Wider economic impact assessment with a S-CGE model

High Speed Two (HS2) Limited

March 2022

Contents

1. Executive summary

1.1. Scope of the report

High Speed Two (HS2) is Britain's largest infrastructure project. It will significantly increase capacity on the UK's rail network, including by freeing up capacity on existing lines, and improve connectivity by connecting city regions in the Midlands and North of England with faster, more frequent, and more reliable services. A strategic goal of HS2 is to be a catalyst for sustained and balanced economic growth across the UK and support the Government's 'Levelling Up' agenda. HS2 will be an environmentally sustainable form of transport, offering significantly lower carbon emissions per passenger kilometre than cars (including electric vehicles) and domestic air travel.

The business case for HS2 includes in the Economic Case estimates of the Benefit-Cost Ratio (BCR) for the HS2 scheme. The benefits within the BCR estimates are derived from direct transport user and capture a wider range of economic benefits, such as increased agglomeration. However, the BCR does not necessarily capture the full transformational impact and additional benefits that are expected to be realised from HS2.

PricewaterhouseCoopers LLP (PwC) was appointed by High Speed Two Limited (HS2 Ltd) to examine and provide estimates of these wider economic impacts that could be realised due to the improved connectivity and increased capacity that will be delivered by the HS2 railway when operational. These estimates were generated using a Spatial - Computable General Equilibrium (S-CGE) model. The purpose of this report is to provide a summary of the methodology and the results of the analysis derived from the S-CGE model simulations of those wider economic impacts. The S-CGE modelling considers the economic impacts of HS2 when HS2 services are operational and excludes the economic impacts that will be realised during the design and construction of HS2, such as spending and employment in the HS2 construction supply chain.

For the purposes of this study, we group the Phases of the HS2 scheme into the following three categories that we use when modelling the wider economic impacts of the scheme:

- **Phases One and 2a combined** – linking London to the West Midlands and Crewe.
- **Phase 2b Western Leg increment** – linking Crewe to Manchester, with a connection to the West Coast Mainline for HS2 services to Scotland. This assumes that Phases One and 2a are operational.
- **Phase 2b Western Leg Full Network** – combining Phases One, 2a and 2b Western Leg.

HS2 Ltd commissioned and financed this work, provided key inputs for modelling, and commented on all relevant aspects. Other key stakeholders, such as the Department for Transport (DfT), have provided commentary and input. We have also benefited from the advice of Dr Adolf Stroombergen, Chief Economist of Infometrics; and Dr James Laird, Director of Peak Economics and Visiting Research Fellow of the University of Leeds who provided independent peer review of our work. This advice is available in conjunction with this report. However, this report and the analysis within it represents the independent work of PwC.

This report is designed to inform the Business Case for HS2 and is intended to be one input into a broader assessment of both the benefits and wider economic impacts of the scheme. Its findings should be considered alongside evidence from other sources. This report has been written for readers who have a general understanding of economics and who are looking to explore the underlying parameters in the model and drivers of results.

1.2. Background and context

In the HS2 business case, the Economic Case appraises the value-for-money of the scheme, as quantified in BCR estimates, while the Strategic Case sets out the strategic context, case for change and the scheme objectives and benefits. In 2019, the independent review of HS2, led by Sir Douglas Oakervee concluded that 'the published evidence on HS2 has considered the impacts of HS2 in line with the HM Treasury and the DfT's Transport Appraisal Guidance (TAG)' but 'does not capture the full view of the expected benefits of HS2 given the prominence in the strategic case to deliver transformational land-use change' [1].

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[1] Oakervee, D (2020) '*Oakervee Review of HS2*'

Furthermore, the Department for Transport (DfT) updated its Transport Analysis Guidance to cover wider economic impacts (September 2016); and HM Treasury's Green Book guidance has been refreshed and additional emphasis has been placed on the increased importance of place-based analysis to help drive the 'levelling-up' agenda, with the core BCR only forming one part of the decision-making process (November 2020).

In response to the findings HS2 Ltd and the DfT reviewed the technical feasibility and the timescales for the delivery of a Supplementary Economic Model to assess the wider economic benefits of HS2 to inform both the Strategic Case for HS2 and the assessment of benefits in monetary terms in the Economic Case. It was determined that the use of a Spatial Computable General Equilibrium (S-CGE) Model was technically feasible and consistent with the model robustness criteria outlined in guidance on Supplementary Economic Modelling in DfT's TAG.

Modelling and valuing wider economic benefits that result from transport investments is complex and cannot currently be done with a high degree of precision or certainty, particularly when attempting to estimate the wider economic impacts of large national transport schemes. Using S-CGE modelling to estimate wider economic benefits is innovative and is not routinely used to estimate the impacts of major transport investments. S-CGE is intended to supplement the evidence base on the economic impacts of HS2, rather than replace standard appraisal methods used in the HS2 business case.

1.3. Methodology

Our S-CGE model is a large-scale numerical model that simulates the key economic interactions of households, businesses, and the Government within an economy. Our model uses regional accounts data from various sources to capture economic interactions and inter-regional trade flows within the UK economy.

These data provide a snapshot of the economy in a single year, which is used as a starting point or a baseline to produce a picture of the UK economy prior to the operation of HS2. The underlying data is linked together through a set of equations that governs how the economy evolves over time. These equations, which are based on the economic theory of general equilibrium, ensure supply and demand for goods and services and factors of production in the economy are balanced.

The- S-CGE model is then 'shocked' with the first round of productivity impacts of HS2 when HS2 services are operational. This enables us to measure the difference between the HS2 scenario and the baseline, providing estimates for the changes in economic metrics such as Gross Domestic Product (GDP), household consumption, exports, imports, and investment.

There are three categories of 'productivity shock' used within our S-CGE model to capture the first round of productivity impacts of HS2, as inputs into the S-CGE model: benefits to business users of rail and the wider transport network; static agglomeration effects from improved opportunities for businesses and people to interact with each other over distance; and labour supply benefits from improving access to job opportunities. These are the direct or 'first round' effects of improvements in productivity associated with a reduction in business operating costs and an increase in the effective density of economic centres due to reduced journey times and improvements in connectivity and capacity between city regions.

Our S-CGE model captures the subsequent or 'second round' effects of these 'productivity shocks' through the investment and employment channels. Better connectivity reduces obstacles to efficient land-use, resulting in a change in the location of firms and households who see the potential of higher productivity and reduced business costs. The development of new economic clusters attracts inflows of investment and leads to changes in land use as these clusters mature. Furthermore, these changes reduce labour market frictions, which alters the composition of employment across regions and creates new opportunities to access jobs.

Our S-CGE model also estimates the 'subsequent round effects' as they diffuse and ripple across the economy by adjusting prices and dampening increases in output in response to increases in private investment and household spending. The extent to which prices and output change is dependent on the level of employment of labour and capital. Our S-CGE model also makes allowances for the displacement of economic activity from one part of the economy to another. This process will continue until all the effects of the 'productivity shocks' have flowed through the entire economy. At this point the total wider economic impacts of HS2 can be assessed.

1.4. Limitations and Uncertainties

S-CGE modelling has not routinely been used to estimate the impacts of major transport investments. S-CGE modelling differs from the standard approach to appraising transport schemes, as set out in DfT's Transport Analysis Guidance. This makes it difficult to compare S-CGE modelling with standard transport modelling approaches used in the HS2 business case.

S-CGE modelling measures the benefits to the wider economy in response to economy-wide improvements in productivity. Standard transport modelling focuses on the benefits to individual transport users and the perceived benefit from journey time savings, improvements in the frequency and reliability of services and reduced levels of crowding based on estimates of the 'value of time'. Both approaches are rooted in economic theory. However, we would argue that both measures capture different (although to some extent overlapping) elements of the gains from HS2, and therefore, are not directly comparable.

There are sources of uncertainty specific to our S-CGE model. The main sources of uncertainty are the allocation of S-CGE model inputs; and uncertainty over the model parameters that seek to represent the behavioural responses of firms and households.

Productivity Inputs and Allocation

The inputs into the S-CGE model are essential in determining the magnitude of the economic impacts of HS2. The estimates of business user benefits, agglomeration and labour supply effects that make up the first-round 'productivity shock' that is input into the S-CGE model were provided by HS2 Ltd. These inputs were then allocated to one of 8 sectors of the UK economy within one of 10 regions.

The S-CGE model includes a spatial element that seeks to capture the regional impacts of HS2. We worked with HS2 Ltd to consider a variety of methods to estimate these regional impacts. Given the experimental nature of the regional analysis and limitations in the underlying data on UK regional economies, the estimates produced for the regional economic impacts of HS2 are subject to a greater level of uncertainty than those produced for the national UK economy.

Model Parameters

Our S-CGE model is an industry standard model, characterised by commonly used functional forms, structures, assumptions, and parameter values. We have made some adjustments intended to induce greater economic realism to the model, consistent with established S-CGE modelling conventions. A range of multipliers from academic literature is used to provide a range around the point estimates from the S-CGE modelling for the GDP impacts of HS2.

1.5. Key findings

The S-CGE model provides estimates of the impacts of HS2 on the level of output in the UK economy, as measured by GDP. Table 1 below sets out the estimated impacts of HS2 on the level of UK GDP in 2051 (the date within our S-CGE model in which the economy is assumed to have achieved a new long-run steady state following the opening of HS2 services).

The Phase 2b Western Leg Full Network (linking London to Manchester) is forecast to increase the level of UK real GDP by between £1.9 billion and £3.5 billion (2015 prices, undiscounted) compared to baseline GDP in 2051. The point estimate is £2.5 billion. This amounts to an additional 0.1% of UK GDP in 2051.

Table 1 UK real GDP Impacts, 2051, £bn (2015 prices, undiscounted)

£bn, 2015 prices	**Phases One + 2a**	**Phase 2b Western Leg Increment**	**Phase 2b Western Leg Full Network**
Range	1.4 – 2.5	0.5 – 1.0	1.9 – 3.5
Point Estimate	1.7	0.8	2.5

The North West and the West Midlands regions are forecast to be the main beneficiaries of the Phase 2b Western Leg Full Network. In 2051, the S-CGE model estimates that their economies will be 0.24% and 0.11% larger, respectively, compared to the baseline without HS2 services.

The estimated national GDP multipliers generated by the S-CGE model measure the change in the level of UK GDP for every £1 of first-round productivity impacts generated by HS2 (i.e., the ratio of GDP outputs to productivity inputs). These estimated multipliers are provided in Table 2 below.

For the Phase 2b Western Leg Full Network (linking London to Manchester) the estimated GDP multiplier is 1.32. This implies that for every £1 of static productivity gain (the business user benefits, agglomeration and labour supply inputs into the S-CGE model), UK GDP is estimated to increase by £1.32 over the 64-year appraisal period.

The GDP multiplier for the Phase 2b Western Leg Increment (1.65) is estimated to be larger than that for Phases One and 2a (1.21). These estimates lie within the reasonable range of implied multipliers from comparable studies (between 1.0 and 1.8) and hence the difference in estimated multipliers between phases may reflect uncertainties in the modelling rather than genuine differences in multiplier effects by phase.

Table 2 GDP multipliers by Phase

	Phases One + 2a	Phase 2b Western Leg Increment	Phase 2b Western Leg Full Network
Appraisal Period	31 Dec 2021 – 31 Dec 2081	31 Dec 2025 – 31 Dec 2085	31 Dec 2021 – 31 Dec 2085
Appraisal Length	60 years	60 years	64 years
GDP Multipliers	1.21	1.65	1.32

The total general equilibrium welfare benefits of Phase 2b Western Leg Full Network over a 64-year appraisal period is estimated to be £100.8 billion (2015 Prices, Net Present Value). This estimate is inclusive of £40.2 billion in total user benefits and £12.8 billion in agglomeration and labour supply benefits.

The total general equilibrium welfare benefits are inherently uncertain. We have attempted to reconcile the GDP estimates produced by our S-CGE model with conventional appraisal methods outlined in Transport Analysis Guidance (TAG) to estimate total economic welfare. It is not possible to make a full and accurate reconciliation between GDP estimates and welfare benefits because of differences in the modelling approach, methods and assumptions between conventional appraisal methods and the approach used in our S-CGE model. It should not be assumed that the final welfare estimates are directly comparable to the estimated Level 1 and 2 welfare benefits of HS2 in the HS2 business case.

1.6. Conclusion

Our S-CGE model is intended to capture the economic impacts of HS2 operations on the UK economy using a general equilibrium framework. The use of an S-CGE model to assess the wider economic impacts of a major transport infrastructure scheme in this way is innovative and intended to add to the evidence base on the economic impacts of HS2 and inform the HS2 business case.

2. Purpose of this report

2.1. Report commissioning

PricewaterhouseCoopers LLP (PwC) was appointed by High Speed Two Limited (HS2 Ltd) to carry out a Spatial - Computable General Equilibrium (S-CGE) model simulation to examine how wider economic impacts could be realised due to the improved connectivity and increased capacity that will be delivered by the High Speed Two (HS2) railway when operational. This report presents PwC's findings based on the analysis derived from the S-CGE model simulations of those wider economic impacts.

HS2 Ltd commissioned and financed this work, provided key inputs for modelling, and commented on all relevant aspects. Other key stakeholders, such as the Department for Transport (DfT), also provided commentary and input. However, the report represents the independent analysis of PwC. Our analysis has also benefited from the advice of two independent peer reviewers, who have carried out a peer review of the S-CGE model and results.

The decision to proceed with HS2 was confirmed by Government on the 14th April 2020. HS2 will look to provide a long-term solution to overcrowding on the rail network, be a major contributor to the Government's objective of levelling up the UK economy and, finally, support the UK's 2050 net zero carbon objective.

Readers may use this report to understand the net incremental economic benefits that could arise with the development of the HS2 network. Our S-CGE model output shows that HS2 will stimulate economic growth (GDP), household income, trade between different parts of the UK and overseas, along with private sector investment and additional employment opportunities. As described in more detail in Chapter 5 of this report, net exports in our model are assumed to remain constant, and so our economic impacts modelled show both imports and exports growing in parallel as international trade grows.

This report is designed to inform the Business Case for the Western Leg of Phase 2b of HS2 and is intended to be one input into a broader assessment of both the benefits and wider economic impacts of HS2. Its findings should be considered alongside evidence from other sources. This report has been written for readers who have a general understanding of economics and who are looking to explore the underlying parameters in the model and drivers of results.

