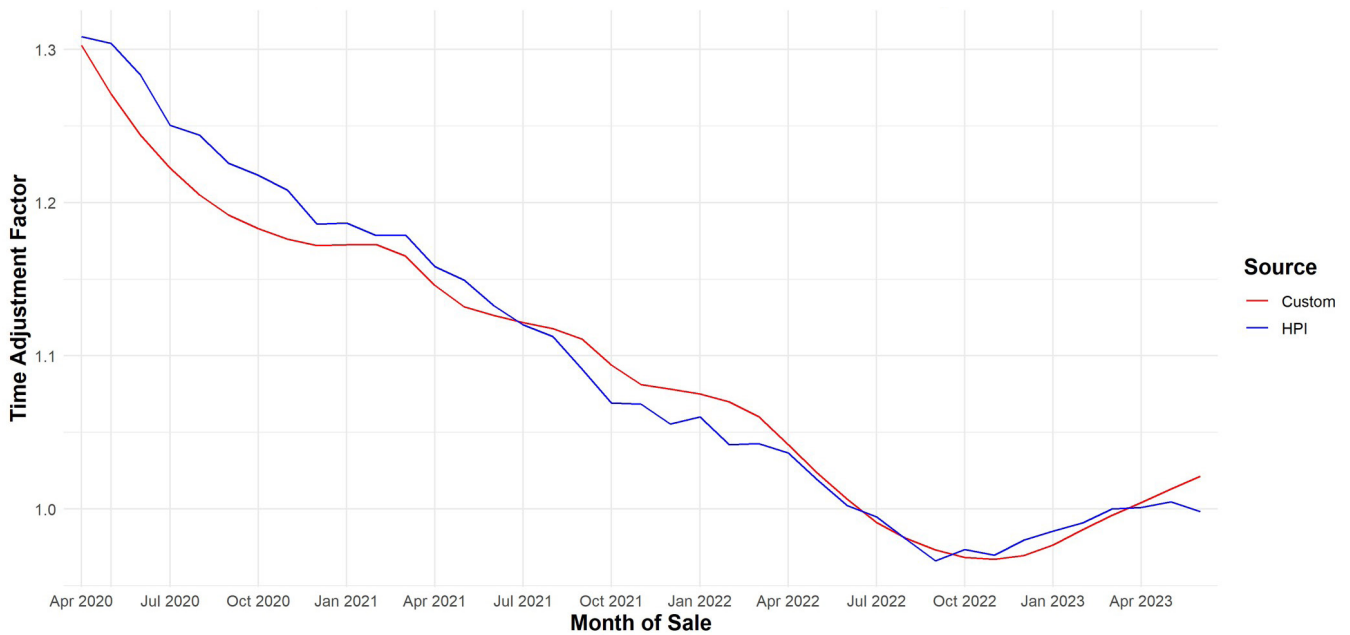
These two trends for detached properties are not statistically significantly different by the Granger Causality Test ($p = 0.324$) and the Wilcoxon Signed Rank Test ($p = 0.271$).

FIGURE 7: COMPARISON OF TIME TRENDS FOR FLAT PROPERTIES

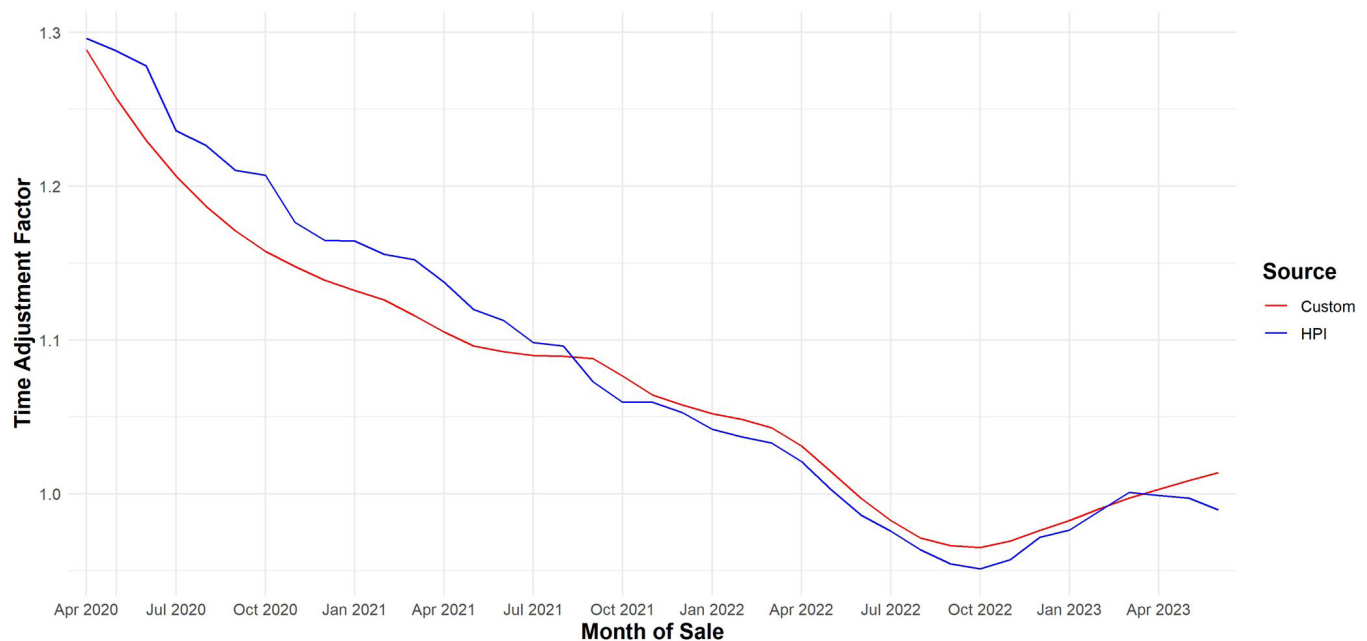


These two trends for flat properties are not statistically significantly different by the Granger Causality Test ($p = 0.440$) and the Wilcoxon Signed Rank Test ($p = 0.654$).

FIGURE 8. Comparison of Time Trends for Semi Properties



These two trends for semi properties are not statistically significantly different by the Granger Causality Test ($p = 0.978$) and the Wilcoxon Signed Rank Test ($p = 0.220$).

FIGURE 9. Comparison of Time Trends for Terrace Properties

These two trends for terrace properties are not statistically significantly different by the Granger Causality Test ($p = 0.506$) and the Wilcoxon Signed Rank Test ($p = 0.189$).

Not only do the HPI trends closely follow the custom trends for each dwelling type group but also, by two statistical measures, the HPI trends are not statistically significantly different from the custom trends at the 5% level.

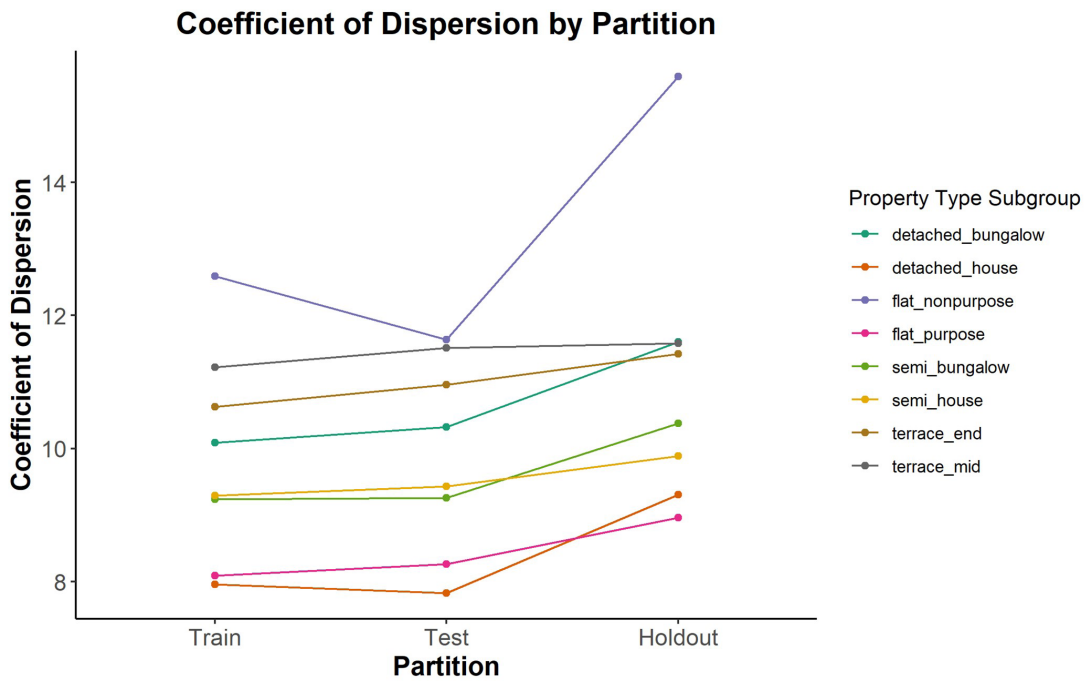
Therefore, this decision to base the trends on the HPI is well-founded. HPI trends closely mirror custom-developed price trends by dwelling type group and have the advantage of being determined by an independent entity.

Comparisons of Coefficient of Dispersions by Partition

As illustrated in **Figure 10**, the comparable sales model values show a reasonable degree of deterioration when moving from the training sample to the holdout sample. This indicates that model overfitting on the training sample was kept relatively low, a good model development practice. The presence of this small degree of deterioration also instills more confidence in using the holdout sample values as a reasonable way to ascertain true model performance.

The same conclusions are reached when using the mixed model and the spatial random fields (SRF) model. The VOA rightly and correctly employed the useful holdout sample technique as a way to guard against model overfitting and provide an unbiased set of sales with which to assess the performance of competing models. The rest of this report uses the holdout sample.

FIGURE 10. Comparison of CODs from the Comp Model by Partition for Each Dwelling Type Group



Comparison of Various Models

As was previously described, three different models were considered by VOA as a part of the model development process, a mixed model, a spatial random fields (SRF) model, and a comparable sales model which was based on the SRF model. This section compares the three models statistically in many different ways.

FIGURE 11. COD by Model Type

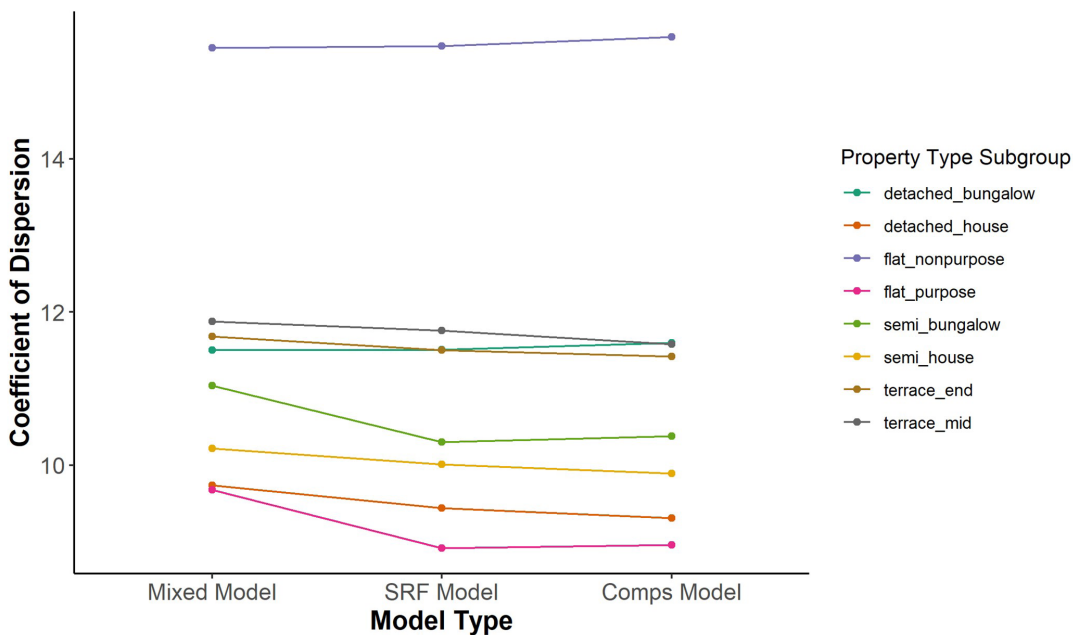


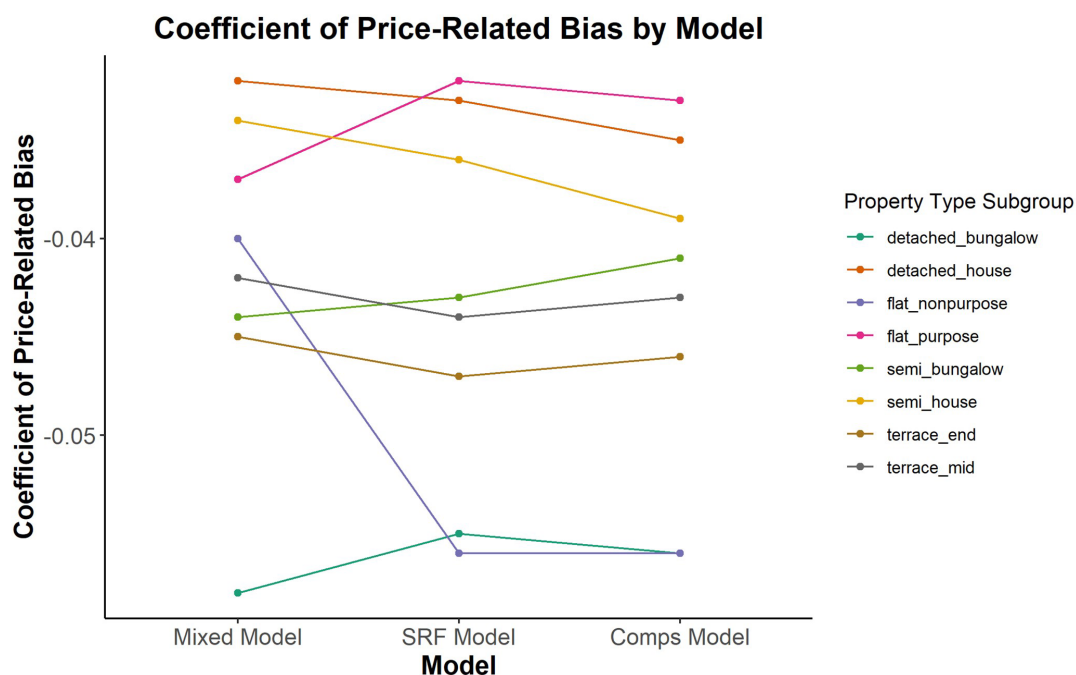
Figure 11 shows that the COD tends to get marginally better when moving from the mixed model to the spatial random fields model and then from the spatial random fields model to the comparable sales model, but this is not true for every dwelling type subgroup. The table in **Figure 12** tests whether these differences are statistically significant.

FIGURE 12. COD Permutation Tests of Comparing Models by Dwelling Type Subgroup

Model Comparison via COD Permutation Test P-Values for each Property Type Subgroup			
	Comps vs. SRF	SRF vs. Mixed	Comps vs. Mixed
Detached Bungalow	0.998	0.286	0.275
Detached House	0.685	0.012	0.004
Non Purpose-Built Flat	0.728	0.044	0.019
Purpose-Built Flat	0.155	0	0
Semi Bungalow	0.921	0.934	0.983
Semi House	0.376	0.004	0
Terrace End	0.653	0.048	0.016
Terrace Mid	0.891	0.164	0.124

These results show the following:

- The COD for the comparable sales model is not statistically significantly different than the COD for the spatial random fields (SRF) model alone for each of the eight dwelling type subgroups. This result should make VOA discuss internally whether it is worth the effort to produce the comparable sales model values as a step beyond the spatial random fields model.
- The COD for the spatial random fields model is statistically significantly lower than the COD for the mixed model for detached houses, non purpose-built flats, purpose-built flats, semi houses, and terrace end units.
- The COD for the comps model is also statistically significantly lower than the COD for the mixed model for detached houses, non purpose-built flats, purpose-built flats, semi houses, and terrace end units.

FIGURE 13. PRB by Model Type

There is no general relationship between the model and the PRB by dwelling type subgroup as shown in **Figure 13**. PRBs for each dwelling type subgroup fall in the acceptable range.

This next set of analyses is a comparison of the failure rate at meeting IAAO standards for the median and COD of the three models when these statistics are computed by middle layer super output area (MSOA) and dwelling type group.

FIGURE 14. COD Comparison Table

Comparison of the Failure Rate at Meeting the Coefficient of Dispersion Standard			
Across All Middle Layer Super Output Areas with at least 10 Sales			
	Mixed Model	SRF Model	Comps Model
Detached	0%	0%	0%
Flat	4.1%	0%	0%
Semi	0.7%	0.7%	0.7%
Terrace	4.9%	4.3%	3.1%
Overall	2.1%	1.5%	1.2%

Figure 14 confirms failure rates are low for all three models, both by dwelling type group and overall. However, failure rates for the comp model are slightly lower overall than the other two models. This is one indication that the comp model is indeed the best of the three competing models, although by a marginal amount that may not be statistically significant.

Figures 15 - 22 display analysis comparing the percentage of each model's values that were within a certain percentage of the sale price. A model is more accurate than another model the higher the percentage of its values fall within a given percentage window of the sale price.

FIGURE 15. Percentile Comparison for Detached Bungalows

Model Percentile Comparison for Detached Bungalow Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	25.8%	27.8%	28.4%
Within 10%	49.2%	49.9%	49.9%
Within 15%	66.6%	68.8%	68.3%
Within 20%	78.9%	80.9%	79.8%
Within 30%	92.5%	93.4%	92.7%
Within 50%	98.7%	98.4%	98.6%

FIGURE 16. Percentile Comparison for Detached Houses

Model Percentile Comparison for Detached House Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	29.2%	32.7%	34.4%
Within 10%	55.9%	59.1%	60.2%
Within 15%	73.9%	76.9%	76.3%
Within 20%	85.3%	86.9%	86.9%
Within 30%	94.4%	95.2%	95.3%
Within 50%	98.9%	99.2%	99.2%

FIGURE 17. Percentile Comparison for Purpose-Built Flats

Model Percentile Comparison for Purpose-Built Flat Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	27.5%	32.8%	34.6%
Within 10%	48.6%	58.8%	61.4%
Within 15%	66.8%	75.3%	77.6%
Within 20%	77.6%	85.3%	87.5%
Within 30%	91.7%	94.3%	94.5%
Within 50%	98.4%	98.9%	98.9%

FIGURE 18. Percentile Comparison for Non Purpose-Built Flats

Model Percentile Comparison for Non Purpose-Built Flat Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	18.4%	18.4%	21.3%
Within 10%	33.8%	37.9%	40.1%
Within 15%	51%	54.1%	54.3%
Within 20%	64%	69.3%	68.6%
Within 30%	80.7%	83.6%	84.1%
Within 50%	92.3%	94.7%	95.7%

FIGURE 19. Percentile Comparison for Semi Bungalows

Model Percentile Comparison for Semi Bungalow Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	27.7%	27.7%	28%
Within 10%	53.1%	55.3%	57.5%
Within 15%	75.8%	74.5%	73%
Within 20%	84.6%	84%	83.3%
Within 30%	94%	94.3%	94%
Within 50%	98.7%	97.8%	97.8%

FIGURE 20. Percentile Comparison for Semi Houses

Model Percentile Comparison for Semi House Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	29.6%	30.5%	32.5%
Within 10%	52.8%	55.7%	57.1%
Within 15%	70%	74.3%	75.3%
Within 20%	83.2%	85.4%	85.6%
Within 30%	93.7%	94.8%	94.6%
Within 50%	98.6%	98.6%	98.6%

FIGURE 21. Percentile Comparison for Terrace End

Model Percentile Comparison for Terrace End Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	24.3%	27.7%	30.3%
Within 10%	48.4%	51%	51.3%
Within 15%	65.6%	67.8%	68.6%
Within 20%	77.5%	80.1%	79.9%
Within 30%	89.5%	91.3%	91.5%
Within 50%	97.2%	98%	97.7%

FIGURE 22. Percentile Comparison for Terrace Mid

Model Percentile Comparison for Semi House Properties			
	Mixed Model	SRF Model	Comps Model
Within 5%	29.6%	30.5%	32.5%
Within 10%	52.8%	55.7%	57.1%
Within 15%	70%	74.3%	75.3%
Within 20%	83.2%	85.4%	85.6%
Within 30%	93.7%	94.8%	94.6%
Within 50%	98.6%	98.6%	98.6%

For each of the eight dwelling type subgroups, the comps model has the highest percentage of values fall within both 5% and 10% of the sale price, although differences tend to be slight. If you look across all dwelling type subgroups, the comps model is the preferred model, although only by a marginal amount.

Across the eight dwelling type subgroups, the comparable sales model typically has slightly better performance than the spatial random fields model across all metrics discussed in this section. The two CODs, however, are not statistically significant for any of the eight dwelling type subgroups.

By virtue of the fact that the comparable sales model has a list of comparable sales built into the approach and these are used inside the office, the comparable sales model has the advantage of explainability over the spatial random fields model and swings the weight of the evidence more in its favor. Both the spatial random fields model and the comparable sales model compare more favorably to the mixed model across all metrics discussed in this section.

Considering the above points, it is clear the VOA was justified in selecting the comparable sales model for the valuation in Wales.

Statistics by Dwelling Type

The following statistics for the median sales ratio, the coefficient of dispersion (COD), and the coefficient of price-related bias (PRB) were found by dwelling type are displayed in **Figures 23 - 25**.

FIGURE 23A. Median Sales Ratio by Dwelling Type

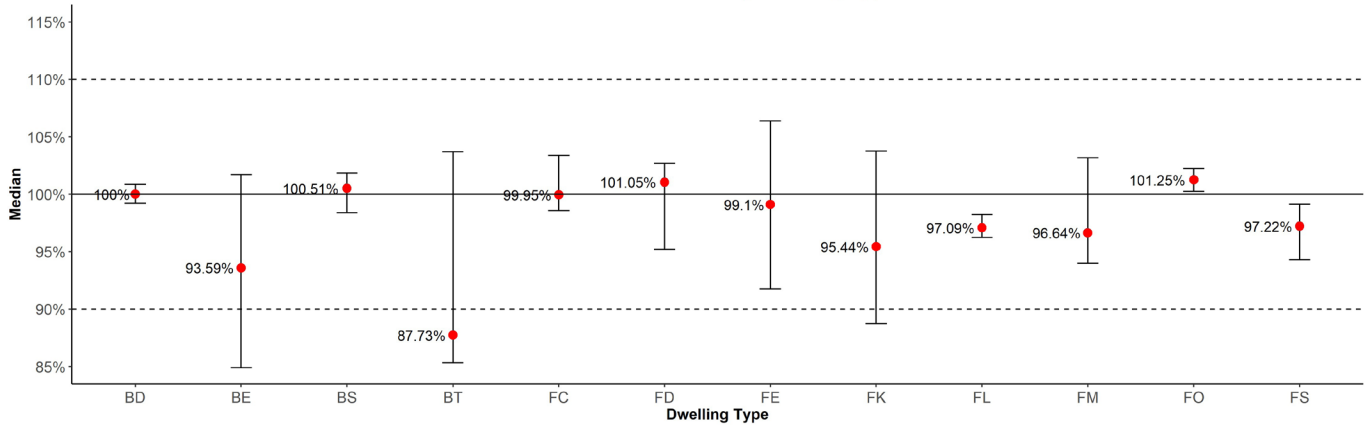
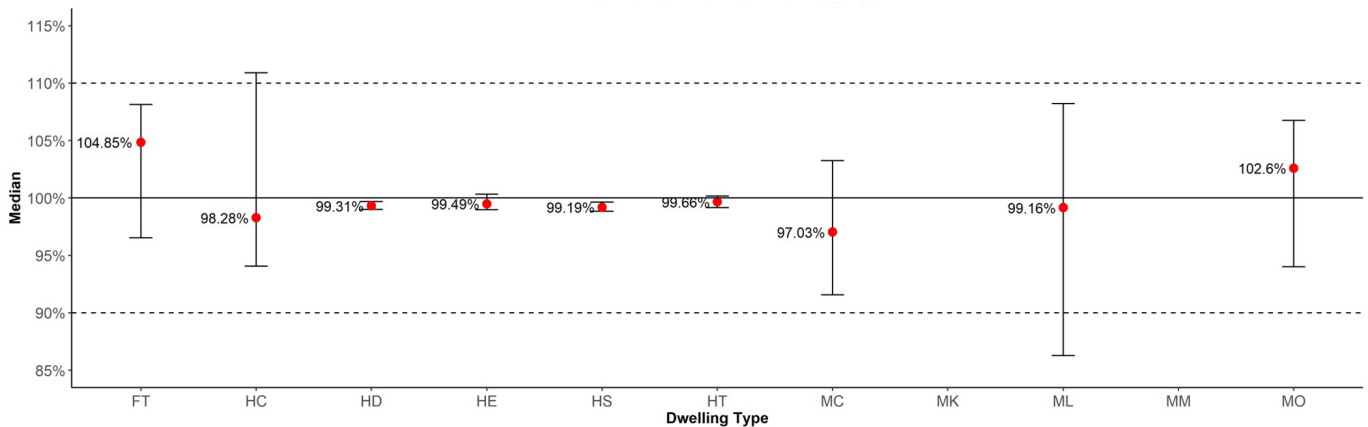


FIGURE 23B. Median Sales Ratio by Dwelling Type



The median sales ratio for each dwelling type is within the range of 90% to 110% when considering its confidence interval, meaning that the IAAO standard is met.

FIGURE 24A. COD by Dwelling Type

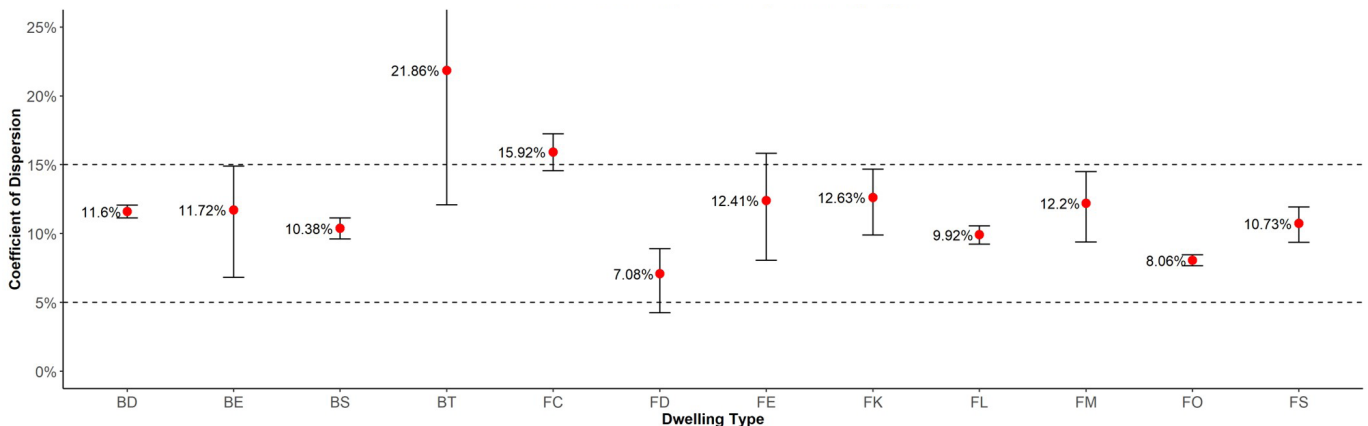
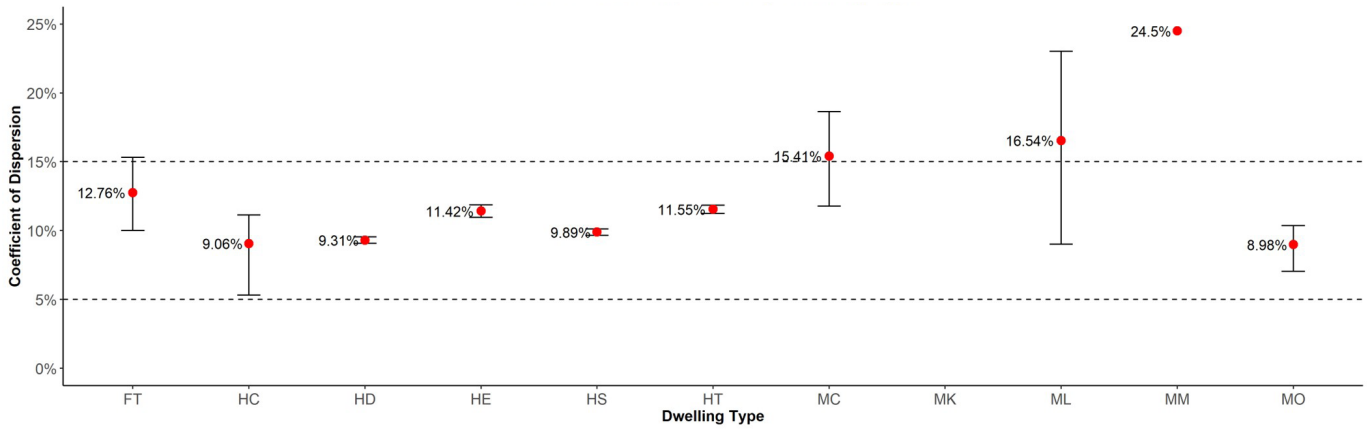


FIGURE 24A. COD by Dwelling Type



The COD for each dwelling type is within the range of 5% to 15% when considering its confidence interval (or does not have an adequate sample size to compute a confidence interval), meaning that the IAAO standard is met.

FIGURE 25A. PRB by Dwelling Type

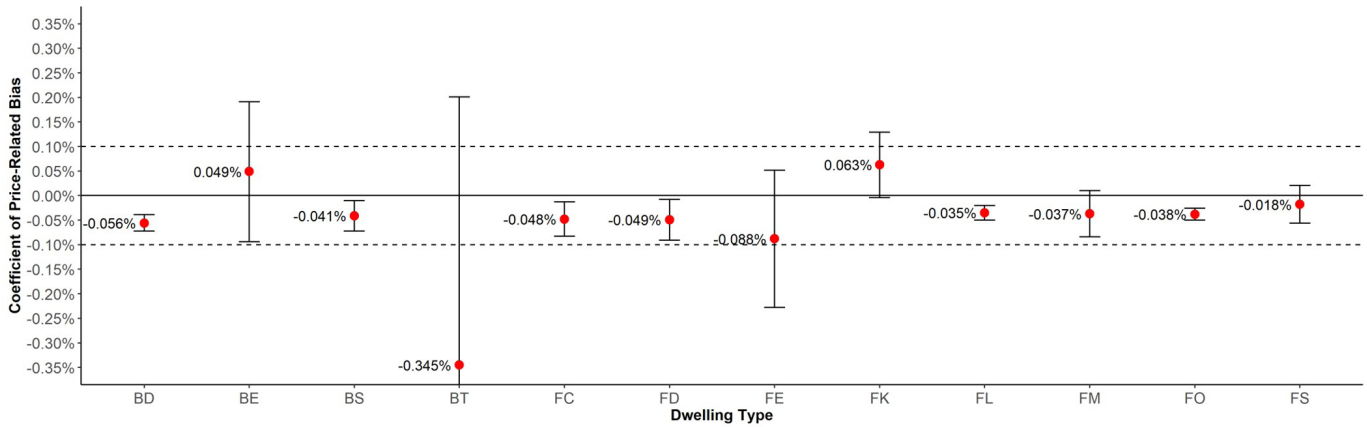
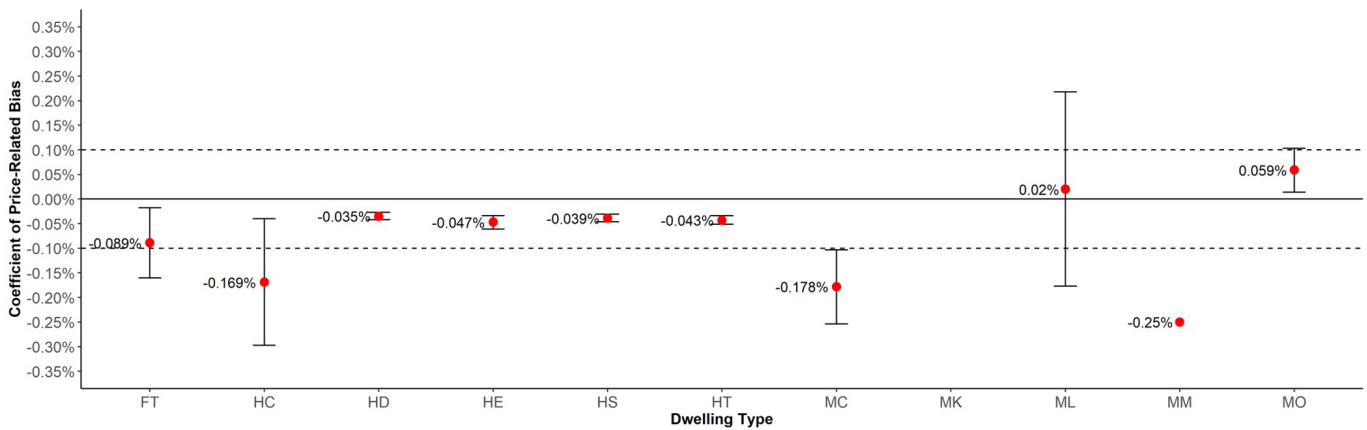


FIGURE 25B. PRB by Dwelling Type



With one exception, the PRB for each dwelling type is within the range of -10% to 10% when considering its confidence interval (or does not have an adequate sample size to compute a confidence interval), meaning that the IAAO standard is met. The one exception is dwelling type “MC” whose PRB value of -0.178 is more regressive than the Standard permits.

Statistics by Dwelling Type Subgroup

Dwelling type subgroup is an important stratifying variable because it allows us to look at performance by dwelling type but in a way that is aggregated yet still specific. Several analyses will be conducted in this section including the calculation of the median sales ratio, COD, and PRB as well as decile bar charts.

Decile bar charts allow one to analyze vertical inequity graphically because they depict how the median sales ratio varies over the range of the market value proxy, with the market value proxy calculated as in the PRB. The sales are broken into deciles, with the lowest decile meaning the sales with the lowest 10% of the market value proxy values and the highest decile meaning the sales with the highest 10% of the market value proxy values.

The following statistics for the median sales ratio, the coefficient of dispersion (COD), and the coefficient of price-related bias (PRB) were found by dwelling type subgroup are displayed in **Figures 26 - 28**.

FIGURE 26. Median Sales Ratio by Property Type Subgroup

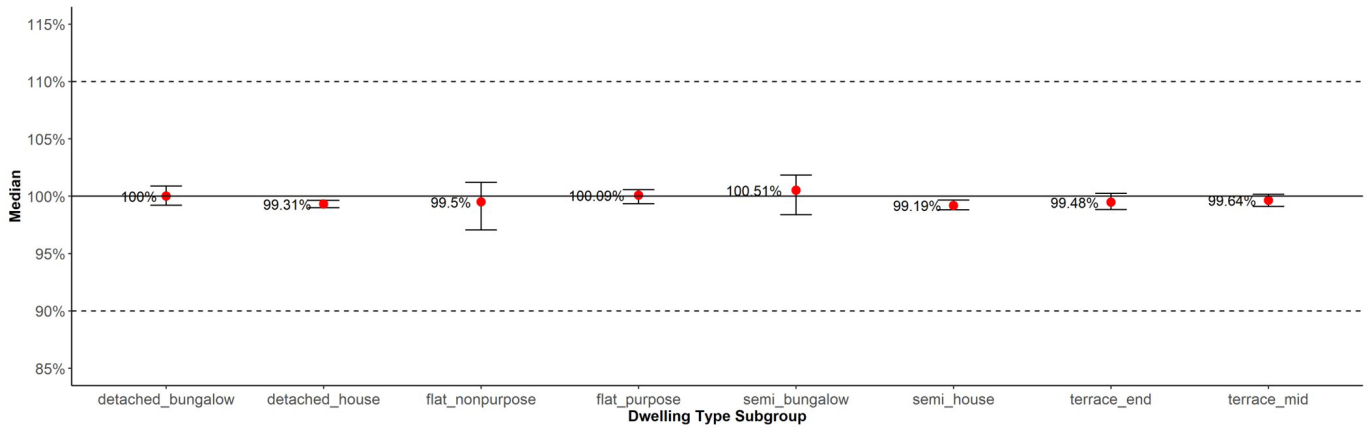


FIGURE 27. COD by Property Type Subgroup

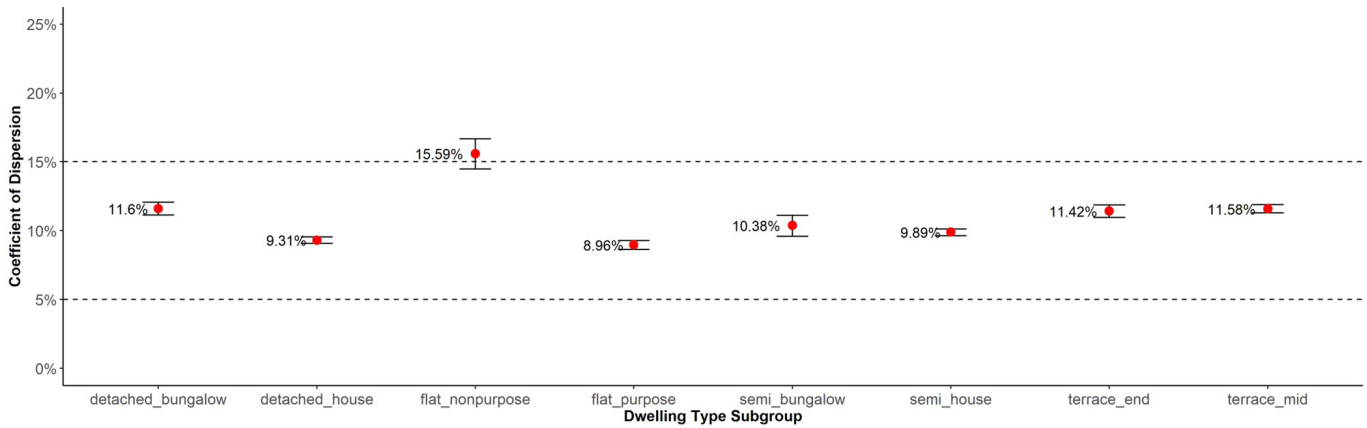
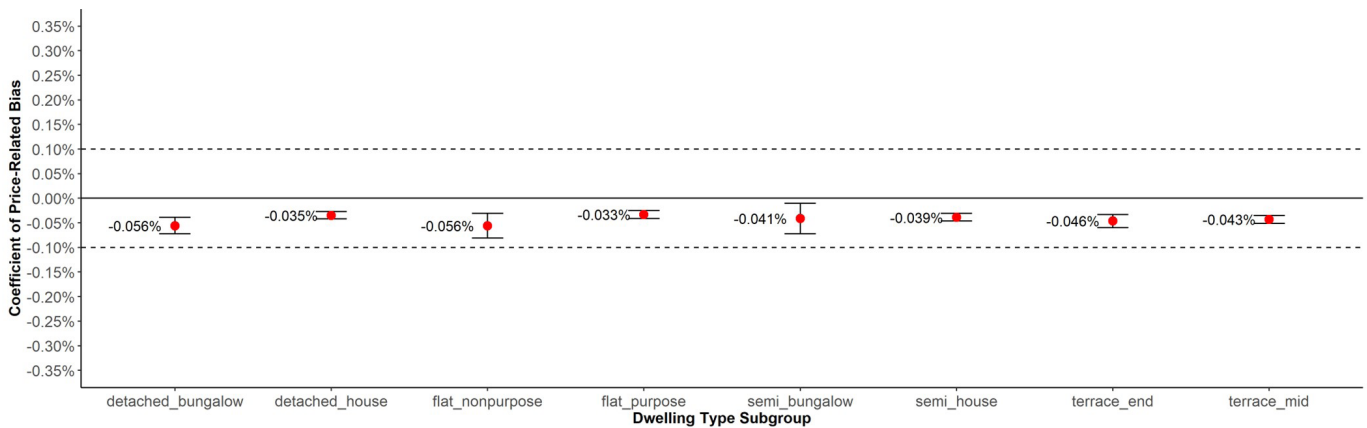


FIGURE 28. PRB by Property Type Subgroup



The IAAO standards for the median sales ratio, COD, and PRB are met in each individual dwelling type subgroup. Additionally, the indicated values of each statistic tend to be quite consistent across each dwelling type subgroup, particularly in the case of the median sales ratio. Overall, these are considered very good results.

The following **Figures 29 – 36** are the decile bar charts for each dwelling type subgroup.