2.2. Project governance and quality assurance

To ensure that the work was carried out as effectively, efficiently, and transparently as possible and in a way which is consistent with the wider project objectives, PwC engaged with HS2 Ltd to discuss inputs into the modelling process and conducted workshops with key stakeholders from HS2 Ltd and the DfT to present emerging results.

PwC was responsible for:

- reviewing and challenging its own emerging work;
- providing transparency around the work and promoting robustness of the approach;
- promoting timely provision of data;
- monitoring progress against the project plan ensuring the timely delivery of outputs;
- aligning methodology with DfT Transport Analysis Guidance (TAG) where relevant; and
- facilitating and running key workshops on progress, including the CGE model's results, sensitivity testing and other identified findings (with key stakeholders).

Additionally, to help promote the robustness of our analysis, HS2 Ltd appointed two peer reviewers to review our analysis:

- Dr Adolf Stroombergen, Chief Economist of Infometrics; and
- Dr James Laird, a Director of Peak Economics and Visiting Research Fellow of the University of Leeds.

They have both been consulted throughout the project for their subject matter expertise.

2.3. Project background

HS2 is a new high speed rail network linking London, the Midlands, and the North of England. The new railway is being delivered in phases:

- **Phase One** linking London and the West Midlands;
- **Phase 2a** linking the West Midlands to Crewe;
- **Phase 2b Western Leg** linking Crewe to Manchester, with a connection to the West Coast Mainline (WCML) for HS2 services to Scotland; and
- **HS2 East** linking the West Midlands to East Midlands Parkway with a design to allow trains to reach the existing stations in Nottingham and Derby, and to be capable of future extension. Government will also look at the most effective way to run HS2 trains to Leeds, as set out by Government in the Integrated Rail Plan for the North and Midlands.[2]

The S-CGE analysis focuses on the wider economic impacts of Phases One, 2a and the 2b Western Leg.

For the purposes of this study, we group Phases One and 2a and analyse the incremental impact of the Phase 2b Western Leg. This gives us the following three categories that we use when modelling the benefits of the Phases:

- **Phases One and 2a combined** – Linking London to the West Midlands and Crewe.
- **Phase 2b Western Leg increment** – Linking Crewe to Manchester, with a connection to the West Coast Mainline for HS2 services to Scotland. This assumes that Phases One and 2a are already operational.
- **Phase 2b Western Leg Full Network** – Combining Phases One, 2a and 2b Western Leg.

Each Phase of the HS2 project has a different timeline for development and delivery. For the purposes of this S-CGE analysis, the relevant aspects of these timings for each phase of HS2 include the opening year of operational services and the point at which scheme development is sufficiently mature to provide a clear signal of confidence to firms and households that the scheme will be delivered.

The S-CGE modelling considers the economic impacts of HS2 when HS2 services are operational, and the opening year is the starting point for the realisation of user benefits for passengers and agglomeration impacts due to improved rail connectivity. The date of signalling commitment to delivery for each Phase is proxied by the actual/forecast dates of Royal Assent for its hybrid Bill to secure powers for construction. These dates have been provided to us for the purposes for the model by HS2 Ltd, and are outlined below:

- **Phases One and 2a:** Royal Assent has been granted for Phase One in February 2017 and Phase 2a in February 2021. It should be noted that for modelling purposes, it is assumed that the start date for operational services for both Phases 1 and 2a is 2034 as a simplifying assumption.
- **Phase 2b Western Leg:** The Royal Assent date for the Phase 2b Western Leg is assumed for modelling purposes to be December 2024. The assumed start date for operational services on the Phase 2b Western Leg is 2038.

The S-CGE model combines economic data and a system of equations to capture economic interaction between three major economic agents – households, firms, and the Government. CGE modelling is a widely used tool by national governments and other leading international agencies. Each economic agent is defined and linked through labour market or capital market flows, household consumption, intermediate product demand, taxes, or government transfers.

These mechanisms enable CGE models to assess the impacts of different policies, or to investigate the effects of various economic scenarios. For the purposes of this study, we have developed a Spatial CGE (S-CGE) model which splits the UK economy into regions for a more granular analysis that is bespoke for HS2. This enables the model to include trade linkages between regions and sectors.

This report presents the results from our analysis, which covers projected impacts for indicators such as GDP, investment, and welfare through three impact channels: business user benefits, agglomeration, and labour supply

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[2] DfT (2021), Integrated Rail Plan for the North and Midlands, accessible at: https://www.gov.uk/government/publications/integrated-rail-plan-for-the-north-and-the-midlands

effects. We note that our analysis and the results presented in this report are in part based on the business user benefits and agglomeration and labour supply inputs provided by HS2 Ltd. PwC has not been able to quality assure these inputs. As such they are the responsibility of HS2 Ltd.

The S-CGE model assesses the wider economic impacts that are expected to be generated by HS2 services until 2059, after which we project figures forward using steady state growth rate assumptions. These projected impacts are relative to a do-minimum scenario in which HS2 services do not operate.

This S-CGE modelling project started in April 2021 and completed in December 2021. During that period PwC has shaped the model to represent linkages between the 8 identified sectors and 10 UK regions, which are defined later in the report. The scope was agreed with HS2 Ltd to capture some of the key transmission mechanisms by which HS2 will impact the UK economy when operational.

The remaining sections of this chapter explore our sector and region selection process.

2.3.1. Defining the regions

To assess the spatial distribution of wider economic impacts it is necessary to define the regions for our analysis. This section explains how we have defined the regions to be assessed and sets out the key features of these chosen regional economies. This was a process conducted in collaboration with HS2 Ltd.

Defining the appropriate number of regions and the extent of spatial granularity to be included in our wider economic impact assessment involved assessing the availability of key economic data and the practicalities and constraints associated with developing a S-CGE model at different levels of geographic disaggregation.

We defined regions based on the Nomenclature of Territorial Units for Statistics (NUTS) codes of the United Kingdom (Table 3). Out of the three levels of granularity, we disaggregated the UK geographically using NUTS1 regions, which splits the country into Scotland, Wales, Northern Ireland, and 9 statistical regions in England.

We agreed with HS2 Ltd that we would consider ten regions in the model to provide geographic disaggregation to assess the economic impacts of HS2 at a regional level. Three NUTS-1 regions (Northern Ireland, Wales, and the South West), were grouped into one territory termed the 'Rest of the UK'.

Table 3: Regional allocations in the HS2 S-CGE model

Region Name	NUTS 1 Region Code	Description
North East England	UKC	North East (UKC)
North West England	UKD	North west (UKD)
Yorkshire and the Humber	UKE	Yorkshire and the Humber (UKE)
East Midlands	UKF	East Midlands (UKF)
West Midlands	UKG	West Midlands (UKG)
East of England	UKH	East of England (UKH
South East	UKJ	South East (UKJ)
London	UKI	London (UKI)
Scotland	UKM	Scotland (UKM)
The Rest of the UK (Northern Ireland, Wales and South West)	UKN, UKL and UKK	Northern Ireland (UKN), Wales (UKL) and South West (UKK),

Each of the ten regions we assessed exhibit distinctive economic and demographic traits, which are likely to have implications for the potential economic impacts of HS2. This section compares the ten regions to understand their economic and demographic landscape.

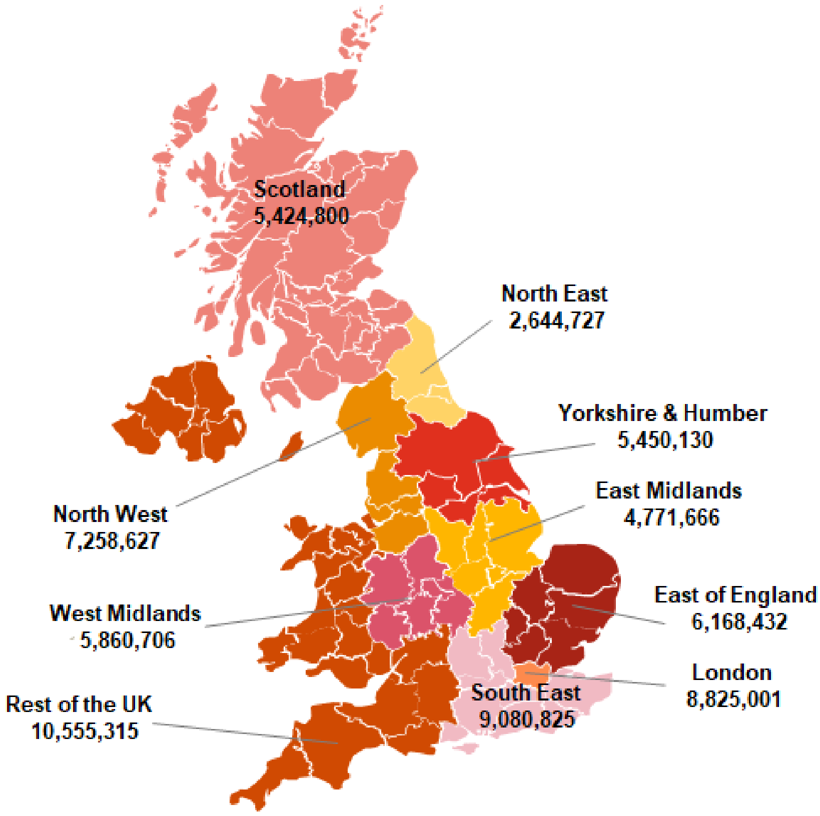
2.3.2. Structure of the economy

Population

We set out each region's total population in Figure 1 below. The North East has the smallest population of 2.6 million people according to 2017 ONS figures. By contrast, London has around 8 million people and the South East has a population of around 9 million. This highlights that the ten regions differ in terms of size of their potential labour force.

Throughout this Section (2.3) we refer to 2017 data to be consistent with the assumptions that underpin our CGE model, which uses 2017 data.

Figure 1: Population of different regions, 2017

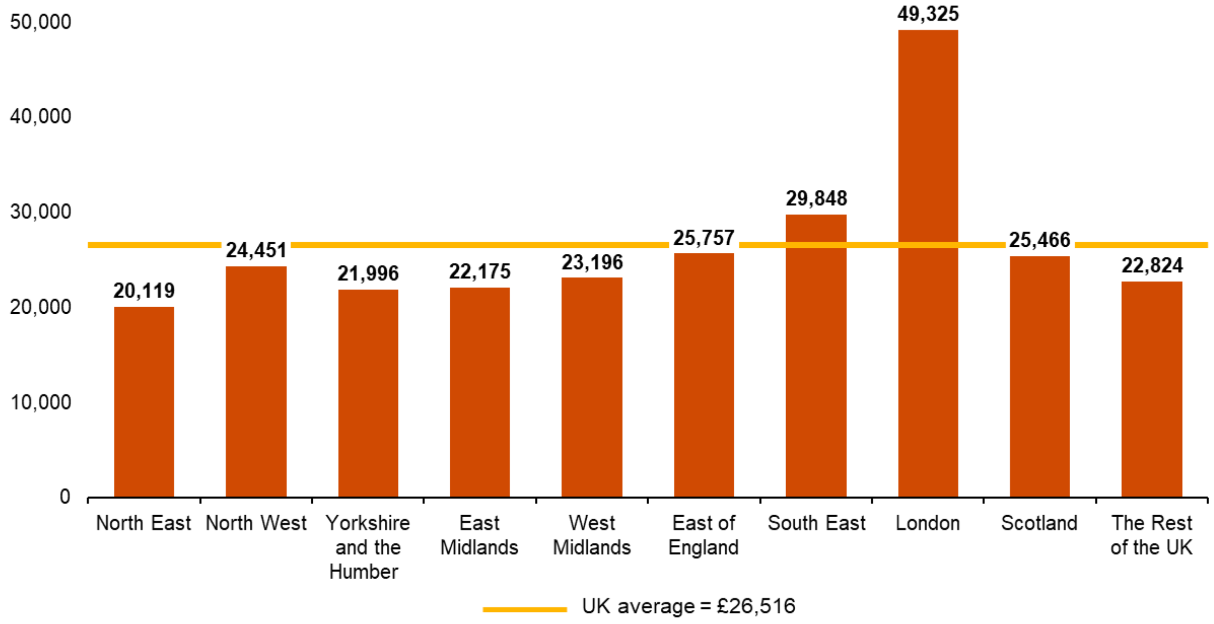


Source: ONS and PwC analysis

GVA per capita

Gross Value Added (GVA) per capita measures the average contribution of an individual to economic output and gives an indication of income and the standards of living in each region. Figure 2 provides a comparison of GVA across each of the regions, as well as how each region compares to the UK average. Only London and the South East regions have GVA per capital levels above the UK average.

Figure 2: GVA per capita per region, 2017, £

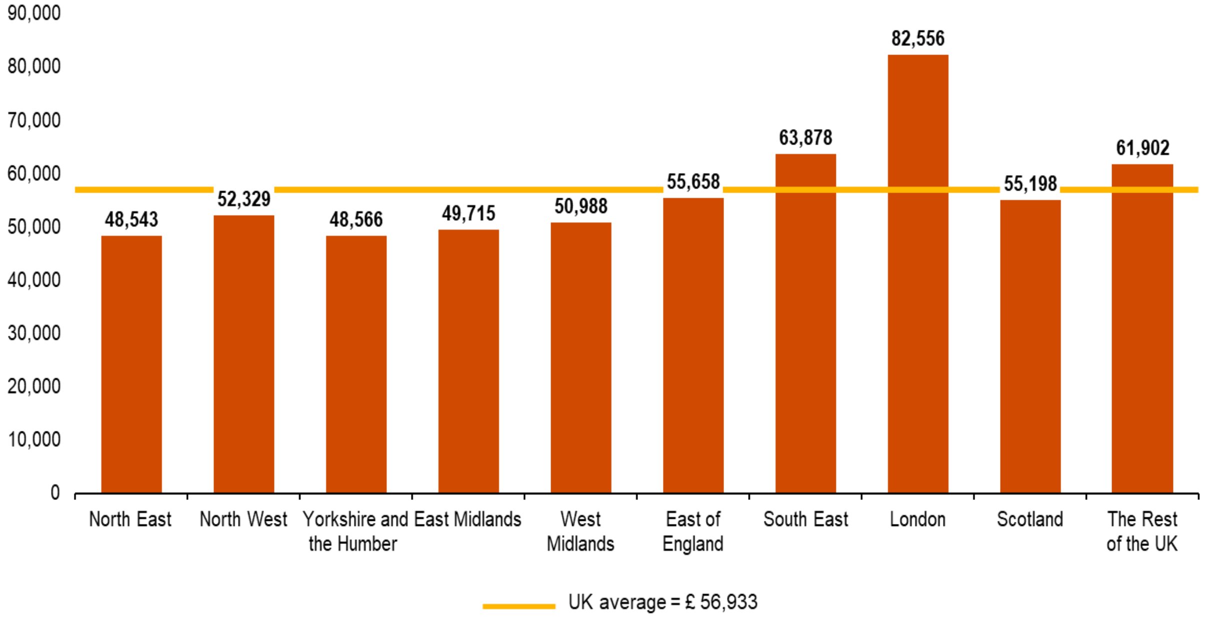


Source: ONS Annual Population Survey and PwC analysis

Productivity

GVA per worker provides a measure of productivity and Figure 3 shows a clear disparity in productivity levels across our selected regions.

Figure 3: GVA per worker, workplace population, 2017, £

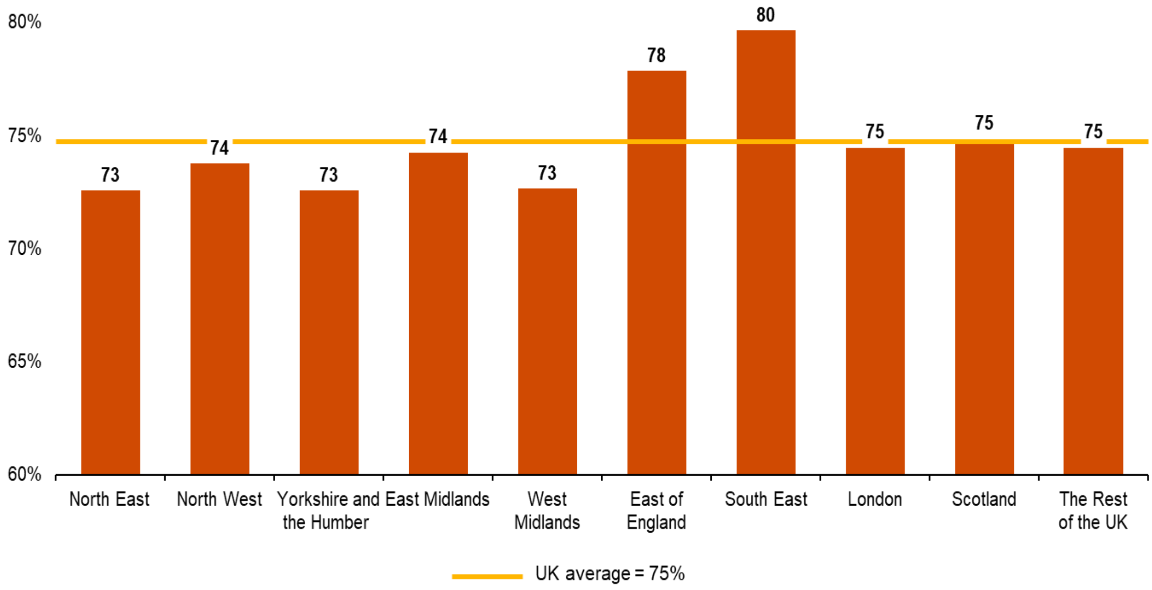


Source: ONS and PwC analysis

Employment

Employment rates are calculated by the ONS as the number of employed people aged from 16 to 64 divided by the population aged from 16 to 64. Population is the sum of employed plus unemployed plus inactive.[3] The rest of the UK is a weighted average of the relevant regions, according to the aggregation shown in Table 3. Figure 4 displays the results of this regional comparison of employment rates for December 2017. The South East and East of England are markedly higher and the North East, Yorkshire and the Humber and West Midlands lower than the UK (weighted) average.

Figure 4: Employment rates by region, as of December 2017, %



Source: ONS and PwC analysis

2.3.3. Defining the sectors

To assess the wider economic impacts of HS2, it is necessary to define appropriate sectors for our analysis. We used the Standard Industrial Classification (SIC) to group key industries into eight sectoral groups. SIC is the standard approach used by the Office for National Statistics (ONS) in classifying economic activities, providing the framework for collecting and presenting a large range of statistical data by economic activity.

We agreed with HS2 Ltd to use 8 sectoral groups in our S-CGE analysis, as summarised in Table 4 below. Appendix (Section A.11) provides a detailed description of the sector allocation.

Table 4: Sector allocations used in HS2 S-CGE model

Sector Grouping	Abbreviation
Advanced manufacturing	ADVMANU
Business services	BUSSERV
Construction	CONSTRUCT
Consumer services	CONSSERV
Digital	DIGITAL
Discretionary consumption	DISCON
Financial services & professional services	FINPROFSERV
Other	OTH

[3] ONS, Regional labour market statistics in the UK: December 2017, release date 13.12.2017, accessed 29.10.2021, accessible: https://www.ons.gov.uk/employmentandlabourmarket/peopleinwork/employmentandemployeetypes/bulletins/regionallabourmarket/december2017

Sectoral makeup

Each of the ten regions considered in our analysis has a different economic structure. Figure 5, below, shows the distribution of employment across sectors and regions.

Figure 5: Sectoral employment distribution, 2017

Source: Office for National Statistics

2.4. Report outline

2.4.1. Part I – Executive summary & Purpose of the report

Chapter 1 Executive summary: This chapter summarises the scope of the report, the strategic objectives of the HS2 programme, project background and context. We also provide an overview of chosen model parameters, methodology and conclude with the summary of our results.

Chapter 2 Purpose of this report: In this chapter we describe the context for this report. We briefly outline the phased delivery of HS2, describe the S-CGE model and how the S-CGE results can be interpreted.

2.4.2. Part II – Overarching approach

Chapter 3 Overarching approach: We outline the approaches to appraising transport projects and explain the rationale for our methodology. Following that, we explore the economic impact themes and provide a high-level description of the methodology that we use in the assessment of HS2.

Chapter 4 Estimating inter-regional trade using the I-O Approach: In this chapter, we provide a high-level description of the input-output (I-O) model and the approach that we use to define and build the I-O table for each local area. Specifically, we outline data sources used as well as assumptions taken to create the bespoke inter-regional I-O table.

Chapter 5 Overview of our S-CGE model: We begin this chapter by providing more detail on S-CGE models, and the relationships within an economy that they capture. This is followed by a literature review covering the use of S-CGE modelling within transport appraisal, our modelling approach and analysis. We then outline the key features of the S-CGE model used in this analysis, followed by a description of the data used in the model.

Chapter 6 Incorporating effects into S-CGE model: In this chapter, we describe in detail how we use the outputs from the models operated by HS2 Ltd as two inputs within our S-CGE framework. We set out our assumptions behind the adjustments made to HS2 Ltd's model outputs for them to be used as inputs into our S-CGE model.

2.4.3. Part III – Results

Chapter 7 Risks and Uncertainties: In this chapter, we consider the main sources of uncertainty, including the uncertainty over S-CGE model inputs (i.e., Business User Benefits, Agglomeration and Labour Supply); and uncertainty over the model parameters such as the behavioural responses of firms and households in the model.

Chapter 8 National Results: In this chapter we present the results of our analysis for the UK economy. We begin this chapter by focusing on the overall projected GDP impact of HS2's Phase 2b Western Leg Full Network (i.e., Phases One, 2a and 2b Western Leg), and outline what drives these impacts. We then present the estimated impacts on economic variables such as GDP and investment.

Chapter 9 Regional Results: This chapter presents results of exploratory analysis at a regional level. This chapter presents the regional distribution of GDP impacts for the HS2 Phase 2b Western Leg Full Network.

Chapter 10 Welfare Impact: This chapter presents the welfare impacts from the S-CGE model including an overview of the consumption-leisure substitution effects modelled.

2.4.4. Part IV – Appendices

Appendix A: Detailed description of the S-CGE model structure. Here we set out in greater detail the key components and assumptions within the S-CGE model, which are introduced in Chapter 5. We outline the structure of the S-CGE model, discuss the role of the government and its influences on consumption and production, and identify the nuances of the model, including the labour market, the dynamic nature of the model, imperfect competition among firms, inter-regional trade, non-resident household expenditure and international trade.

Appendix B: Sensitivity tests. This appendix presents and discusses results from sensitivity testing of the S-CGE model, with tests varying the distance decay factors, alignment of expectations assumption and labour-leisure substitution elasticity.

Appendix C: Assumptions in the model. Here we set out the underlying assumptions for the model.

3. Overarching approach

3.1. Chapter overview

In this chapter, we discuss our modelling approach from two perspectives.

- We first consider the impact channels that have been informed by the Department for Transport Analysis Guidance (TAG) framework in sub-section 3.2 and how our S-CGE modelling exercise should be understood within that framework in sub-section 3.3.
- We then provide an overview of our three-step approach to S-CGE modelling in Section 3.4. This also introduces the more detailed methodological discussions in Chapters 4, 5 and 6.

3.2. Categories of economic impacts

In 2014, the DfT commissioned Professor Tony Venables, Dr James Laird and Professor Henry Overman to review current methods for appraising transport schemes.[4] In their report, Venables et al. (2014) set out the different ways transport investments could affect the wider economy:

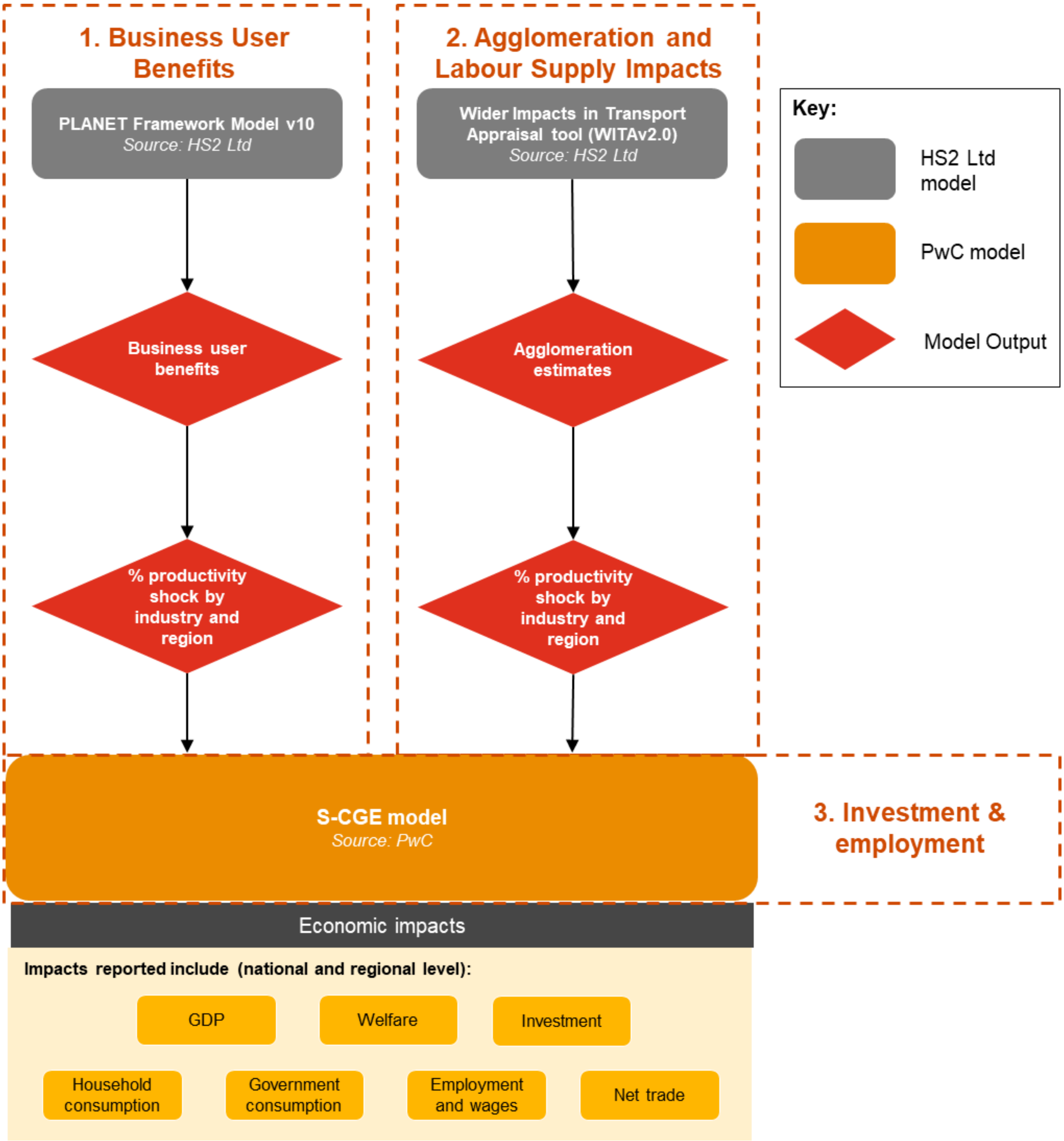
1. **User benefits:** direct effects – such as time savings – that are experienced by the users of the transport scheme because of the investment. We only consider user benefits to businesses in the S-CGE model. From this point forward, 'user benefits' should be taken as shorthand for business user benefits.
2. **Cities and Agglomeration:** productivity gains experienced by workers and businesses because of better connectivity. Using the DfT's terminology, better connectivity can lead to 'static clustering' (workers and firms becoming effectively closer together) and/or 'dynamic clustering' (where labour and firms physically relocate because of a transport scheme).[5] Within this channel, Venables et al. (2014) capture access to economic mass over both short and long distances; and
3. **Investment & employment** could be attracted to places with improved transport. This is often a subsequent effect that results from improvements in user benefits or agglomeration. Care must be taken in ascertaining whether (i) the benefits are additional; and (ii) not displaced from elsewhere in the UK. A large and potentially transformational project like HS2 will affect competition in the markets within which firms operate, the performance of the firms within those markets, the way in which competitors are linked and the use of labour and capital in all these markets – in other words, general equilibrium effects.