FIGURE 29. Decile Bar Chart for Detached Bungalows

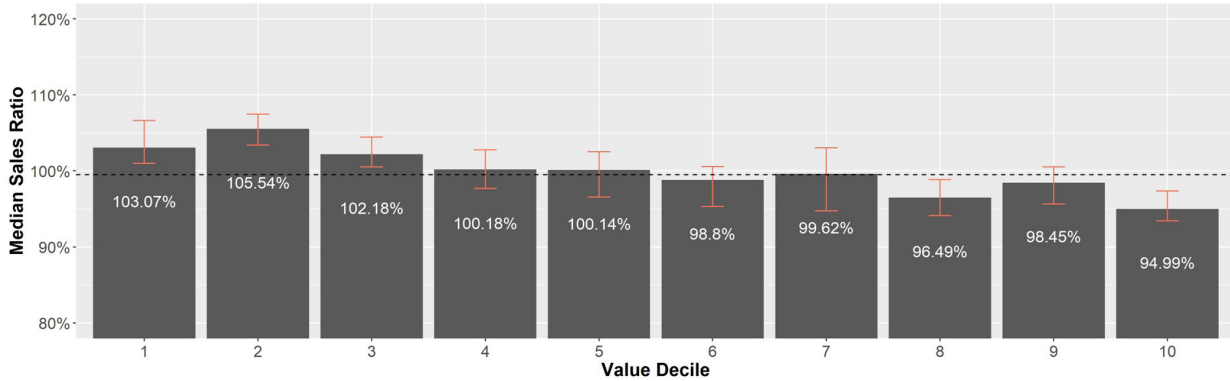


FIGURE 30. Decile Bar Chart for Detached Houses

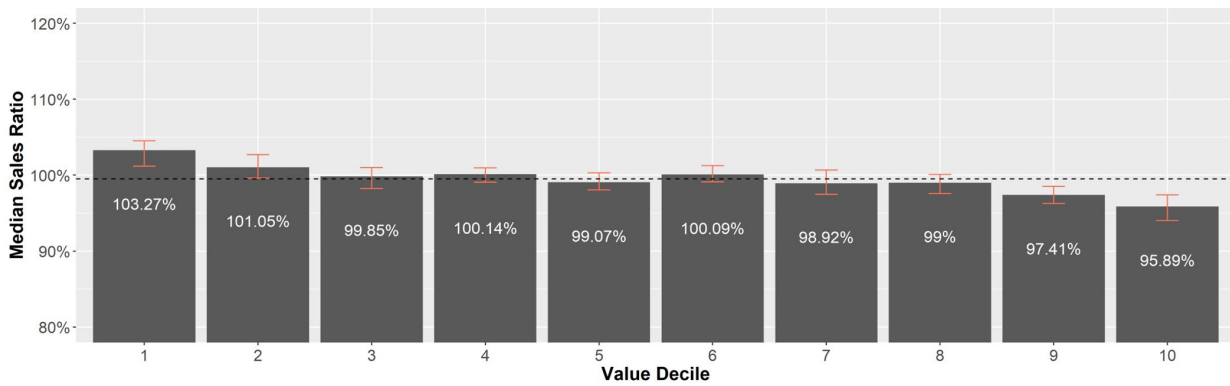


FIGURE 31. Decile Bar Chart for Purpose-Built Flats

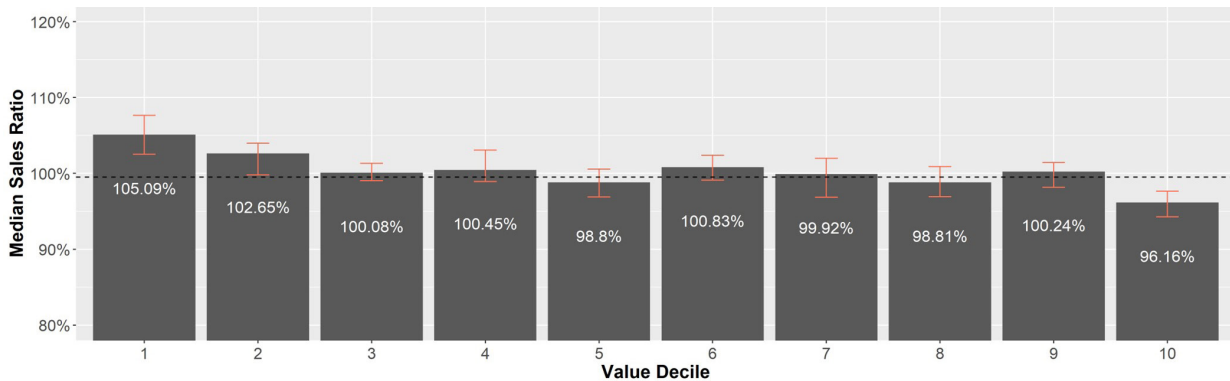


FIGURE 32. Decile Bar Chart for Non Purpose-Built Flats

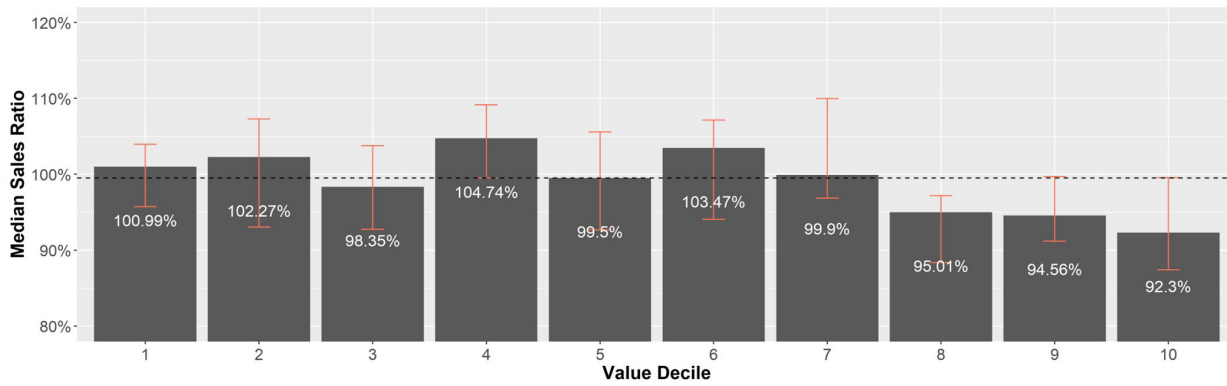


FIGURE 33. Decile Bar Chart for Semi Bungalows

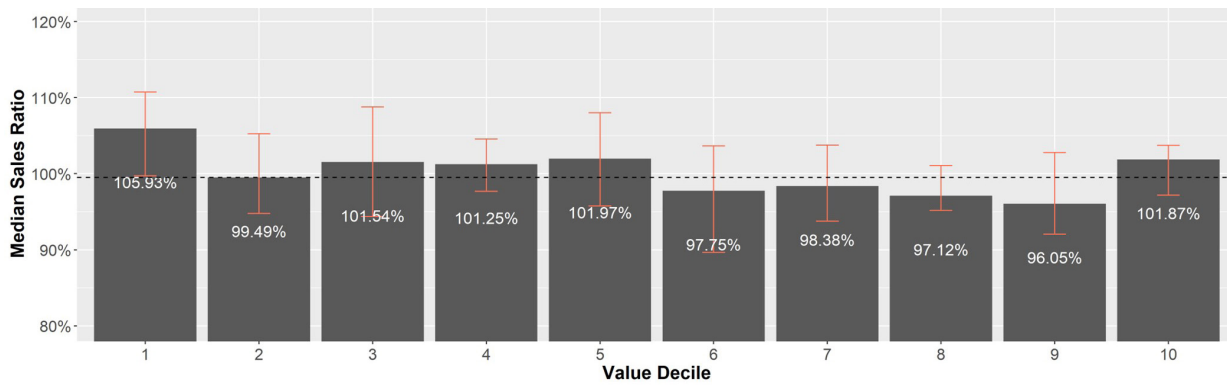


FIGURE 34. Decile Bar Chart for Semi Houses

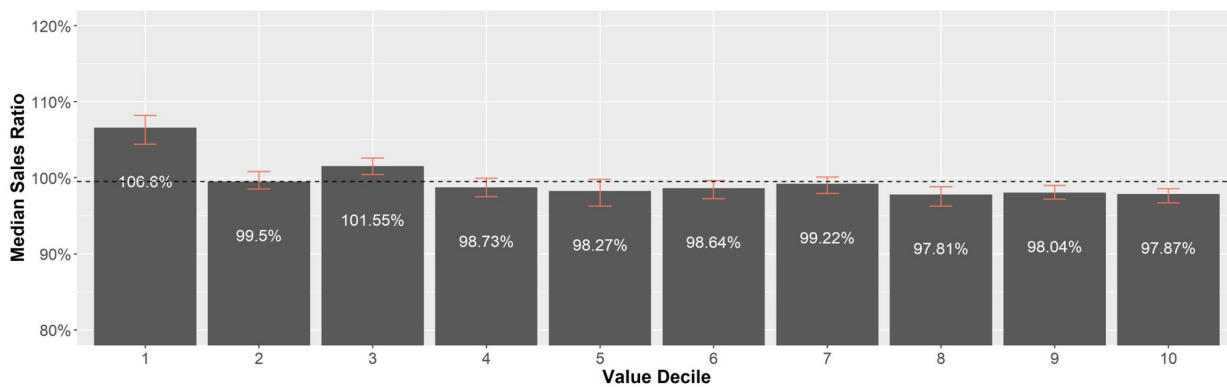
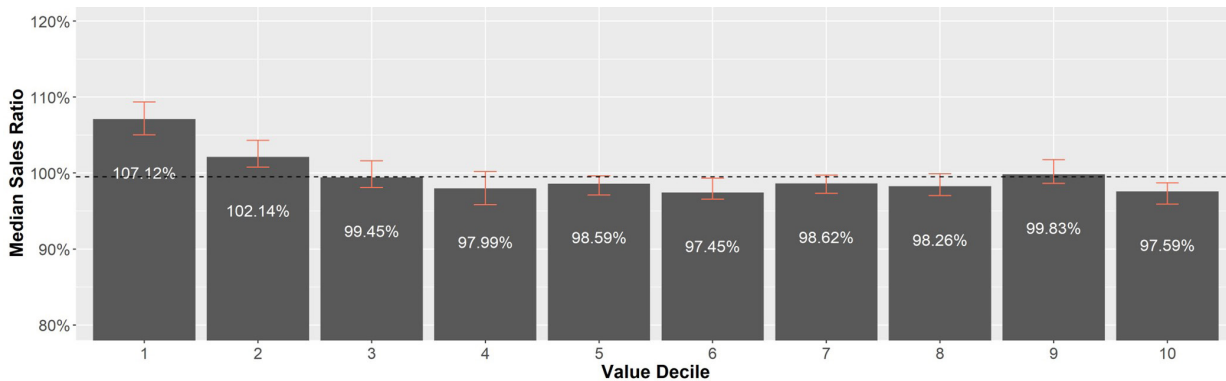
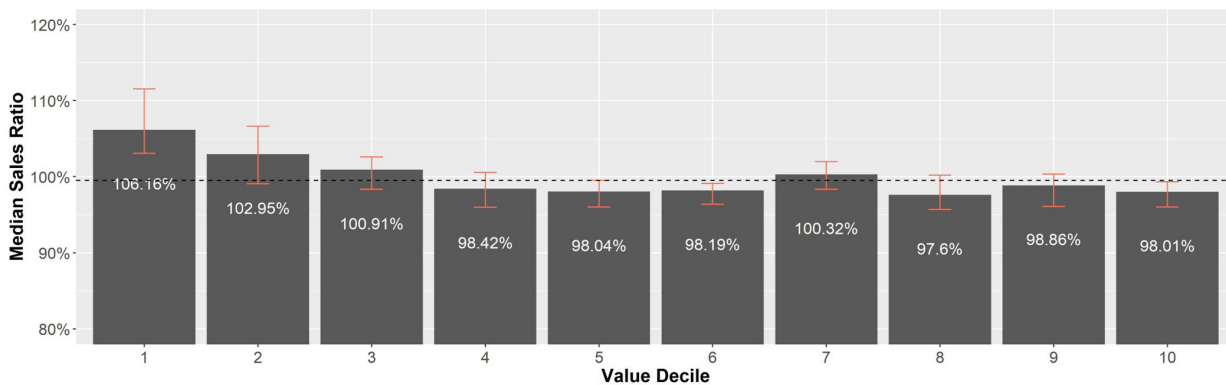


FIGURE 35. Decile Bar Chart for Terrace Mid**FIGURE 36. Decile Bar Chart for Terrace End**

Graphically, the decile bar charts each have the same general pattern and generally indicate a slight degree of regressivity. One test mentioned in the *IAAO Standard on Ratio Studies* (IAAO 2013, p. 17) is that the median sales ratios in each stratum must be within 5 percent of the overall median sales ratio.

In the decile bar charts, the overall median sales ratio is represented by the dashed horizontal line and the confidence intervals of each bar can be compared to this dashed line to see if the confidence interval bounds fall outside of the allowable range (± 5 percent around the median sales ratio). This slight degree of regressivity is not problematic as each of the decile medians for each dwelling type subgroup falls within this allowable range except for just one (the first decile for terrace mid properties).

Statistics by the Combination of Dwelling Type Subgroup and Another Variable

It is important to measure the quality of the comparable sales model value estimates through the combination of dwelling type subgroup and other key variables. This allows one to see how the model performs across each key variable inside of each dwelling type subgroup. Appendix A contains statistics presented in graphical format for the combination of dwelling type subgroup and dwelling area range, age range, bathroom count group, bedroom count group, floor count group, floor level code group, and subsidized status. For the sake of brevity, this section presents a summary of the findings.

The vast majority of the statistics across all categories meet IAAO standards for the median sales ratio, COD, and PRB (where applicable). A very good indication of results. The following strata are exceptions:

- The COD for non purpose-built flats is higher than IAAO standards permit for flats between 60 and 70 square meters (COD = 19.59%) and flats greater than 70 square meters (COD = 19.09%) based on their confidence intervals.
- The COD for detached houses built before 1900 (COD = 17.33%) is higher than IAAO standards permit based on its confidence interval.
- The PRB for detached bungalows with three or more bathrooms (PRB = -0.329) is higher than IAAO standards permit based on its confidence interval.

Another observation from this analysis is that there seems to be slight but meaningful relationship across the house (non-flat) dwelling type subgroups between bathroom count and the median sales ratio. As bathroom count goes up, the median sales ratio tends to go down slightly. This may be for two reasons: 1) because there is not a variable for building quality and higher bathroom counts can often be correlated to some degree with building quality and/or 2) because bathroom count is not included as a variable in this model.

Concerning the second point, VOA had decided not to include bathroom count in the houses model because it did not seem to make a meaningful difference in the goodness of fit statistics. Sometimes, however, a real meaningful difference from adding certain variables can often only be seen in a small number of observations and can be obscured by the bulk of the unaffected properties. Adding bathroom count to the houses model should be explored further.

A similar effect may be present with the flats model and properties with high bedroom counts (three or higher). It is easy to see how adding incremental bedrooms past the second bedroom to a flat may invoke the principle of diminishing marginal utility and may not be as in demand. Adding a variable for high bedroom count to the flats model should be explored further as well.

Step 7: Evaluation and Use of Result

The key findings of the statistical analysis are as follows:

- The decision to base the market trends on the HPI was well-founded. HPI trends closely statistically mirror custom-developed price trends by dwelling type group and have the advantage of being determined by an independent entity.
- VOA rightly and correctly employed the useful holdout sample technique as a way to guard against model overfitting and provide an unbiased set of sales with which to assess the performance of competing models.
- The spatial random fields (SRF) model was generally statistically significantly better than the mixed model. The time and effort spent to develop the SRF model yield improved results.
- VOA was justified in selecting the comparable sales model for the valuation in Wales over other competing models. Although, the improvement in performance over the spatial random fields model alone was slight and not statistically significant, the comparable sales model affords VOA with more practicability and explainability because it automatically includes comparable sales. The comparable sales model was generally statistically significantly better than the mixed model.
- The comparable sales model values meet the IAAO standard for each statistic across all dwelling types and dwelling type subgroups. Decile bar charts confirm that there generally is not a problematic level of vertical inequity present.
- The vast majority of the statistics across all variable categories for each dwelling type subgroup meet IAAO standards for the median sales ratio, COD, and PRB (where applicable).

These findings are more than satisfactory and should lead the VOA to have confidence in the quality of the new valuation project conducted in Wales.

Nevertheless, the following recommendations are made from the statistical analysis to help improve the model development approach in the future:

- VOA should explore adding a variable for bathroom count to both the houses and flats models.
- VOA should explore the potential development and use of property quality and physical condition variables in future revaluation projects. Utilizing those data characteristics in future revaluations will help to ensure property banding reflects the market value of individual property.
- Conduct an independent time trend study and analysis to verify the validity of the continued use of the HPI trend factor in future revaluations.
- VOA should use sales on or after July 1 of the revaluation year, time-trended back to April 1, for after the fact sales ratio studies. After some time passes, this would yield more sales than the 10% holdout sample. In addition, statistics calculated on this sample could be an even better indicator of model performance if the data quality in this sample is more equivalent to the data quality of the population of properties than the official holdout sample.

REFERENCES

1. International Association of Assessing Officers. (2013). *Standard on Ratio Studies*. Kansas City, MO.
2. International Association of Assessing Officers. (2020). *Standard on Verification and Adjustment of Sales*. Kansas City, MO.