We have modified and used a S-CGE model to simulate these general equilibrium effects through investment and employment. This exercise constitutes a supplementary economic model (SEM) under DfT TAG's Unit A2.1, in line with the rationales set out in paragraph 3.4.2 of that Unit of TAG. This allows us to obtain context-specific estimates of welfare impact in the case of a large transport infrastructure scheme of the scale of HS2 (Point (ii)), as well as to estimate sub-national impacts (Point (iv)). This is summarised in Figure 6.

[4] Venables, A., Overman, H. and Laird, J. (2014). 'Transport Investment of Economic Performance', a paper prepared for the DfT.

[5] Department for Transport (2014) Understanding and Valuing the Impacts of Transport Investment.

Figure 6: Economic impact themes



Source: PwC

We would expect HS2 to affect the wider economy across four broad areas of impact (or economic impact themes). We discuss each of these impacts in turn, beginning with the three themes proposed by Venables et al. (2014).

3.2.1. Business user benefits

User benefits are the direct effects experienced by the users of the transport scheme. User benefits arise through a change in generalised travel costs associated with improved transport[6]. These include savings from improvements in:

[6] Venables, A.J., Laird, J., & Overman, H. (2014). Transport investment and economic performance. Paper commissioned for the UK Department of Transport

- **Journey time** – time foregone for travel which could otherwise be used for work, commuting and/or leisure. This includes time spent travelling, as well as time spent walking, accessing public transport and wait time (which is related to the frequency of public transport savings) and the impacts of improvements in the reliability of services; and
- **Other travel costs**[7] – include user charges such as fares, tariffs, and tolls.

Together these categories comprise generalised travel costs.[8] The HS2 scheme is expected to reduce generalised travel costs for both existing and new users of rail, due to faster, more frequent, more reliable, and less crowded HS2 services, as well as due to the capacity that HS2 will release on the conventional rail network (CRN).

Under the DfT TAG framework, the appraiser is to compare traffic forecasts in the absence of transport investment with forecasts post-investment to calculate the change in generalised journey times and composition of journeys. It then draws on extensive empirical research on economic agents' willingness to pay for transport improvements and evidence on cost savings to ascribe monetary values to generalised journey time savings and reductions in other transport costs. TAG also attempts to capture reliability benefits by calculating the standard deviation of journeys and comparing this to mean journey times.[9]

We have not attempted to estimate Business User Benefits directly. Sector-level and region-level estimates of business user benefits were provided by HS2 Ltd derived from the PLANET Framework Model (PFM) version 10. We worked together with HS2 Ltd to allocate these Business User Benefits to individual sectors in each region, which we treated as productivity improvements that drive subsequent economic benefits in the S-CGE model. This is discussed in more detail in Chapter 6.

3.2.2. Cities & Agglomeration

Businesses, workers, and consumers can experience gains through the improved connectivity HS2 will bring without using HS2 services. This includes the two impact channels that we have modelled in this S-CGE exercise.

Firstly, the HS2 scheme will bring businesses in different economic centres along the HS2 line (and lines that benefit from released capacity due to HS2) closer to each other by reducing generalised journey times through faster, more frequent, and more reliable transport connections between economic agents (firms, workers, and consumers), thereby increasing the 'effective density' between firms in their current locations. This allows them to collaborate with and compete against each other more effectively and brings productivity benefits. In this report, this effect is referred to as 'agglomeration'.

In TAG, the estimation of this productivity benefit – also known as static clustering – starts by estimating the effective density for areas affected by a scheme using generalised travel costs between each area under two scenarios: with the scheme and in the absence of the scheme. It then uses elasticities of productivity with respect to agglomeration (effective density), estimated by Graham et al. (2009), to assign productivity impacts to each area[10]. Note that estimates of static clustering assume fixed patterns of land use – that is, that firms and households do not change their location decisions in response to the improved transport connectivity.

Secondly, lower generalised travel costs could affect individuals' labour supply decisions. Transport costs are an important part of individuals' thinking when deciding whether to participate in the labour market[11]. In TAG, the estimation of the benefits associated with this labour supply effect starts with assessing the average change in generalised commuting costs between the scenario in the absence of the scheme and the scenario with the scheme. This is used to proxy the average change in the annual return from working. Labour supply elasticities with respect to wages are then applied to these changes to derive changes in labour supply.

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[7] DfT (2014) WebTAG UNIT A1.3. User and Provider Impacts, Available at: https://www.gov.uk/transport-analysis-guidance-webtag

[8] User benefits may also include a change in operating costs – (also referenced in WebTAG UNIT A1.3, see above footnote). These are made up of fuel and non-fuel operating cost, where non-fuel costs include stock maintenance and energy related depreciation.

[9] Same reference as footnote 6.

[10] Graham, D. J., Gibbons, S. and Martin, R. (2009). Transport investments and the distance decay of agglomeration benefits. Draft report for the Department of Transport. Paper commissioned for the UK Department of Transport.

[11] The non-exchequer impacts are captured in the calculation of commuter user benefits, as described above.

Both channels could generate subsequent round effects, which are estimated by the S-CGE model. TAG recognises that there are dynamic clustering effects that may result from agents changing their location of economic activities and that they could only be estimated with SEMs such as a S-CGE model.

In this exercise, HS2 Ltd provided us with both agglomeration and labour supply estimates that have been derived from the Wider Impacts in Transport Appraisal (WITA) model, which we used as inputs into the S-CGE model. This is discussed in more detail in sub-section 6.3.2.

3.2.3. Investment & Employment

The final economic impact channel specified by Venables et al. (2014) is Investment & Employment. They note that transport investment can lead to impacts on the local, regional, and possibly national economies because it has the potential to attract inward investment and 'change the spatial distribution of economic activity' (Venables et al. 2014, page 42).

Better connectivity may change the location of economic activity through dynamic clustering. The development of new economic clusters may attract inflows of investment and could lead to changes in land use as clusters mature. Furthermore, these changes may alter employment opportunities across regions. However, the extent to which this impact is additional or displaces activity elsewhere will depend upon local and regional economic conditions (such as employment levels, the skill composition of the labour force and the source of inward investment). These two aspects – additionality and displacement – are discussed below:

- **Additionality** – It is important to understand whether the investment or employment effects would have occurred in the absence of the scheme. Only if the effects would not have occurred in the absence of the scheme can the effect be regarded as additional and therefore be counted as a net economic impact; and
- **Displacement** – Even if a transport investment stimulates truly 'additional' investment, it may use resources that might have been used elsewhere in the UK.

Our S-CGE modelling exercise estimates some of these 'subsequent round effects' as they diffuse and ripple across the economy. For instance, as demand increases because of additional investment and household consumption spending, prices are driven up, which dampens increases in output. These price rises will be larger, and hence have a greater dampening effect on output, where labour and capital are already fully or nearly fully employed, and hence increased activity (whether direct, indirect, or induced) needs to compete for labour and capital with alternative users, and so on.

This process will continue until all markets clear across the whole of the UK economy in a general equilibrium; only once the effects have flowed through the entire economy can the total wider economic impact be assessed.

3.2.4. Summary & Limitations

Table 5 summarises how our S-CGE model captures economic impacts through different channels.

Table 5: A summary of how our modelling captures economic impacts through different channels

Economic impact channels	Source	Model
Business User Benefits	Venables et al. (2014)	Conventional appraisal methods in TAG estimate partial equilibrium welfare estimates of user benefits. HS2 Ltd has provided these for business users. These estimates are inputted into the S-CGE model to assess the subsequent round effects related to business user benefits.

Economic impact channels	Source	Model
Cities & Agglomeration	Venables et al. (2014)	Conventional appraisal methods in TAG only capture the effects of static agglomeration impacts (i.e., it assumes that economic agents and factors of production do not move spatially). These estimates for agglomeration and labour supply effects have been provided by HS2 Ltd. These estimates are inputted into the S-CGE model to assess the general equilibrium effects of static agglomeration impacts. Any dynamic clustering that occurs (i.e., improvements in productivity which accrue because of the physical movement of factors of production and/or economic agents) is captured in the S-CGE model, as changes in factor prices lead to movements of economic agents across sectors and/or regions. The results in this report reflect the effects of both static and dynamic clustering.
Investment & Employment	Venables et al. (2014)	Investment & Employment impacts are not captured by Conventional Appraisal Methods in TAG. Several SEMs, including S-CGE modelling, are recognised in TAG to be useful in estimating these impacts. These effects are captured within the S-CGE model and can be thought of as a subset of the subsequent round effects arising from the User Benefits and Cities & Agglomeration channels.

The net impact estimated by a S-CGE model accounts for both the additionality and displacement effects of the BUBs and agglomeration and labour supply effects that served as inputs into the S-CGE modelling. This is in line with the recommendations in TAG Units A2.1 and M5.3.

3.3. The S-CGE model's place in the Transport Analysis Guidance framework

In this section, we set out this S-CGE simulation exercise in the context of the wider transport appraisal described in the above sub-section.

With reference to the three categories that Venables et al. (2014) has set out:

- **Business User Benefits** have been provided by HS2 Ltd. We have used the Business User Benefits to derive productivity improvements for businesses as inputs into the S-CGE model, which we used to estimate their subsequent round effects. It is worth noting that the user benefits of non-business users (i.e., those of commuters and leisure travellers) are not captured in our S-CGE modelling exercise.
- **For Cities & Agglomeration,** we have used the agglomeration and labour supply effects that have been provided by HS2 Ltd to derive productivity improvements that are then used as inputs into the S-CGE model. These estimates assume fixed land use. We can convert static clustering effects into general equilibrium effects, as well as capturing effects through dynamic clustering and competition under imperfect markets.
- The S-CGE model captures the subsequent round effects through the **Investment & Employment** category, as it captures both feedback loops and relative price changes to provide more comprehensive analysis of economic impacts. In principle, S-CGE models can capture all the economic impact channels that we identified in Section 3.2 and, due to their general equilibrium structure, are particularly well-suited to capturing subsequent round effects.

As Venables et al. (2014) observed, S-CGE models capture the 'ripple effect of changes in demand in one sector on other sectors' as they are 'partially crowded out through price adjustments reflecting supply side constraints. Such models therefore have the ability to capture additionality in output and employment.'

In recognition of this ability, TAG Unit A2.1 has since September 2016 recognised the role of Supplementary Economic Modelling (SEM) such as S-CGE models to capture a broader range of more context-specific wider economic and welfare impacts. As shown in Table 6 below, our S-CGE modelling exercise has been carried out with reference to the robustness criteria that were set out in the TAG Unit M5.3.

Table 6: Summary of the robustness of our S-CGE modelling

Model robustness criterion	Remarks
6.2 Economic Principles	Our S-CGE model is aligned with the economic principles set out in TAG. It assumes full employment in the long run and hence there may be displacement effects. Our model also considers market imperfections.
6.3 Baseline Assumptions	We report the baseline assumptions underlying the 'without scheme' scenario in all our reports and communicate net impacts from a scheme.
6.4 Model Geographic Scope	Our S-CGE model is limited to a specific number of model zones. We have discussed and agreed the regional definition with HS2 Ltd, DfT and peer reviewers during a workshop.
6.5 Transport Accessibility Improvement	In Section 6.4, we set out how the benefits from transport accessibility improvements under the categories we set out are captured in the S-CGE mode to estimate the economy-wide net impacts.
6.6 Macroeconomic Projections	The underlying macroeconomic projections are based on historical economic trends in the ten years prior to the outbreak of the COVID-19 pandemic (i.e., between 2009 and 2019). Hence, we assume real GDP growth of 2.1% p.a. in our underlying projections. This is higher than current DfT TAG assumptions (between 1.5% and 1.7% p.a.). This does not affect the model results in percentage differences from the baseline, only the absolute size of the estimated impacts of HS2 compared to the baseline.
6.7 Model Structure	In Appendix A, we report the key mathematical relationships underpinning the model and provide reasoned explanations for each.
6.8 Model Parameters	In Appendix C, we report the key parameters underpinning SEMs, including sources and evidence indicating the level of uncertainty associated with these estimates.
6.9 Displacement Effects	S-CGE models explicitly estimate displacement effects through price changes and movement of economic activities.
6.10 Estimating Social Welfare Impacts	Our approach to S-CGE modelling allows GDP impacts to be reconciled with impacts on social welfare. We report and discuss these results in Chapter 10.
6.11 Complementary Interventions	We focus on estimating the net effect of the HS2 scheme by holding all other things – including any interventions that may be contemporaneous to the HS2 scheme – unchanged. Therefore, we did not attempt to explicitly account for any complementary interventions.
6.12 Sensitivity Testing	Sensitivity tests have been carried out; their results are set out and discussed in Appendix B.
6.13 Realism Tests	We have carried out realism tests by way of impact multipliers between the magnitudes of the impacts and those of the inputs. We discuss their results in Section 8.1.
6.14 Consistency with Conventional Appraisal Methods	The connection between our S-CGE modelling exercise and more conventional appraisal methods has been discussed earlier in this section.
6.15 Independent Peer Review	Our work has been reviewed by independent peer reviewers commissioned by HS2 Ltd.

It is worth noting that this S-CGE simulation exercise – itself a SEM under TAG Unit A2.1 – is only part of the assessment of the impacts of HS2. For example, TAG considers other factors such as environmental effects and safety, which are outside the scope of this analysis.

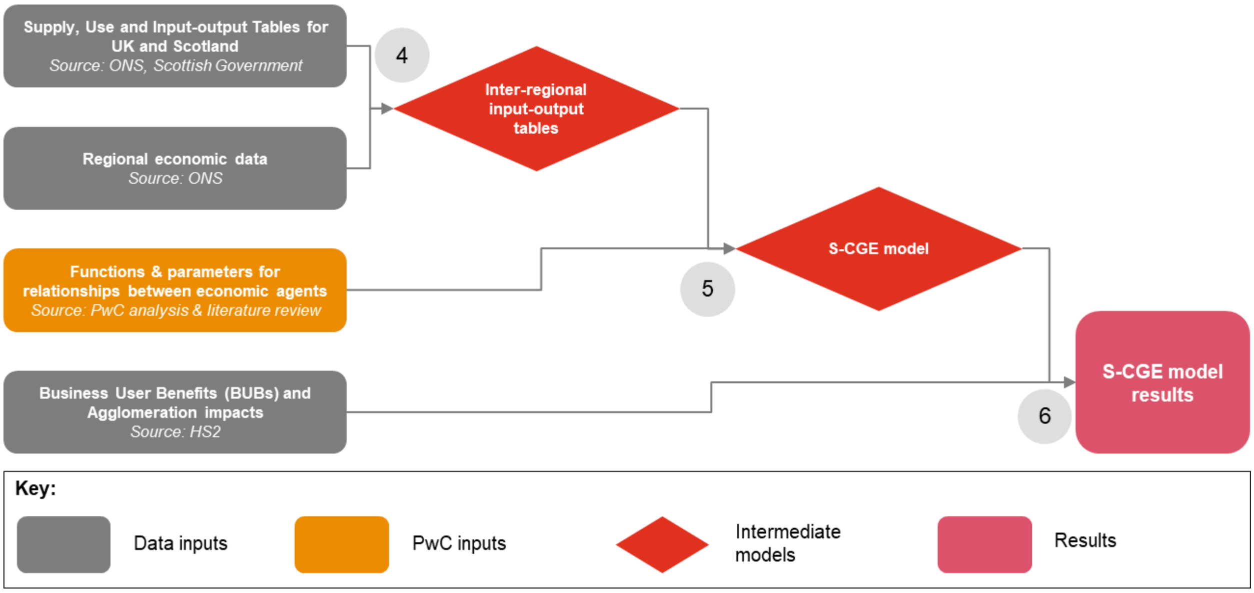
3.4. Approach to modelling

We have taken a three-step approach in this S-CGE modelling exercise for the HS2 scheme:

- Deriving a set of inter-region and inter-sector Supply, Use and Input-Output Tables (SUIOTs) that are tailored to the appraisal method. The SUIOTs describe the economic relationships between businesses in different sectors and between different regions, which are essential ingredients in building and calibrating a S-CGE model. We discuss this in more detail in Chapter 4.
- Building and calibrating a bespoke S-CGE model with these SUIOTs, as well as equations and parameters grounded in academic and policy literature. This specifically tailored S-CGE model is described in more detail in Chapter 5.
- Applying inputs provided by HS2 Ltd into the model to obtain simulation results over a wide range of economic indicators for different regions, sectors, and over time. We discuss this step in more detail in Chapter 6.

This is set out in Figure 7.

Figure 7: Three step approach to S-CGE modelling of HS2



Source: PwC

CGE modelling allows us to account for knock-on impacts, whether they take place within or between regions. It is worth noting that CGE models have become an established tool to evaluate key policy decisions in the UK. For example, HMRC has used a CGE model when assessing the impact of changes in taxation on the UK economy.[12]

More recently CGE models have been used to appraise large transport infrastructure projects both in the UK and internationally. For example, within the UK, we have used a version of our model for the Airports Commission and for the Lower Thames Crossing wider economic impact assessment. CGE models have also been used in the Netherlands and Australia. Such appraisals of transport schemes have used 'Spatial' CGE models, which have been enabled by recent advances in the estimation of economic data at the regional level and have been adopted in our approach to modelling the economic impacts of HS2.

[12] For example, see: HMRC's CGE model documentation, December 2013, accessible at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/263652/CGE_model_doc_131204_new.pdfhttps://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/263652/CGE_model_doc_131204_new.pdf

4. Estimating inter-regional trade using the I-O approach

Prior to S-CGE modelling, it is necessary to determine inter-regional trade flows as the basis for the model to operate on. The inclusion of inter-regional trade flows within the model are required before being able to understand the regional impacts of the productivity impacts. As such, we used input-output analysis across sectors and regions included in our model to determine flows between regions and across sectors for the UK.

4.1. What is an input-output table?

The Input-output ('I-O') table describes how each sector demands inputs from other sectors (and from within the same sector). Input-output tables are constructed by combining and transforming two important data sources – The Use Table and The Supply Table. The Use Table provides data on the products consumed by each sector of the economy, while The Supply Table provides data on the products produced by each sector of the economy.

We construct a bespoke set of supply, use and I-O tables with the purpose of estimating the inter-regional trade across different regions and sectors within the UK, which we then use as an input to our S-CGE model to form the baseline UK economy scenario.

4.2. Our approach

We used official statistics data to build a bespoke economic model that simulates the linkages between the ten regions and eight sectors, described in Chapter 2, to capture the impacts of HS2's operations on the UK economy.

We used a four-step approach to build the inter-regional I-O table; these are:

- **Step 1:** Data collection;
- **Step 2:** Amalgamation of the UK and Scottish I-O tables;
- **Step 3:** Apportioning the aggregated I-O table to the regions; and
- **Step 4:** Applying entropy algorithm to balance the inter-regional I-O table

Step 1: Data Collection

We sourced the following data from the Office for National Statistics (ONS) and the Scottish Government:

- National and regional GVA, consumption and employment data by sector; and
- The UK's and Scotland's I-O tables.

Our analysis used the 2017 UK I-O table published by the ONS and the 2017 Scottish I-O table. The ONS updates the figures approximately every year, with the latest published in 2017. The I-O tables published by the Scottish Government have 105 and 98 sectors. These are the most recently available data at the time of modelling.
We combined these sectors to the eight sectors identified for the purposes of this modelling exercise by summing the constituent rows and columns of the original tables.

We obtained employment data for 2017 sourced from the Office for National Statistics (ONS) Business Register Employment Survey to supplement our analysis of the employment multipliers. We note that the employment data includes employees plus the number of working owners, and, therefore, includes self-employed workers that are registered for VAT or PAYE. A similar sector mapping procedure has been used for the employment estimates.

Step 2: Amalgamation of the UK and Scottish tables

We used data from the Scottish I-O table as a basis on which to derive trade between Scotland and the Rest of the UK, which is then incorporated into the UK I-O table to derive an adapted version of the UK input-output table which ensures that we effectively capture the trade between Scotland and the rest of the UK within our analysis.

We then constructed an I-O table with the two regions (Scotland and the rest of the UK) to give us trade estimates across the 8 sector allocations, for:

- **Scotland's I-O tables:** These estimates are taken from the 2017 Scottish I-O table.
- **Scotland's imports from the rest of the UK:** From the Scottish I-O table, we can source information on the imports by each sector in Scotland from the Rest of the UK. This was then apportioned across the 8 sectors by applying the UK import share for the selected imported product against the total imports of the selected sector. We note that the column sum of this is consistent with the Scottish I-O as the values for Scotland's imports from the rest of the UK is known.
- **Scotland's exports to the rest of the UK:** From the Scottish I-O table, we were able to source information on the exports by sector in Scotland to the rest of the UK. This was then apportioned across the sectors by applying the UK import share for the selected imported product against the total domestic use of the selected sector (total use excluding exports). We note that the row sum of this is consistent with the Scottish I-O as the values for Scotland's exports to the rest of the UK is known.
- **The rest of the UK I-O table:** This is derived from deducting all the above components from the UK 2017 I-O table estimates for the relevant sector and region.

Step 3: Apportioning the aggregated I-O table to the regions

We adjusted the amalgamated UK and Scottish input-output table from Step 2 using public data sources. Three main adjustments were made to apportion the aggregated I-O tables into an extensive inter-regional table that reflects the trade between the 10 regions: (1) Gross Value-Added weights; (2) sector location quotients; and (3) region-to-region inverse distance weightings. We note that adjustments were mainly made to the rest of the UK trade and consumption estimates whilst making minimal changes to the Scottish estimates, such that it is consistent with the Scottish I-O table values. We describe these three adjustments in detail in Appendix C (Section C.4.1).

It is worth noting that, at this stage, the uses and supplies for each sector's products are not yet balanced, as this process is applied in the next step.

Step 4: Entropy algorithm and balancing

We balance the I-O table with an entropy algorithm. For this step we looked to balance the inter-regional I-O table derived from step 3 having conducted apportioning procedures. Firstly, we adjusted Scotland's inter-regional trade and consumption estimates, such that the new summations were equivalent to the original totals prior to adjustments made using the region-to-region inverse distance weighting. We did this by applying a residual ratio between the original estimated totals (before Inverse Distance Weightings (IDW) adjustments) and new estimated totals (post IDW adjustments) for each of the components. Next, we used a cross-entropy methodology within a program where we take the inter-regional I-O table, to which we reconciled by making minimal changes to the matrix using various optimisation techniques. This methodology is described in detail in Appendix C (Section C.4.2).

5. Overview of our S-CGE model

5.1. Chapter overview

In this chapter we set out further details of the S-CGE model developed for this study, and the relationships within an economy that it is intended to capture.

Since emerging in the 1970s, CGE models have become an important tool for analysing the impact of various economic scenarios and policies. They are widely used by international institutions such as the World Bank, IMF, OECD, and European Commission, as well as the UK Government. In 1996, the UK Secretary of State for Transport asked the Standing Advisory Committee on Trunk Road Appraisal (SACTRA)[13] to explore the nature and significance of the relationship between transport provision and economic growth. One of the recommendations made by the Committee was that the Department should undertake further development of CGE modelling of the total economic impacts of transport schemes.

Updates to TAG Unit A2.1 and M5.3 were published in September 2016, recognising the role of Supplementary Economic Modelling (SEM) and non-standard appraisal methods, such as S-CGE models, to capture a broader range of more context-specific wider economic impacts. The assessment of wider economic impacts is complex and inherently uncertain, particularly when attempting to estimate the wider economic impacts of large national transport schemes such as HS2. Given the challenges, SEM and non-standard appraisal methods, such as CGE, are intended to supplement, rather than replace standard appraisal methods.

In this chapter we explain our application of S-CGE modelling in more detail. Specifically, we provide:

- A summary of CGE modelling concepts;
- A literature review of CGE models and their application to the appraisal of transport infrastructure; and
- An overview of the S-CGE model used in this report.

5.2. What is a S-CGE model?

The S-CGE model combines economic data and a system of equations to capture economic interaction between three major economic agents – households, businesses, and the Government. Each economic agent is defined and linked through labour market or capital market flows, household consumption, intermediate product demand, taxes, or government transfers (see Figure 8 for a highly simplified representation of these interactions).

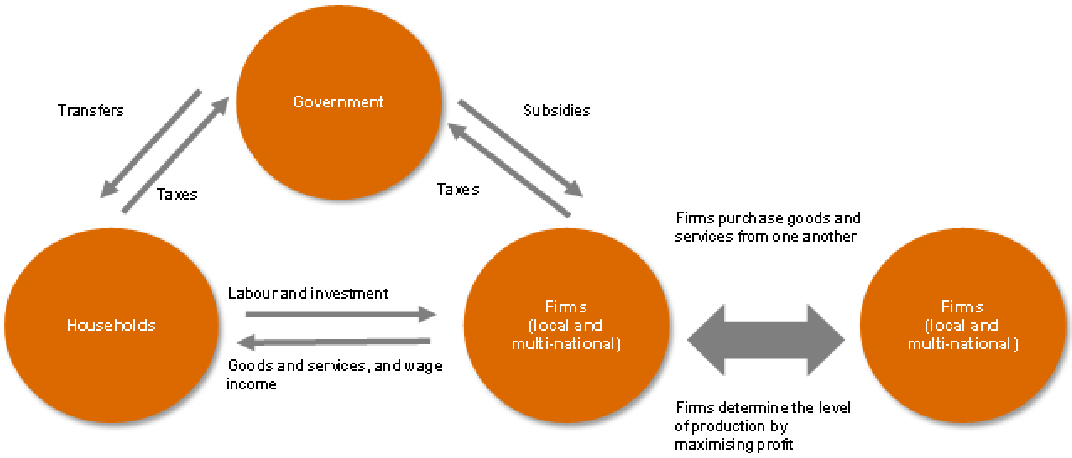
The economic linkages and systems that S-CGE models' proxy are complex. The multiple households and businesses that are defined in each model engage in repeated local and regional microeconomic interactions. This, in turn, gives rise to macroeconomic relationships affecting variables such as employment, investment and GDP. These macro relationships also feed back into the determination of local micro interactions. Because of this relationship, CGE models are often referred to as micro-macro models (Sue-Wing and Balistreri, 2012).[14]

These mechanisms and relationships enable S-CGE models to assess the impacts of different policies, or to investigate the effects of various economic scenarios.

[13] SACTRA, (1999). Transport and the economy: full report.

[14] Sue-Wing, I., & Balistreri, E.J. (2012). Computable General Equilibrium Models for Economic Policy Evaluation and Impact Analysis. Working Paper, Department of Earth and Environment, University of Boston.

Figure 8: Main relationships captured in a CGE model



Source: PwC

5.2.1. How does a regional model differ from a national model?

A national UK-level CGE model views the UK economy as one whole region, whereas a regional model breaks the economy down into various regions. As we have added a regional dimension to our analysis, we refer to our model as a Spatial-CGE model, or S-CGE model. As we set out in sub-section 2.3.1 and agreed with HS2 Ltd, we have divided the UK economy into ten geographical regions to be able to model the economic impact within and between areas (see Table 3).

One of the benefits of using the S-CGE model specification is that it includes trade linkages between regions. This enables the model user to understand trading patterns between regions and provides a fuller picture of behaviours in the UK economy. Though trade patterns between regions may vary over time in our model, the net exports for the UK are assumed to remain constant over time. This is due to the long timeframe that the model covers, as over the appraisal period it would likely be infeasible to assume that either: (1) the UK could maintain a permanently higher trade deficit on foreigners' largesse; or (2) the UK could improve its trade position permanently (implying another nation has a permanently higher trade deficit).

The S-CGE model also captures the effects of movements in economic activity across regions. In this case, if wages in the West Midlands were to rise, and workers from other regions were to migrate to the West Midlands due to the attraction from higher wages, then (all else equal) the level of GDP in other regions is likely to fall as economic activity leaves these areas. The model captures the effects of labour and capital movements between regions and models the net effects on the UK economy. This is known as the 'dynamic clustering' effects referred to by Venables et al. (2014).

5.2.2. Key factors to consider when specifying S-CGE models

In this sub-section, we summarise three factors – identified by Partridge and Rickman (2010)[15] – which define best practice and should be taken into consideration when constructing S-CGE models:

- **A S-CGE model should be specified based on regional economic theory, instead of borrowing specifications from other national or international CGE models.** This allows the model to capture information at the regional level that is not explicitly portrayed in national models. For instance, nuances in inter-regional trade and migration, and the role of government stimuli in influencing regional economic activity. For example, an S-CGE model can assume a central government providing services to the whole country, or a devolved government system providing services to a defined region with region-specific public finances. We assume a single central government in our model.
- **S-CGE models should account for a region's influence on national trade patterns.** In general, national, and international models assume that international trade terms and patterns are not influenced by activity in a region within the country (known as the 'small country' assumption).[16] However, within a country, large regions can

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[15] Partridge, M.D., & Rickman, D.S. (2010). Computable general equilibrium (CGE) modelling for regional economic development analysis. Regional Studies, 44(10), 1311-1328.