APPENDIX

Additional Statistics

APPENDIX: Additional Statistics

1. Statistics by Dwelling Type Subgroup and Dwelling Size Range

FIGURE 1.1

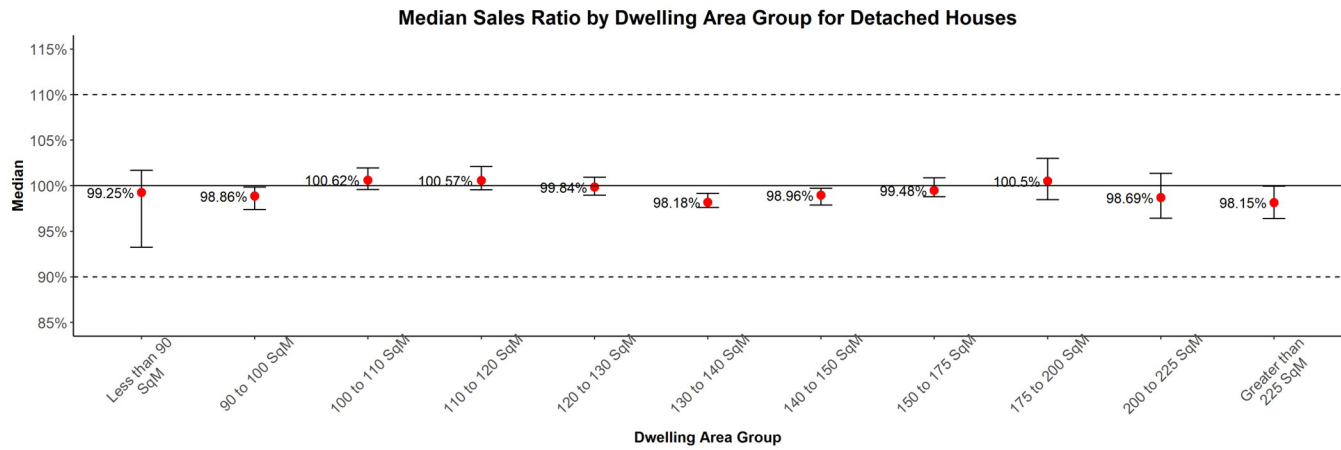


FIGURE 1.2

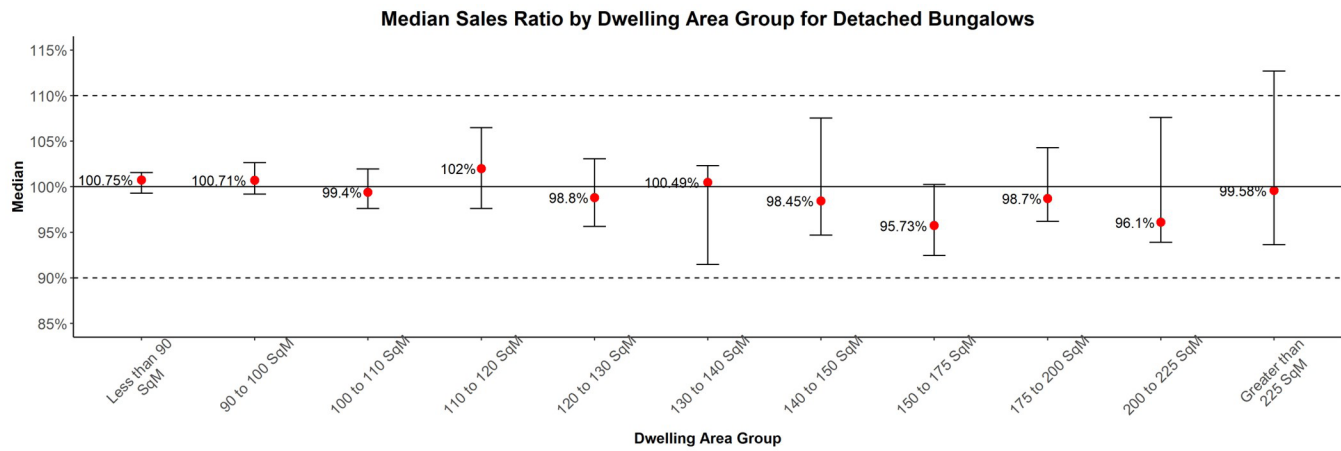


FIGURE 1.3

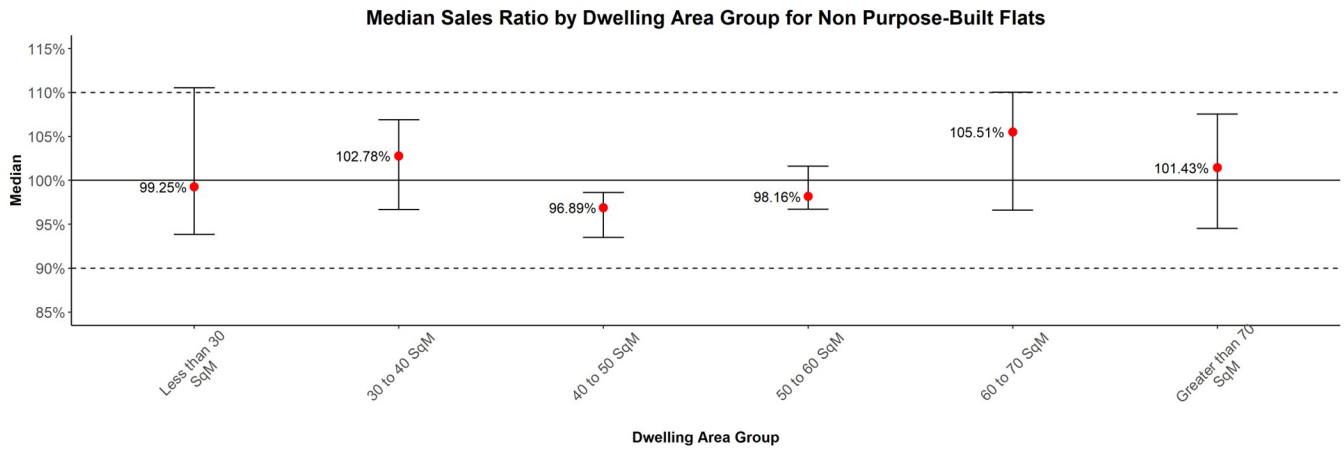


FIGURE 1.4

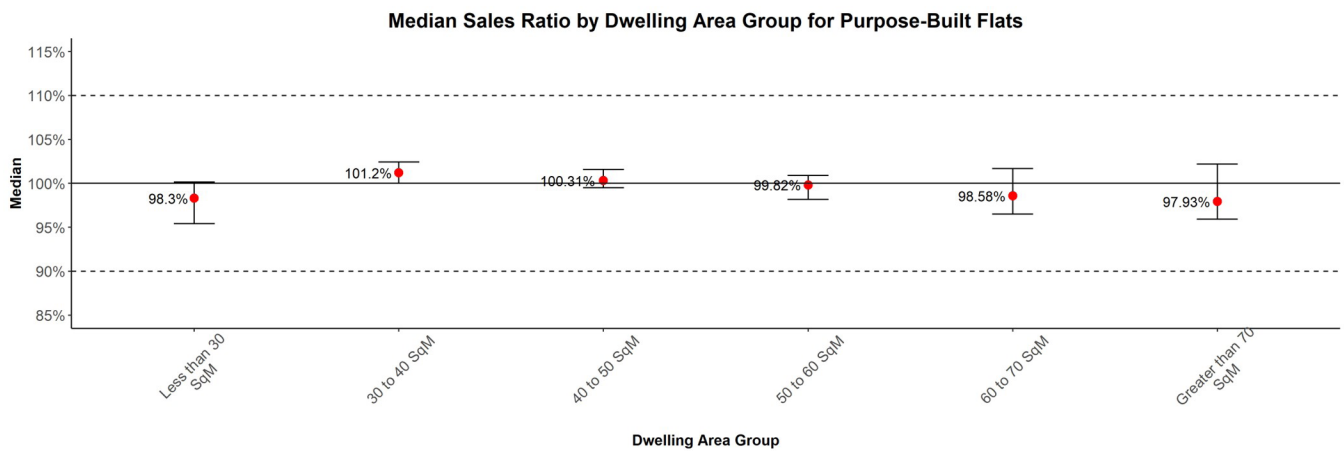


FIGURE 1.5

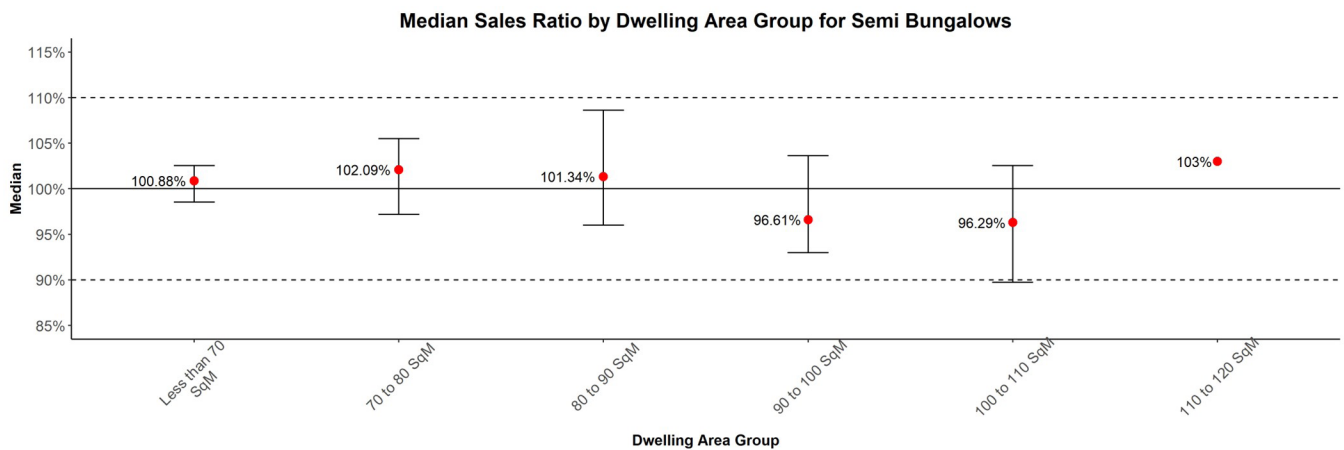


FIGURE 1.6

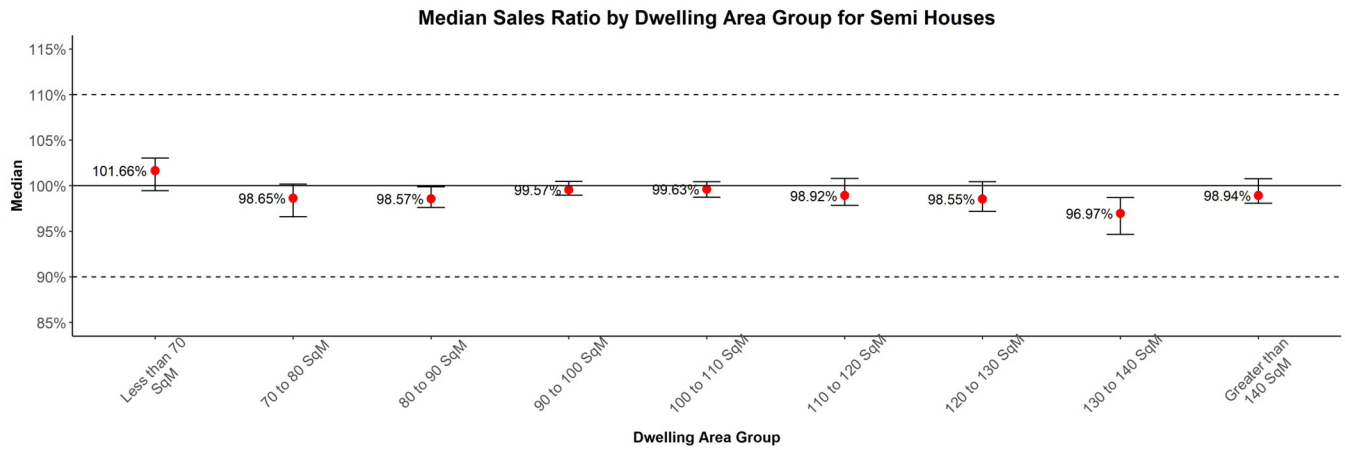


FIGURE 1.7

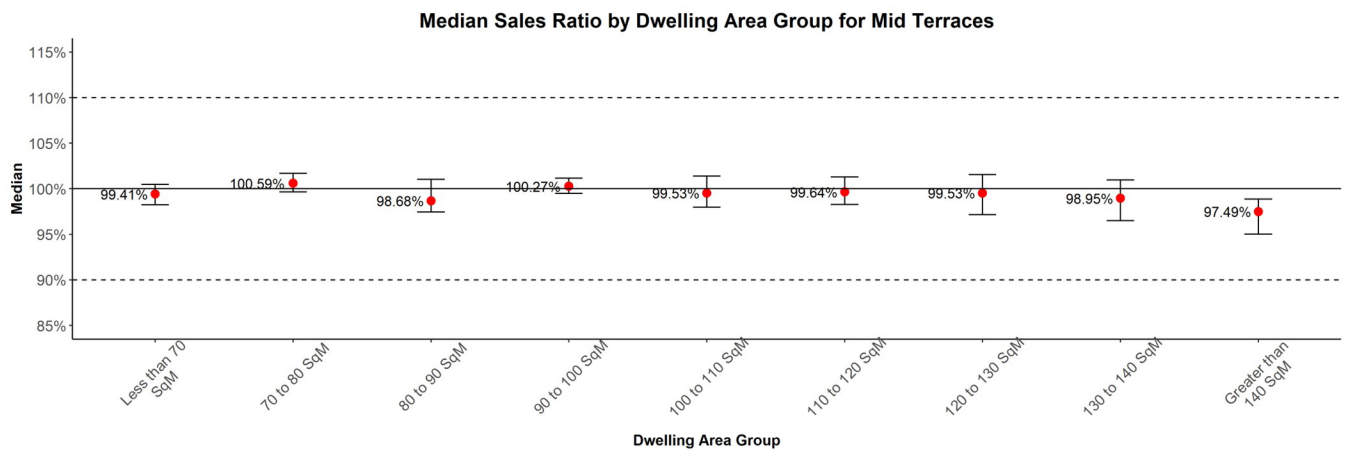


FIGURE 1.8

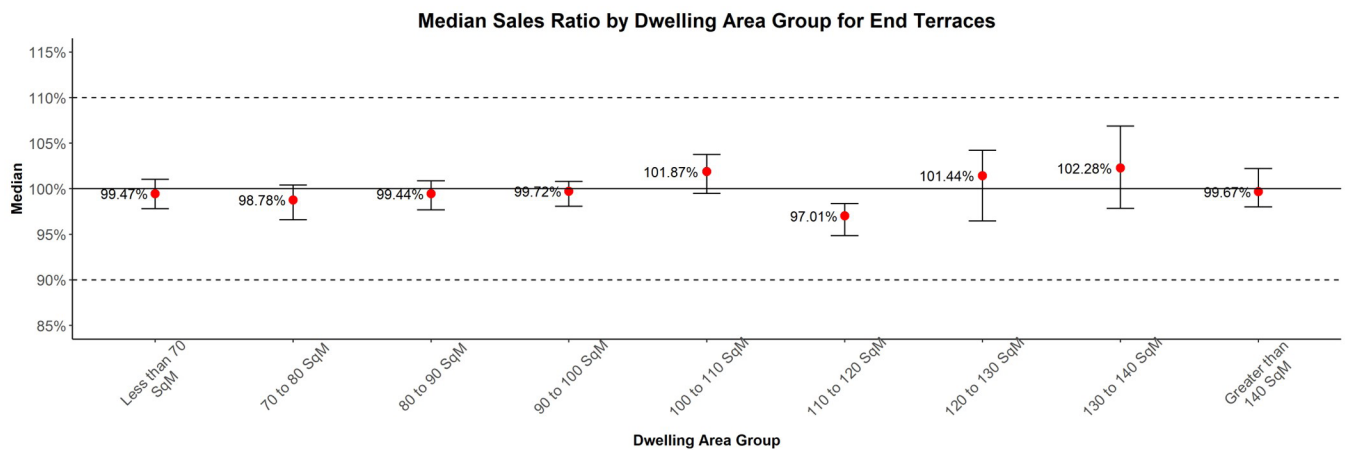


FIGURE 1.9

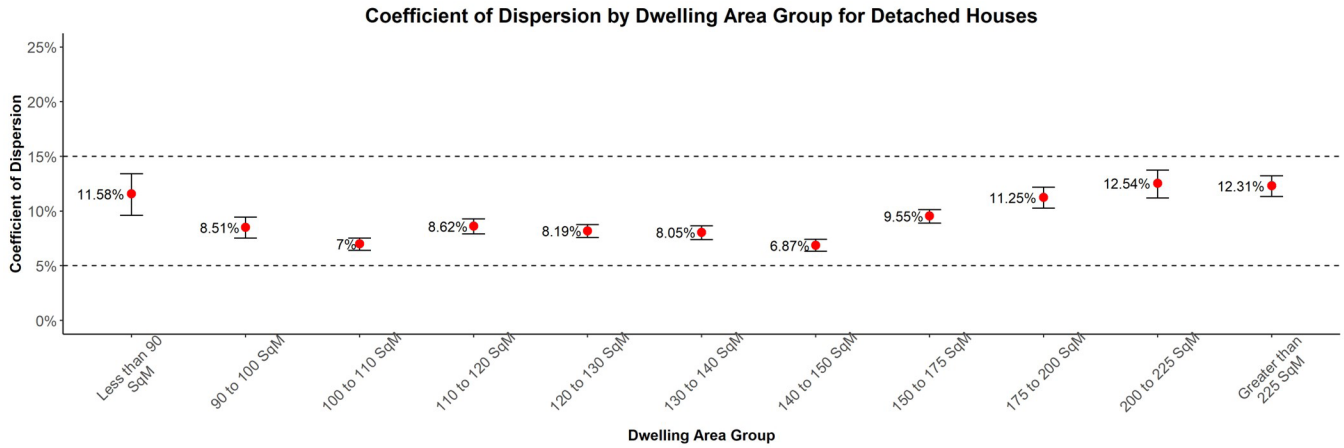


FIGURE 1.10

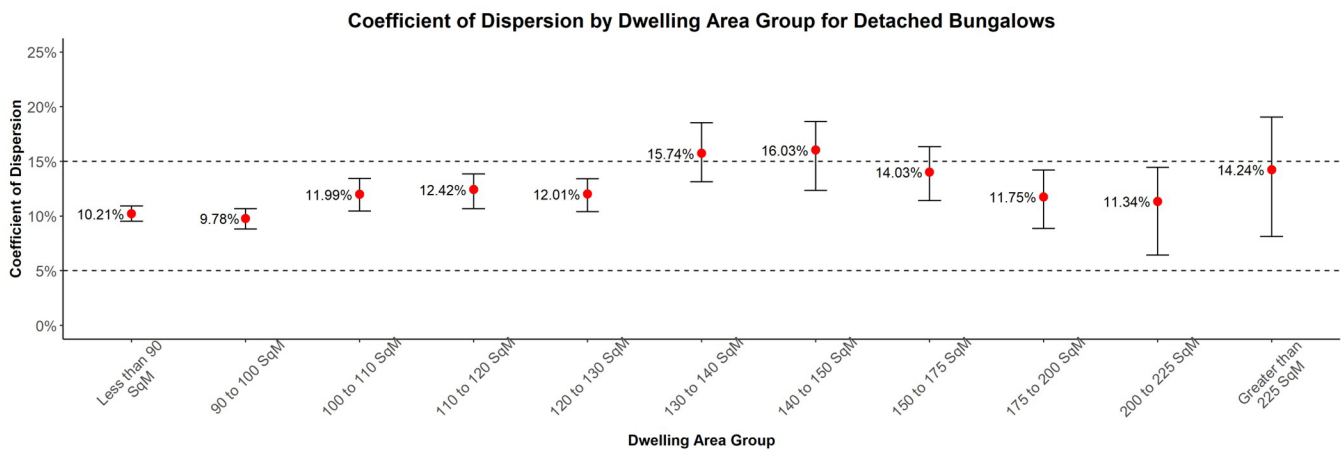


FIGURE 1.11

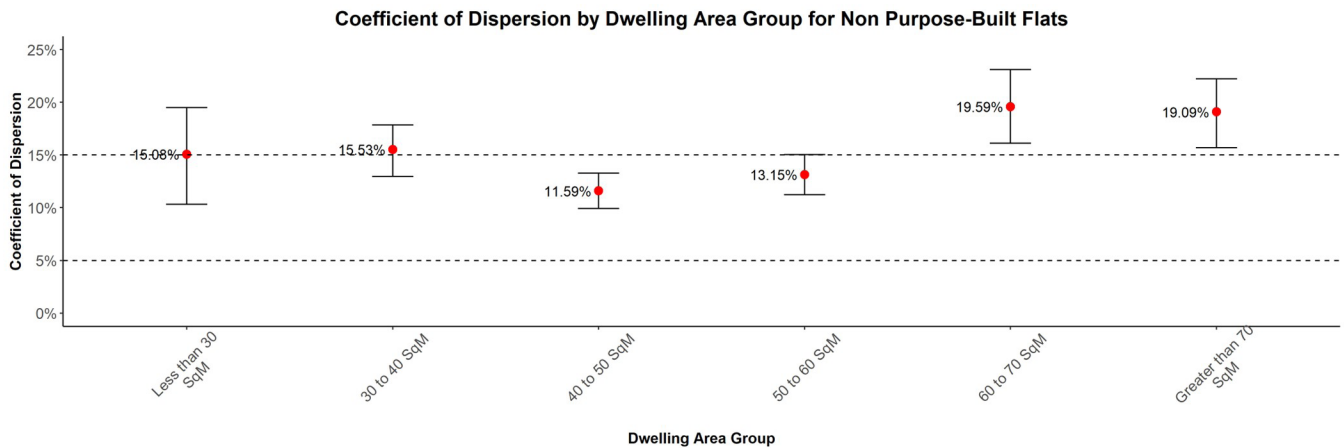


FIGURE 1.12

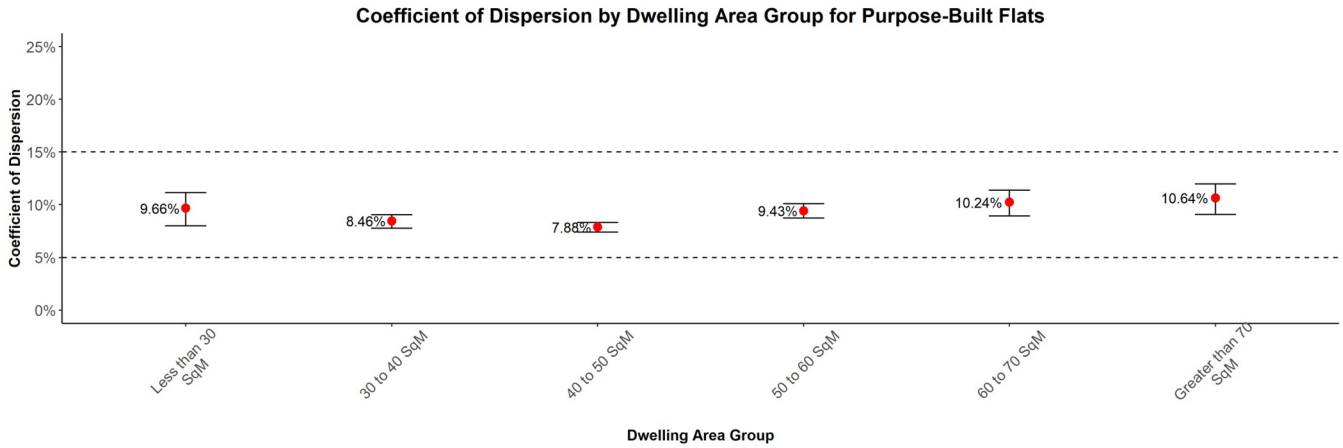


FIGURE 1.13

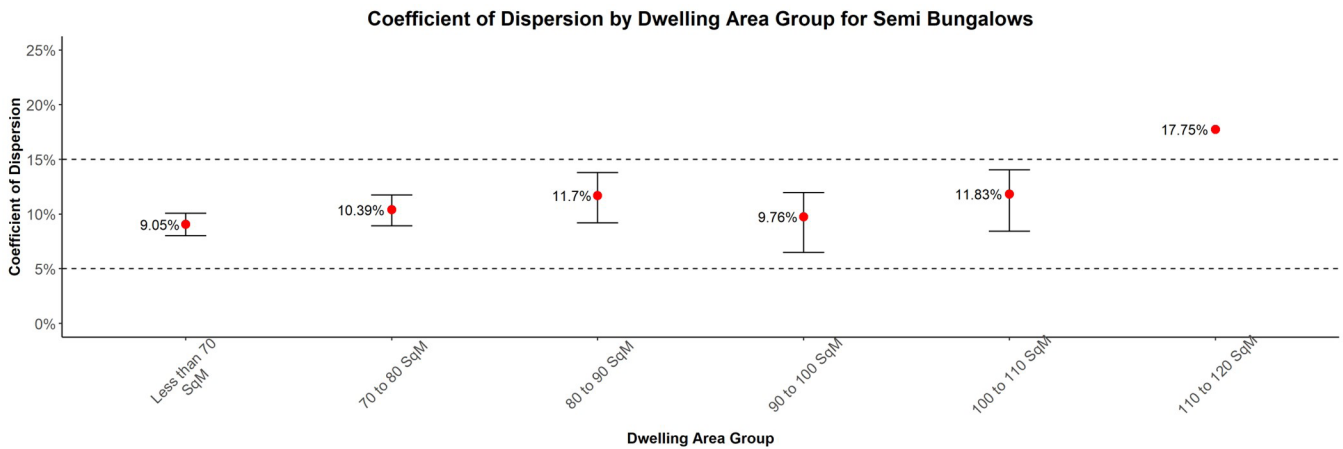


FIGURE 1.14

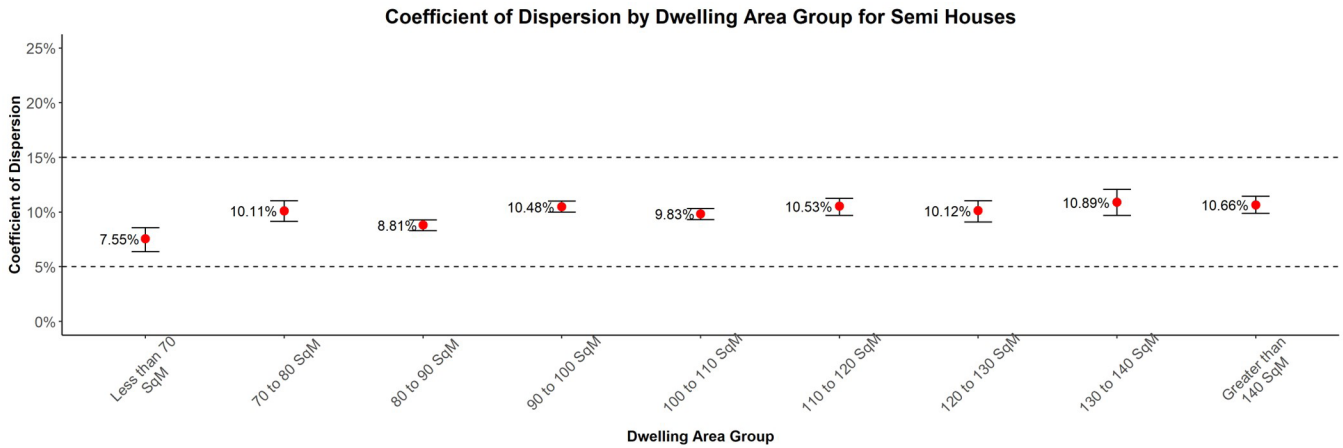


FIGURE 1.15

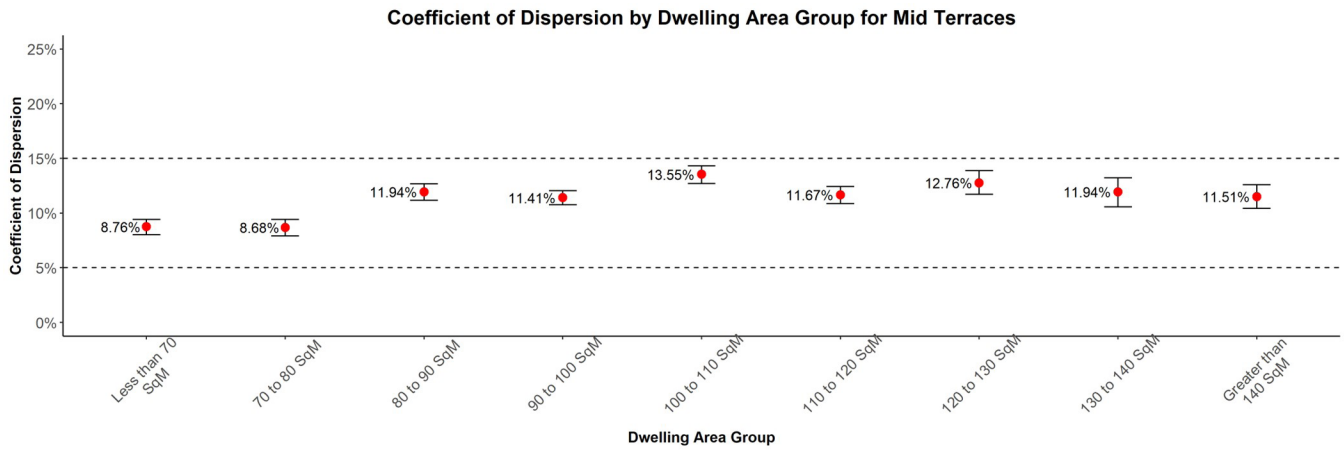
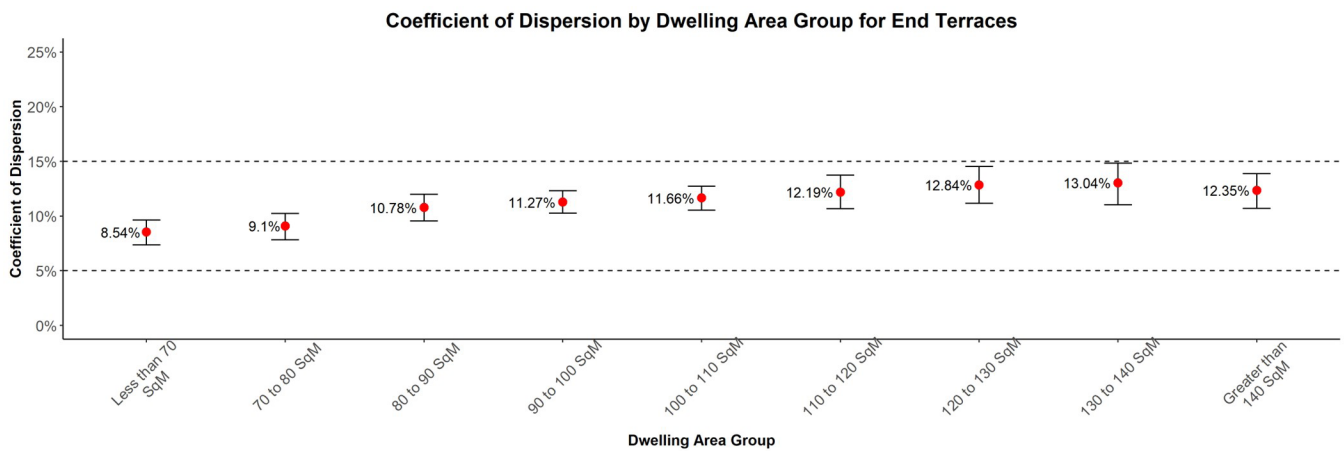


FIGURE 1.16



2. Statistics by Dwelling Type Subgroup and Age Range

FIGURE 2.1

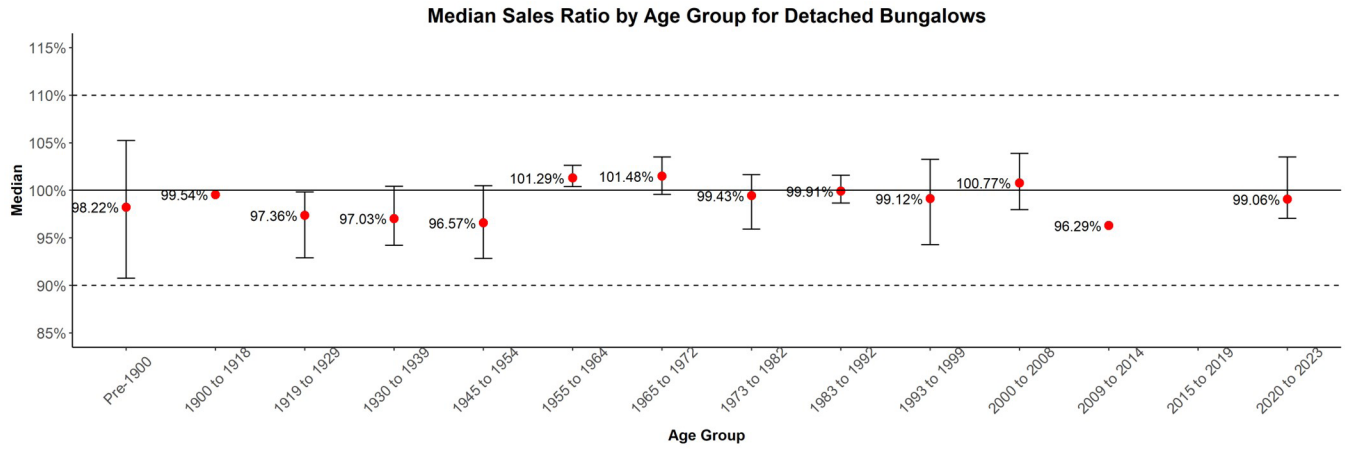


FIGURE 2.2

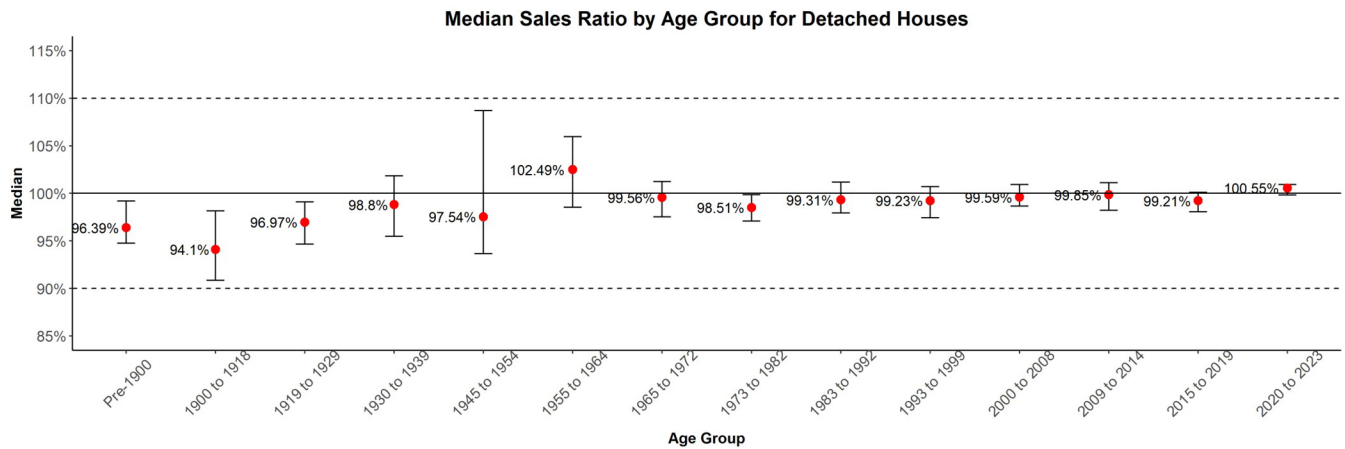


FIGURE 2.3

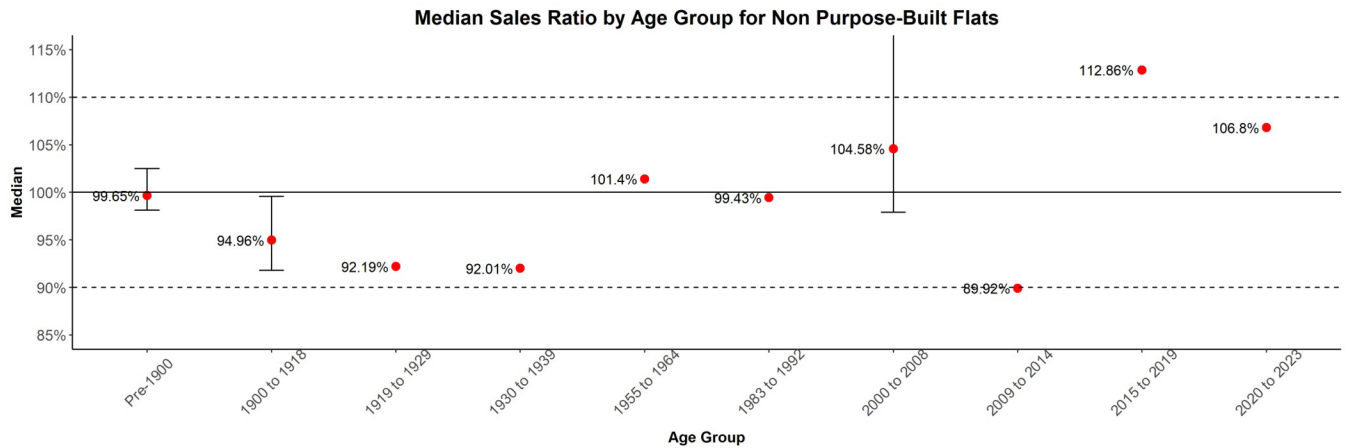


FIGURE 2.4

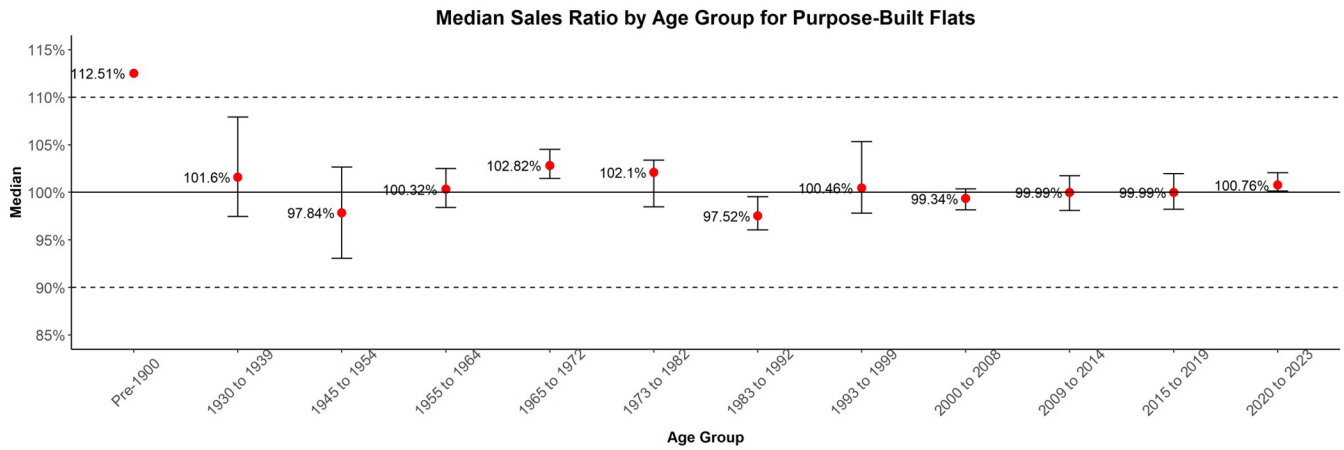


FIGURE 2.5

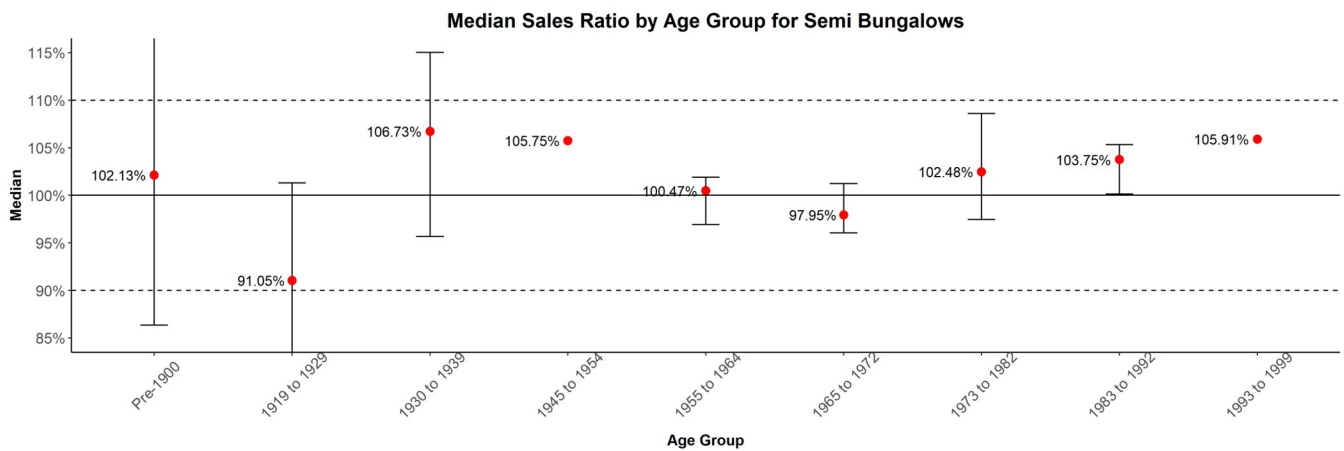


FIGURE 2.6

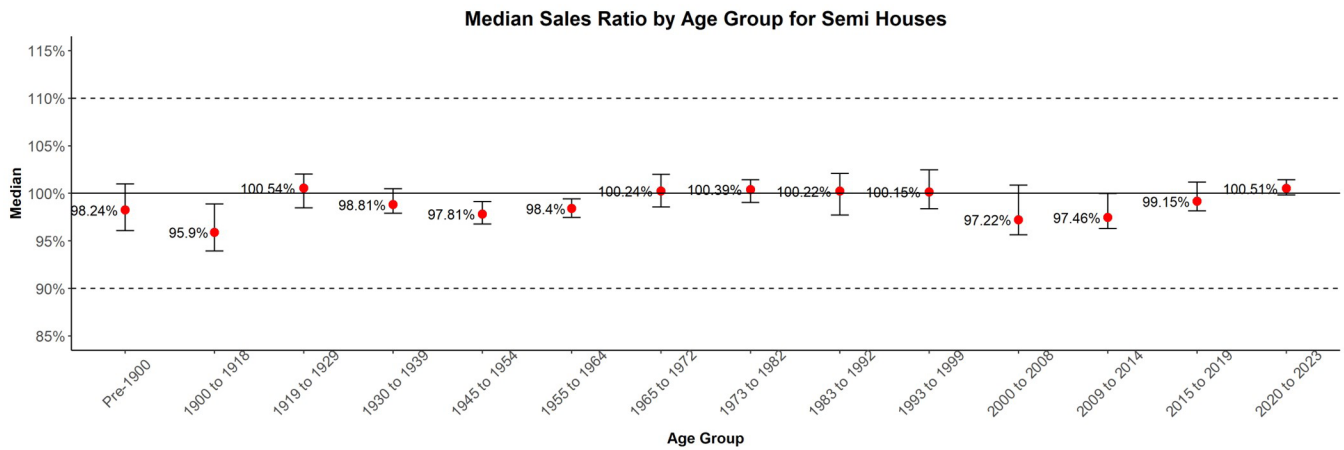


FIGURE 2.7

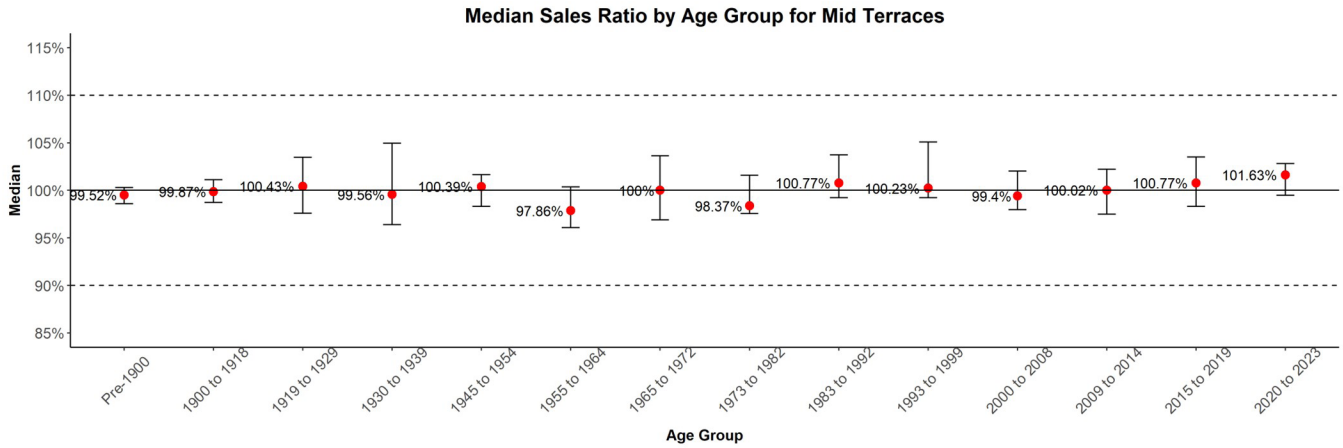


FIGURE 2.8

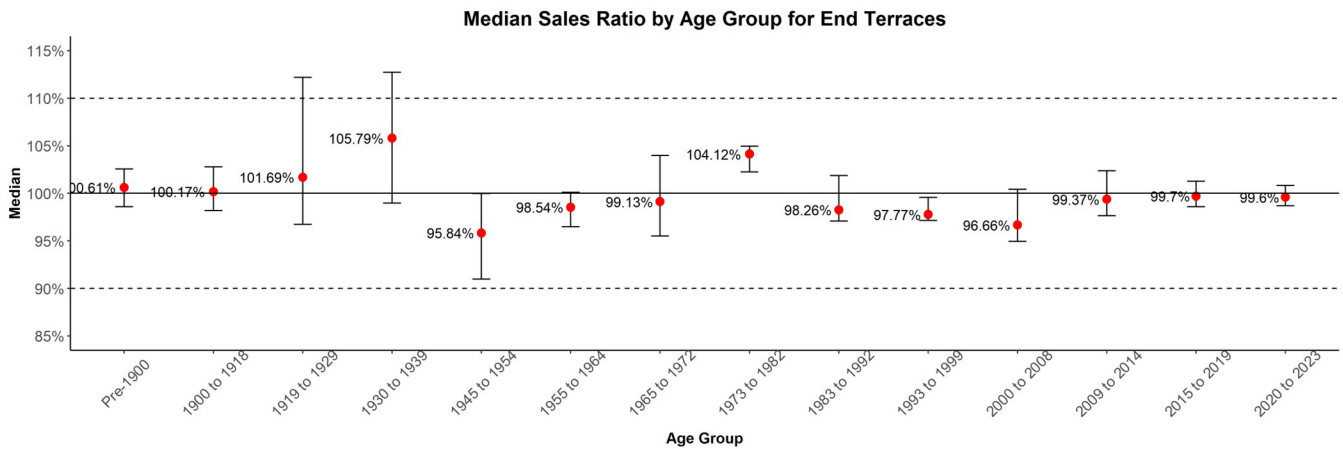


FIGURE 2.9

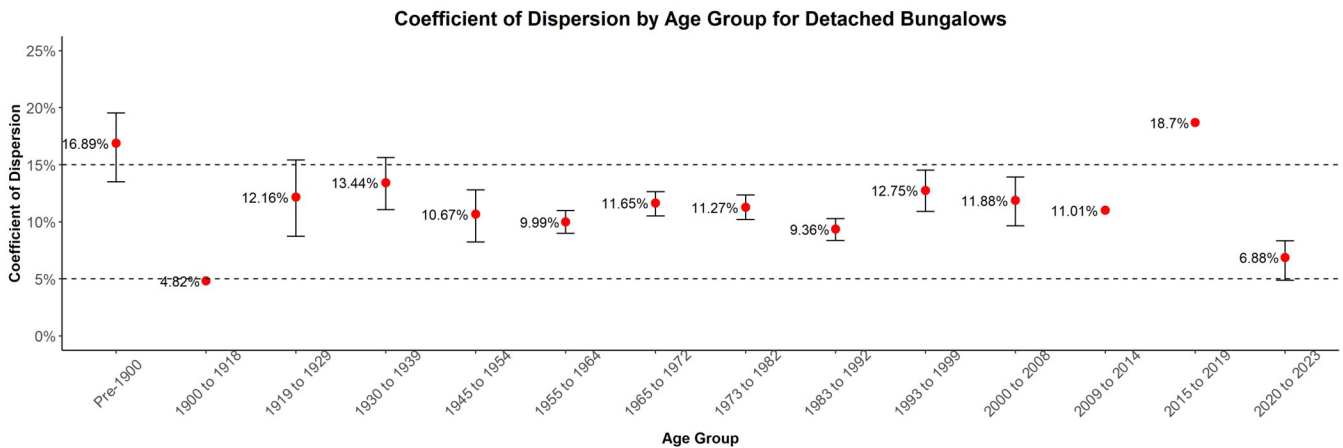


FIGURE 2.10

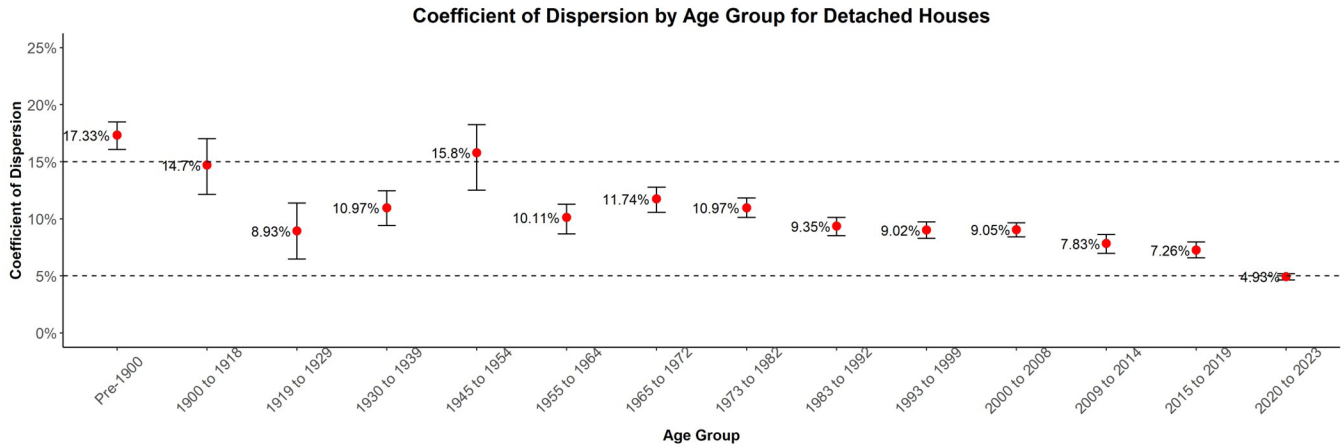


FIGURE 2.11

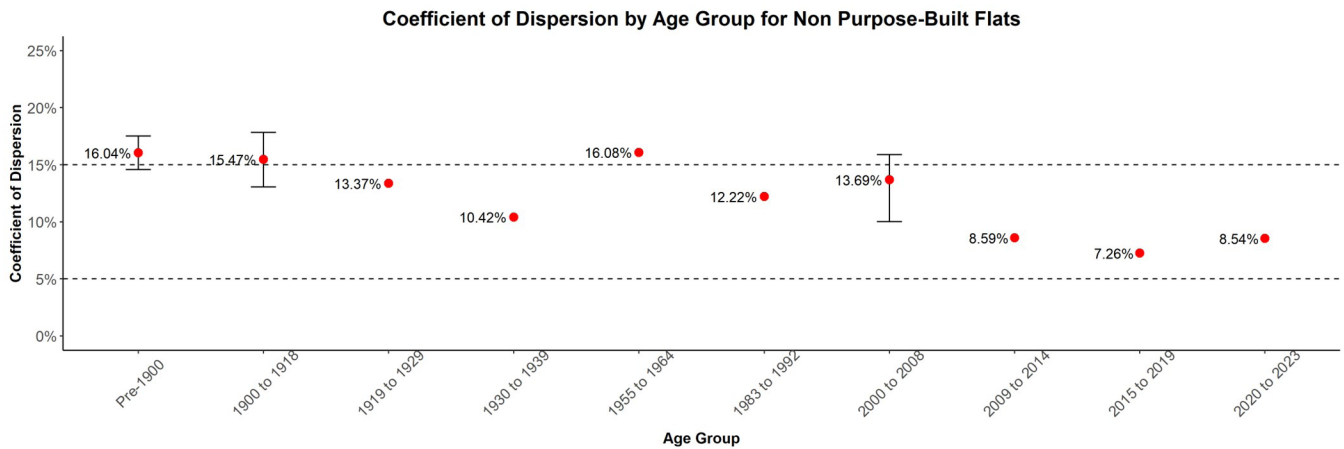


FIGURE 2.12

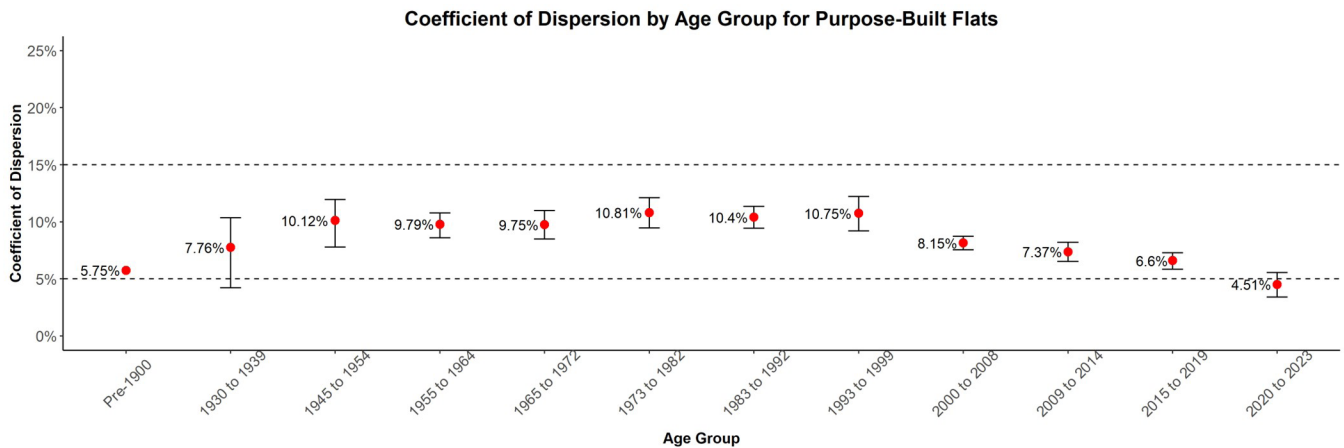


FIGURE 2.13

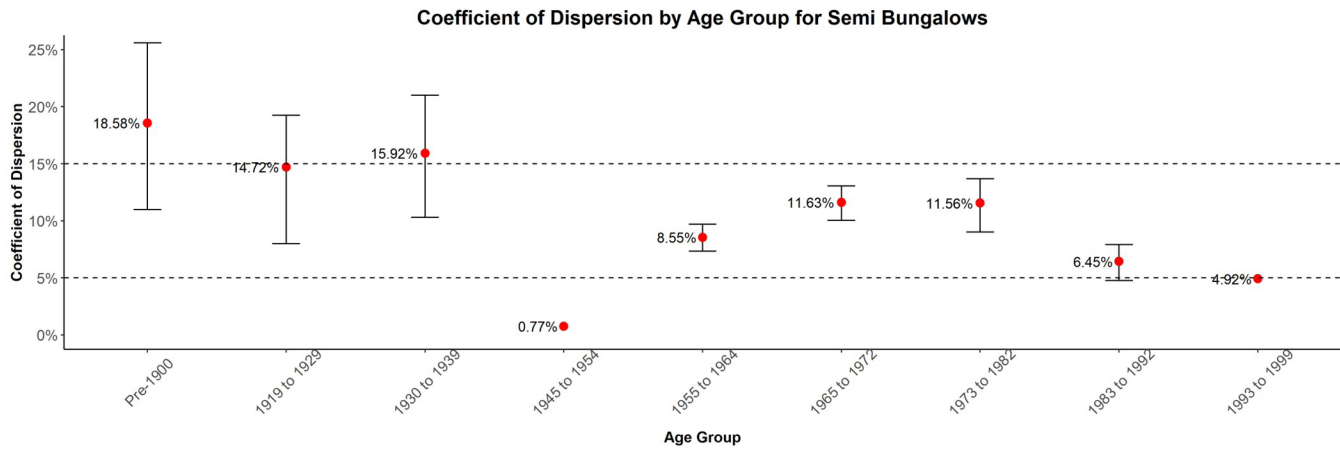


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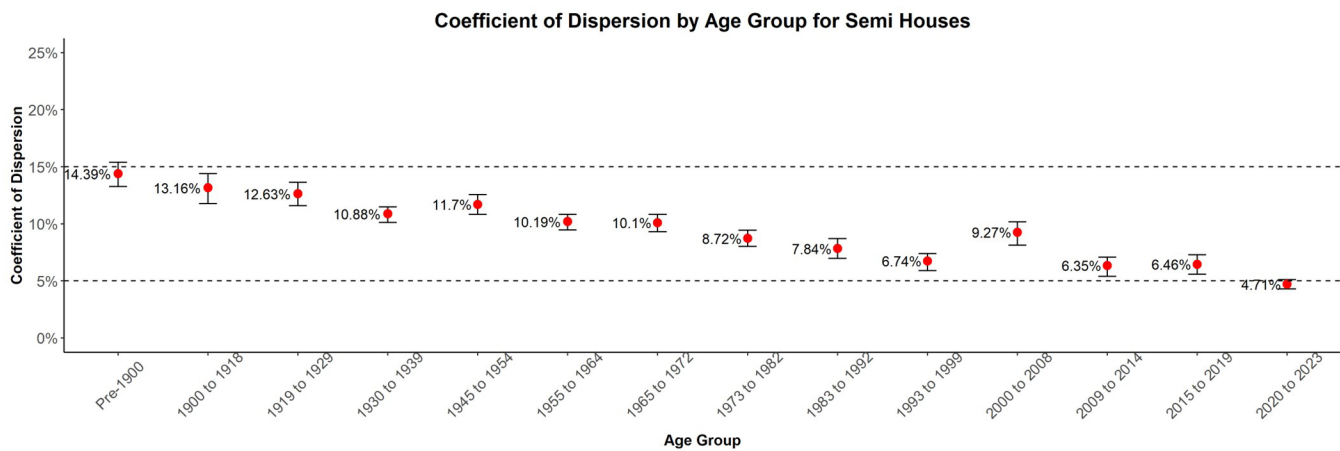


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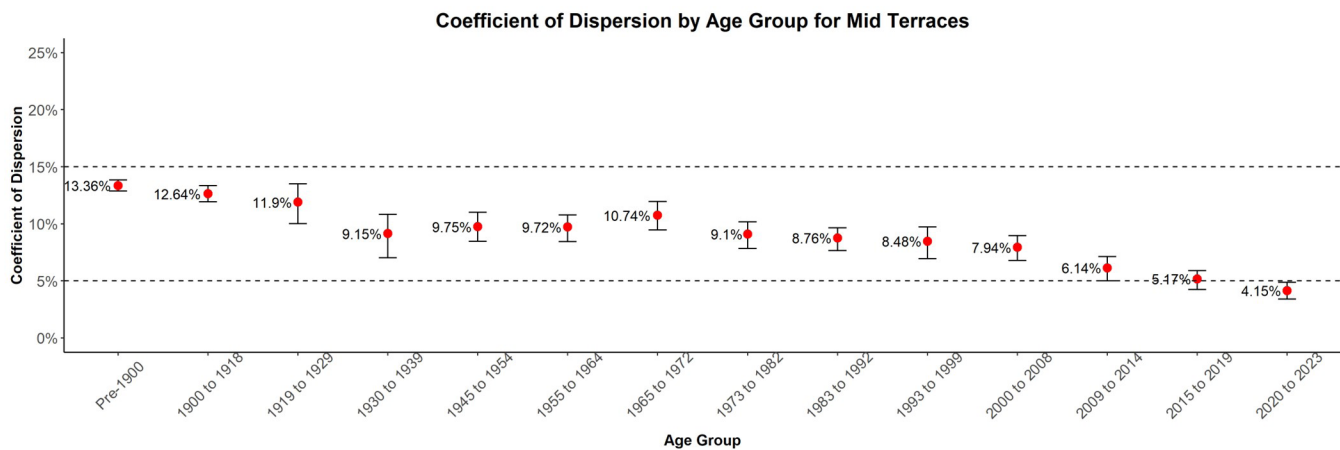