[16] Harris, R. (1983). Applied general equilibrium analysis of small open economies with scale economies and imperfect competition. Working Papers 534, Queen's University, Department of Economics.

influence trade terms and patterns between regions, and this must be captured when specifying an S-CGE model. In our S-CGE modelling of HS2, each of the ten regions has its own set of inter-regional trade flows, and businesses can buy goods and services from any of the other regions.

- **S-CGE models should contain a time element.** By including a time element, the dynamic impact of economic scenarios can be assessed. Partridge and Rickman (2010)[17] note that, until recently, regional models either ignored a time element or mimicked the dynamic behaviour of national models. They recommend that the parameterisation of an S-CGE model should be consistent with the dynamic behaviour of the regional economy. As dynamic-regional models use national parameters, they may produce adjustment periods inconsistent with empirical evidence on regional dynamic adjustment. However, external data is not usually available to calibrate the model based on a benchmark time period, or the external data is usually out-of-date. More complex approaches such as Bayesian estimation approaches (Adkins et al. 2003)[18] or using calibration based on several years of data (Kehoe et al. 1995)[19] have been implemented, but the use of such approaches is not widespread. In addition, the limitations of the UK data prevent this approach. As a result of these limitations, our S-CGE model of HS2 contains a time element that is based on national dynamic behaviour.

5.3. Literature review

This section summarises the literature that was used to inform our modelling approach and analysis. Given the scope of the study, and the breadth of research which has been undertaken in this area using similar methodologies, this list should not be considered as an exhaustive list. The literature summarised below is the most relevant to the modelling and analysis set out in this report. The review consists of the following two sections:

- Use of CGE modelling in transport infrastructure projects; and
- Use of CGE modelling in rail infrastructure projects.

5.3.1. Use of CGE modelling in transport infrastructure projects

5.3.1.1 UNITE project

The objective of the UNITE project, according to Nash (2003),[20] was to support policy makers in setting charges for the use of transport infrastructure by providing empirical evidence and appropriate methodologies. Both partial and general equilibrium models[21] were used to undertake the analysis of the impact of transport infrastructure use and cost. The general equilibrium model consisted of using a CGE model to analyse the impact of different approaches to transport pricing on the economies of Belgium and Switzerland.

Nash compared analysis of the effects of alternative pricing approaches from both partial and general equilibrium models. It is noted that a general equilibrium approach provided a greater understanding of the link between financing transport investment and the rest of the economy, for example, by capturing the increased taxation revenue required if the project is financed through public funds.

In addition, it was found that general equilibrium modelling was better able to track the impact of a policy change on the utility of different individuals. While general equilibrium modelling was viewed as the best approach for handling macroeconomic aspects, it was noted that the partial equilibrium approach provided a greater degree of detail around the specific impact on the transport sector.

5.3.1.2 The RAEM model

One of the most prominent S-CGE models used to evaluate the impacts of specific infrastructure projects is the RAEM model. This model was developed for the Netherlands by Transport and Mobility Leuven[22] and can be viewed

[17] See footnote above.

[18] Adkins, L.C., Rickman, D.S., & Hameed, A. (2003). Bayesian estimation of regional production for CGE modelling. Journal of Regional Science, 43(4), 641-661.

[19] Kehoe, T.J., Polo, C., & Sancho, F. (1995). An evaluation of the performance of an applied general equilibrium model of the Spanish economy. Economic Theory, 6(1), 115-141.

[20] Nash, C. (2003). Final Report for Publication. UNITE Project (1999-AM. 11157).

[21] Forsyth, P. (2013). Air capacity for Sydney. International Transport Forum Discussion Paper.

[22] A Dutch research organisation which specialises on modelling the economic impact of transport infrastructure.

as a best-in-class example of an S-CGE model. The S-CGE model we have used in this analysis of HS2, and the principles on which it is based, are strongly influenced by the RAEM model.

The RAEM model was developed by dividing the Dutch national input-output framework into 40 interacting regional matrices. The regional goods markets within the model are linked through the estimation of inter-regional transport costs. It is a dynamic model in the sense that it assesses changes in economic variables over time, and is broken down into 14 sectors, as in the 1993 Dutch SBI classification.[23] Markets operate under conditions of monopolistic competition, consumers are assumed to have utility functions which include a love of variety, and the labour market is based on the search-and-matching theory set out in Pissarides' (2000)[24] (for more detail on the model structure and assumptions see van Oort, Thissen & van Wissen, 2005[25]).

The RAEM model has been applied in several different studies and academic papers.[26] One such example is a study by Thissen, Limtanakool, & Hilbers (2011)[27] which analysed the impact of a congestion charging scheme on agglomeration effects. This paper uses a two-stage approach to evaluate the impact of additional infrastructure investment and changes in road pricing. The first of these stages involves calculating the 'partial direct effect'. This uses the demand curve to determine the partial direct effect of additional transport infrastructure investment. This is calculated through a shift in the supply curve, which maps the relationship between the number of trips taken and the willingness to pay for each journey. The second stage uses the S-CGE model to calculate additional indirect effects, which are not captured within partial equilibrium modelling.

5.3.1.3 The Lower Thames Crossing

The Lower Thames Crossing (LTC) wider economic impact assessment conducted by PwC presented findings on a wide range of key economic variables to be influenced by the Lower Thames Crossing options that were assessed.

PwC built upon the work of Venables et al. (2014) that considered 'User benefits', 'Cities and agglomeration' and 'Investment and employment' by adding two additional economic impact themes: 'Construction and funding' and 'Subsequent round effects'. These economic impact themes were then captured in a CGE model, supported by econometric analysis and other modelling techniques to produce a model capable of factoring in second round effects across multiple regional economies stemming from the development of the LTC.

The model reconciled TAG[28] partial equilibrium welfare estimates with general equilibrium welfare analysis, with results demonstrating that general equilibrium GDP impacts were roughly 30% higher than TAG partial equilibrium estimates, with welfare impacts being 15% lower, stemming from imperfect competition and the disutility of working respectively and highlighting the benefits of S-CGE modelling in evaluating welfare impacts over partial equilibrium analysis.

5.4. Further use of CGE modelling in rail infrastructure projects

Several studies have attempted to evaluate the impact of investment in rail transport infrastructure upon the wider economy. They have involved several modelling approaches including multiregional input-output and econometric models, spatial price equilibrium models, endogenous price multi-sector models, transportation network models, and multiregional CGE models[29]. Kim, Hewings and Hong (2004)[30] divide the various approaches into two categories:

[23] Available at: http://www.cbs.nl/en-GB/menu/methoden/begrippen/default.htm?ConceptID=646

[24] Pissarides, C.A. (2000). The Economics of Search. Manuscript. London School of Economics, London.

[25] van Oort, F., Thissen, M., & van Wissen, L. (2005). A survey of spatial economic planning models in the Netherlands. Theory, application and evaluation. Rotterdam: NAi-Uitgevers.

[26] See, for example, van Oort, Thissen & van Wissen (2005) and Knaap, T., & Oosterhaven, J. (2011). Measuring the welfare effects of infrastructure: A simple spatial equilibrium evaluation of Dutch railway proposals. Research in Transportation Economics, 31(1), 19-28.

[27] Thissen, M., Limtanakool, N., & Hilbers, H. (2011). Road pricing and agglomeration economies: a new methodology to estimate indirect effects applied to the Netherlands. The Annals of Regional Science, 47(3), 543-567.

[28] Ibid.

[29] Van de Bergh, J., Nijkamp, P., & Rietveld, P. (1996). Spatial equilibrium models: a survey with special emphasis on transportation, Recent Advances in Spatial Equilibrium Modelling.

[30] Kim, E., Hewings, G., & Hong, C. (2004). An Application of an Integrated Transport Network-Multiregional CGE Model: a framework for the Economic Analysis of Highway projects. Economic Systems Research, Vol. 16, No.3, September 2004.

1. Sequential models which combine separate spatial economic models and transport network models which forecast changes in the travel patterns;[31] and
2. Non-sequential models which estimate the economic effect of transport investment without an explicit transport model.

The approach used in our analysis is sequential, i.e., we use the transport model outputs that HS2 Ltd provided to us in our S-CGE modelling. This allows us to investigate changes in the economy due to changes in generalised journey times and travel patterns. However, it does not account for any feedback loops between the economic changes and travel patterns, which an integrated model would explicitly account for.

5.5. Our S-CGE modelling approach – Key features

The model built for this project is a single-nation dynamic model for the UK, based on 2017 data. We split the UK into ten regions. Each region interacts with the others through the movement of goods and services, labour, and capital. Each region is further broken down into eight sectors. The model also allows for the introduction of frictions which affect the ease with which capital, labour and traded goods and services move between different regions or overseas.

To analyse the implications of various rail infrastructure projects in the UK, we have calibrated the equations in our model to UK economic data and created a baseline view of the economy. A new scenario is then imposed on this baseline to reflect the impacts of HS2 when operational. The S-CGE model measures the difference between the new scenario and the baseline. This provides estimates for the changes in the main economic metrics such as GDP, household consumption, exports, imports, investment, and tax receipts. The S-CGE model can also be tailored to generate more detailed results at the sector and household level.

Our results are given in 'net' form, in that the results in each scenario are compared to a 'Do Minimum' baseline scenario for the UK economy. In the baseline it is assumed that HS2 is not operational.

The S-CGE model uses a mixture of regional accounts data published by the ONS, the Scottish Government and Her Majesty's Revenue and Customs' (HMRC) sector tax data to capture the complex transactions in the economy. These data provide a snapshot of the ten regional economies in a single year, which is used as a starting point for comparing HS2 scenarios against a baseline scenario. The number of sectors and household types are constrained by the availability and consistency of data across the ten regions. We have programmed the model using a General Algebraic Model Software ('GAMS')[32] software with the Mathematical Programming Software for General Equilibrium ('MPSGE')[33] interface. The number of equations and amount of data used are also constrained by the ability of this software to solve such a model. We note that GAMS/MPSGE is a standard programming tool for CGE models.

5.6. Representing transport costs in S-CGE analyses

Transport costs are a key factor in determining changes in the pattern of activity within an economy resulting from transport interventions. Most S-CGE analyses take transport costs into account by modelling the impact of transport interventions through a reduction in transport costs. In this exercise, we modelled the productivity benefits that such a reduction in transport costs would bring about via three channels: Business User Benefits (BUBs), agglomeration and labour supply. This is discussed in more detail in Section 6.3.

5.7. Data used in the model

5.7.1. National accounts and survey data used in the model

Our S-CGE model is based in the UK's Supply and Use Tables (SUTs). CGE models are often based in SUTs and/or Input-Output Tables (I-O tables) derived from SUTs that form part of the National Accounts. SUTs and I-O Tables provide data on sector-level output, consumption, business costs and taxation. They are 'balanced' in the sense that income equals expenditure, which is a fundamental property of CGE models.

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[31] Transportation Network Analysis is concerned primarily with the spatial, but also the temporal, nature of the movement of people and freight across land, where the movement is channelled onto roads or railways.

[32] More information on the GAMS software package can be found at: http://www.gams.com

[33] More information on the MPSGE software interface can be found at http://www.MPSGE.org

Both SUTs and I-O tables are part of a country's Social Accounting Matrices (SAMs), which also contain more detailed labour market and household consumption data. They often also include more detail on trade balances between households and governments and the rest of the world. However, in the UK, SAMs are published less regularly and have structures that vary from one publication to another.

The ONS publishes both SUTs and I-O tables for the UK economy. Estimates for SUTs are generally published once every year with the most recent edition, at the point at which the analysis was conducted, was published for 2018. For I-O tables, figures are updated approximately every year. The most recent, at the point at which the analysis was conducted, was for 2017. Regional governments in Scotland and Wales have also started to produce SUTs and I-O tables. The Scottish Government published a SUT for the Scottish economy for 2017 while the Welsh Economy Research Unit (WERU) produced an I-O table for Wales for 2007 (both the latest at the time of analysis).

The following description builds on Chapter 4, where we describe a 10-region SUT dataset for the year 2017. This split allows for the disaggregation of the national impact of improved transport connectivity due to HS2 into relevant regional impacts. We used data from the National Accounts and a range of other ONS publications, such as the Annual Business Survey.

Data are taken from a single year period, 2017, and provide a snapshot of the economy in that year. Details of the key data sources used in this project are provided in Table 7 below.

Table 7: Key data sources used in the S-CGE model of HS2

Data	Time	Source
UK Supply-Use Tables	2017	ONS[34]
Scotland Supply-Use Tables	2017	The Scottish Government[35]
Regional Gross Value Added	2017	ONS[36]
Gross operating surplus by sector and UK region	2017	ONS
Compensation of employees by sector and UK region	2017	ONS
UK self-employment income by sector and UK region	2017	ONS
UK production tax by sector and UK region	2017	ONS
Regional household income and consumption by UK region	2017	ONS[37]
UK household population	2017	ONS[38]
Inter-regional trade data by sector and UK region	2017	PwC calculations
UK Income Tax	2017	HMRC[39]
UK Corporation Tax and subsidies	2017	HMRC[40]

[34] ONS (2017). Input-Output Supply and Use Tables, 2017 Edition. Available at: http://www.ons.gov.uk/economy/nationalaccounts /supplyandusetables/datasets/inputoutputsupplyandusetables

[35] Scottish Government (2017). Supply, Use and Input-Output Tables: 1998-2017, 2017 Edition. Available at: https://www.gov.scot/publications/input-output-latest/

[36] ONS (2017). Regional gross value added (balanced) by industry: all ITL regions, May 2021. Available at: https://www.ons.gov.uk/economy/grossvalueaddedgva/datasets/nominalandrealregionalgrossvalueaddedbalancedbyindustry

[37] ONS (2017). Effects of taxes and benefits on household income, May 2021. Available at: https://www.ons.gov.uk/peoplepopulationandcommunity/personalandhouseholdfinances/incomeandwealth/datasets/theeffectsoftaxesandbenefitsonhouseholdincomefinancialyearending2014

[38] ONS (2017). Families and households in the UK Statistical bulletins, March 2021. Available at: https://www.ons.gov.uk/peoplepopulationandcommunity/birthsdeathsandmarriages/families/bulletins/familiesandhouseholds/previousReleases

[39] HMRC (2016). HMRC tax receipts and National Insurance contributions for the UK. Available at: https://www.gov.uk/government/statistics/hmrc-tax-and-nics-receipts-for-the-uk

[40] HMRC (2017). HMRC (2017). Corporation Tax Statistics 2019. Tables 11.1A to 11.10, and A.5, A.6. Available at: https://www.gov.uk/government/statistics/corporation-tax-statistics-2019

UK Capital Allowances	2017	HMRC[41]
UK National Insurance Contributions	2017	ONS[42]

Following the collection of data from the sources listed above, the data were organised to fit into the SUT framework.[43] It is essential that income equals expenditure in a SUT because the National Accounts reflect this balancing principle. However, data drawn from a range of sources collected using different methodologies will inevitably demonstrate some inconsistency, which can prevent the SUTs and National Accounts from balancing. The standard approach taken in such circumstances is to balance the matrices using a computer algorithm that preserves the core structure of the data but makes iterative adjustments to preserve the income-expenditure relationship. There are a range of algorithms that can be used to undertake these adjustments and they are used as standard practice to balance SUTs.[44] The ONS use the RAS (row and column sums) approach to balance the UK SUTs and this is also the approach used in our S-CGE modelling of HS2,[45] as set out in the previous chapter.