FIGURE 2.16

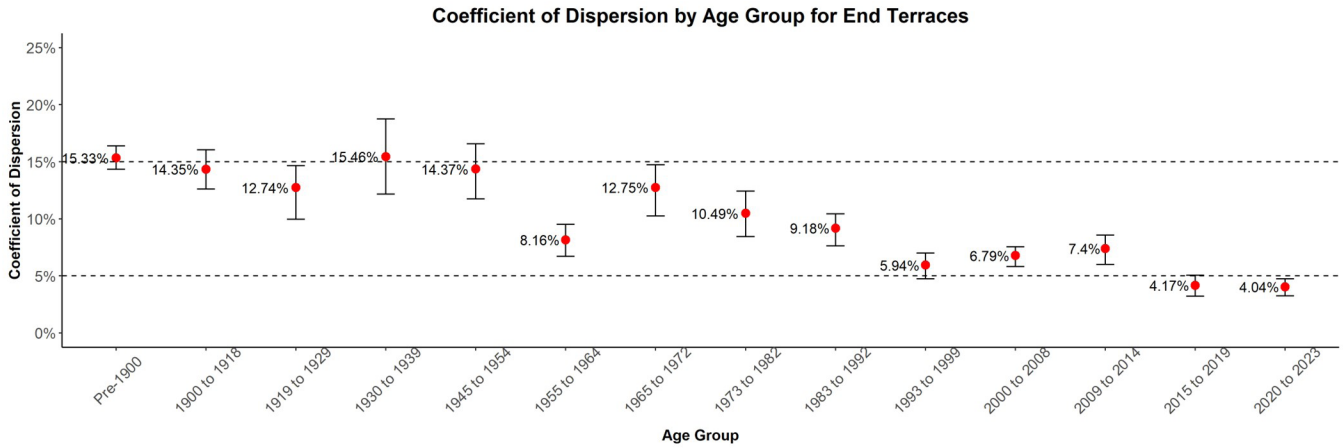


FIGURE 2.17

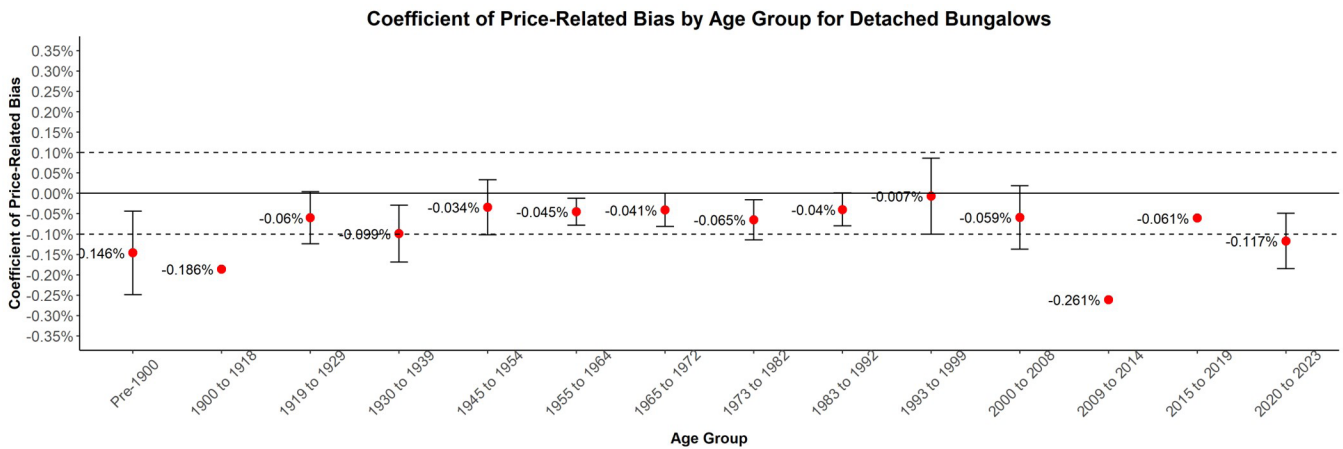


FIGURE 2.18

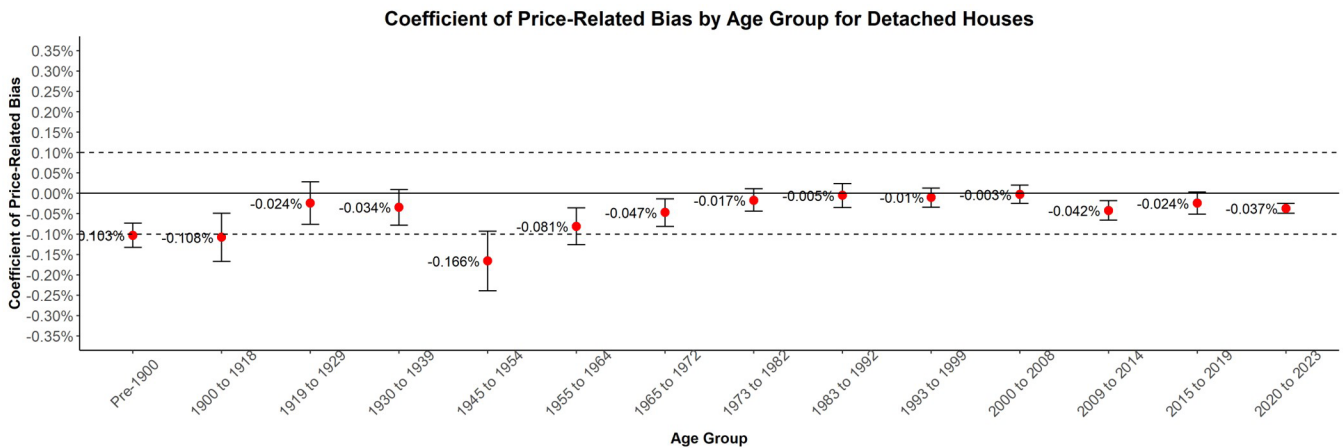


FIGURE 2.19

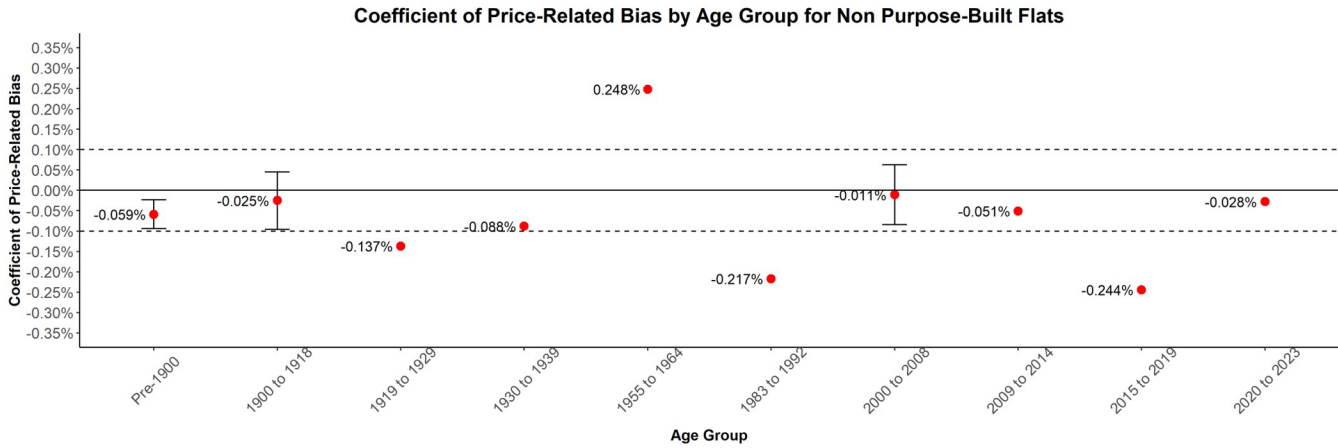


FIGURE 2.20

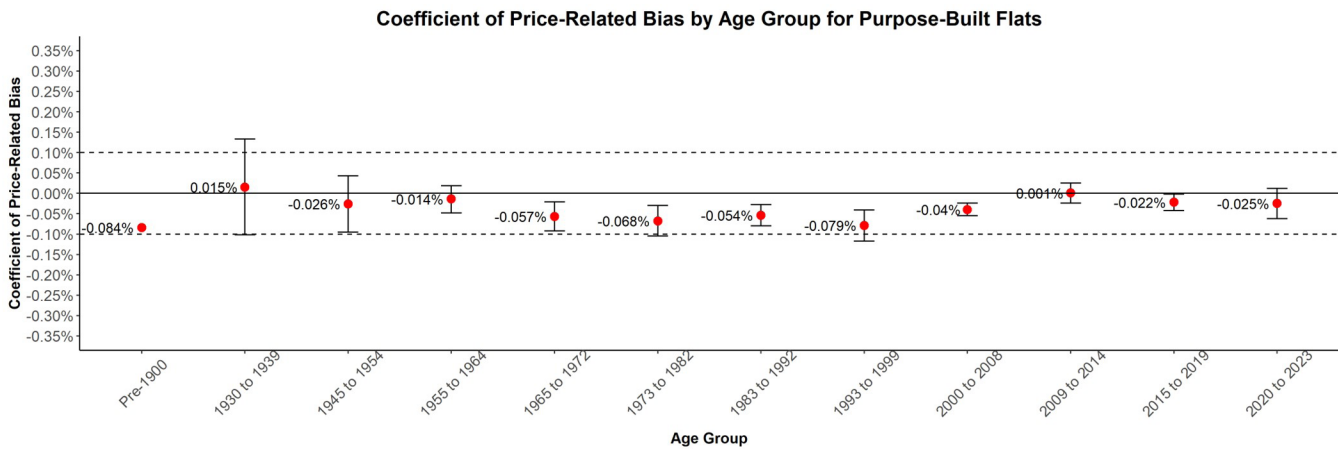


FIGURE 2.21

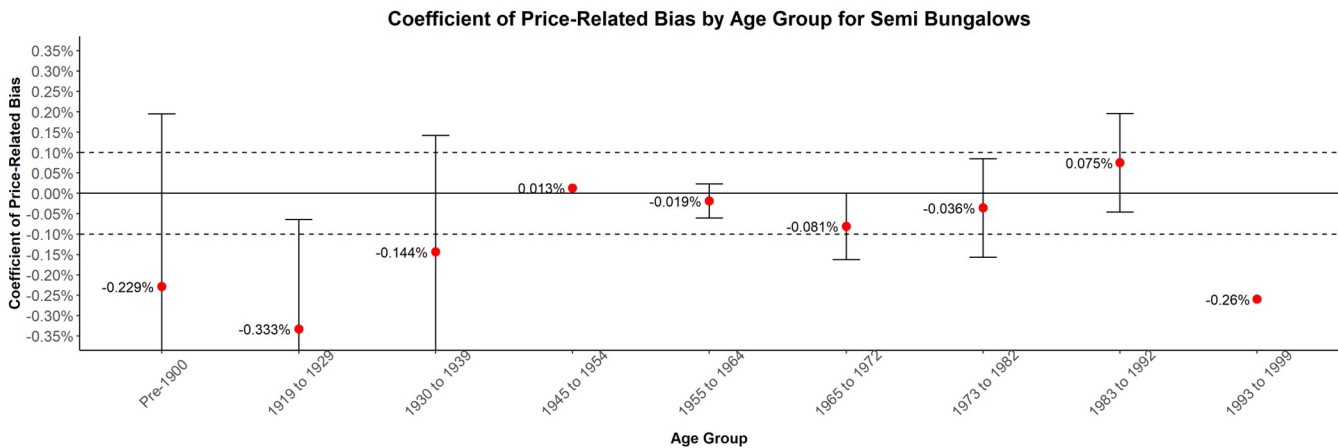


FIGURE 2.22

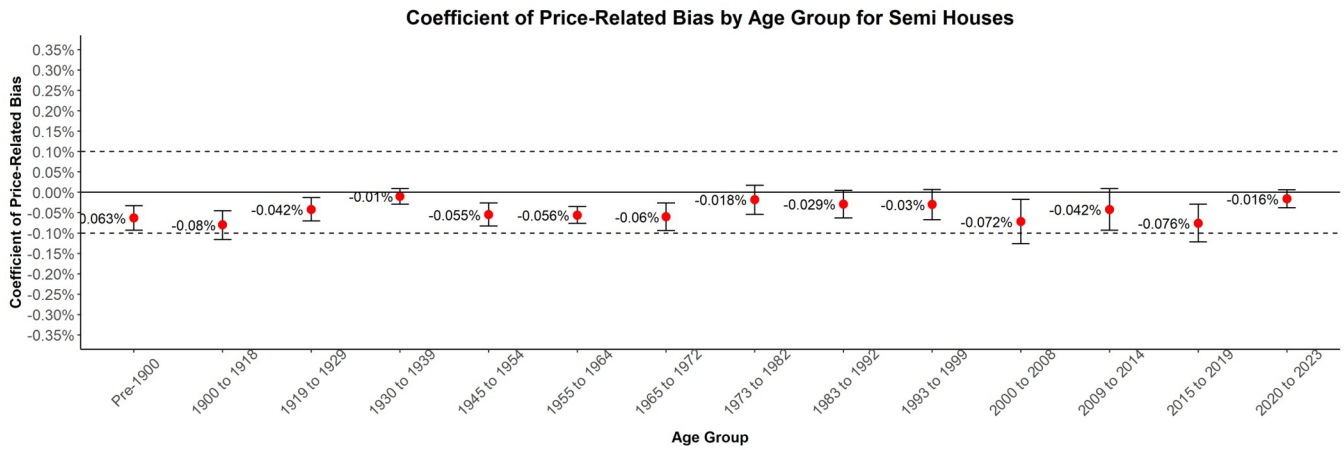


FIGURE 2.23

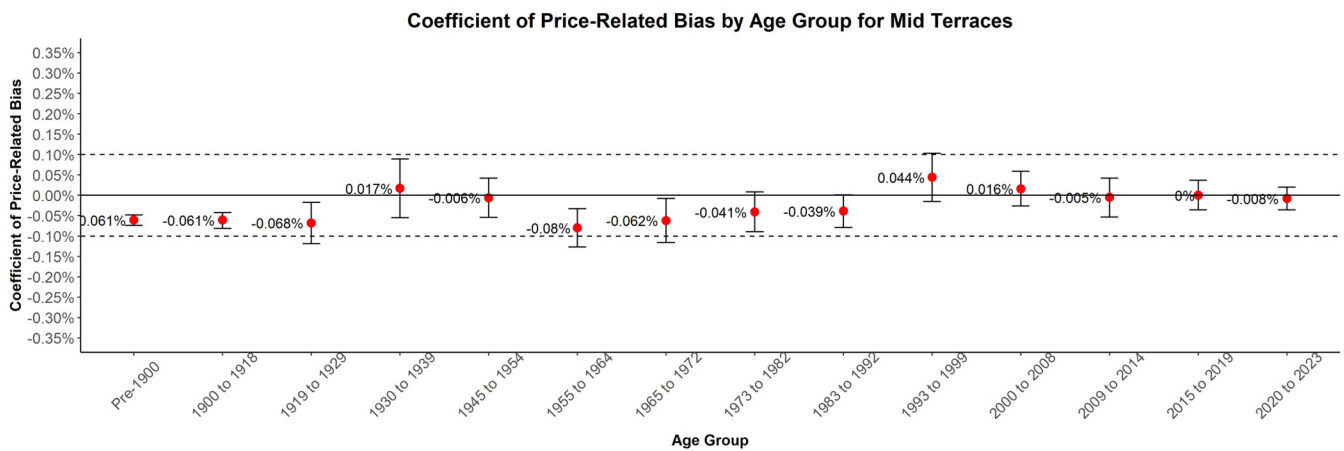
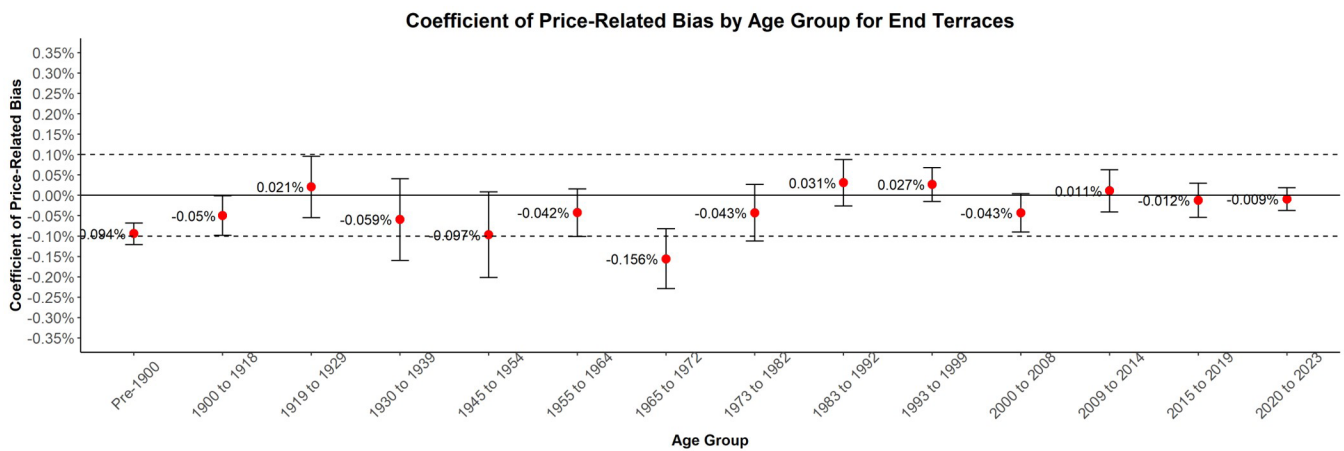


FIGURE 2.24



3. Statistics by Dwelling Type Subgroup and Floor Count Range

FIGURE 3.1

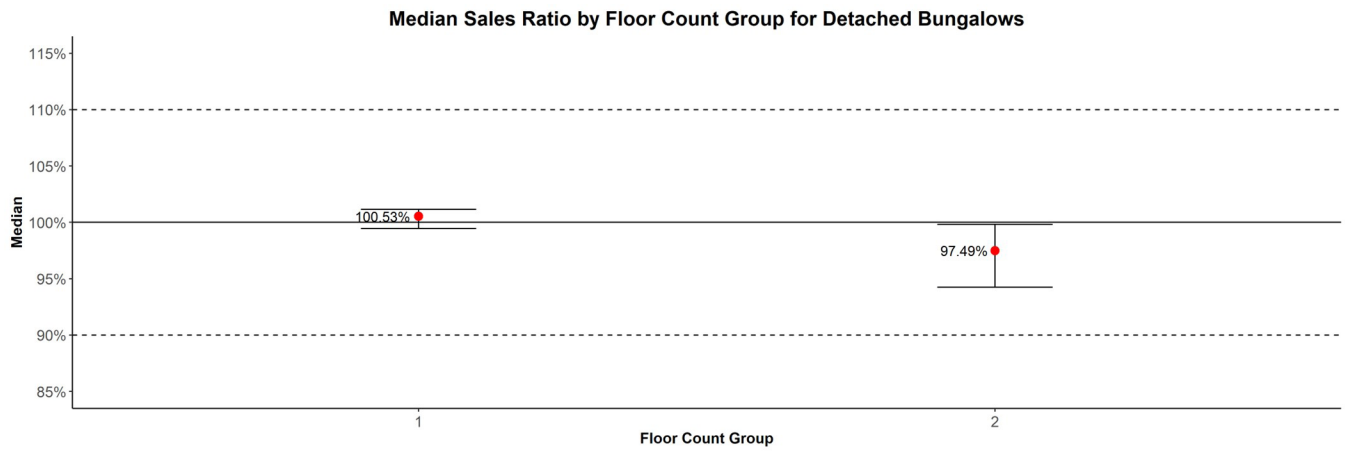


FIGURE 3.2

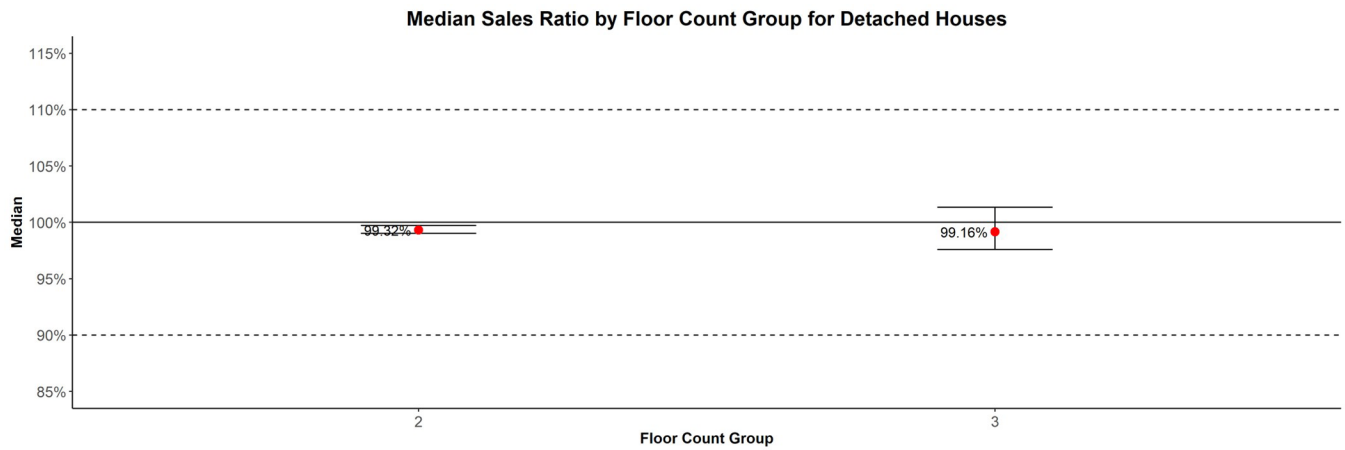


FIGURE 3.3

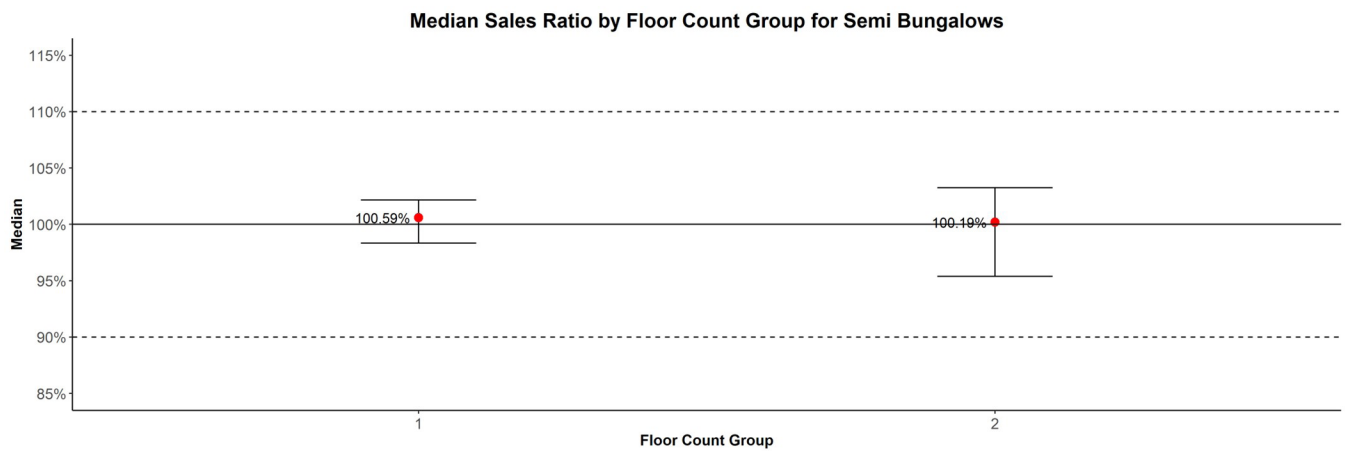


FIGURE 3.4

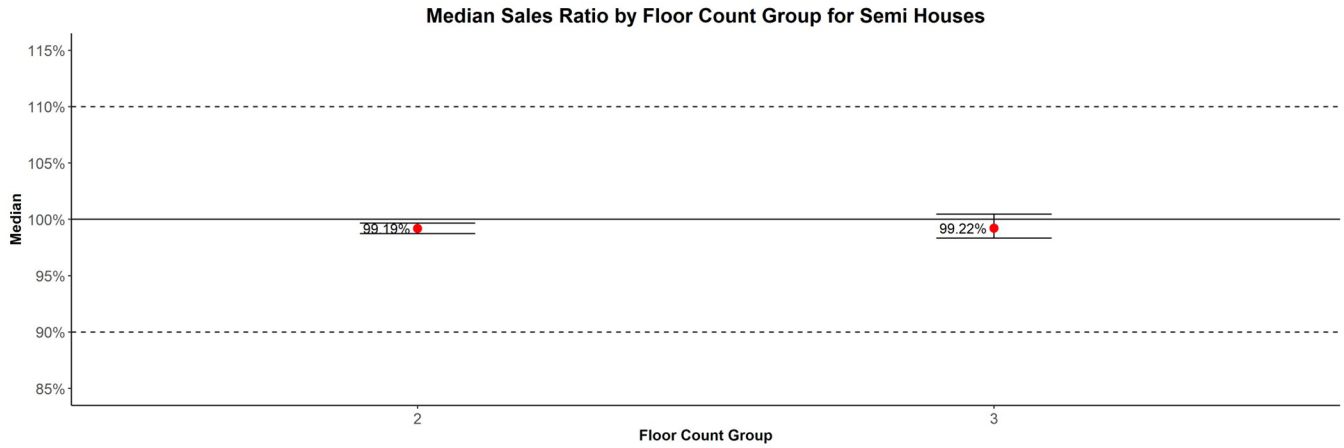


FIGURE 3.5

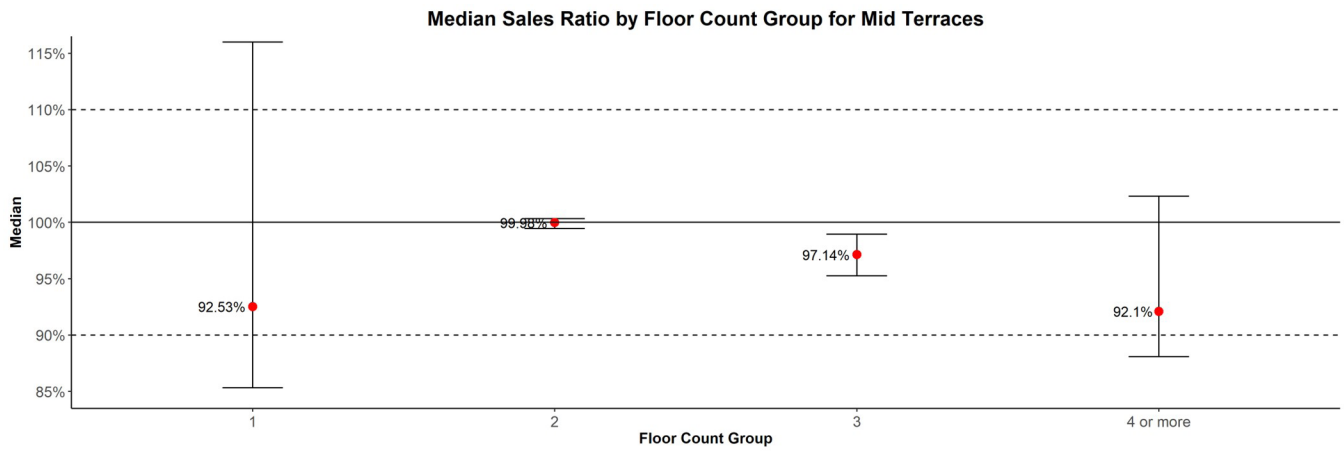


FIGURE 3.6

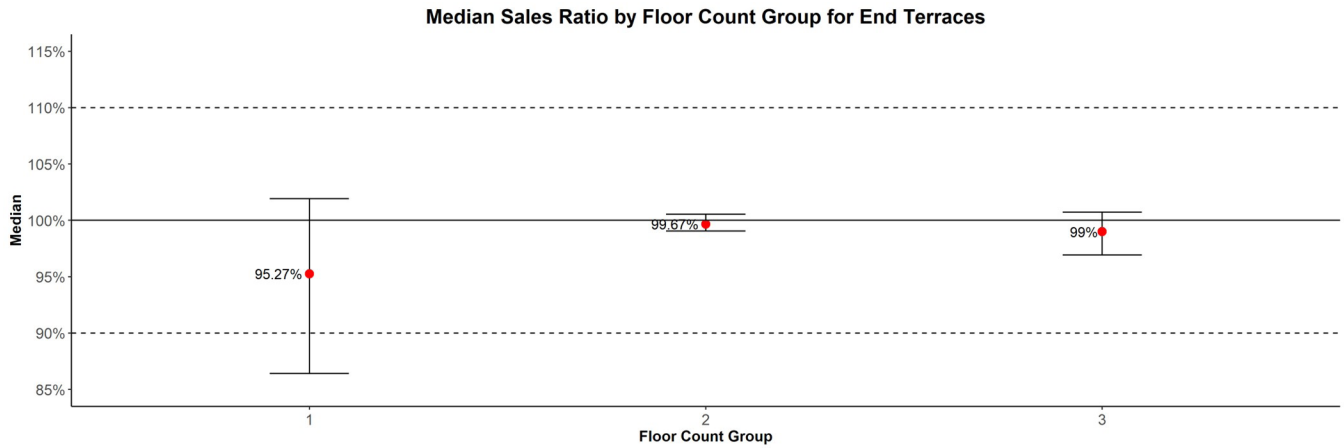


FIGURE 3.7

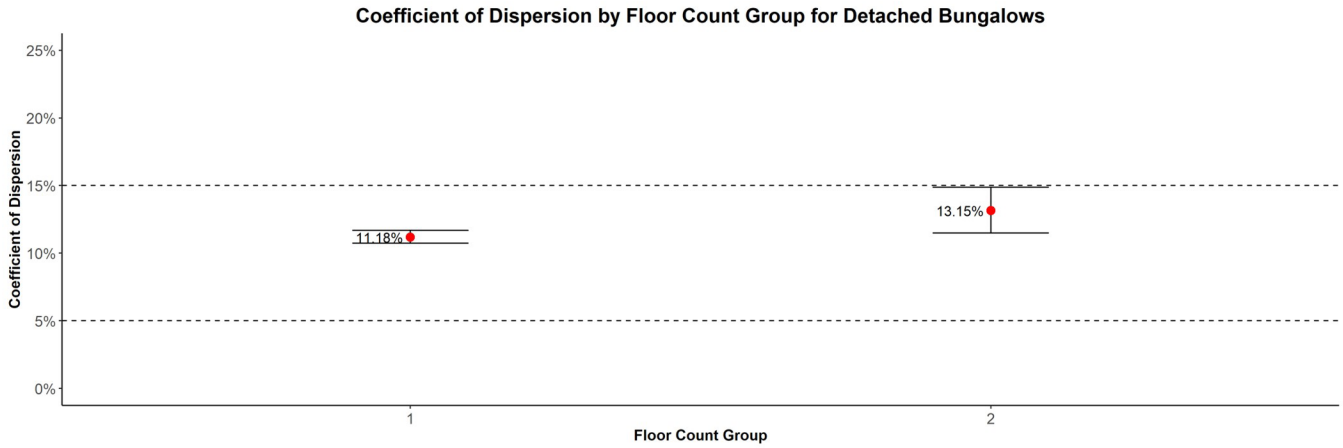


FIGURE 3.8

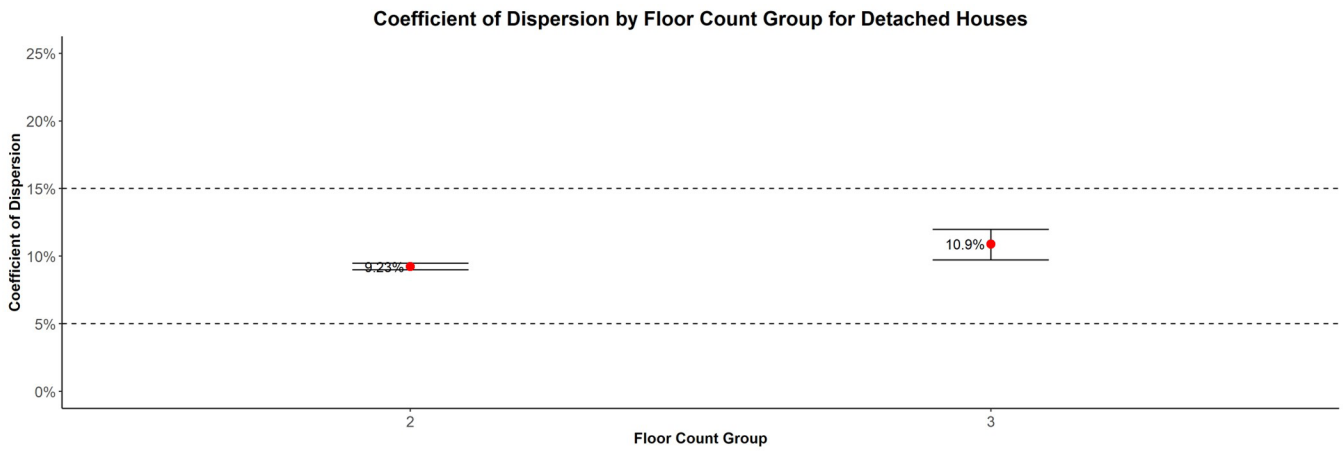


FIGURE 3.9

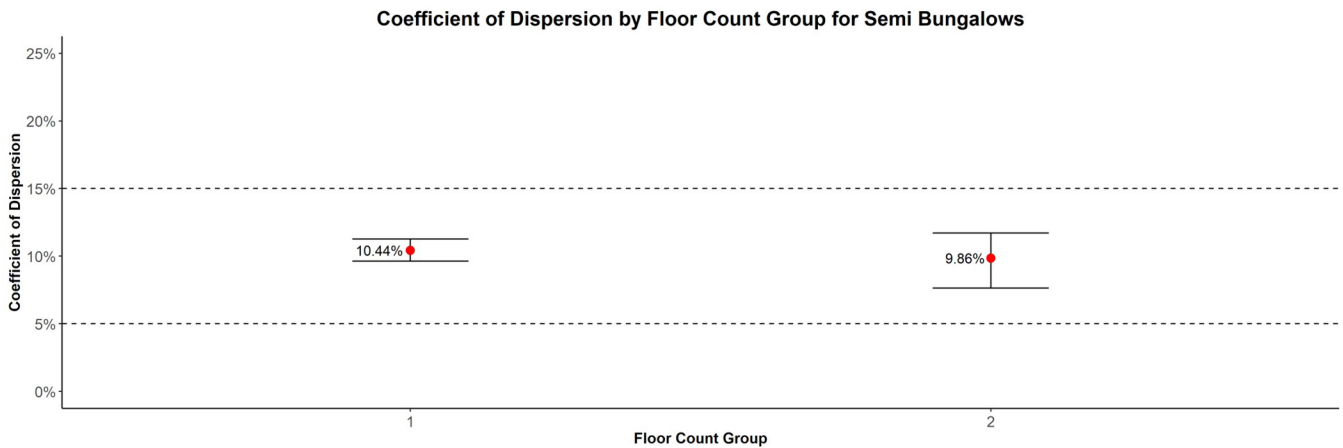


FIGURE 3.10

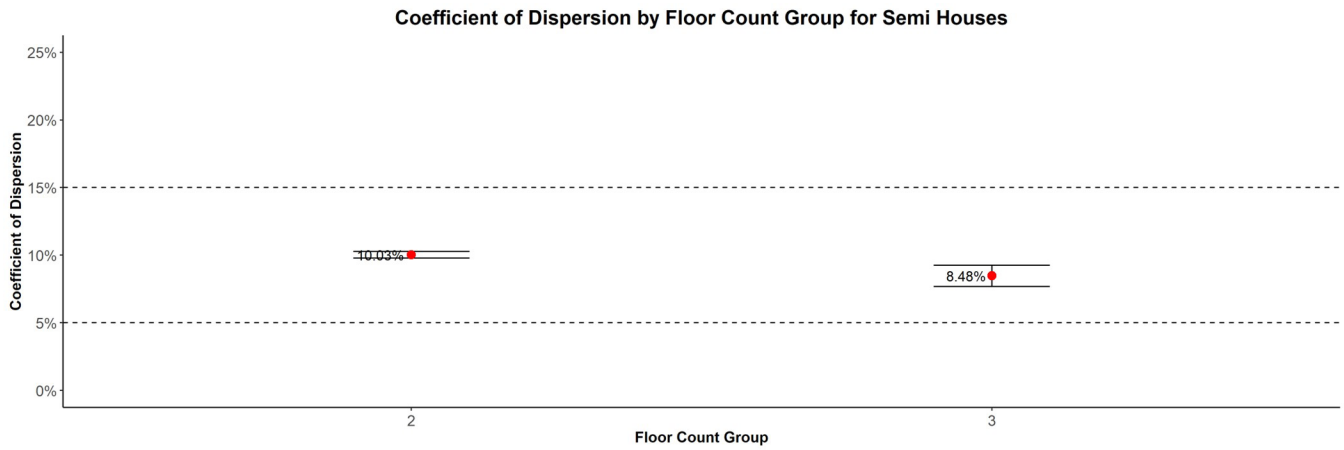


FIGURE 3.11

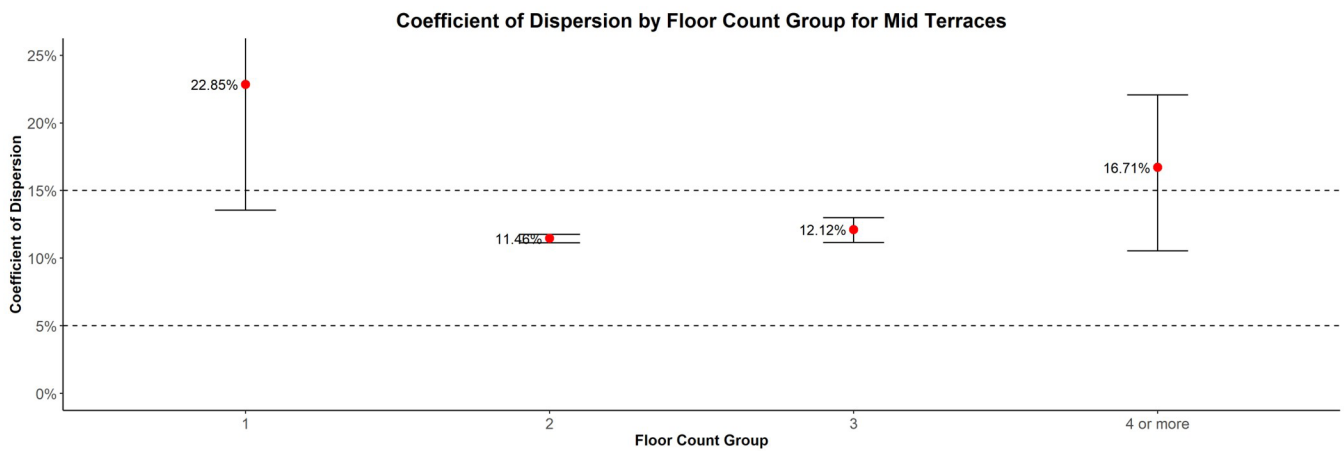


FIGURE 3.12

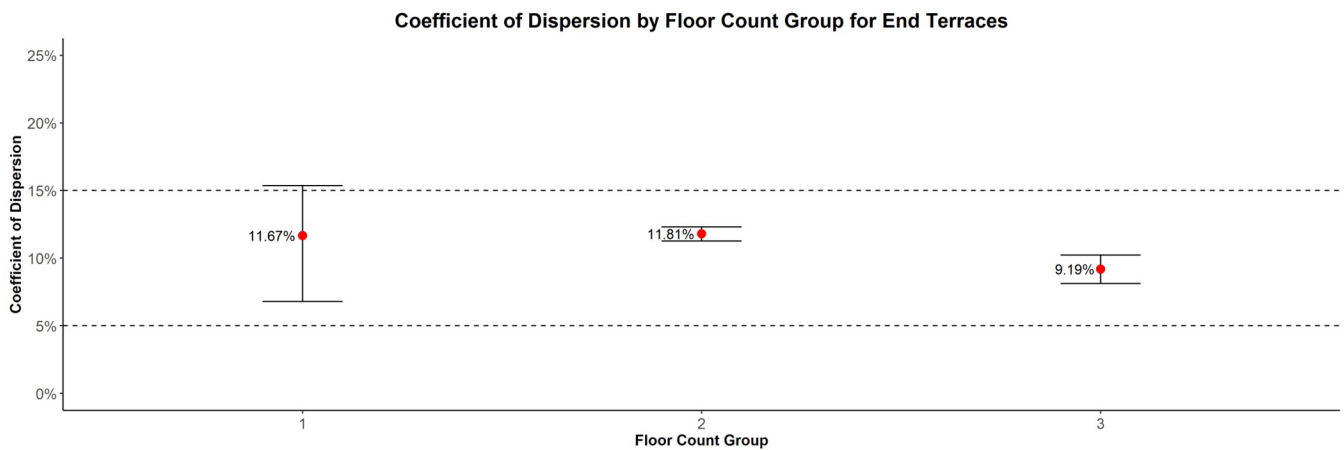


FIGURE 3.13

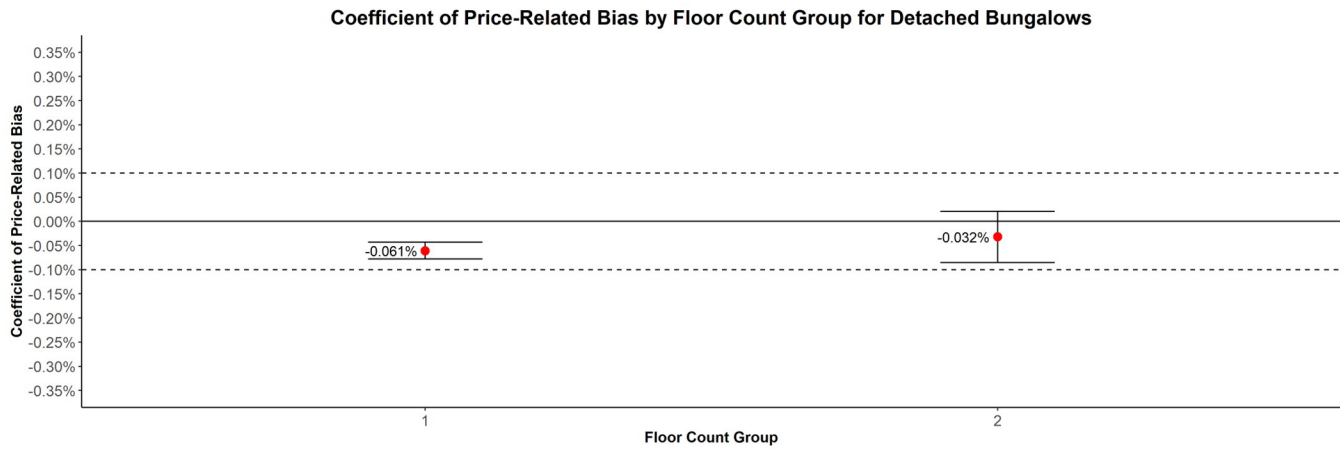


FIGURE 3.14

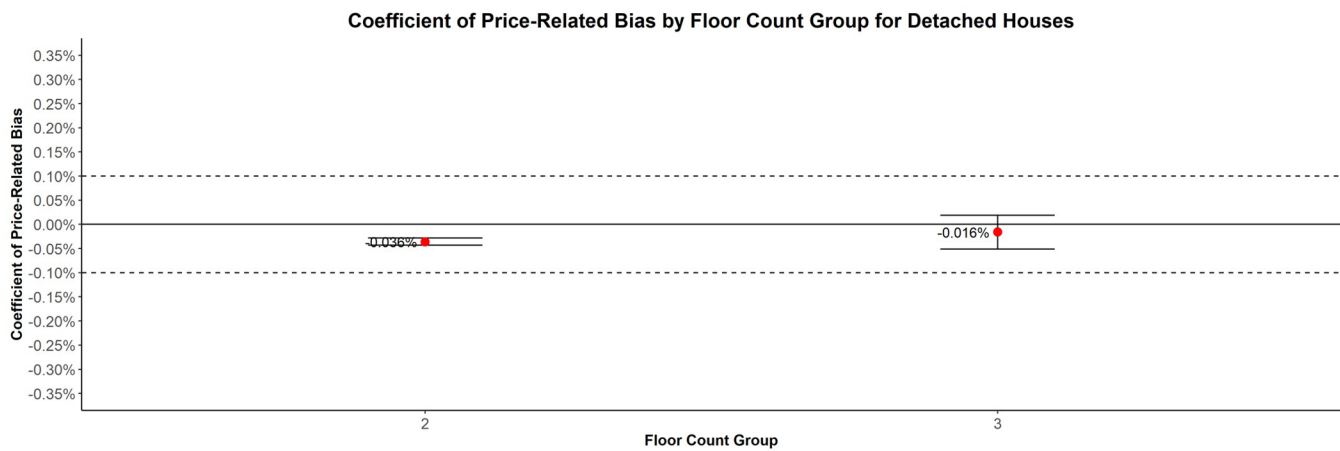


FIGURE 3.15

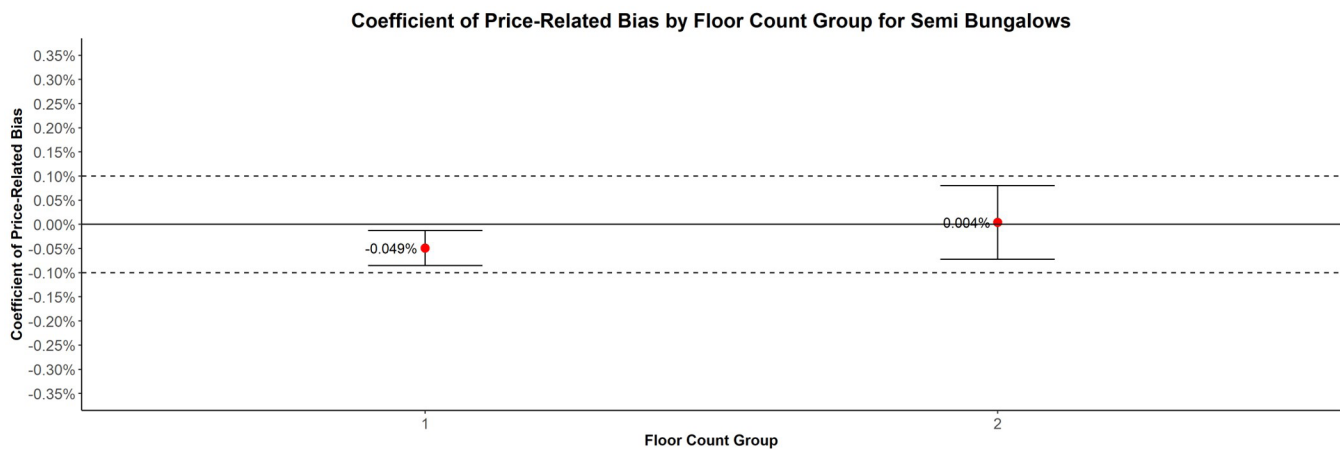


FIGURE 3.16

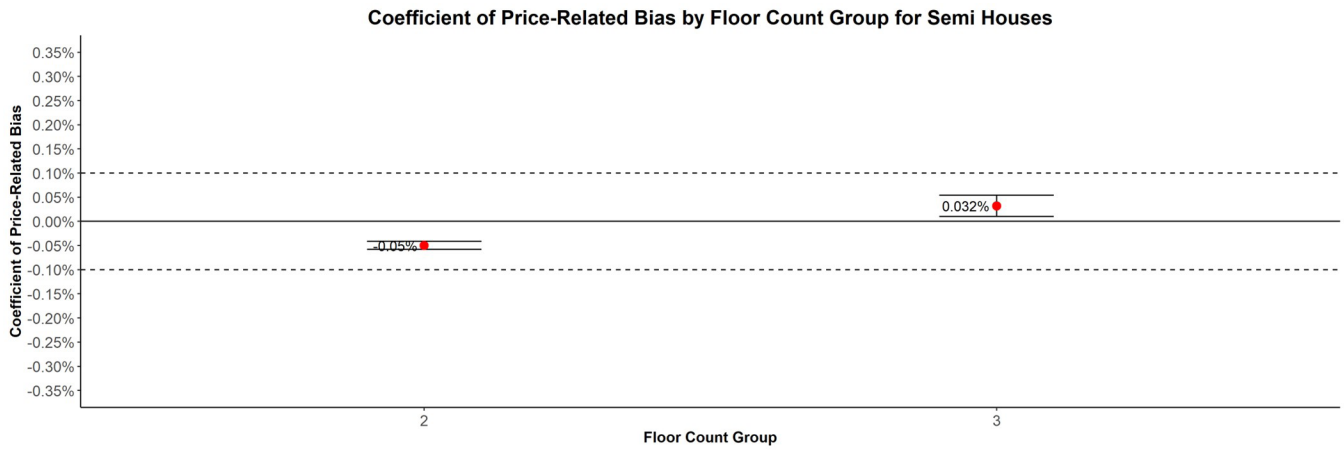


FIGURE 3.17

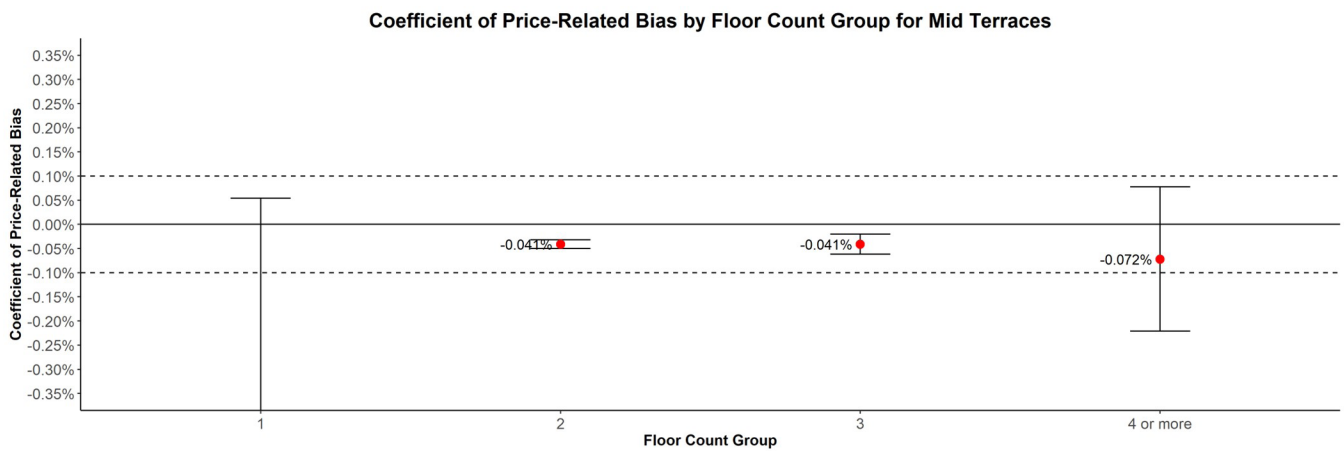
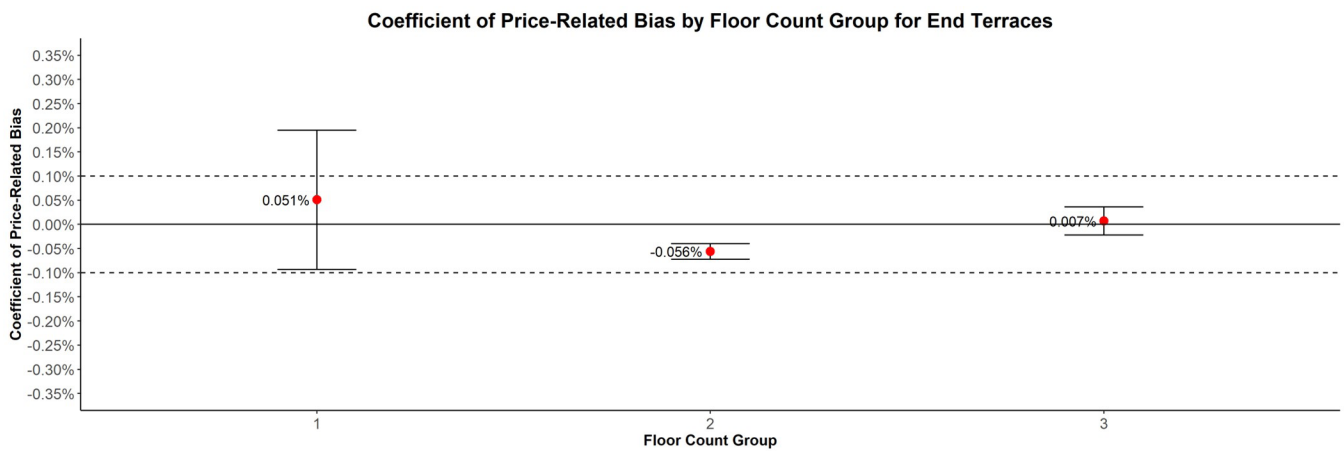


FIGURE 3.18



4. Statistics by Dwelling Type Subgroup and Floor Level Code Group

FIGURE 4.1

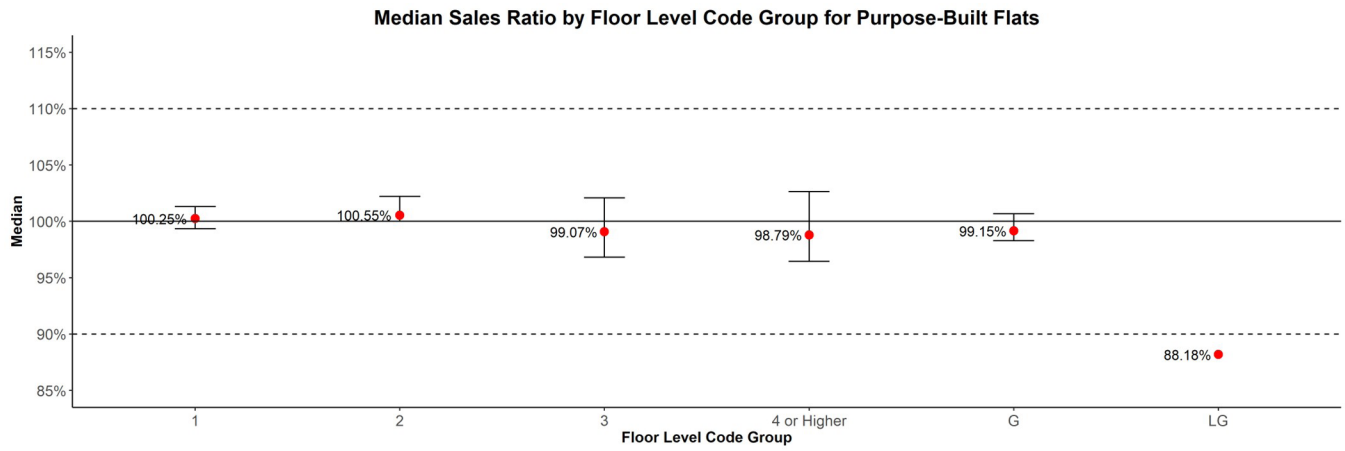


FIGURE 4.2

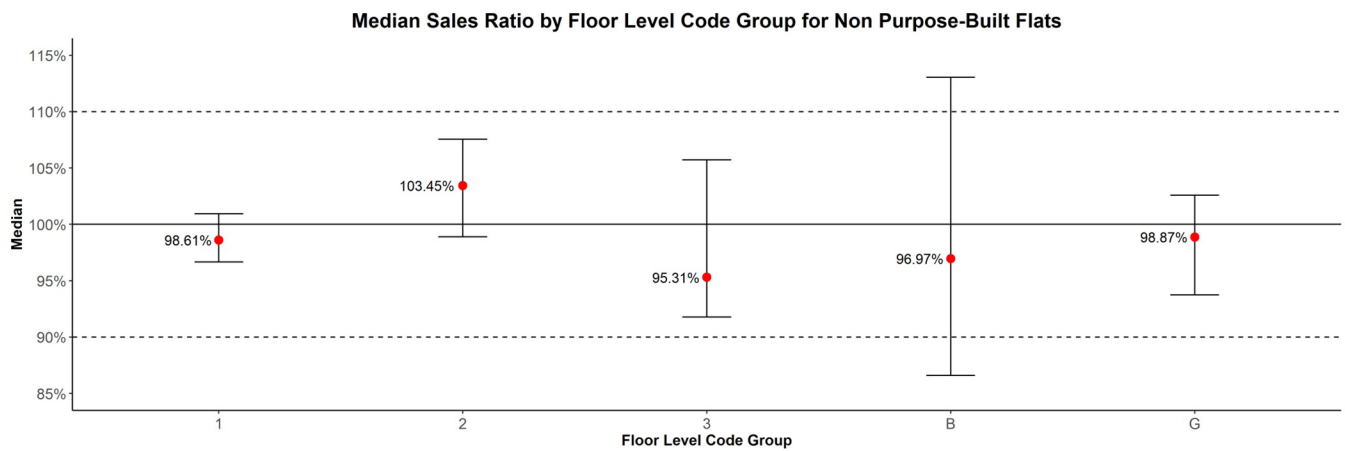


FIGURE 4.3

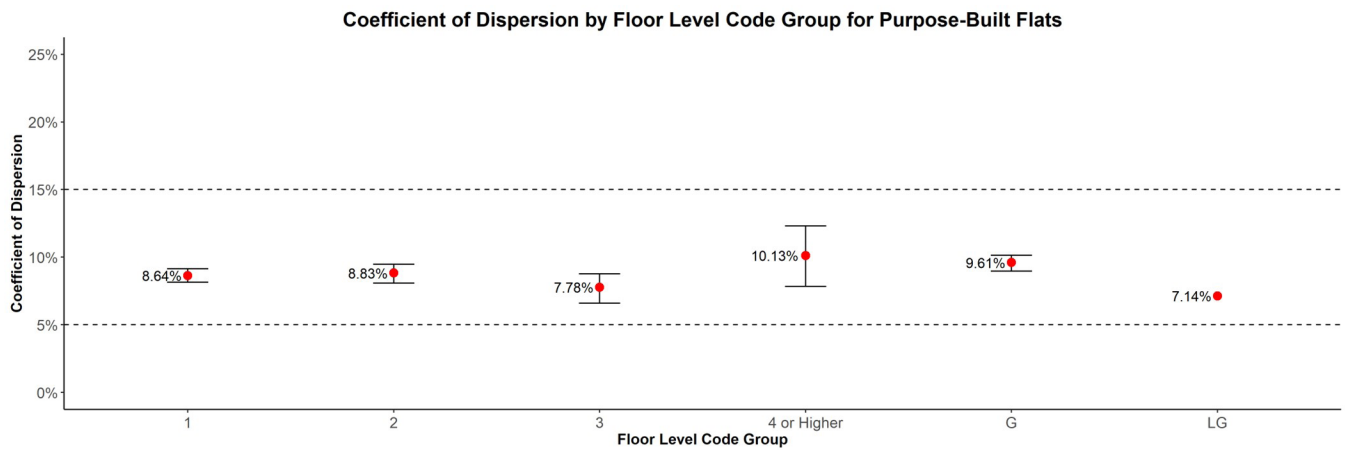


FIGURE 4.4

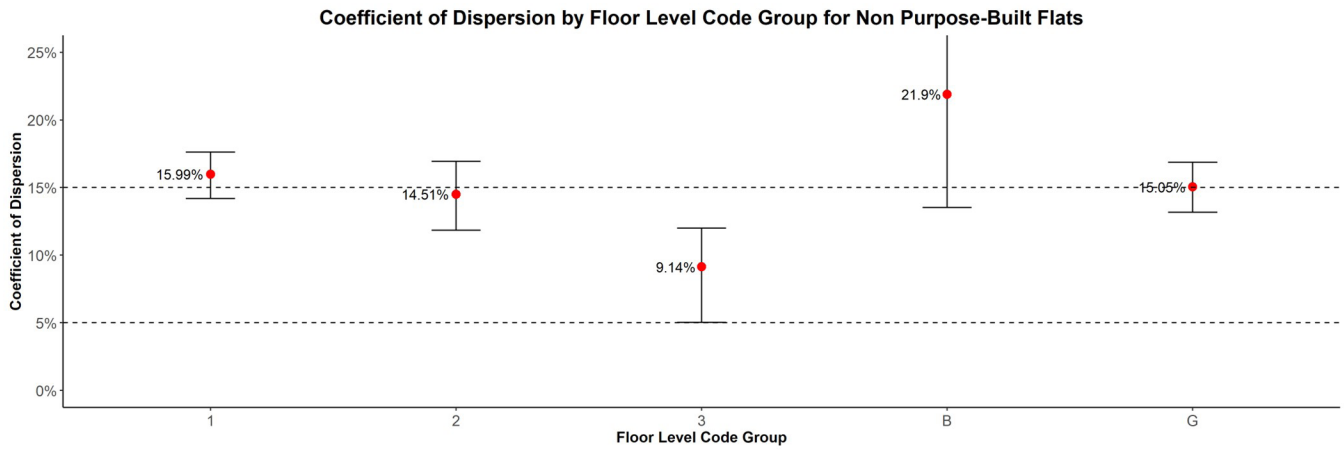


FIGURE 4.5

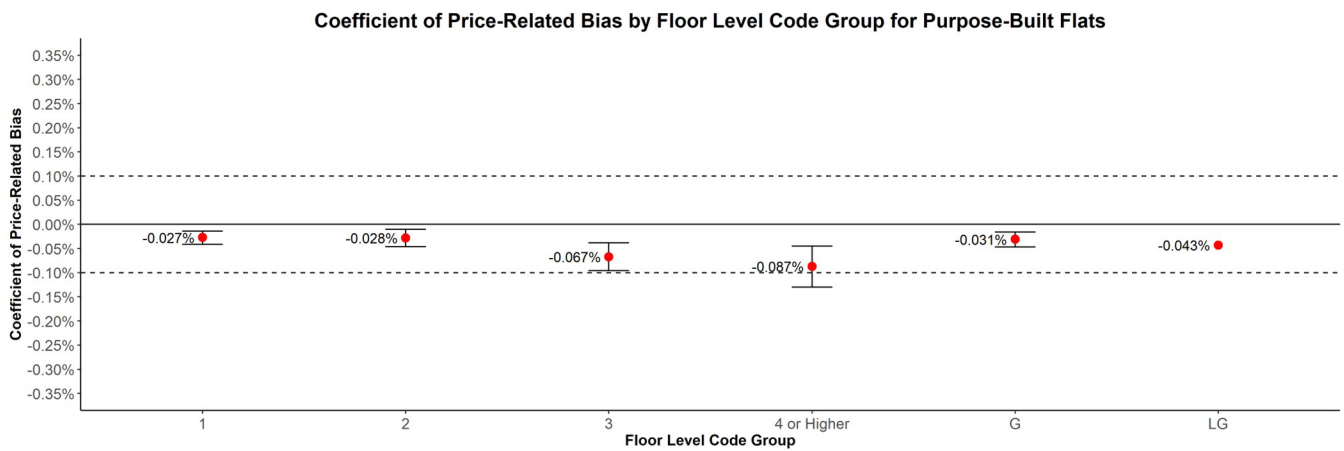
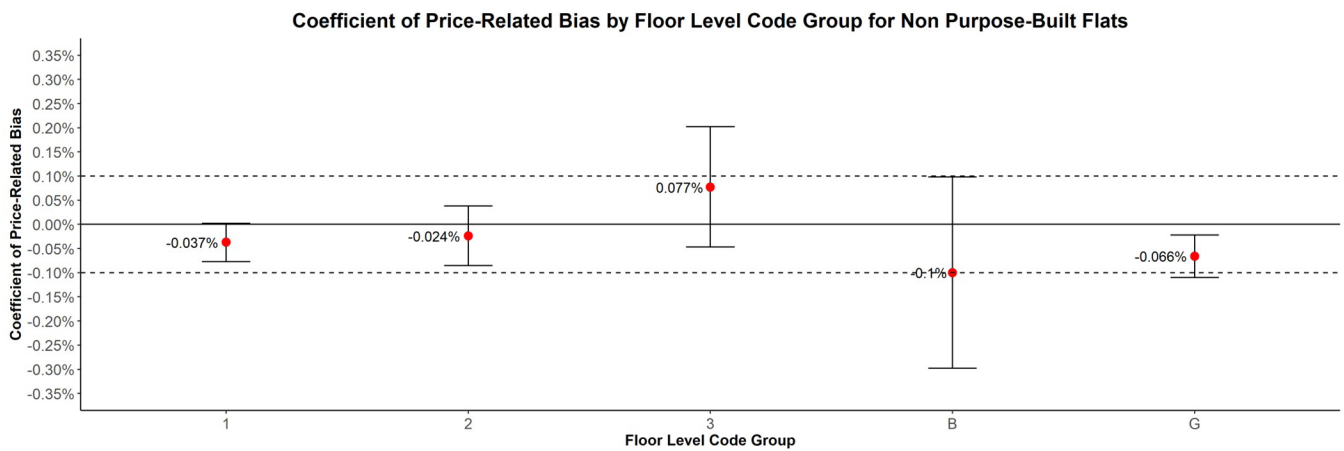