5.7.2. Derived national account estimates

In the S-CGE model of HS2, the baseline levels of GDP and employment are based on the 2017 I-O table. The model then projects this baseline over a 50-year horizon. As shown in Table 8, in 2017 London is the largest of the ten regions in our model, accounting for 23.7% of UK GVA and 17.3% percent of UK employment. The North East is the smallest region, accounting for 2.9% of UK GVA.

All the results presented in this report are calculated as changes from a baseline in which the UK real GDP grows at 2.1% per annum in the long run. It should be noted that any change in the baseline will be inherently second order in nature and are highly unlikely to be material when assessing the incremental economic impact of HS2.

Table 8: Gross Value Added (GVA) and employment by region, 2017

Region	GVA (£m)[46]	Share of UK GVA	Employment	Share of UK employment
North East	53,208	2.9%	1,096,110	3.6%
North West	177,482	9.7%	3,391,625	11.1%
Yorkshire and the Humber	119,879	6.5%	2,468,390	8.1%
East Midlands	105,812	5.8%	2,128,390	7.0%
West Midlands	135,945	7.4%	2,666,205	8.7%
East of England	158,881	8.7%	2,854,610	9.4%
South East	271,049	14.8%	4,243,250	13.9%
London	435,294	23.7%	5,272,725	17.3%
Scotland	138,150	7.5%	2,502,800	8.2%
The Rest of the UK	240,911	13.1%	3,891,828	12.8%

[41] HMRC (2017). HMRC (2017). Corporation Tax Statistics 2019. Tables 11.1A to 11.10, and A.5, A.6. Available at: https://www.gov.uk/government/statistics/corporation-tax-statistics-2019

[42] ONS (2017). CG: Resources: Employees National Insurance Contributions (NICs) CP NSA, October 2020. Available at: https://www.ons.gov.uk/economy/grossdomesticproductgdp/timeseries/gcse/bb

[43] Further details on the structure of the SUT data used in the model are included in Appendix A

[44] Eurostat (2007). Manual of Supply, Use and Input-Output Tables. (2007). Eurostat Methodologies and Working Papers Series ISSN 1977-0375. Available at: http://epp.eurostat.ec.europa.eu/cache/ITY_OFFPUB/KS-RA-07-013/EN/KS-RA-07-013-EN.PDF

[45] Akers, R., & Clifton-Fearnside, A. (2008). Blue Book 2008-Balanced Estimates of Gross Domestic Product using a Supply and Use Approach. ONS website. Available at: file:///C:/Users/901792/Downloads/BlueBook2008SUT_tcm77-144039.pdf

[46] This excludes Extra-Regio GVA. This comprises compensation of employees and gross operating surplus, which is not assigned to regions within the ONS data.

Source: ONS and PwC analysis

Table 9 and Table 10 show that the ‘Other’ sector constitutes the largest sector in the model, accounting for 43.9% of UK GVA and 38.1% of UK employment in 2017. The Consumer Services sector is the 2nd largest by GDP and employment, accounting for 12.7% of GVA and 14.8% of employment.

Table 9: Regional GVA by sector, 2017 (£m)

	North East	North West	Yorkshire and the Humber	East Midlands	West Midlands	East of England	South East	London	Scotland	The Rest of the UK
Advanced Manufacturing	5,040	16,920	8,660	9,404	17,617	11,560	15,253	3,695	7,339	20,238
Business Services	3,653	15,517	9,541	9,317	11,327	16,278	29,299	54,754	11,773	18,167
Construction	6,174	24,017	16,271	16,352	20,201	23,529	38,940	39,604	16,439	31,158
Consumer Services	3,022	10,408	7,409	7,098	8,247	13,844	17,751	19,863	8,560	16,032
Digital	2,940	8,254	4,563	3,762	5,957	8,236	24,468	53,078	5,603	9,427
Discretionary Consumption	2,190	7,980	4,579	3,806	5,430	6,242	12,232	24,245	6,255	10,642
Financial Services and Professional Services	3,051	13,383	8,569	4,287	8,458	9,564	16,674	88,143	11,755	15,849
Other	27,138	81,003	60,287	51,786	58,708	69,628	116,432	151,912	70,426	119,398
Total	**53,208**	**177,482**	**119,879**	**105,812**	**135,945**	**158,881**	**271,049**	**435,294**	**138,150**	**240,911**

Source: ONS and PwC analysis

Table 10: Regional employment by sector, 2017

	North East	North West	Yorkshire and the Humber	East Midlands	West Midlands	East of England	South East	London	Scotland	The Rest of the UK
Advanced Manufacturing	82,750	193,000	140,750	157,000	210,250	132,500	162,500	41,400	101,500	224,549
Business Services	131,500	454,000	343,000	279,500	358,000	498,000	625,000	1,055,500	319,500	460,580
Construction	186,000	617,500	423,250	381,000	510,000	531,000	804,000	768,000	422,000	671,563
Consumer Services	49,000	163,000	120,000	99,000	117,000	162,000	241,000	208,000	147,000	198,044
Digital	31,450	99,500	62,750	58,950	69,000	101,100	240,500	421,000	69,500	141,271
Discretionary Consumption	117,500	311,000	230,250	190,250	243,000	271,500	426,000	544,000	267,000	445,567
Financial Services and Professional Services	41,000	202,000	126,250	72,250	111,000	130,000	183,000	597,000	130,000	174,035
Other	456,910	1,351,625	1,022,140	890,440	1,047,955	1,028,510	1,561,250	1,637,825	1,046,300	1,576,220
Total	**1,096,110**	**3,391,625**	**2,468,390**	**2,128,390**	**2,666,205**	**2,854,610**	**4,243,250**	**5,272,725**	**2,502,800**	**3,891,828**

Source: ONS and PwC analysis

6. Incorporating productivity effects into the model

6.1. Chapter overview

In this chapter, we describe in detail how we use the outputs provided by HS2 Ltd, drawn from HS2 Ltd's transport model (version 10 of the PLANET Framework Model) and the Wider Impacts in Transport Appraisal tool (WITAv2.0) developed by DfT and operated by HS2 Ltd, as three 'effects' within our S-CGE framework. We set out:

1. Assumptions behind the adjustments made to HS2 Ltd's model outputs to provide inputs for the S-CGE model; and
2. How we incorporate each of the inputs into the S-CGE model, including any assumptions made.

6.2. Adjustments to HS2 Ltd model outputs

This section explains the assumptions and adjustments made to the outputs of the HS2 Ltd models for use as inputs for the S-CGE model. We focus on the three 'effects', which are business user benefits, agglomeration, and labour supply effects. For each, we note the information we received from HS2 Ltd before describing the method we used to adjust these into our model inputs.

6.2.1. Business user benefits

User benefits are the direct effects experienced by the users of the transport scheme. User benefits arise through a change in generalised travel costs, which includes generalised journey time savings.

The generalised journey times savings accrue to three user types by journey purpose:

- Business Users (whose journeys are made in the course of work);
- Commuting Users (who are travelling to and from their normal place of work); and
- Other Users (who are making other non-work trips, for leisure purposes for example).

We focus on the journey times savings that are accrued by business users only. Business users affect the real economy through improvements in productivity. They generate additional welfare and GDP improvements.

The relationship between the real economy and other user types is more complex. If an individual saves time and spends that time working more hours, economic output is likely to rise. Conversely, if the individual spends the time saving on leisure activities, the impacts on the real economy may be small or very difficult to identify through formal economic modelling, although there may be an impact on welfare. For this reason, commuter and leisure benefits are not considered equivalent to GDP.

We do not examine other types of user benefits such as fuel cost savings, non-fuel cost savings, and resilience benefits. This data is not available nor the focus of the project scope. Further work to examine these effects would be beneficial and it would be possible to include these estimates into our S-CGE model.

The total business user benefits were derived from HS2 Ltd's PLANET Framework Model (PFM) version 10.
We understood that HS2 Ltd allocated these business user benefits to each of the 10 sectors based on each sector's share of National Travel Survey (NTS) intercity (>50km) rail journeys between 2010 and 2019 (as reported in 2020). HS2 Ltd. also made a single adjustment to the share attributed to the Digital sector to account for its small sample size reported in the NTS. We also agreed with HS2 Ltd to allocate these business user benefits across regions in line with the regional allocation of agglomeration impacts from WITA, as provided to us by HS2 Ltd.

Regional and sectoral allocation method

We applied a cross-entropy method to fit the matrix of sector/regional inputs to the underlying profile of sectors by region in the UK. This adjusted the share of user benefits allocated to a sector within a region to account for the regional variations in sector mix, i.e., HS2 is assumed to not create new sectors where there are none previously. The approach undertaken comprised the following five steps:

Step 1: We extracted the user benefits in each sector and each region in absolute levels. We expressed them in percentages of the total economic outputs in their respective sectors and regions.

Step 2: For each cell in the sector-by-region matrix, we assigned a percentage based on the average of the sector it relates to; and the region it relates to. The arithmetic mean was used. A geometric mean was tested; however, this does not make a material difference to the result.

Step 3: The absolute levels of user benefits in each cell were derived by multiplying the percentages from Step 2 by the level of economic outputs of each sector in each region. These are pre-balancing numbers. While the whole matrix adds up to the total user benefits for a given scenario/year, the row sums and column sums do not add up to those generated in Step 1.

Step 4: We fed the target column sums and row sums from Step 1, as well as the matrix pre-balancing user benefits from Step 3, into a Cross-Entropy Algorithm. This adjusts the matrix so that it fits the target column sums and row sums with minimal changes. This is a method widely used in balancing social accounting matrices and is the method we used in deriving an inter-regional I-O table. More details are provided in the Appendix C (Section C.5).

Step 5: The updated matrix is then converted into percentages of the total economic outputs in the respective sectors and regions before being inputted into the CGE model.

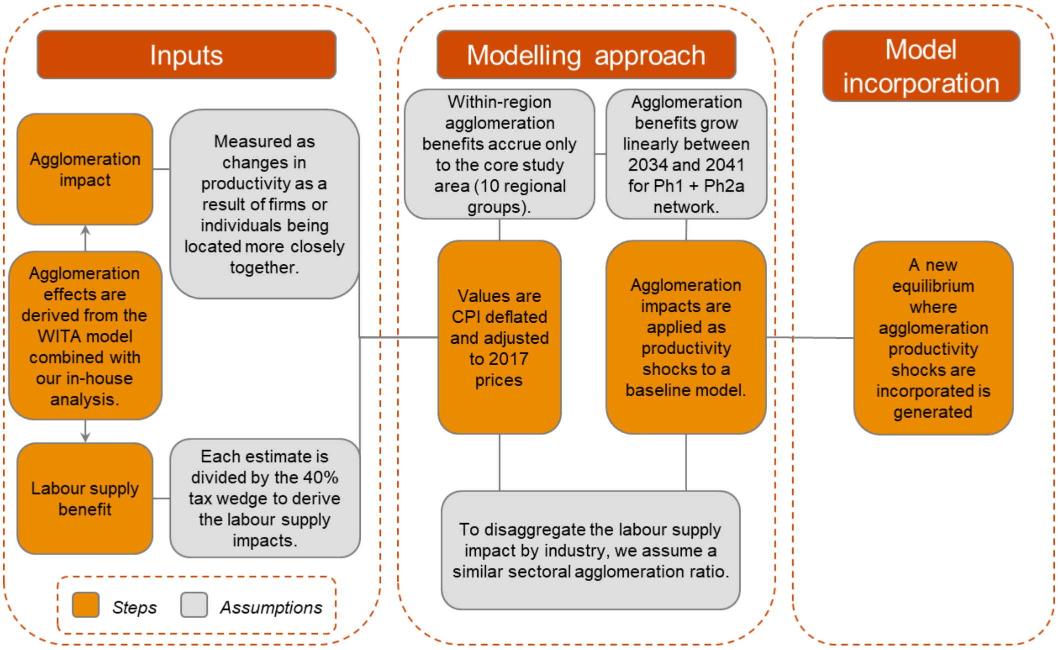
6.2.2. Agglomeration and labour supply effects

Agglomeration and labour supply effects occur when firms or individuals are located more closely to each other by reducing generalised journey times through faster, more frequent, and more reliable and less crowded transport connections between economic agents (firms, workers, and consumers), thereby increasing the 'effective density' between firms in their current locations. This allows them to collaborate with and compete against each other more effectively and brings productivity benefits. In this report, this effect is referred to as 'agglomeration'.

The productivity benefits – also known as static clustering – and labour supply effects have been provided by HS2 Ltd. This has been fed into the S-CGE framework to understand their general equilibrium and dynamic clustering effects. This sub-section details what has been undertaken to generate the static clustering inputs for use in the S-CGE model.

The approach in estimating the 'static' productivity effects from agglomeration and labour supply is illustrated in Figure 9 below.