FIGURE 4.6



5. Statistics by Dwelling Type Subgroup and Bathroom Count Range

FIGURE 5.1

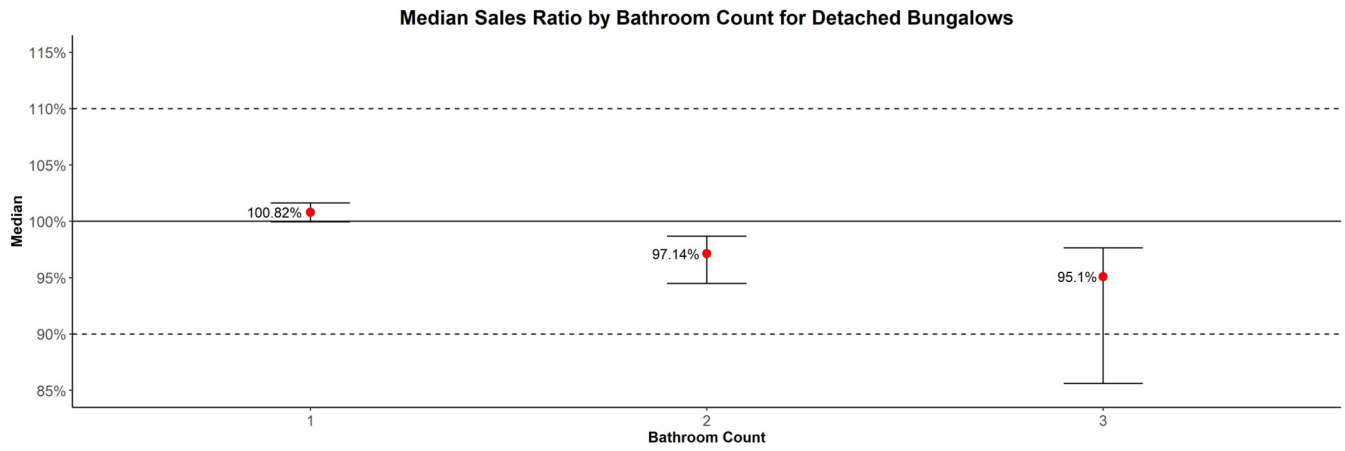


FIGURE 5.2

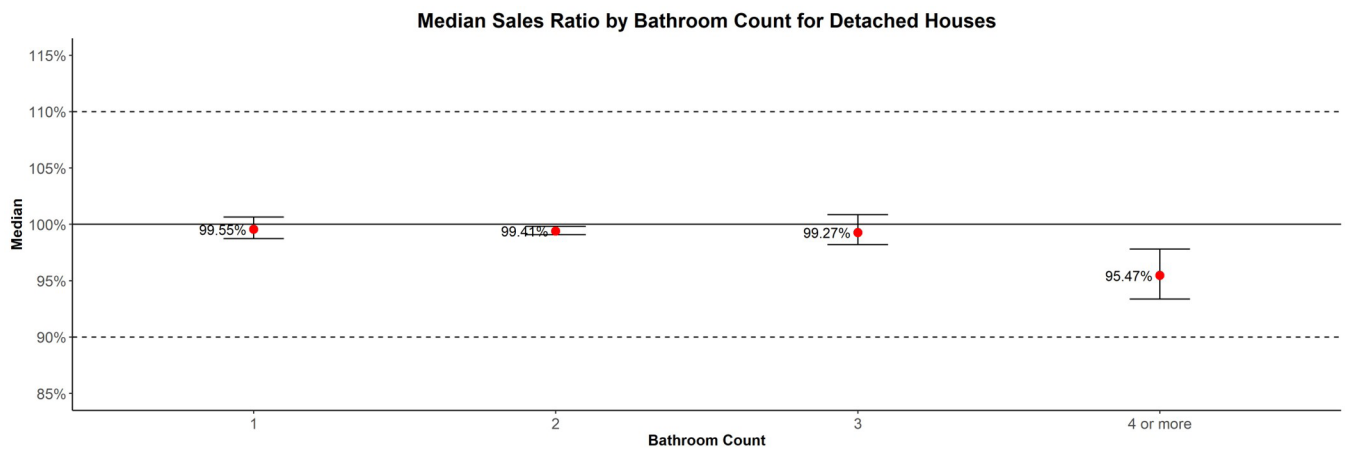


FIGURE 5.3

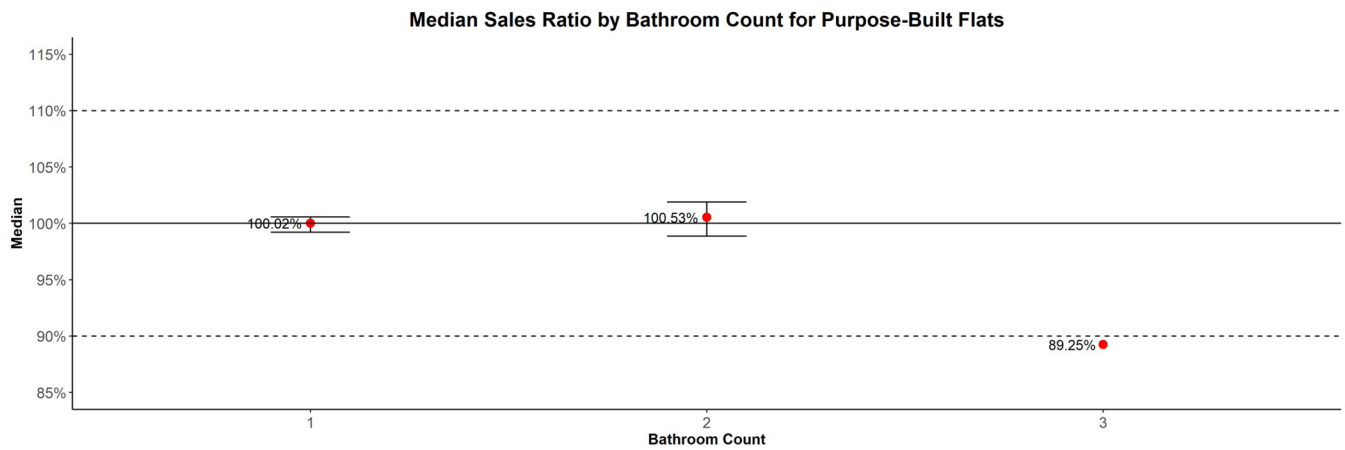


FIGURE 5.4

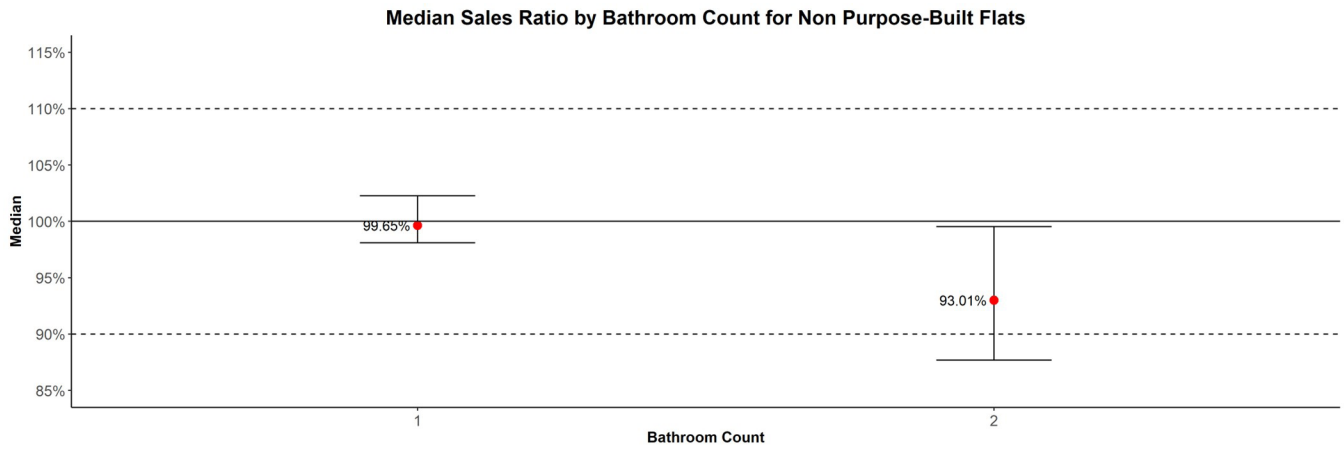


FIGURE 5.5

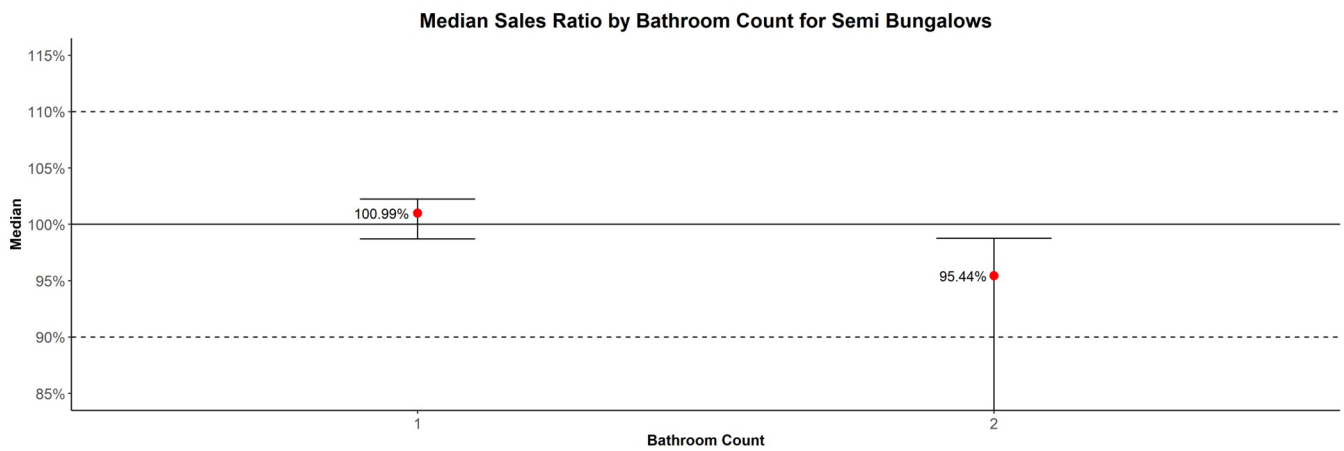


FIGURE 5.6

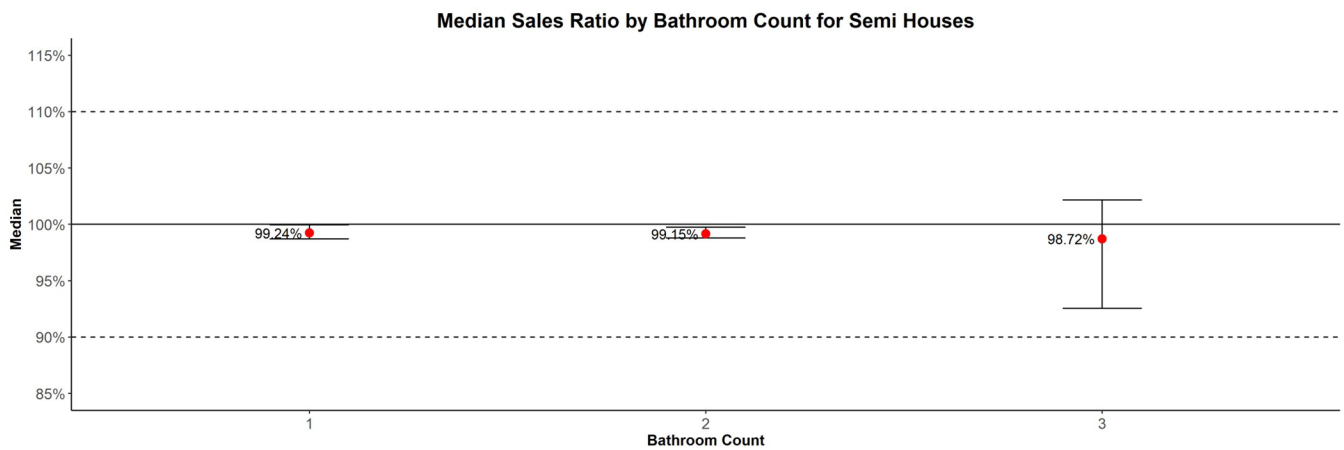


FIGURE 5.7

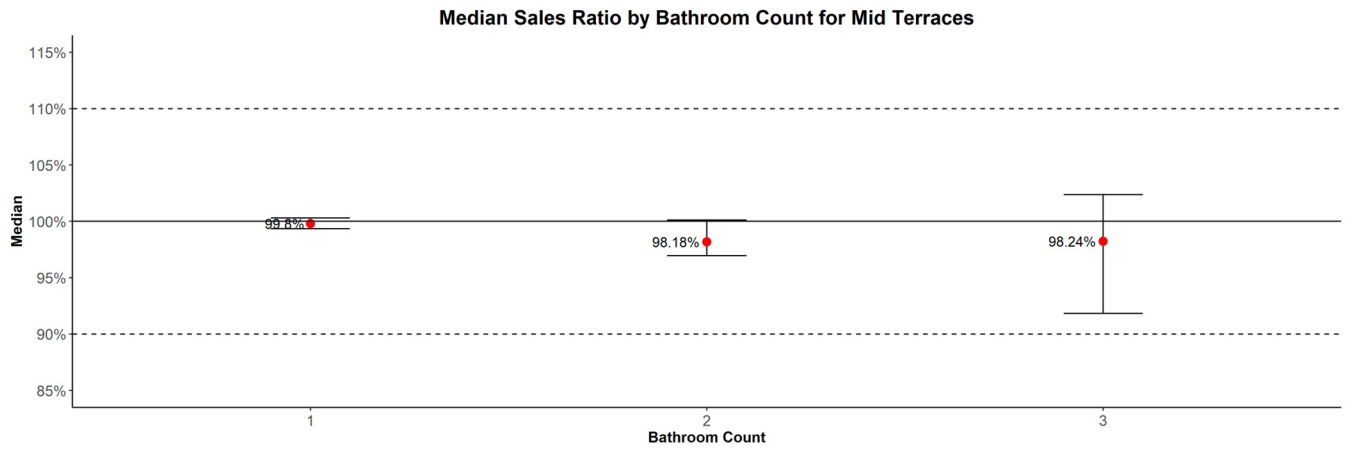


FIGURE 5.8

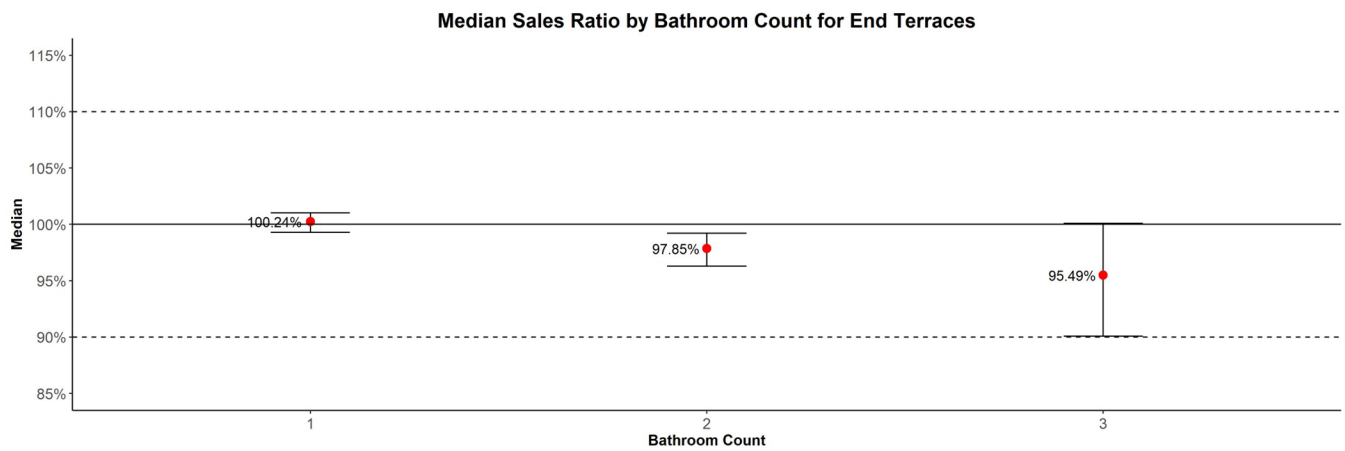


FIGURE 5.9

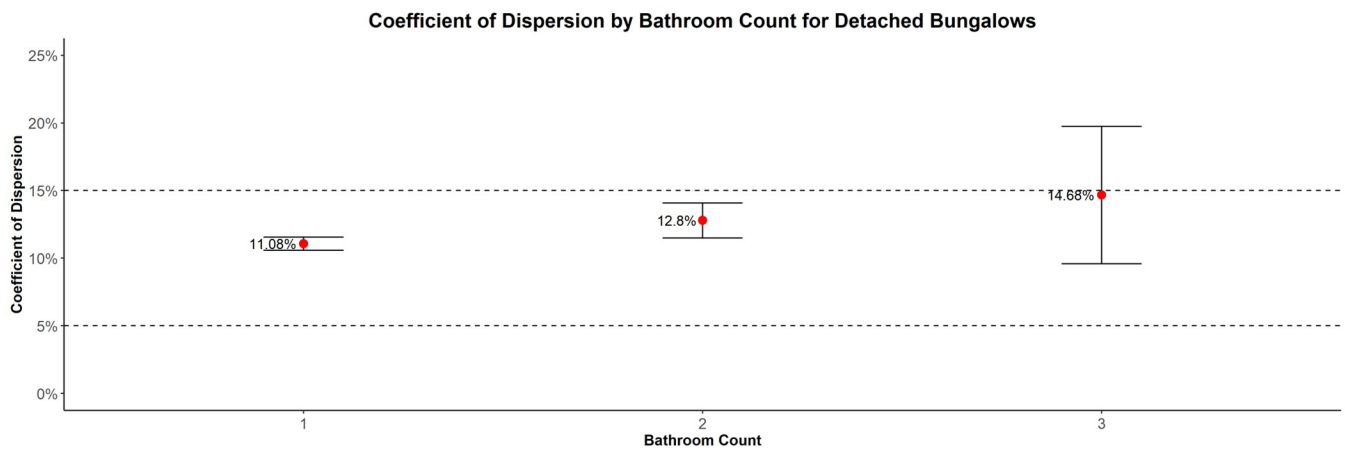


FIGURE 5.10

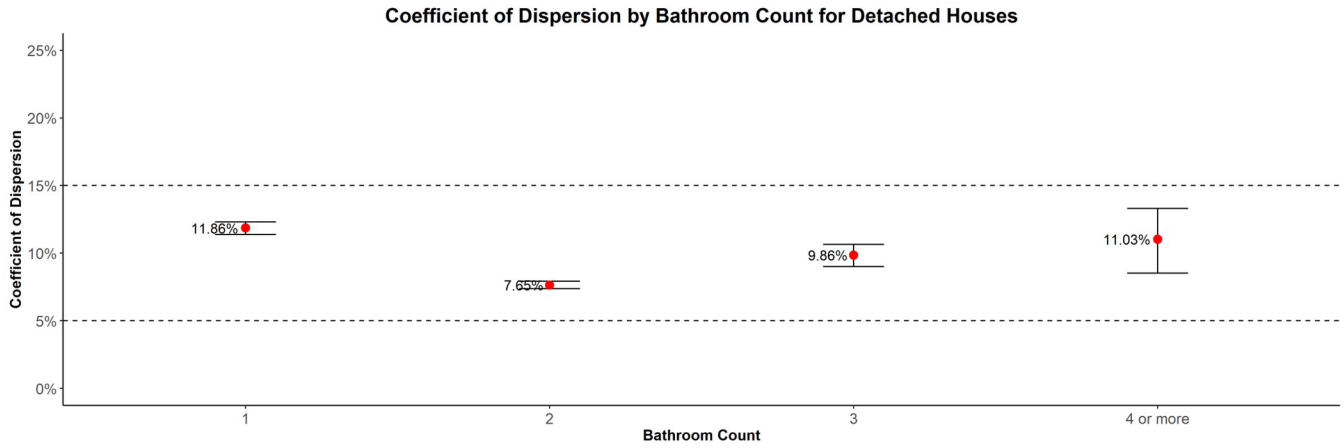


FIGURE 5.11

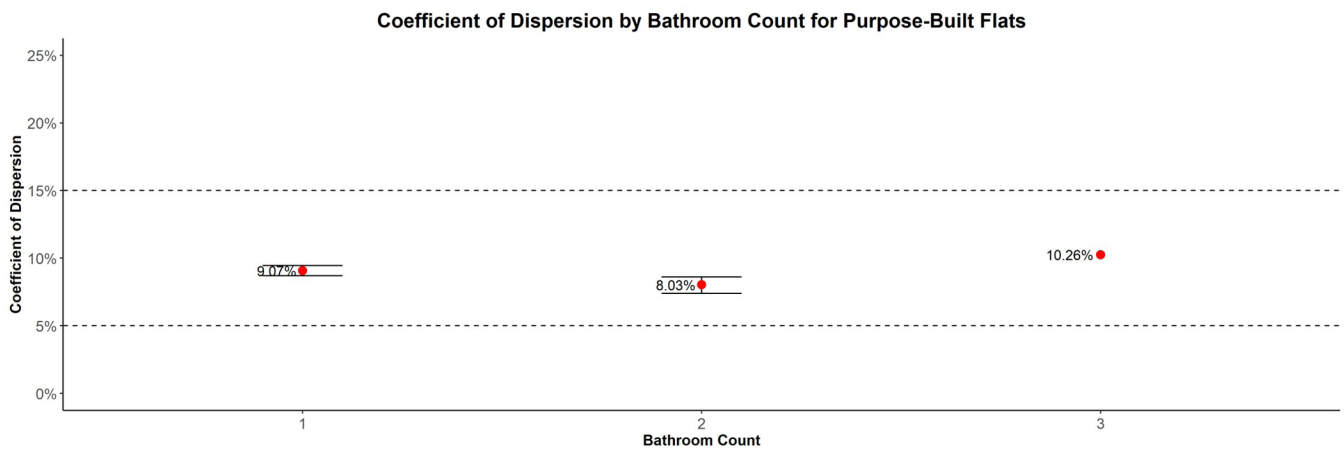


FIGURE 5.12

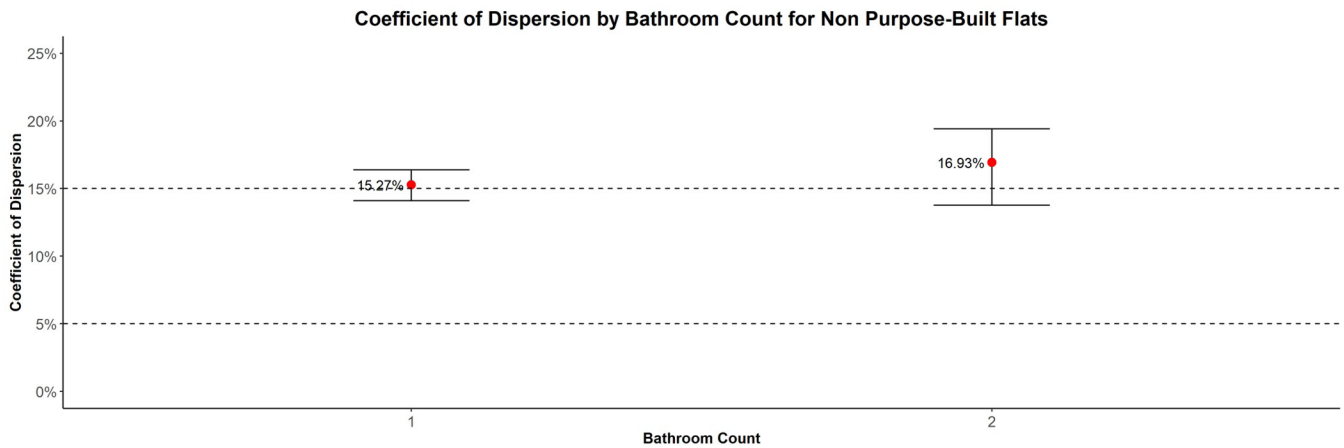


FIGURE 5.13

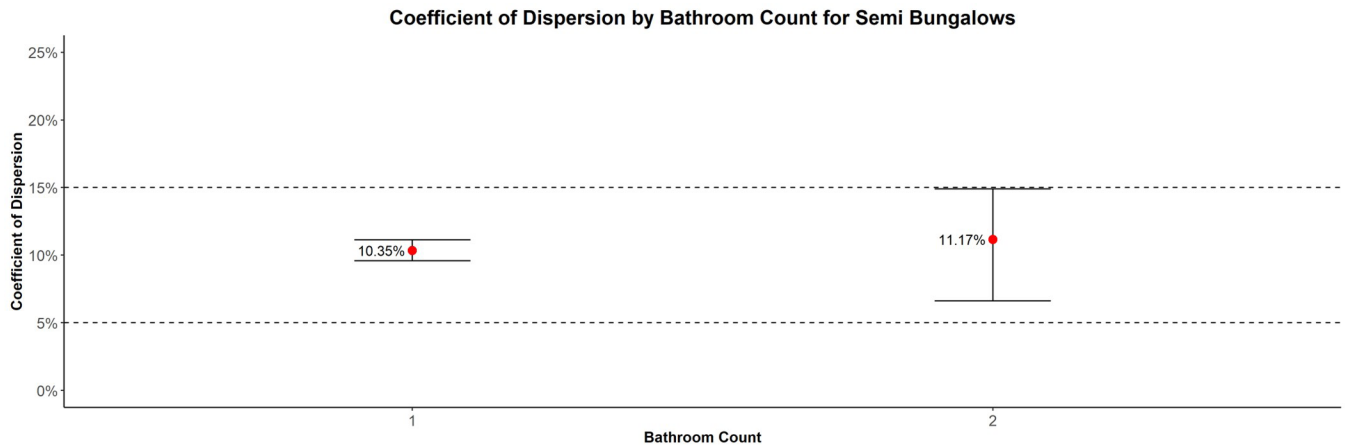


FIGURE 5.14

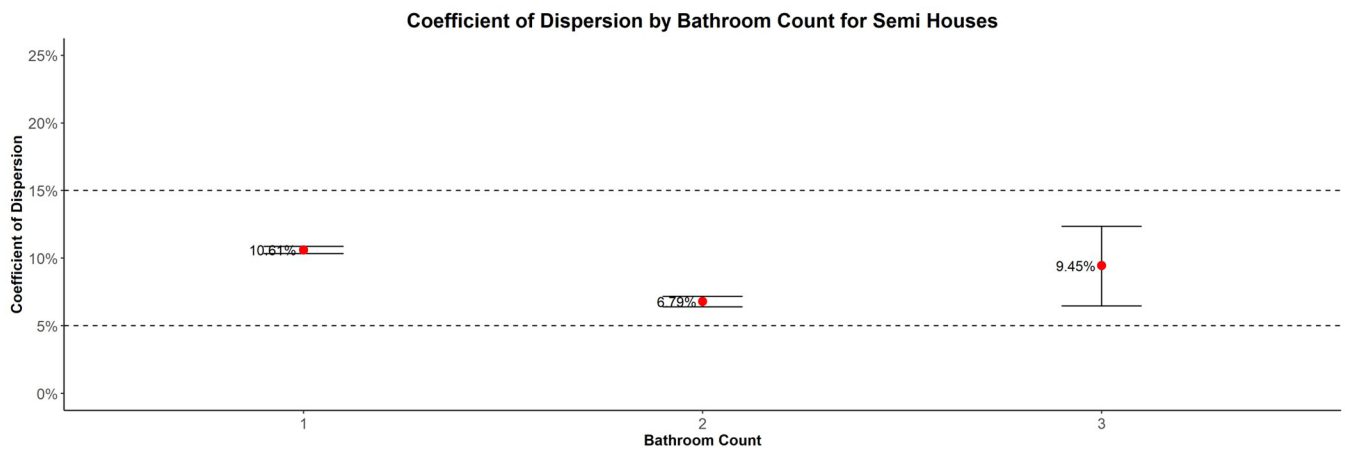


FIGURE 5.15

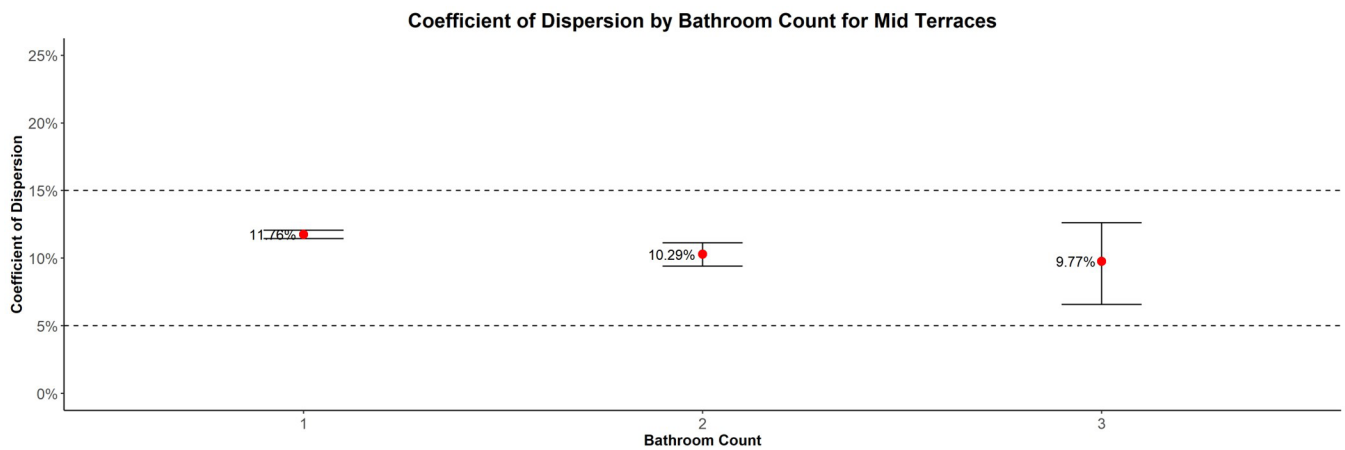


FIGURE 5.16

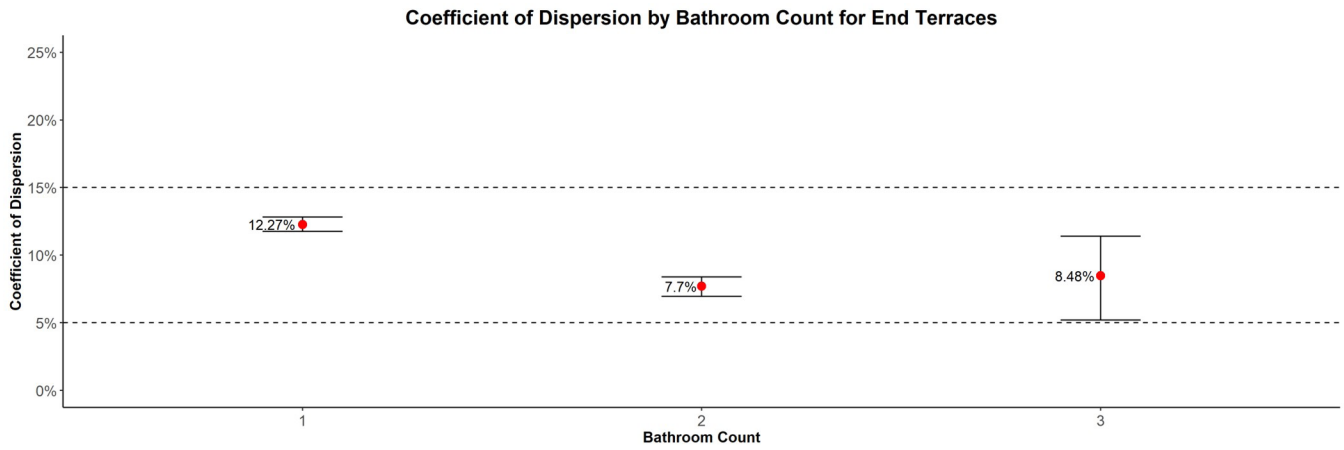


FIGURE 5.17

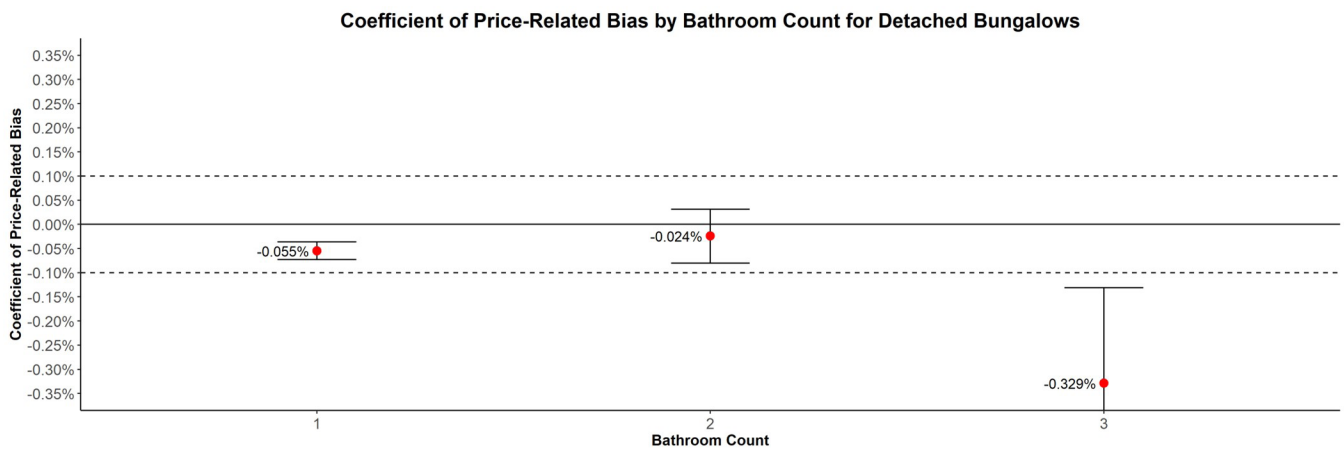


FIGURE 5.18

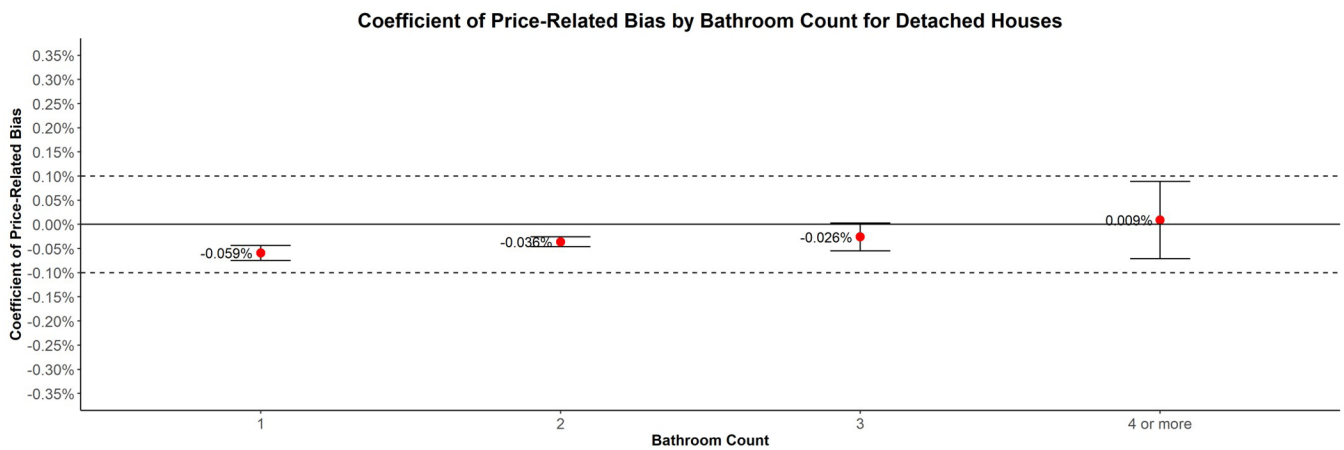


FIGURE 5.19

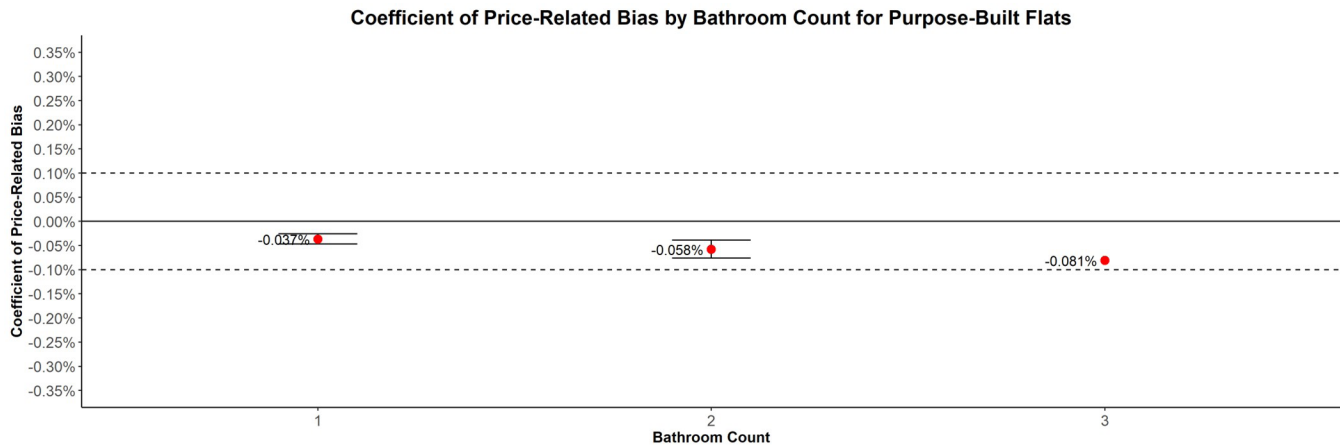


FIGURE 5.20

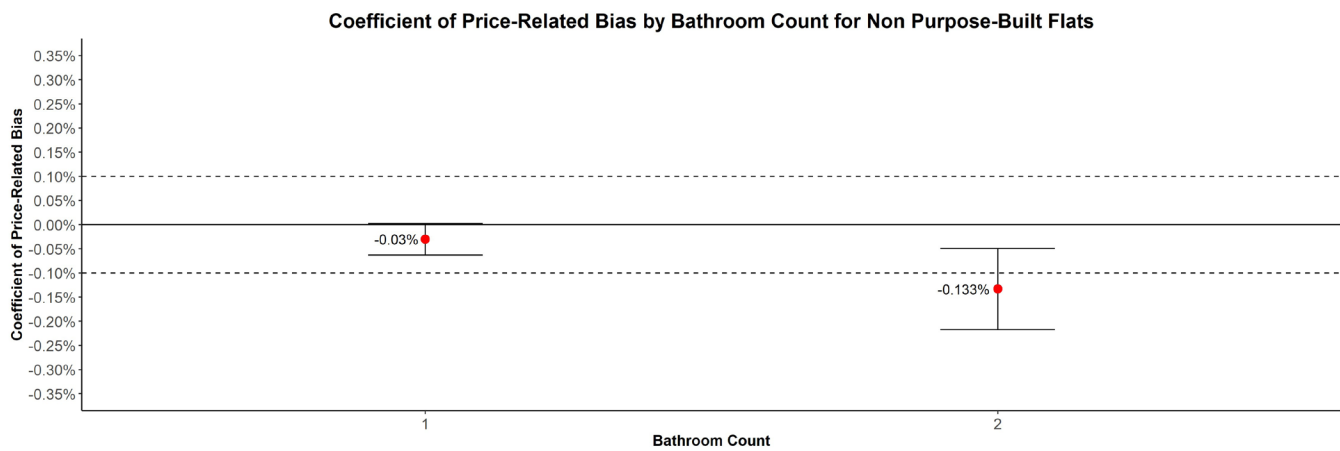


FIGURE 5.21

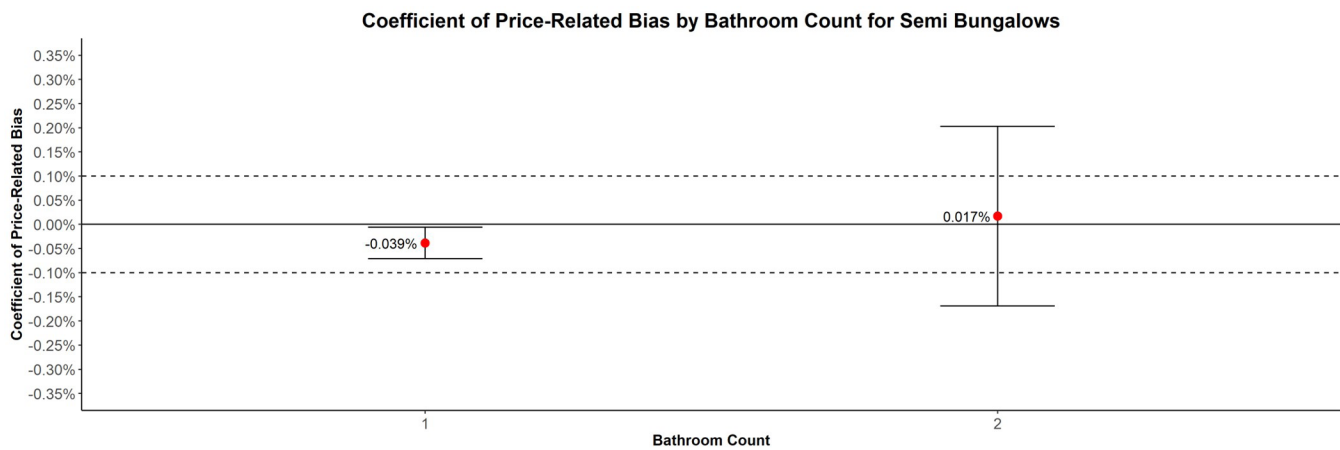


FIGURE 5.22

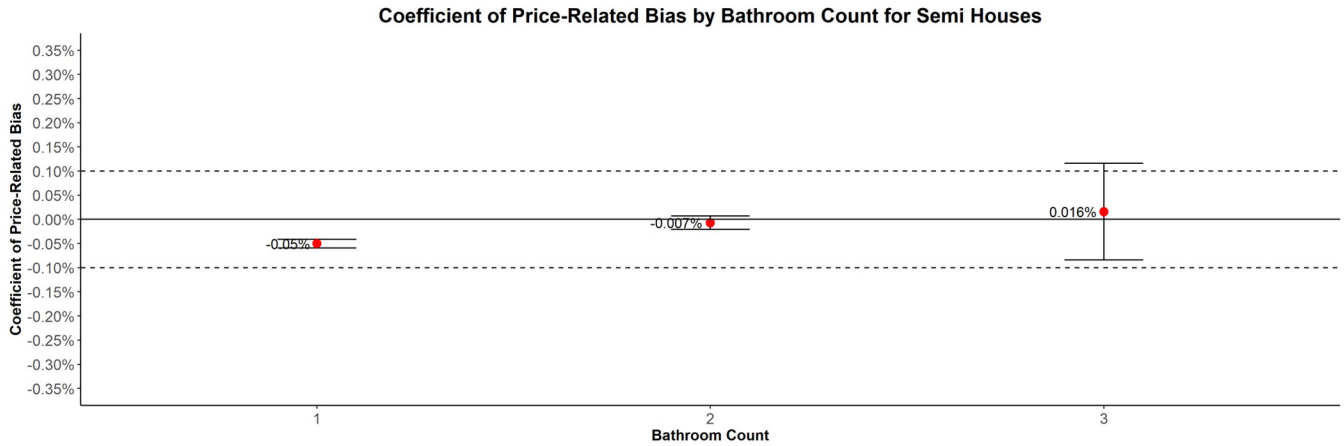


FIGURE 5.23

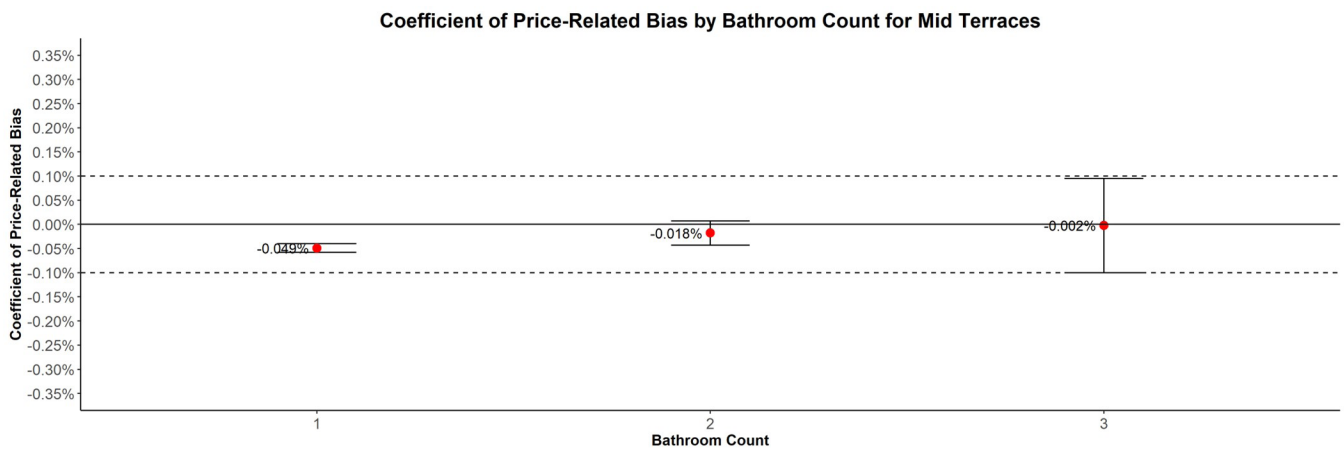
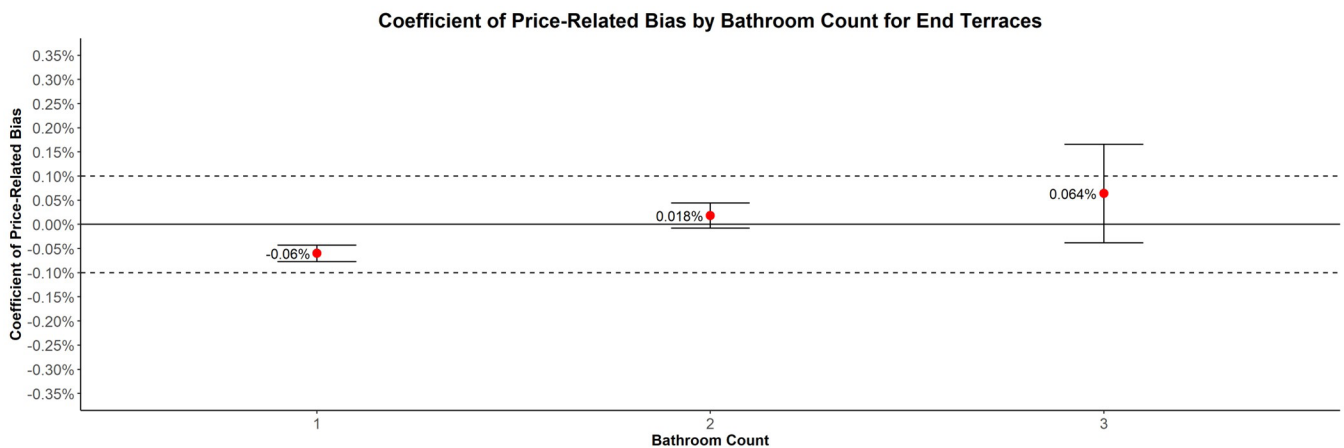


FIGURE 5.24



6. Statistics by Dwelling Type Subgroup and Bedroom Count Range

FIGURE 6.1

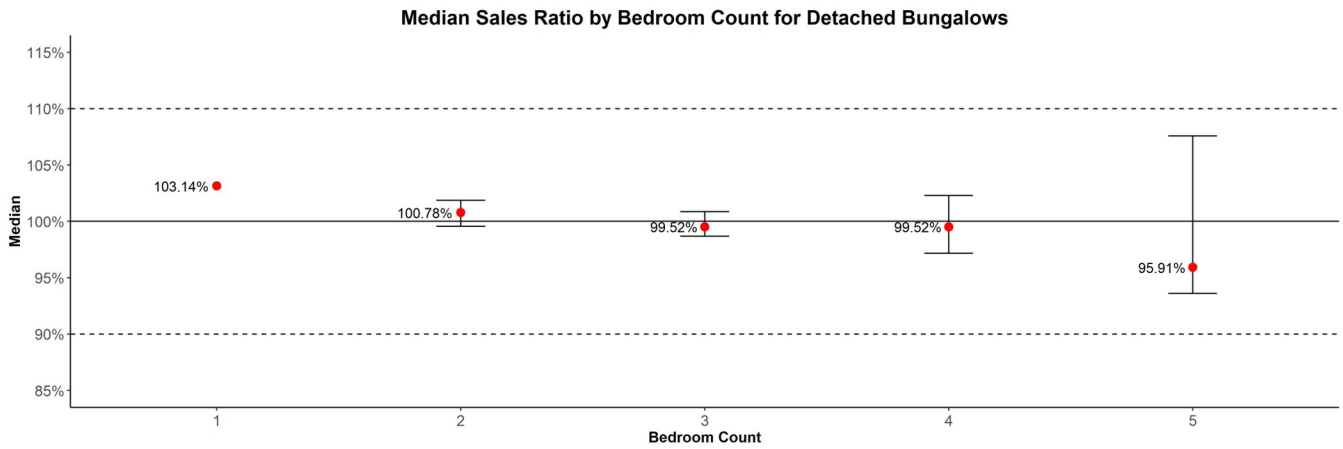


FIGURE 6.2

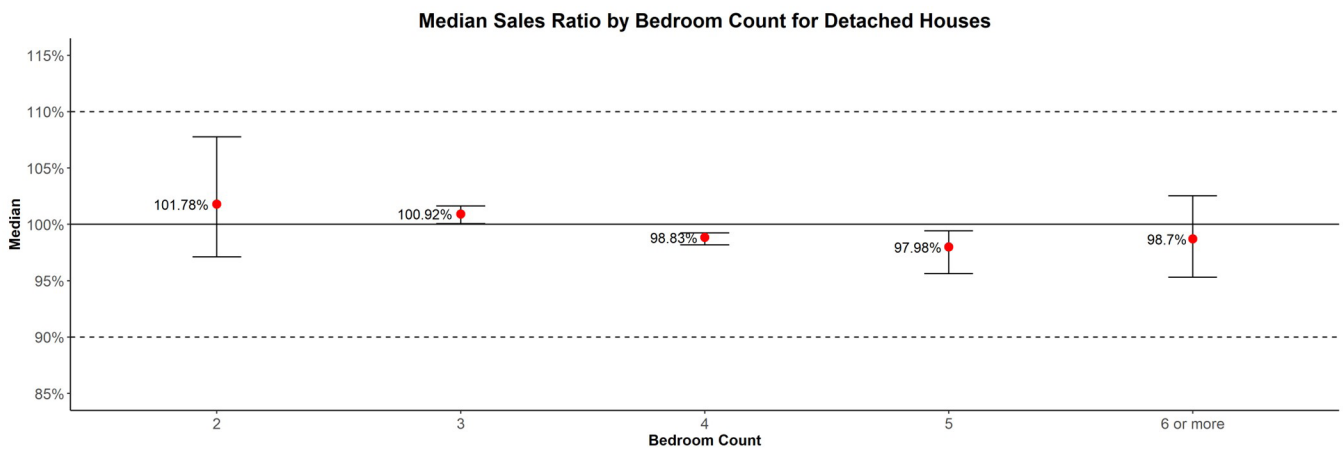


FIGURE 6.3

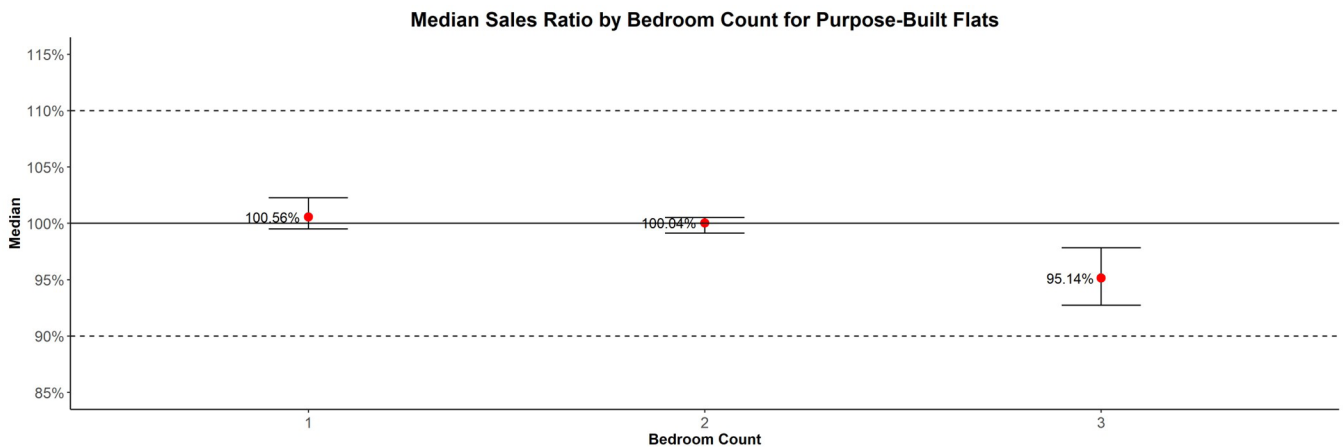


FIGURE 6.4

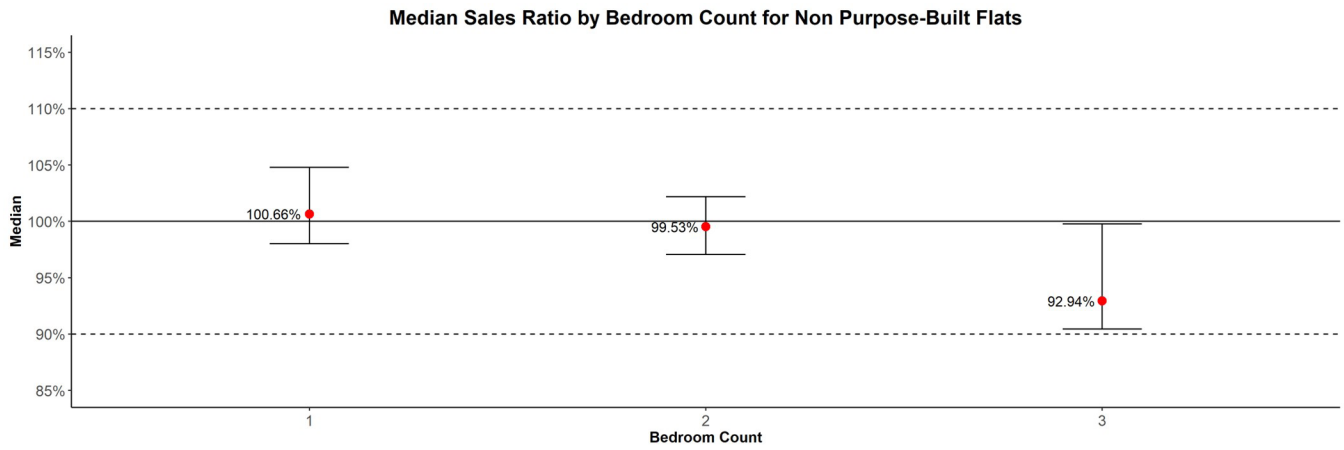


FIGURE 6.5

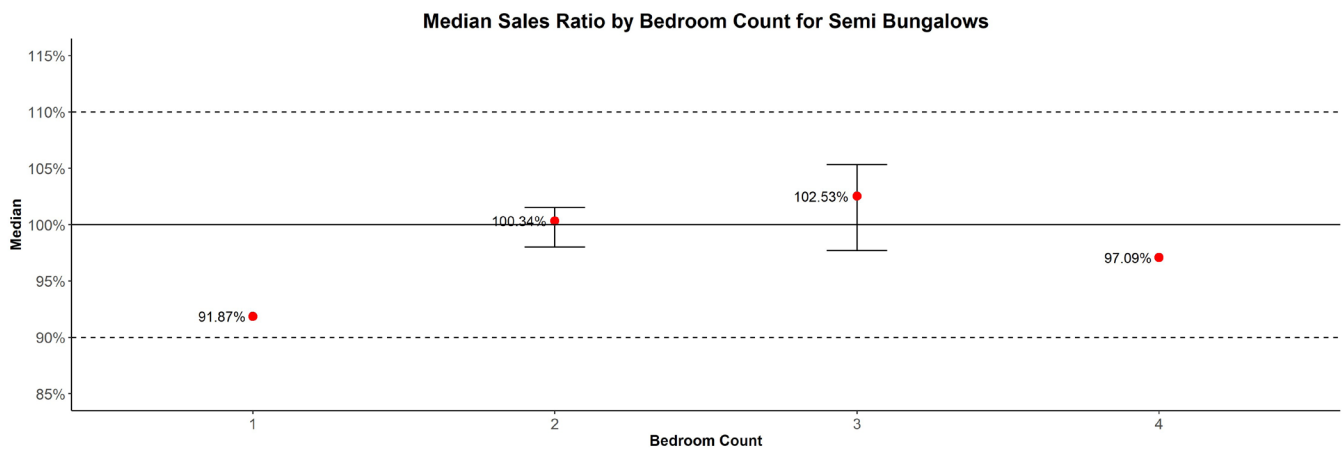


FIGURE 6.6

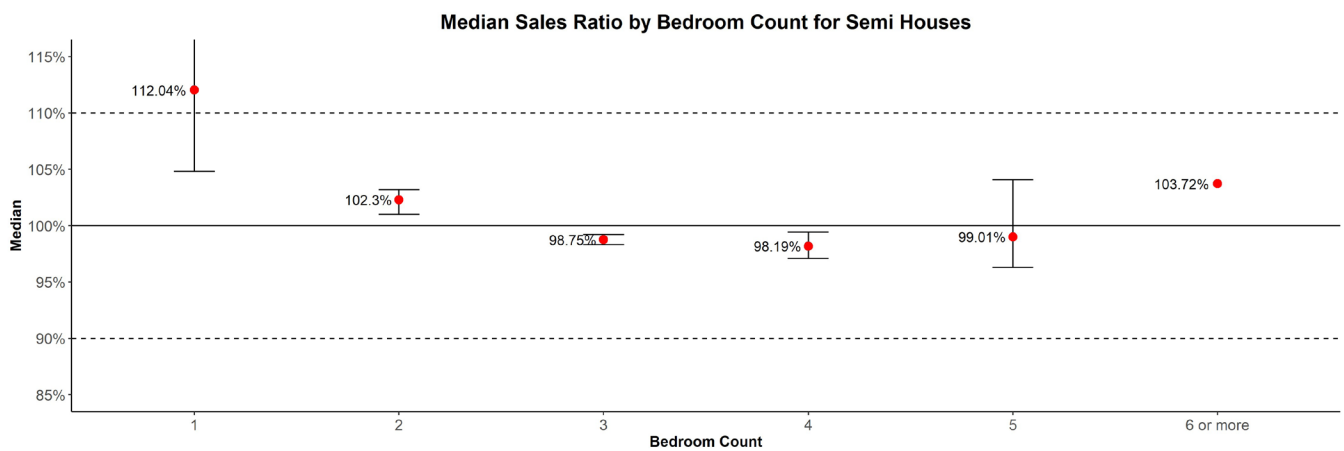


FIGURE 6.7

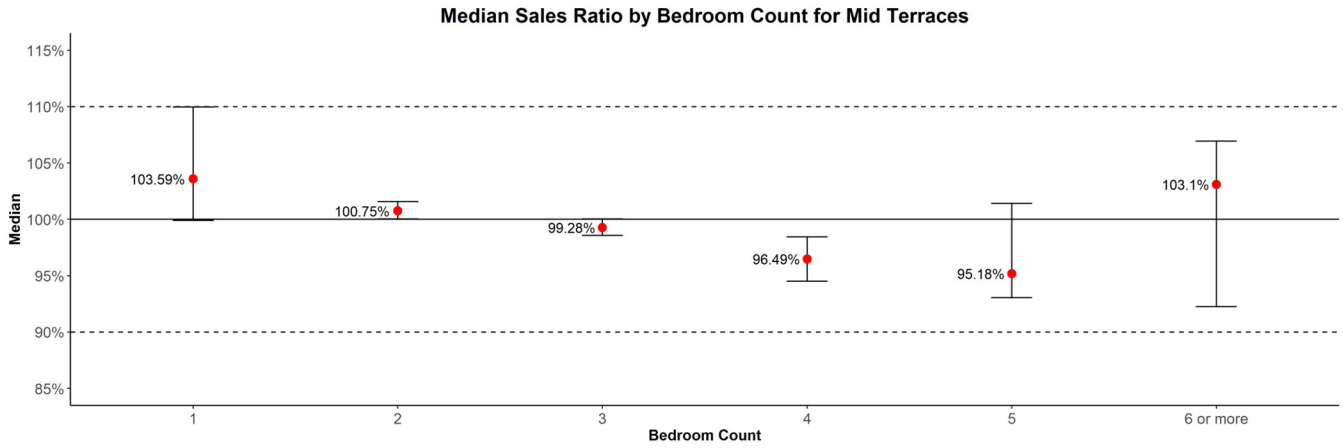


FIGURE 6.8

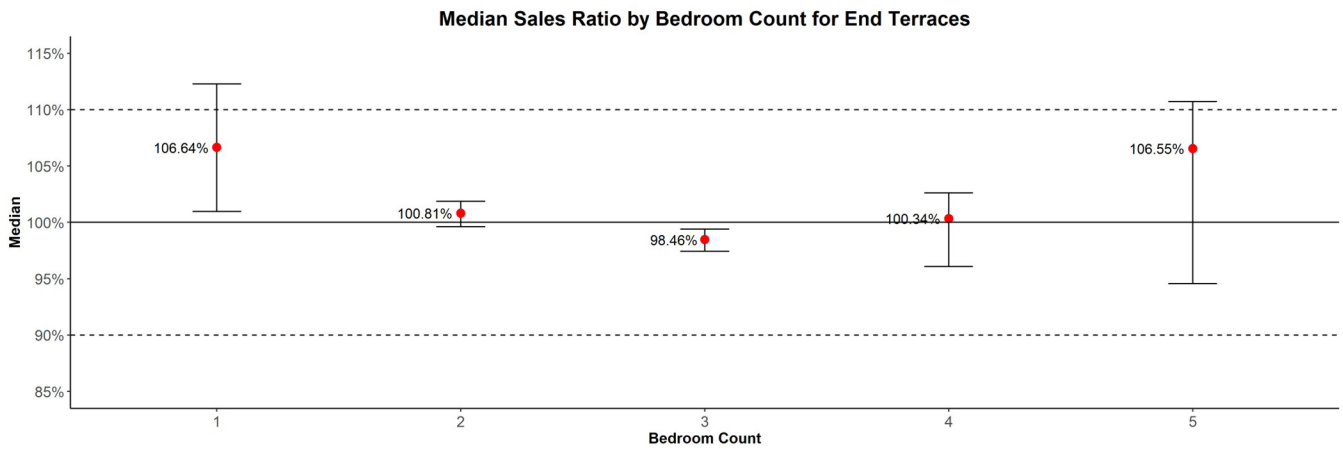


FIGURE 6.9

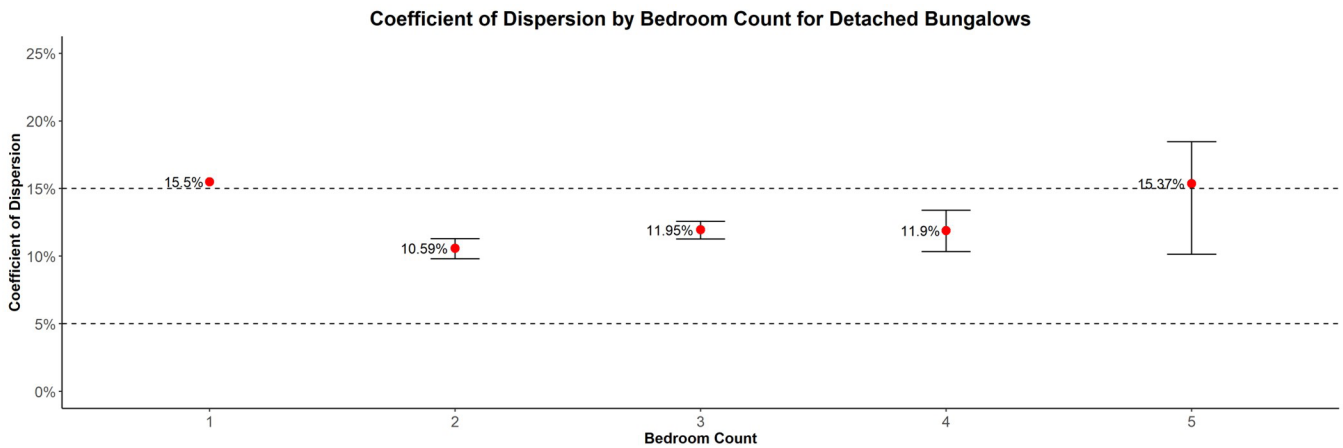


FIGURE 6.10

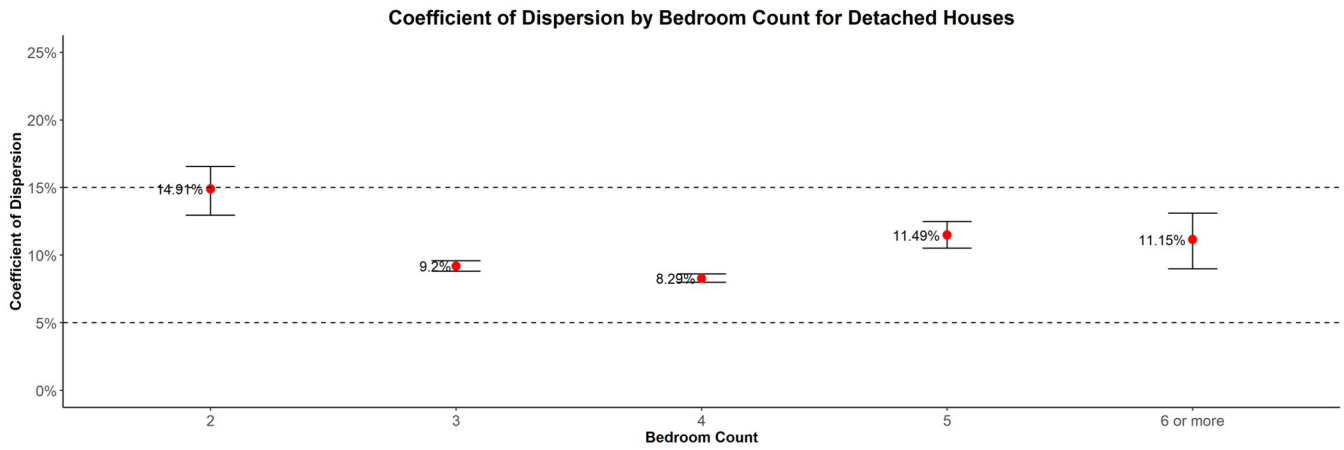


FIGURE 6.11

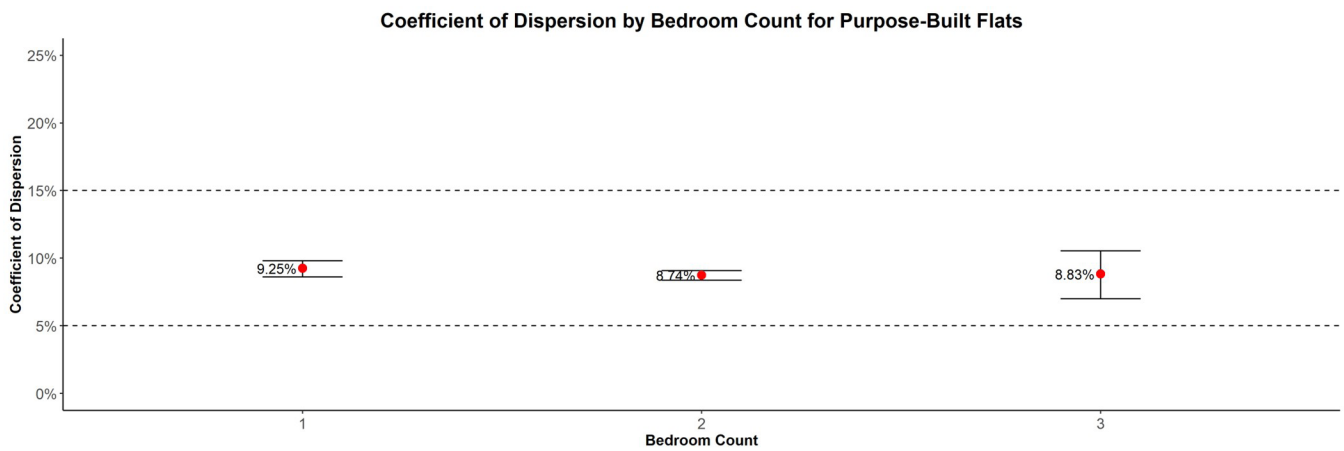


FIGURE 6.12

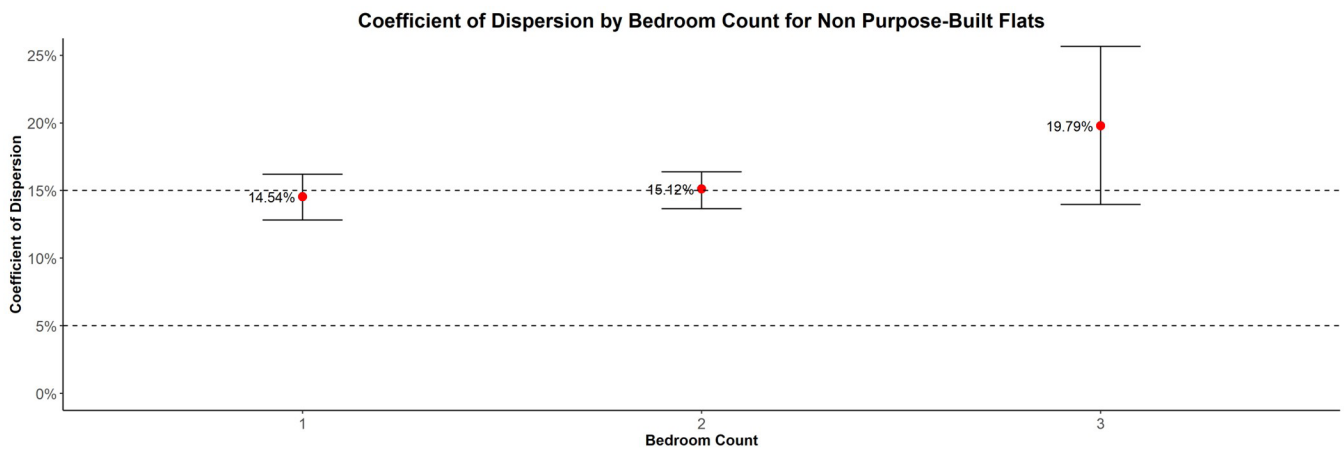


FIGURE 6.13

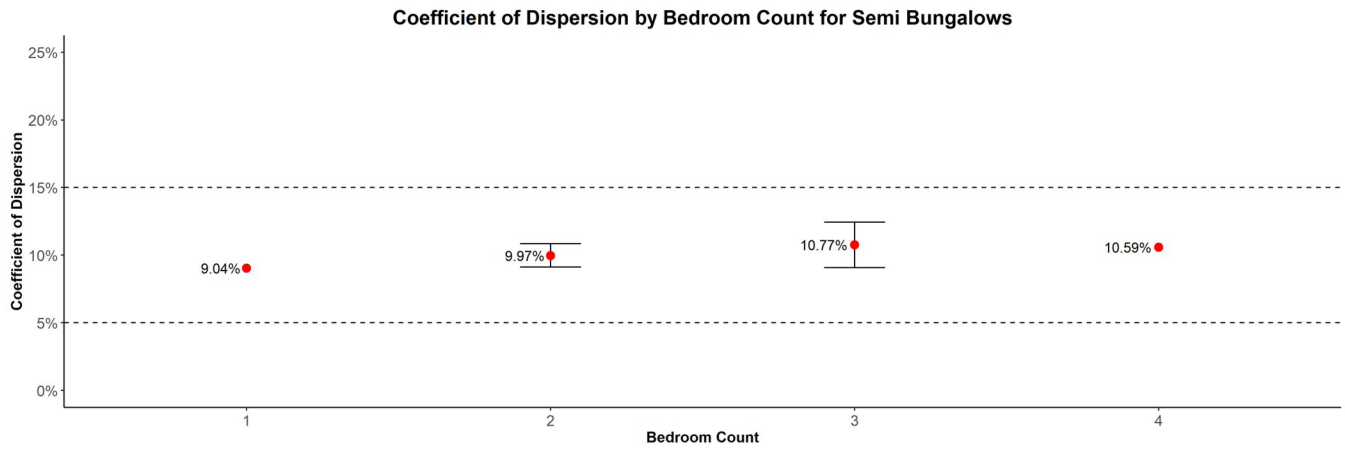


FIGURE 6.14

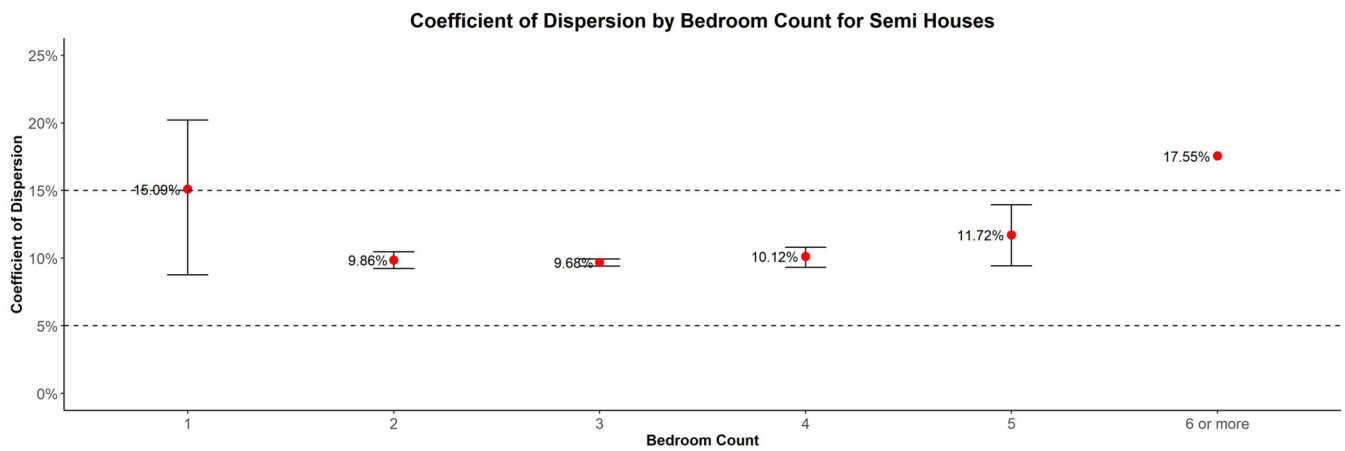


FIGURE 6.15

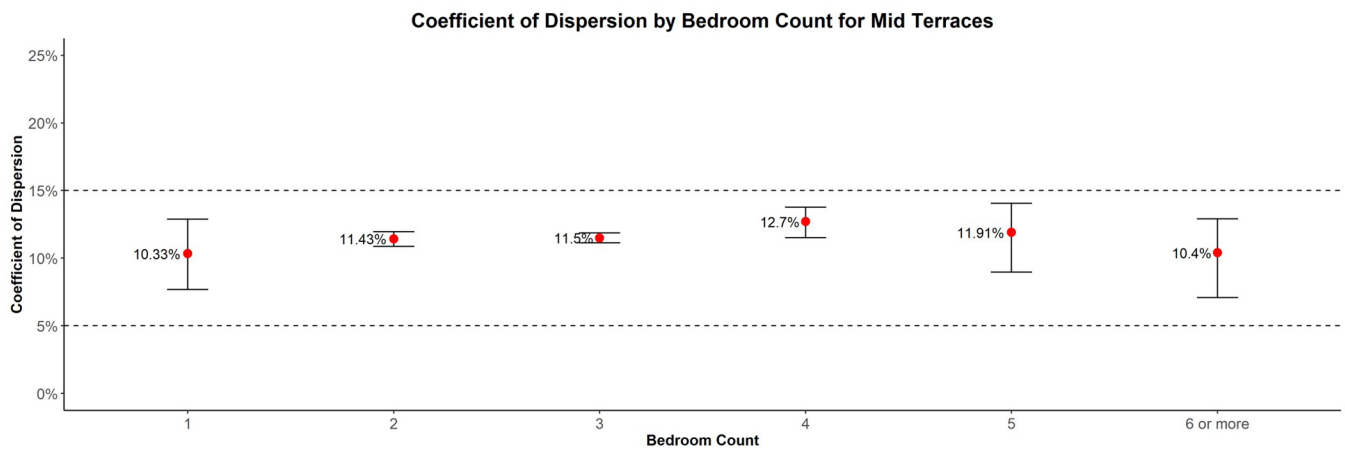


FIGURE 6.16

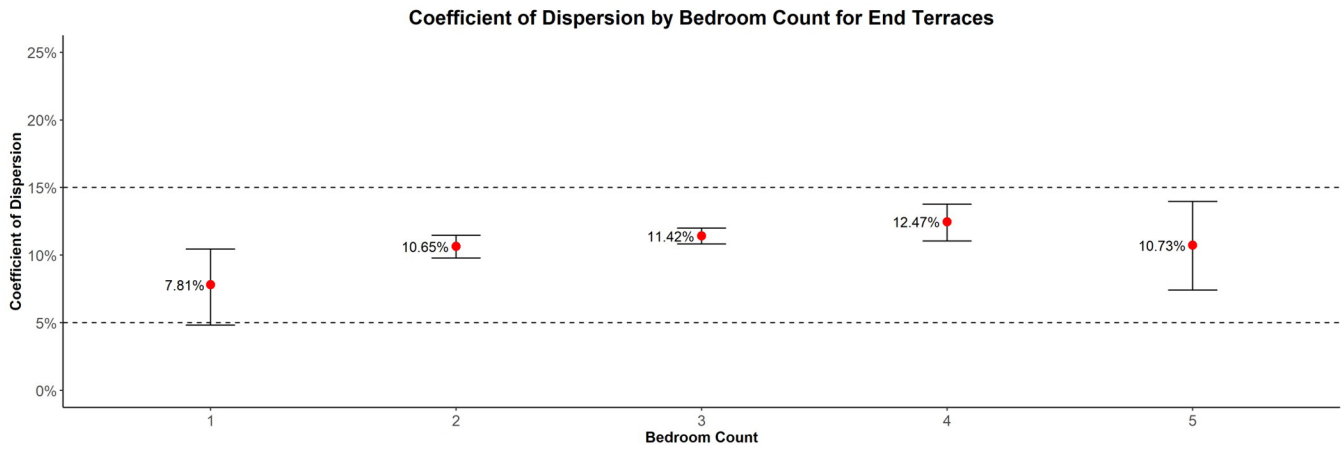


FIGURE 6.17



FIGURE 6.18

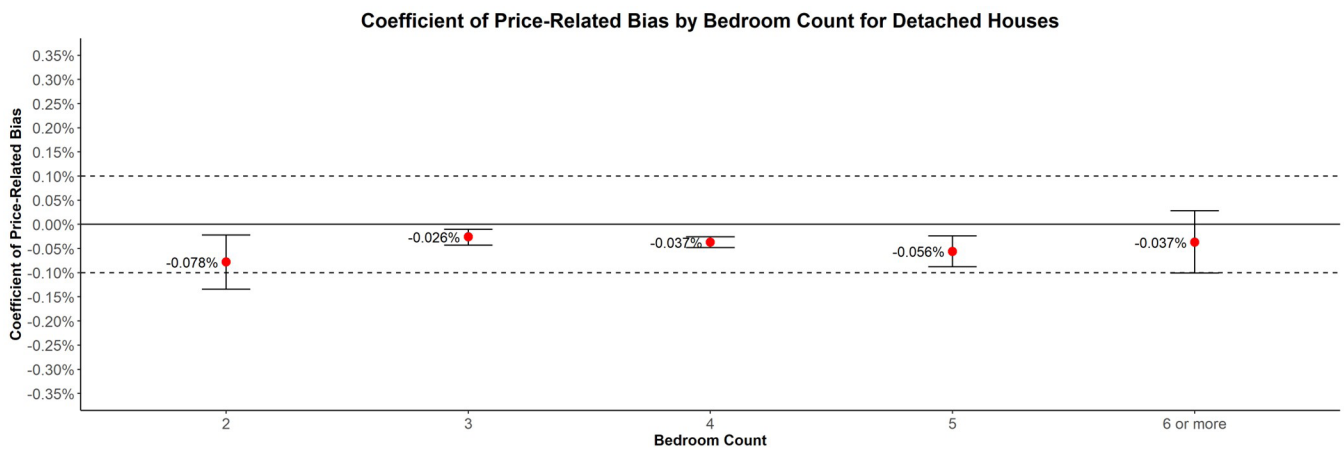


FIGURE 6.19

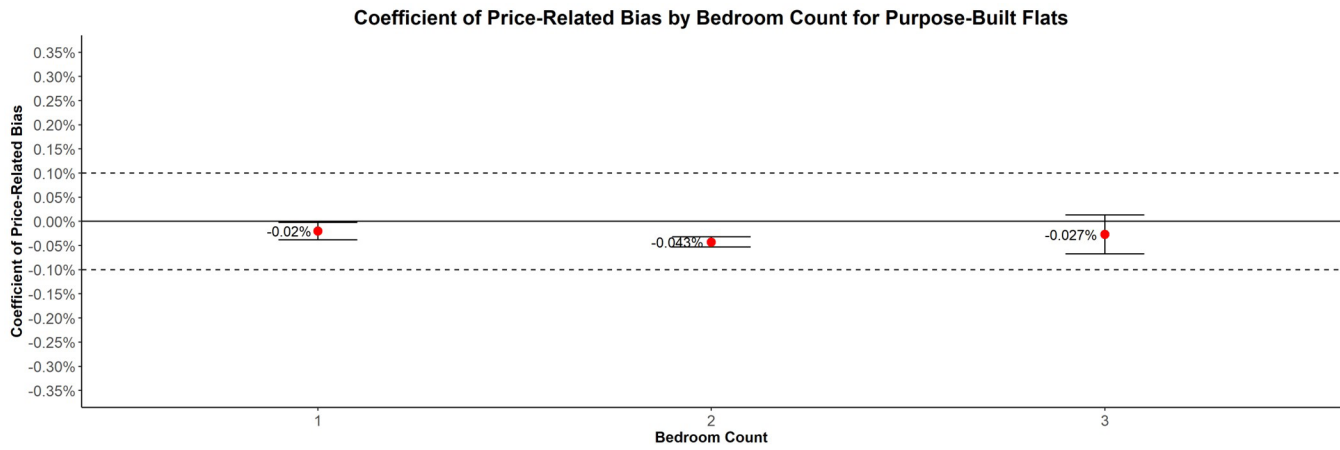


FIGURE 6.20

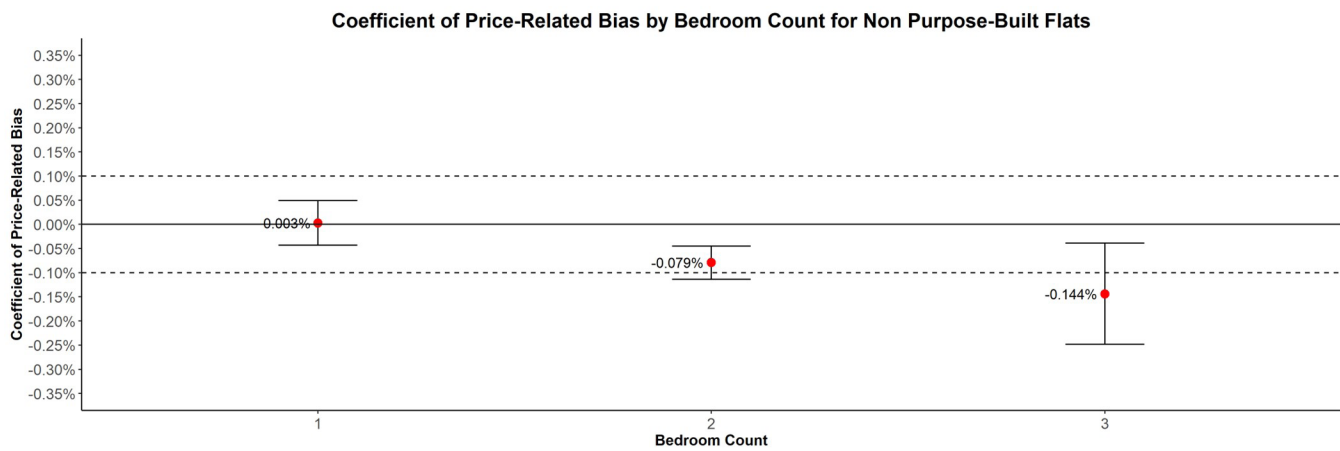


FIGURE 6.21

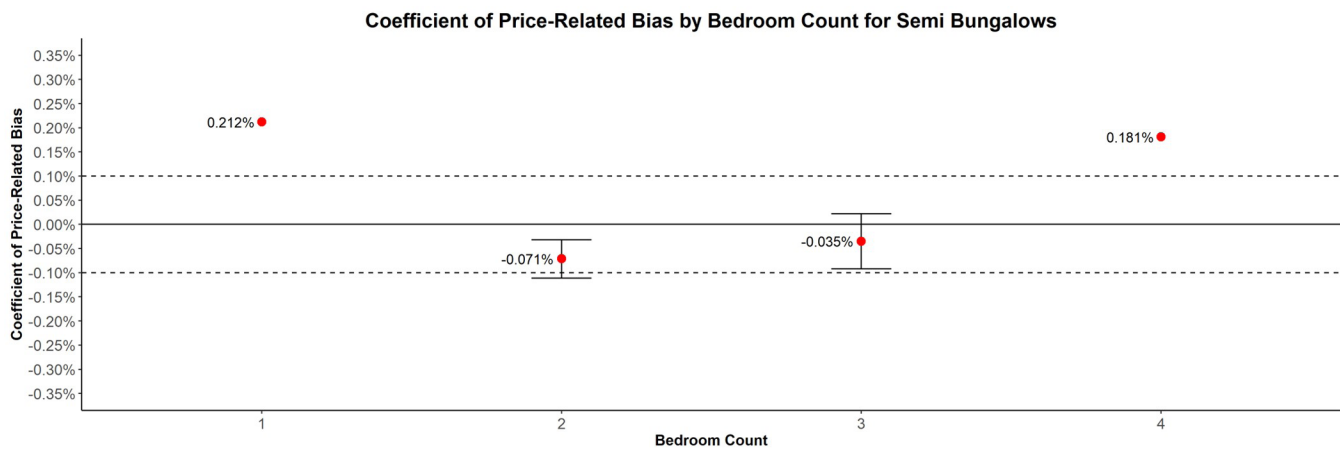


FIGURE 6.22



FIGURE 6.23

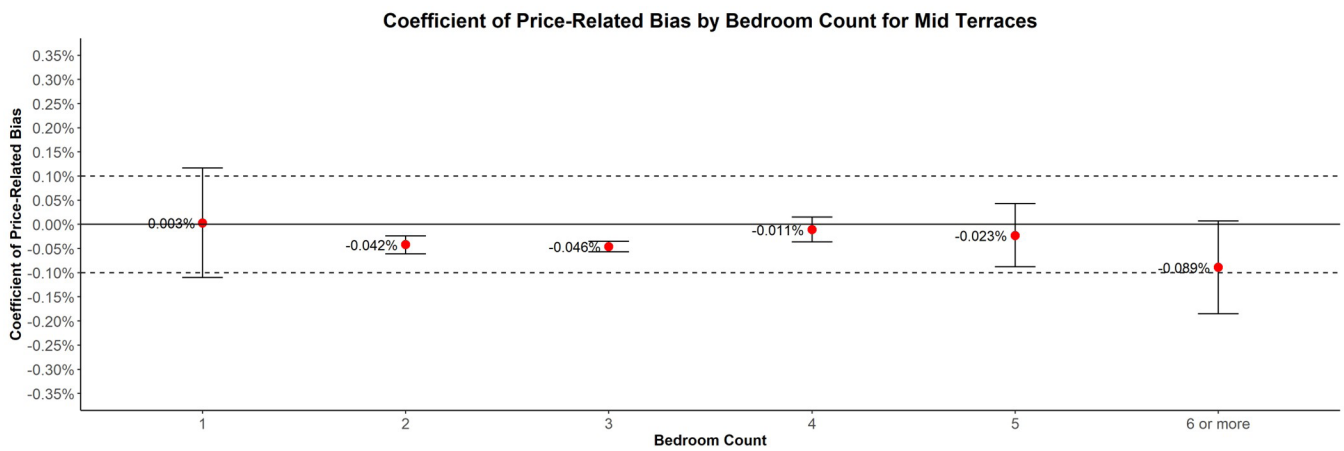
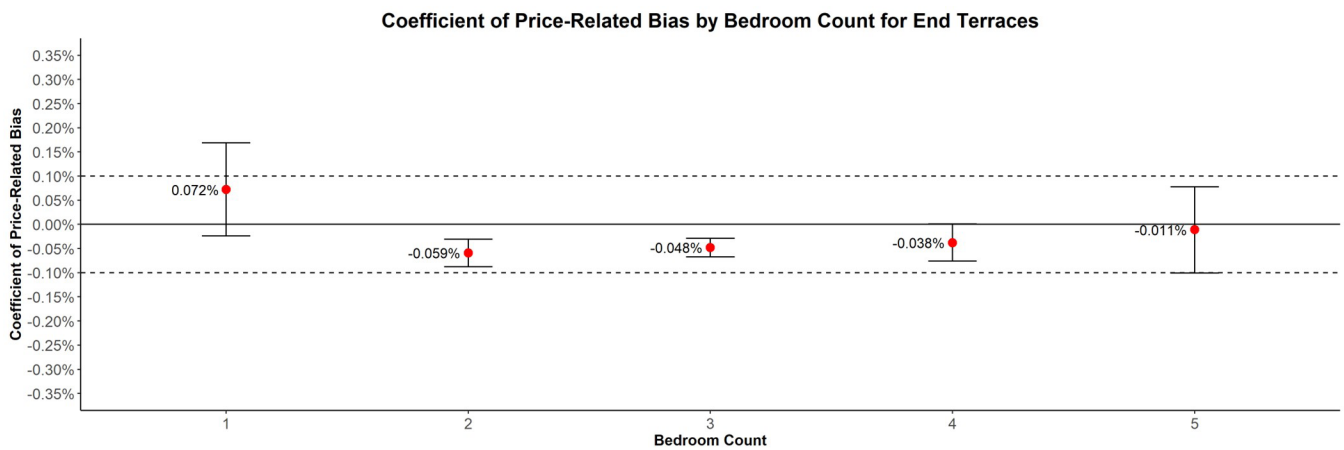


FIGURE 6.24



7. Statistics by Dwelling Type Subgroup and Subsidized Status

FIGURE 7.1

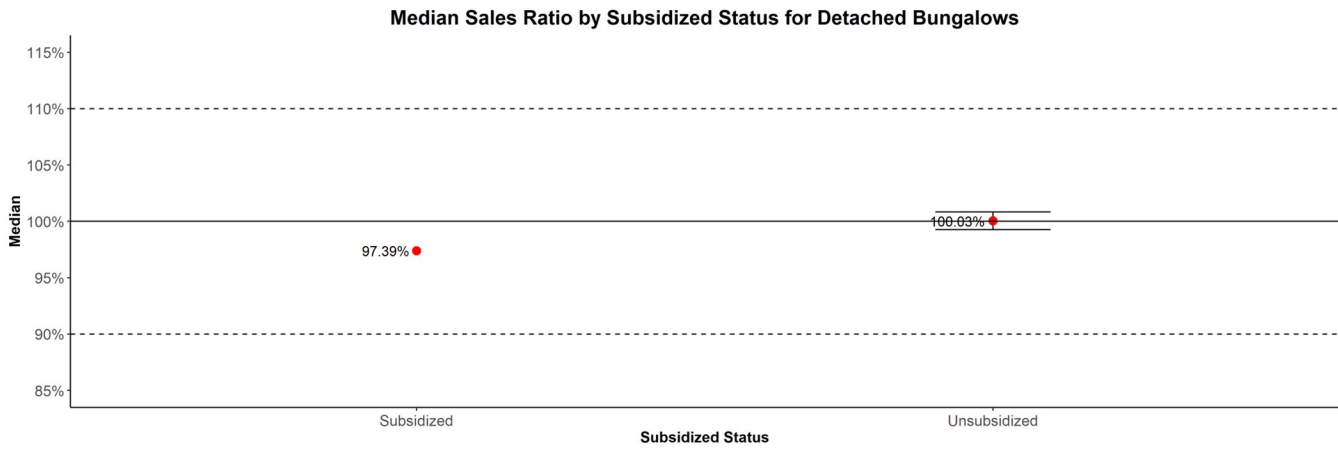


FIGURE 7.2

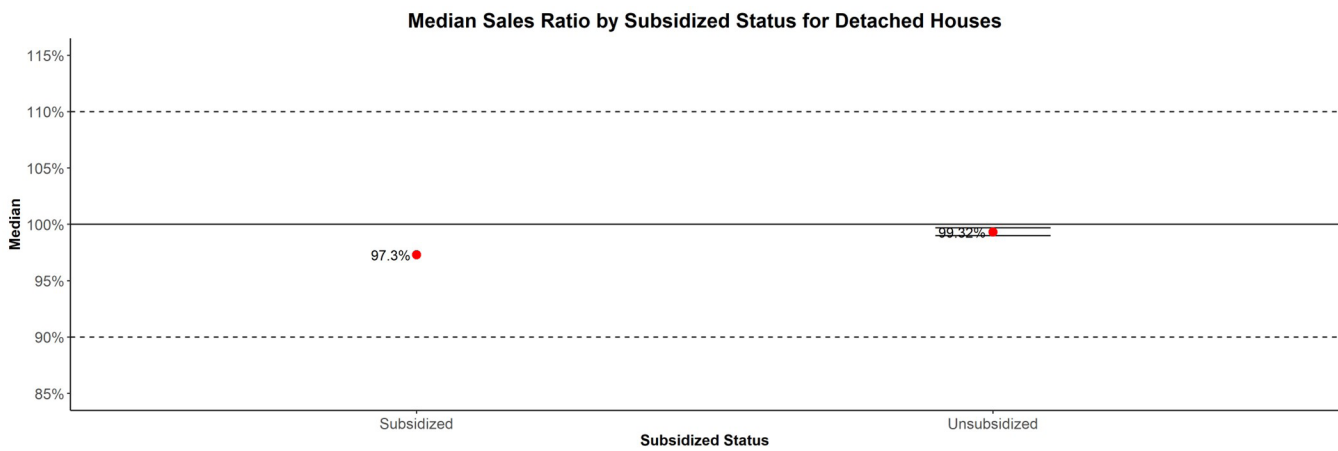


FIGURE 7.3

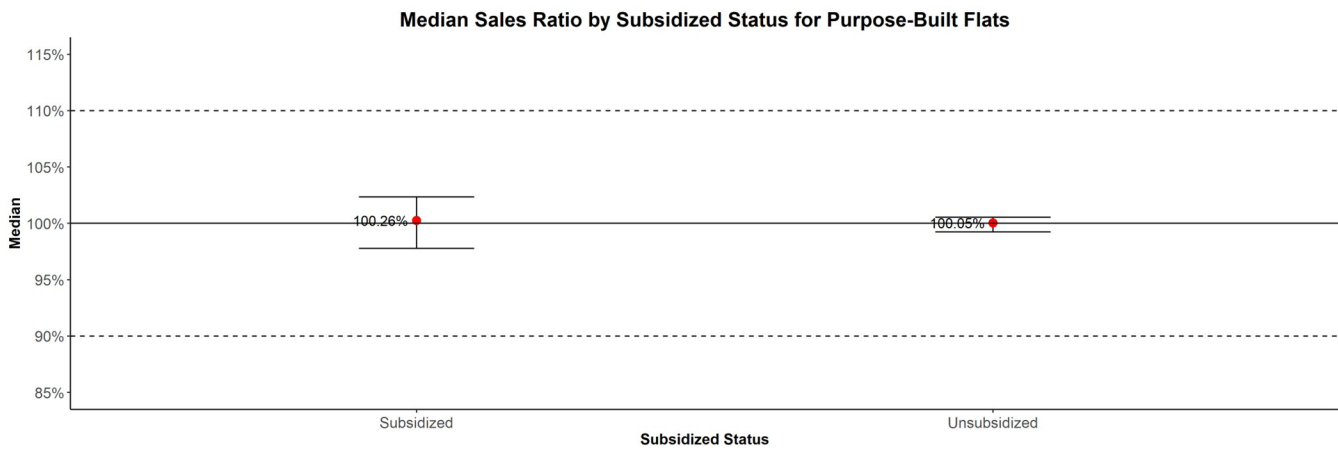


FIGURE 7.4

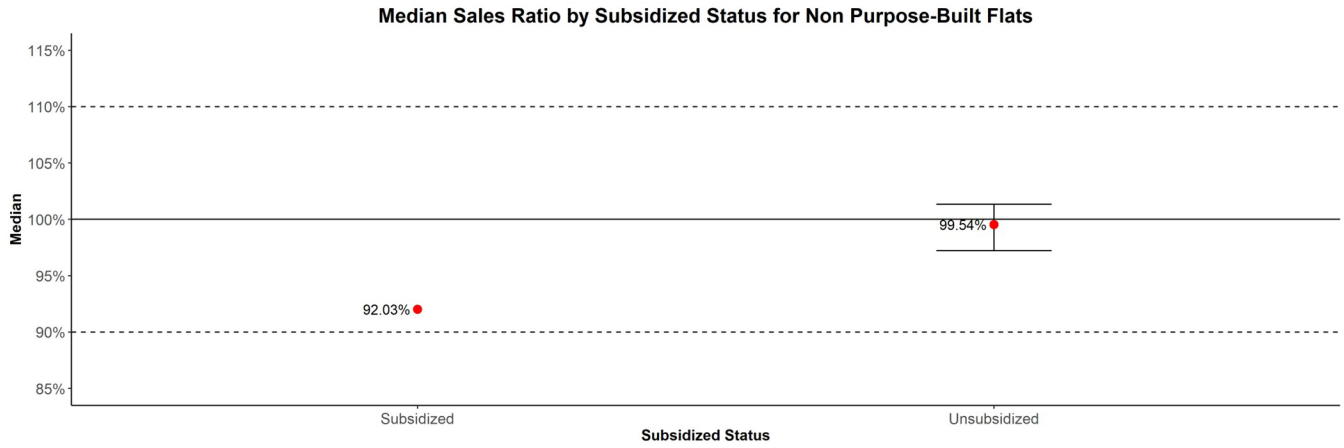


FIGURE 7.5

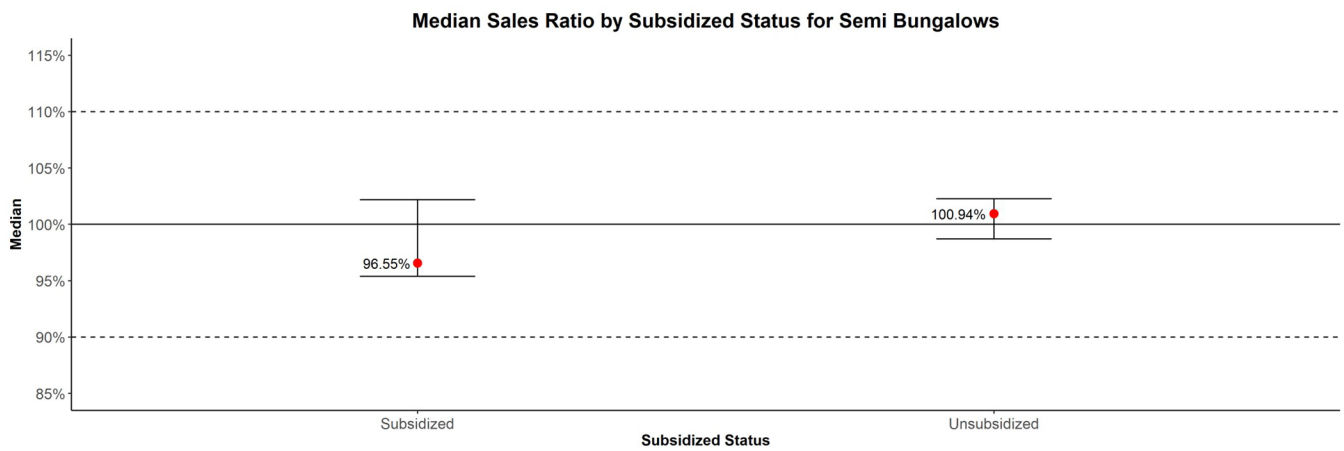


FIGURE 7.6

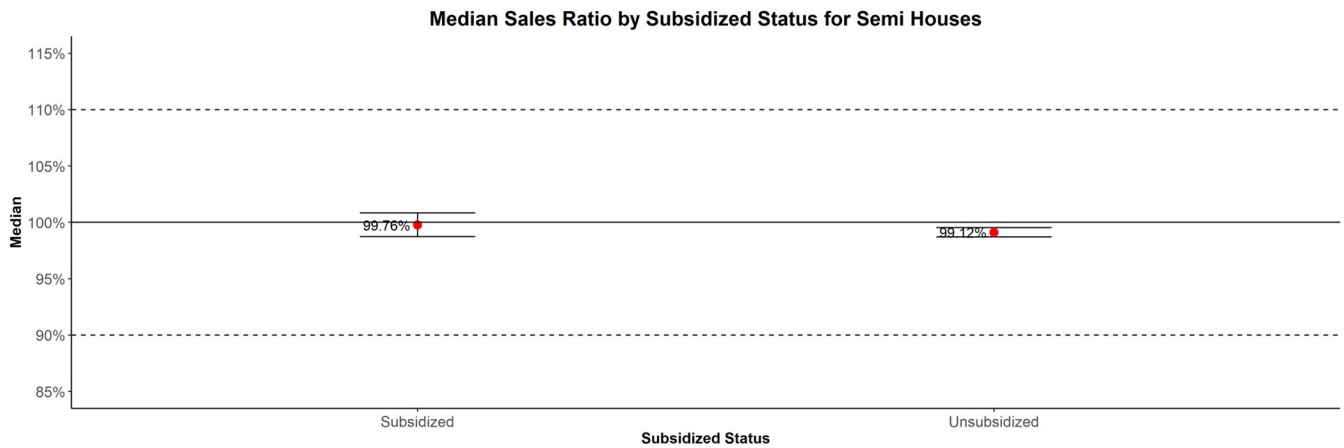


FIGURE 7.7

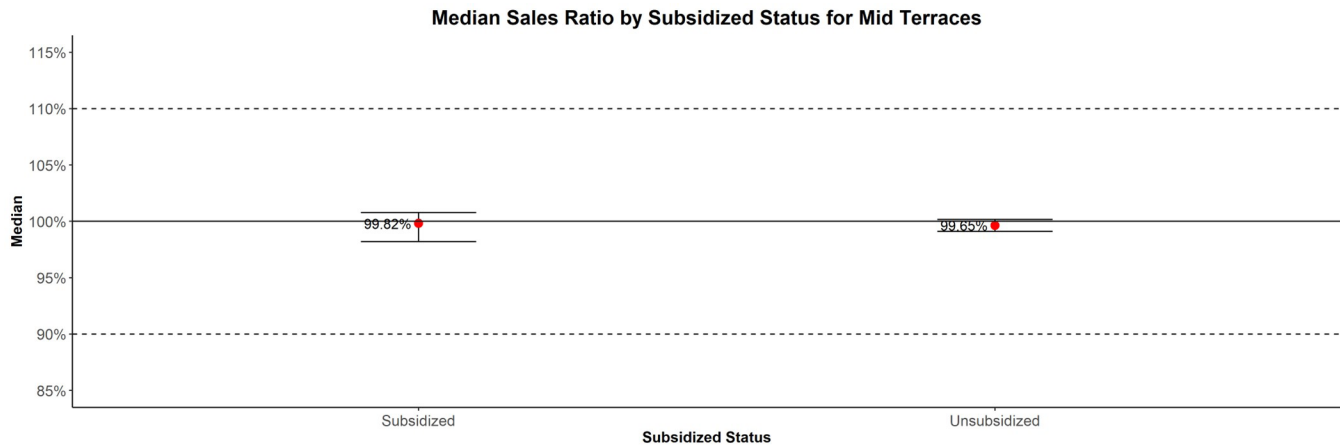


FIGURE 7.8

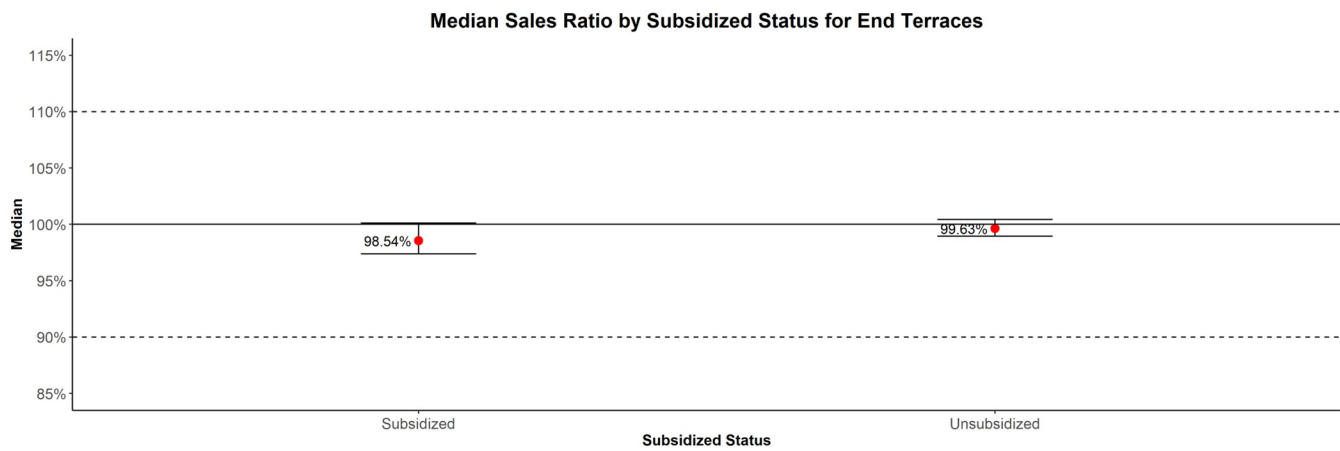


FIGURE 7.9

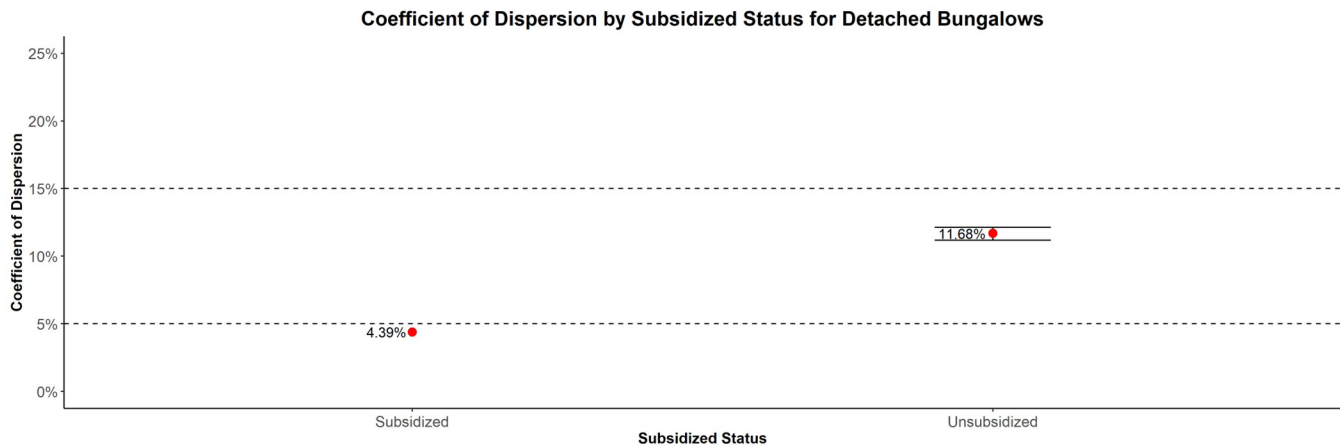


FIGURE 7.10

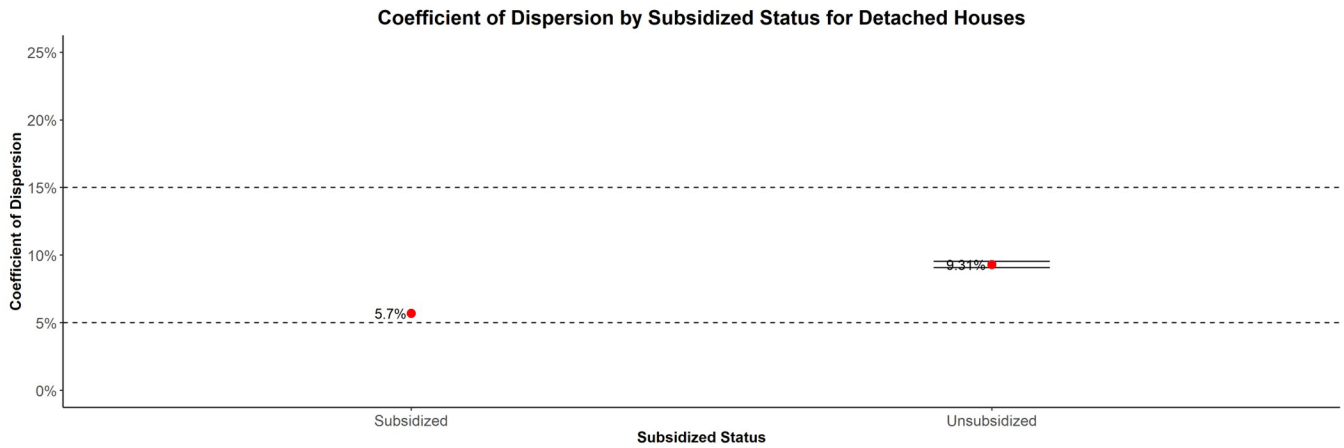


FIGURE 7.11

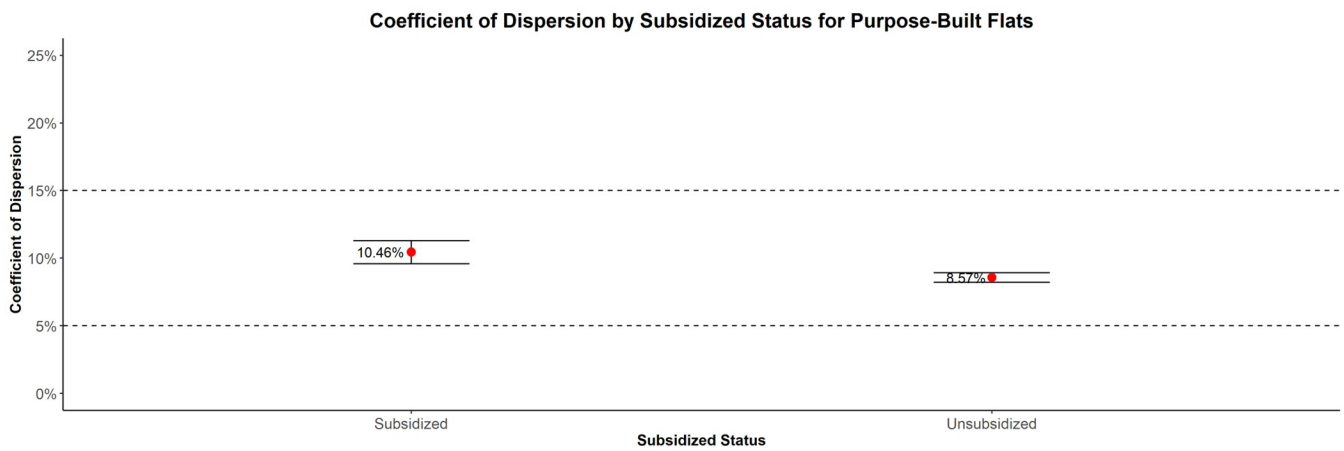


FIGURE 7.12

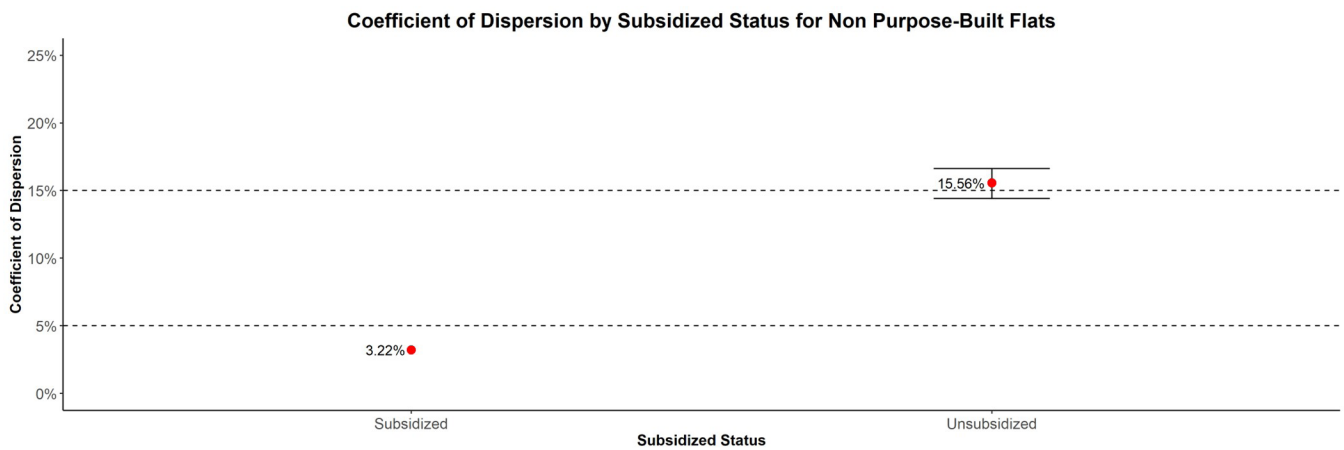


FIGURE 7.13

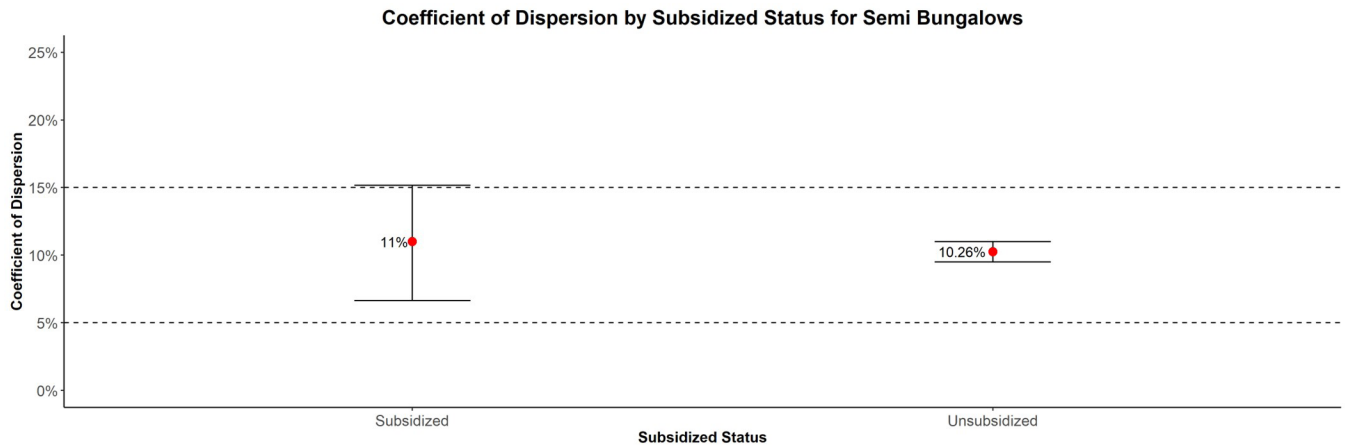


FIGURE 7.14

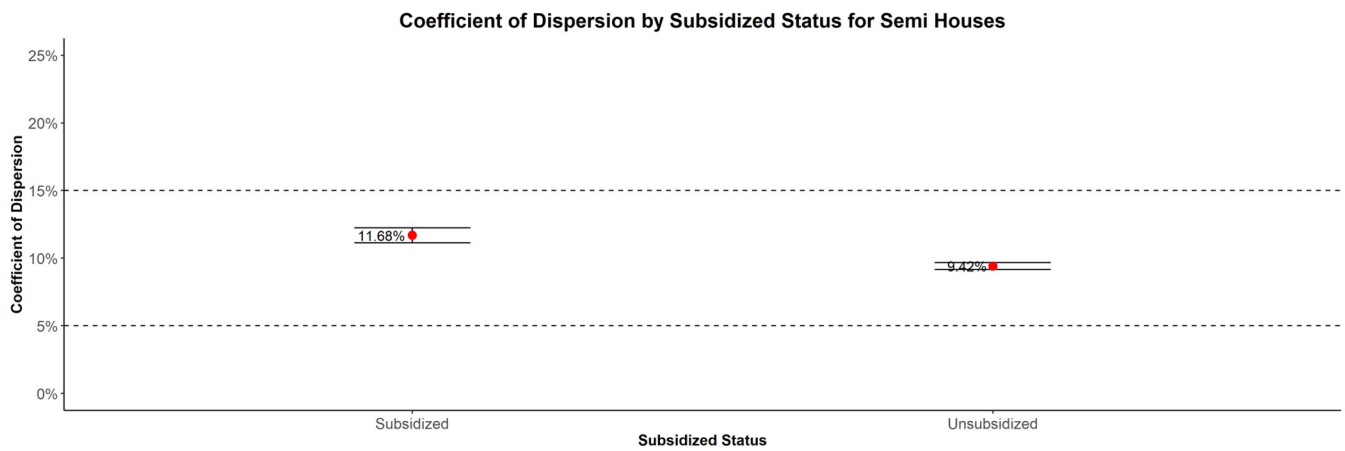


FIGURE 7.15

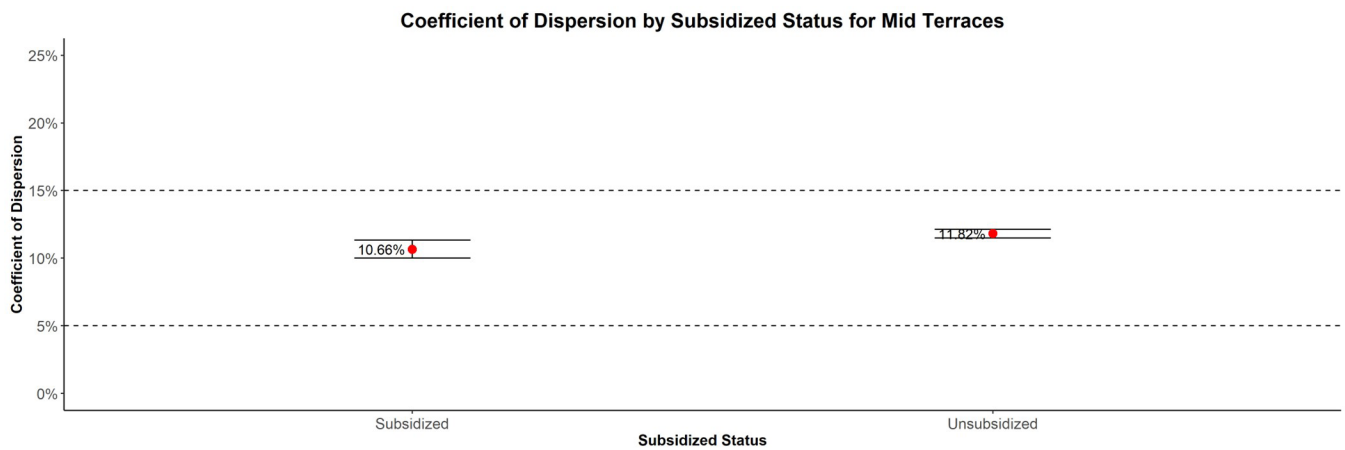


FIGURE 7.16

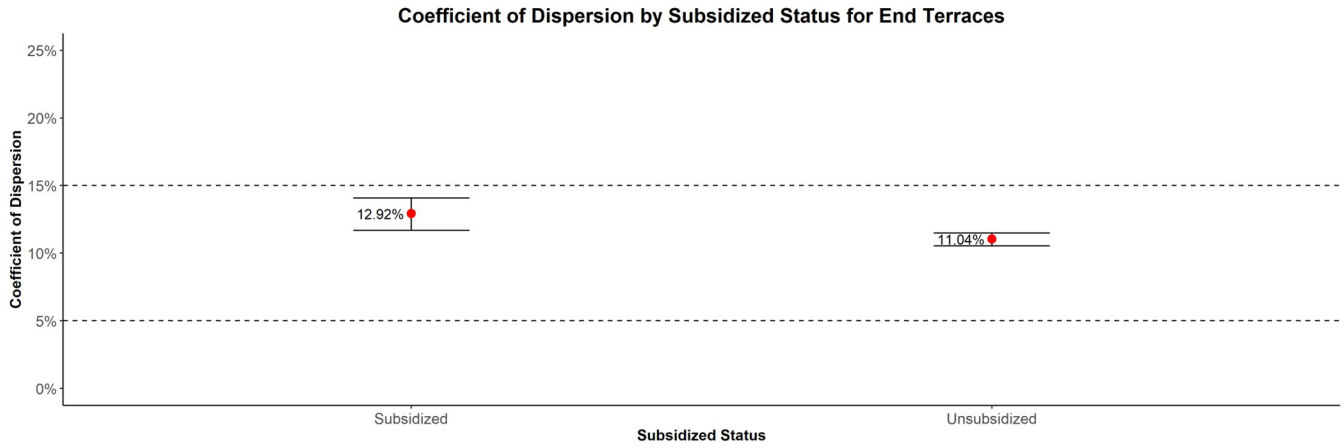


FIGURE 7.17

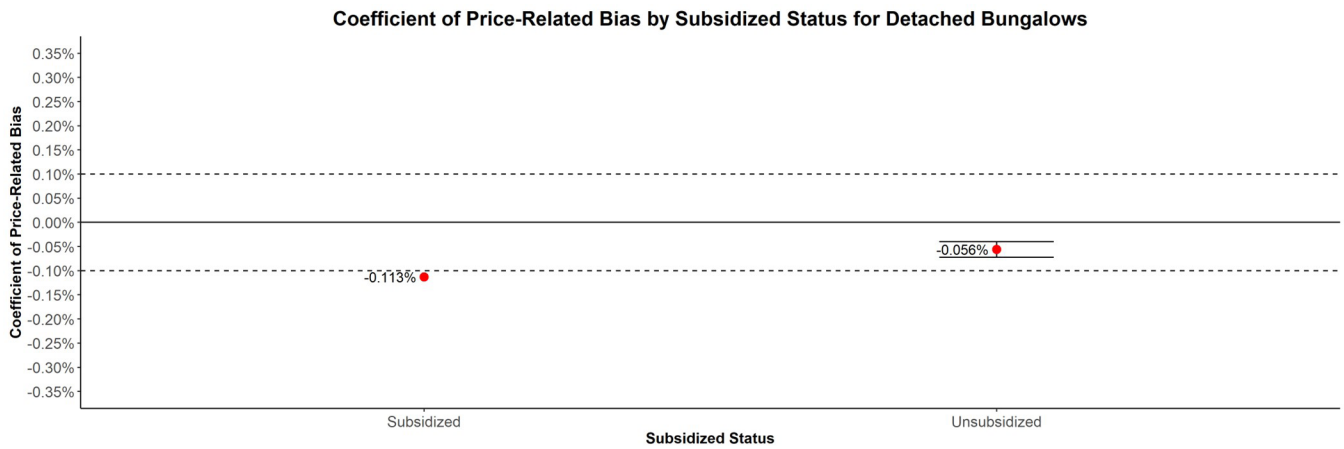


FIGURE 7.18

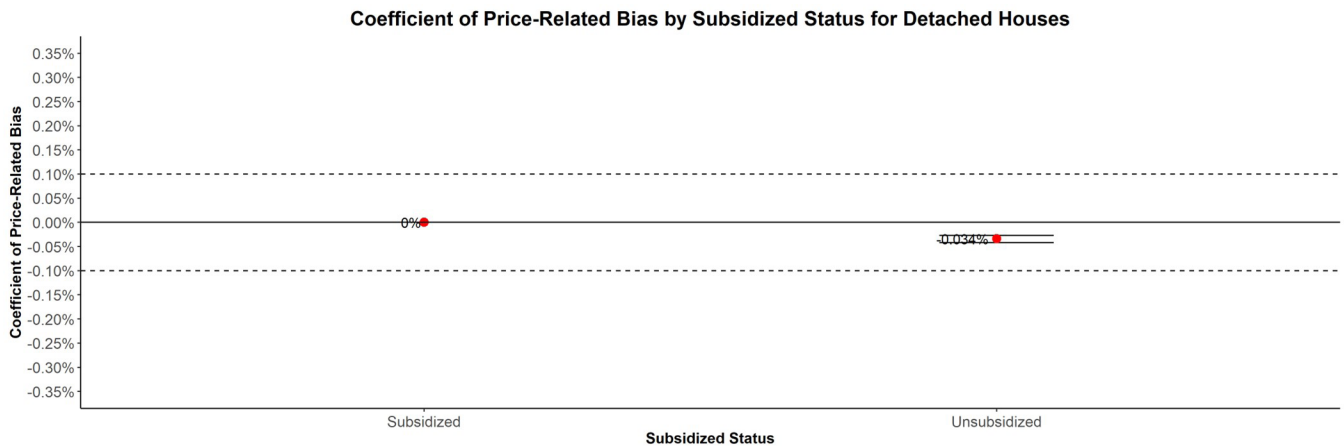


FIGURE 7.19

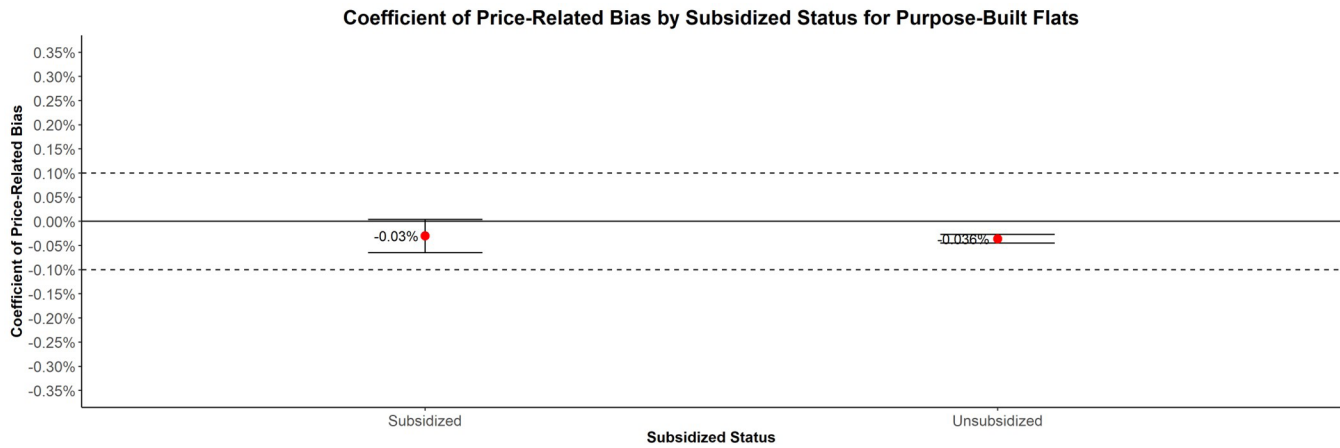


FIGURE 7.20

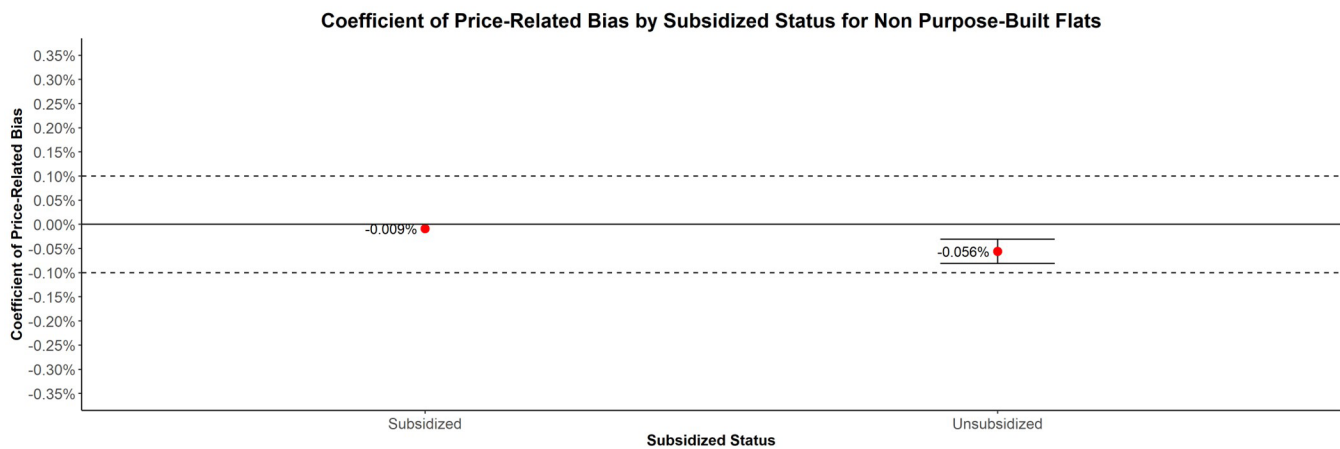


FIGURE 7.21

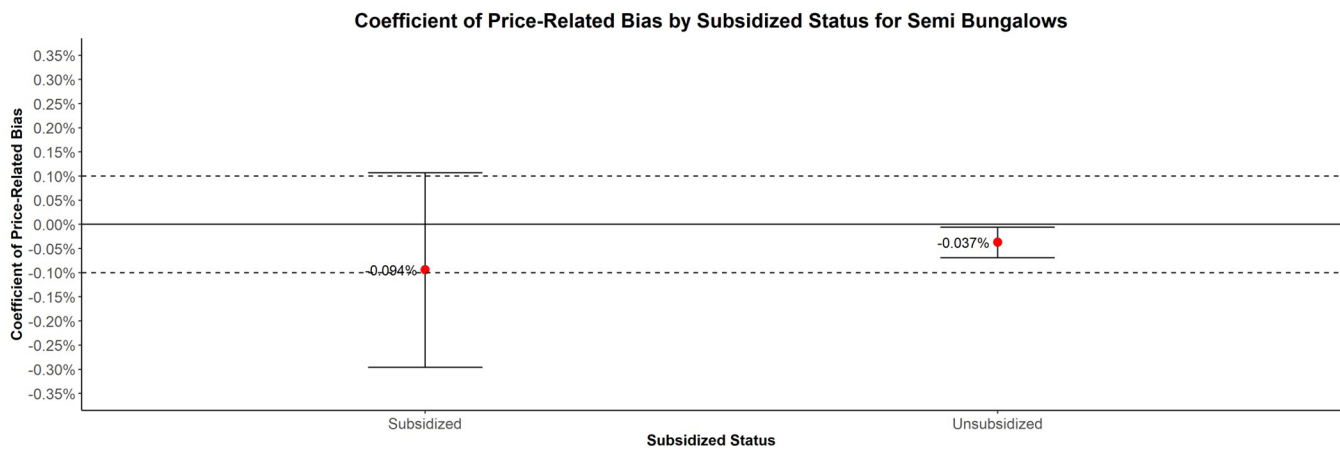


FIGURE 7.22

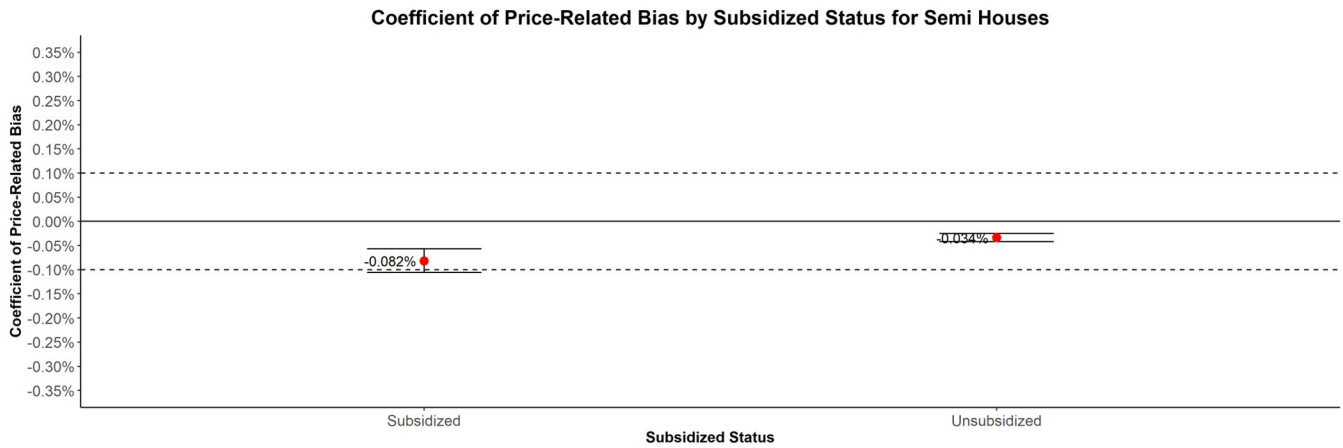


FIGURE 7.23

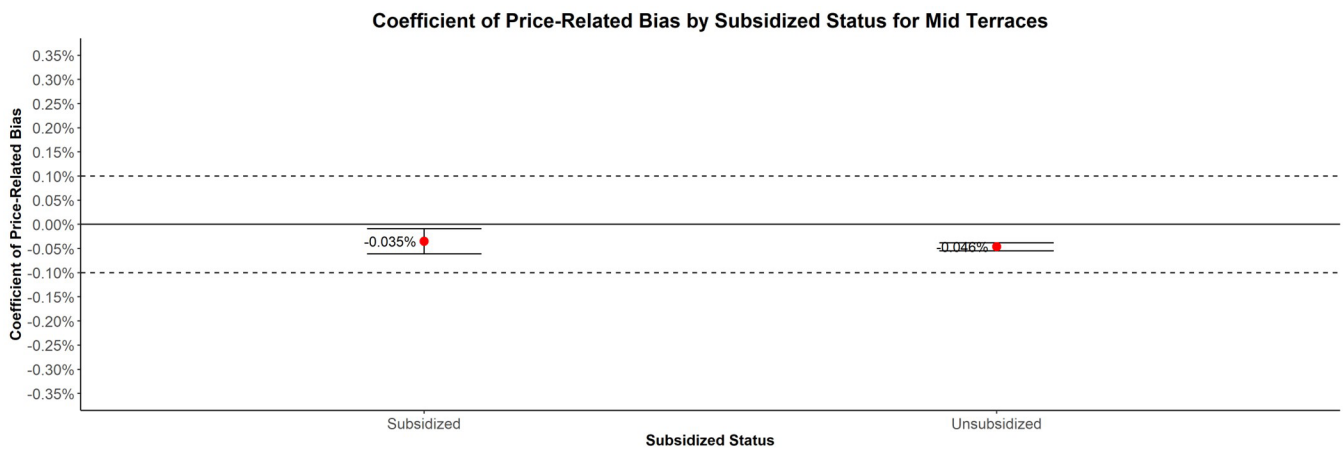
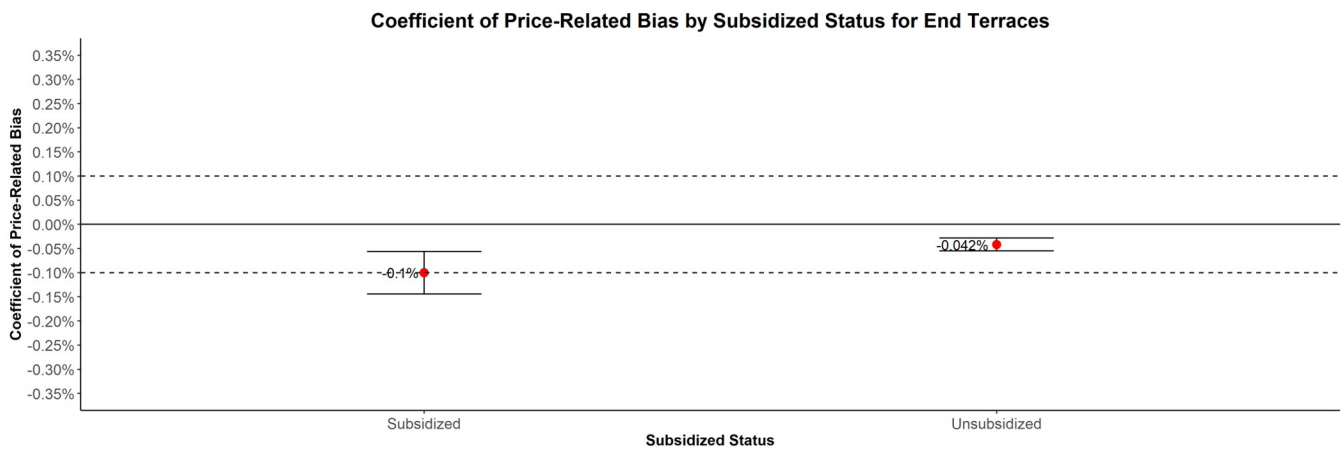
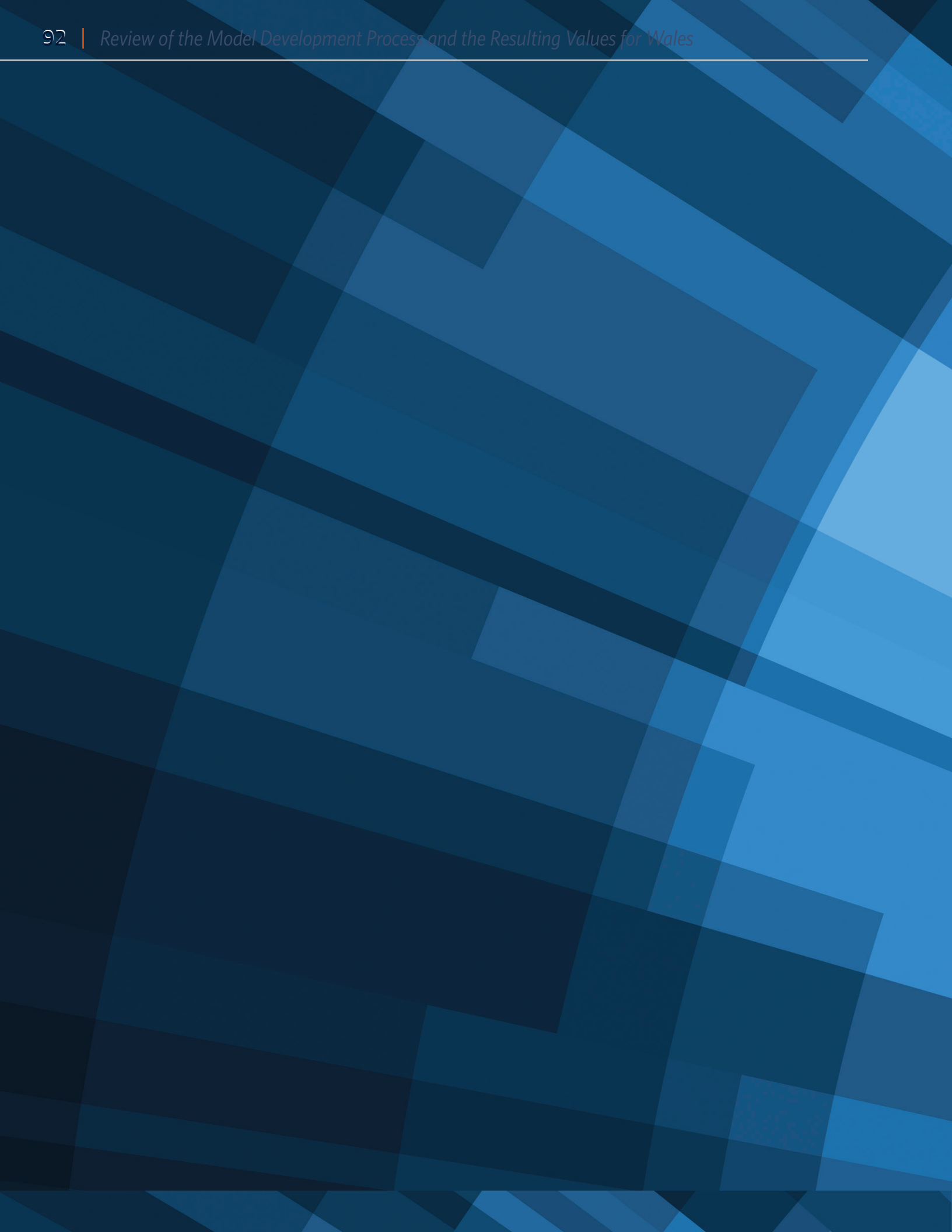


FIGURE 7.24







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