Figure 9: Main relationships captured in a CGE model



Source: PwC

Using the inputs of agglomeration and labour supply derived from the Wider Impacts in Transport Appraisal tool (WITA version 2.0) developed by DfT and operated by HS2 Ltd, we assigned the input estimates to the relevant CGE sectors. We aligned the four WITA sectors (Manufacturing, Construction, Consumer Services and Producer Services) to 7 of our 8 CGE sectors as closely as possible, as shown in Table 11.

We set the agglomeration and labour supply input estimates of the CGE 'Other' sector equivalent to zero, as the 'Other' sector consists largely of agriculture, mining, and public sector activities; all of which are unaccounted for in the WITA modelling guidance. We note that this approach does not capture any impact of HS2 on agglomeration and labour supply in non-advanced manufacturing activities (which is within 'Other') and hence may under-estimate the overall economic impact of HS2 via these transmission mechanisms. However, since non-advanced manufacturing is not a heavy user of High-Speed Rail, we assume this under-estimate to be immaterial.

Table 11: Mapping of the 4 WITA sectors into the 10 S-CGE sectors

WITA model sectors	Our CGE sectors
Manufacturing	Advanced manufacturing
Construction	Construction
Consumer Services	Consumer services
	Digital
	Discretionary consumption
Producer Services	Business Services
	Financial services and professional services
N/A (set to zero)	Other

Source: PwC

For labour supply estimates we uplifted the values by dividing them by a factor of 40% to account for the 40% tax wedge which had been applied to estimate the total labour supply effects to convert these from real economy impacts into a welfare benefit. To disaggregate the labour supply effect by region, we apportion the impact in line with the sectoral agglomeration effects.

We adjusted the values from 2010 prices to 2017 prices using the consumer price index. This ensured that the values for agglomeration and labour supply are in the same price-base as total output estimates derived from the I-O table. This was then used to configure the productivity shocks of agglomeration and labour supply as a percentage of economic output in the relevant region and sector, which can be fed into the S-CGE model as an input.

6.3. How the effects feed into the S-CGE model

In this section, we describe how we incorporate each of the 'effects' into the S-CGE model, including the assumptions made. We consider user benefits and agglomeration and labour supply effects in turn.

6.3.1. Business user benefits – generalised journey time savings

We assume that the reduction in generalised travel costs for business users is equivalent to a reduction in the operating costs of those businesses. The reduction in business user costs can be thought of as an increase in the productivity of those businesses, or a more efficient use of inputs. Total factor productivity is assumed to increase until the value of the rise is equal to the forecast business user benefits, as provided by HS2 Ltd.

As reductions in generalised travel costs accrue to businesses which use rail links, we assume that the increase in productivity is distributed across sectors in line with the distribution of each sector's use of long-distance rail transport. In this way, sectors which utilise long-distance rail transport are more likely to benefit from the new railway and hence experience a disproportionate share of the productivity increase. We also distribute the productivity benefit across regions according to the regional distribution of agglomeration benefits, provided by HS2 Ltd and estimated using the WITA model.

The change in productivity is calculated as the value of the reduction in generalised travel costs as a share of all business purchases. This is done using the formula below, for a specific sector and region:

$$\frac{\text{Cash value of business user benefits}}{\text{Sum of output}} = \text{\% increase in output}$$

Businesses have a range of options for how to respond to this increase in efficiency. Profit-maximising firms produce more output with the same level of inputs or reduce their use of inputs to produce the same level of output to generate greater profits. Greater profits encourage new entry, i.e., the improved transport infrastructure attracts new businesses or employees to the region to take advantage of the increase in productivity. Due to new entry, the level of output increases and prices decrease. All these changes take place within the S-CGE model.

6.3.2. Agglomeration and labour supply effects

We model agglomeration effects within our S-CGE model as an increase in productivity. This is equivalent to businesses experiencing benefits by becoming more efficient in their use of inputs. To transform the agglomeration impact figure into this increase in productivity, we use the following formula:

$$\frac{\text{Cash value of agglomeration benefit}}{\text{Sum of output}} = \text{\% increase in output}$$

Agglomeration benefits arise from firms using the resources they employ more productively. To value this input as an increase in productivity, we compare this benefit as a percentage of total use of all resources employed. This value is specific both to each region and each sector.

The agglomeration benefits cover the direct effect of the scheme, in this case referred to as static clustering by existing businesses. A second effect could also occur in the form of dynamic clustering, whereby businesses relocate because of the increase in productivity brought about by HS2. This would lead to further impacts on the economy, as economic density would increase in some areas and decrease in others as capital and labour move across regions. For regions where economic density increases this would provide all businesses in the area with further agglomeration benefits. These effects occur within the S-CGE model, which allows businesses and individuals to relocate in response to higher wages and mark-ups. As a result, these dynamic clustering effects are not captured within the initial input described above.

The Labour Supply effect uses a similar approach to the agglomeration effects, as the impact on productivity is estimated as follows:

$$\frac{\text{Cash value of labour supply impacts}}{\text{Sum of output}} = \text{\% increase in output}$$

These percentage increases in output from business user benefits, agglomeration and labour supply effects are then fed into our CGE modelling inputs.

7. Risks and uncertainties

7.1. Chapter overview

There are sources of uncertainty inherent to any modelling exercise, particularly those attempting to estimate the wider economic impacts of large national transport schemes. This uncertainty has led to the analysis of these impacts being characterised as 'level 3' impacts in the Department for Transport's Transport Analysis Guidance (TAG). This implies that assessments of wider economic impacts may inform the value-for-money assessment of a scheme, but not be used in benefit-cost ratios that are based on level 1 and level 2 benefits.

In this chapter we consider the main sources of uncertainty specific to this S-CGE model, including the uncertainty over S-CGE model inputs (i.e., Business User Benefits, Agglomeration and Labour Supply); and uncertainty over the model parameters such as the behavioural responses of firms and households in the model (i.e., consumption-leisure elasticities).

The impacts of uncertainty on S-CGE model outputs are presented as ranges around the central estimates. These ranges are derived by adjusting the impact multipliers estimated by the S-CGE modelling, i.e., the ratio of overall GDP impact relative to the model inputs. The distribution of impact multipliers is informed by implied multipliers from estimates in the academic and policy literature.

In addition, several scenarios have been run to reflect either different assumptions about key S-CGE model inputs or different modelling parameters and assumptions. The results of these sensitivities are discussed in Appendix B.

7.2. Uncertainty in S-CGE model inputs

The inputs into the S-CGE model play an important role in determining the magnitude of the economic impact of HS2 on the UK economy.

The Business User Benefits, Agglomeration and Labour Supply effects have been provided by HS2 Ltd. They are drawn from HS2 Ltd's transport model (version 10 of the PLANET Framework Model) and the Wider Impacts in Transport Appraisal tool (WITA version 2.0) developed by DfT and operated by HS2 Ltd. The models are subject to on-going assurance using industry best practice. The models are compliant with DfT's Transport Analysis Guidance (TAG). For further information on the model, assurance, and validation, please see the PLANET Framework Model Description Report.

We assume all three effects are general productivity effects that improve the efficiency of using labour, capital and intermediate goods and services, rather than specific effects affecting a single factor of production i.e., we have assumed the productivity gains are neutral to inputs – they are a 'Hicks-neutral technical change' in that they do not benefit the use of certain inputs more than others. The general productivity assumption is consistent with most economic literature, as summarised in Melo et al. (2013).

The allocation of business user benefits effects to sectors was based on the National Travel Survey (NTS) and profiled by each industry's usage of rail services with trip lengths greater than 50km. The approach and the data source used is, in our view, appropriate and warranted. The allocation of user benefits to a sector within a region was adjusted using a cross-entropy algorithm. The use of a cross-entropy algorithm is appropriate and is widely used to interpolate imperfect data. For example, the ONS uses an algorithm of the same family to interpolate data in its published UK I-O table.

7.3. Uncertainty in the S-CGE model parameters

7.3.1. The source of regional data for the S-CGE model

There is no up-to-date inter-industry trade data that is published at a regional level for the UK. A notable exception being Scotland, which maintains its own set of Supply, Use and Input-Output Tables (SUIOTs). We have amalgamated the UK-wide and Scottish data and interpolated inter-regional trade flows according to the methods we set out in Chapter 4 and Appendix C.

The interpolation of inter-regional trade flows depends on published regional data on industry GVA and assumptions we made with regards to the extent to which inter-regional trade flows are affected by industry mixes and distances.

This introduces additional uncertainty to our regional results but – as the first sensitivity test in Appendix B shows – does not substantially affect the national results.

To understand the impact of these trade flows in our model, we have proxied the change in regional trade flows by adjusting the distance decay factor in the model, as set out in Section B.1 of Appendix B. We drew different values for the distance decay parameter from Linders (2005) to proxy three sets of inter-regional input-output tables: one is the central case of our model (distance decay = 0.78), one has a high distance decay value (= 1.0, proxying less regional trade) and one has a lower distance decay value (=0.5, proxying more regional trade). We found that the national results from the S-CGE modelling of HS2 do not differ materially in each of the cases. This is set out in more detail in Appendix B (Section B.1).

Our approach is consistent with industry standard practice when multi-regional trade flow data do not exist. A similar approach was used by PwC in a report commissioned by HM Treasury and HMRC in 2012 to build a model on their behalf to examine the devolution of tax policy. During that development of the S-CGE model we consulted with ONS and HM Treasury officials on best practice and agreed an approach. To the best of our knowledge, no new relevant datasets have been published that could be used to improve this analysis.

7.3.2. The S-CGE model's structure

Our S-CGE model is characterised by commonly used functional forms, structures, assumptions, and parameter values. The core model is an industry standard fully intertemporal optimised S-CGE model i.e., its underlying set of equations are derived from standard microeconomic optimisation principles and the widely used family of elasticity constraints known as the Constant Elasticity of Substitution (CES) production function. Similarly, the maximisation of the household's utility function in the Dixit-Stiglitz elasticity form is the commonly used approach in CGE modelling (Shoven and Whalley, 1988).

We have calibrated the industry standard S-CGE model to reflect the specific requirements of this project. We sourced parameters on the labour market and capital adjustment costs through a review of academic literature. The values used for these parameters are discussed below. We also explored the sensitivity of our outputs to these parameters, the results of which are presented in Appendix B.

7.3.3. Key parameters and elasticities in the S-CGE model

We sourced values for elasticity parameters from industry standard databases and international academic and policy literature. Most notably, the core assumptions that underpin the firms' production function and the households' demand of goods and services are sourced from the Global Trade Analysis Project (GTAP) database. GTAP is an international database coordinated by Purdue University with inputs from across the world. GTAP provides industry standard assumptions and values for all S-CGE models used globally.

We have made some adjustments to the industry standard S-CGE model. The adjustments are intended to induce greater economic realism to the model e.g., capital has installation costs that increase quadratically (e.g., building things in haste is more expensive) and our labour market function allows for unemployment and assumes that workers maximise not only their consumption of goods and services but also leisure.

The inclusion of the additional parameters and their values will increase uncertainty in the model's calibration and performance. There is only limited empirical literature to draw appropriate estimates from. We have provided the results of sensitivity tests relating to the key structural features of the model. The parameter value and ranges are set out in Table 12. The results of these tests are summarised below and presented in Appendix B. The results are in line with our expectations given the economic theory of these parameters.

Table 12: Summarises the main sources of uncertainty

Parameter	Definition	Values	Range
Intertemporal elasticity of substitution	Degree to which households consider consumption in different time periods.	0.9	0.5 – 0.95
Elasticity between consumption and leisure	Degree to which households are willing to substitute leisure with the consumption of goods and services.	0.3	0.2 – 1.0
Quadratic capital adjustment cost coefficient	The degree to which the model imposes a 'penalty cost' on rapid changes in the capital asset stock.	2.0	1.0 – 2.2
Businesses' propensity to return capital to owner	The degree to which businesses are inclined to return capital to owners in contrast to re-investing. This is a proxy measure of the degree to which households and businesses' expectation of HS2's benefits are aligned with the ones that are modelled by HS2 Ltd – e.g., using PFM and WITA.	1.0	1.000 – 1.001
Steady state growth rate in the baseline	UK long-run real GDP growth rate	2.1%	1.5% – 2.1%

Varying the key modelling parameters leads to adjustments in the results:

- **Intertemporal elasticity of substitution.** A higher value in this parameter implies that households consider goods, services, and leisure in different time periods to be more substitutable and therefore they would consume more in the post-HS2 periods. We opt for a constant intertemporal elasticity of substitution of 0.9 in our model to align with a common set of assumptions used in most published literature. This includes the work by Marten & Garbaccio (2018), Jorgenson et al (2013); Piazolo (2000); and Devarajan and Go (1998), which use an intertemporal elasticity of substitution of 0.9 or is supportive of this elasticity.
- **Elasticity between consumption and leisure.** We aligned the estimate with academic literature and assumed that households have a substitution elasticity of 0.3 between consumption and leisure. We recognise that there may be uncertainty in this parameter and carried out sensitivity analysis for this parameter. At the higher elasticity of labour-leisure substitution (1.0), households choose more leisure in the pre-HS2 period, as they smooth their overall utility over time. This results in a 41% reduction in GDP in 2051. At the lower elasticity of substitution (0.2), households choose less leisure in the pre-HS2 period, as they choose to consume more in the short-term. This results in a 53% increase in GDP 2051. Additional discussion and sensitivity analysis can be found in Appendix B.
- **Quadratic capital adjustment cost coefficient.** In line with the Tobin's q theory of capital asset pricing, we assumed that there is some adjustment cost in changing the level of capital investment. A higher value of this parameter is associated with more anticipatory investment. We carried out a literature review and found that there is a wide range of estimates in academic literature. We chose to use a value of 2, as it is an appropriate value within the range. While we recognise the level of uncertainty may be considerable in this variable, we consider that the value chosen is an appropriate one with which to derive a reasonable central estimate. Additional discussion can be found in Section C.3 of Appendix C.
- **Degree of alignment between economic agents' expectations and those of the modeller.** Households and businesses' investment and consumption decisions are influenced by their expectations of the future. In our central scenario, we assume that their expectation of HS2's benefits are the same as those forecast by HS2 Ltd (e.g., using PFM and WITA).

  In the S-CGE model, this alignment of expectations is captured by a parameter that governs businesses' propensity to return capital to owners. When these expectations are aligned to forecasts modelled by HS2 Ltd, the parameter has a value of 1. If households and businesses are more pessimistic, the value would be below 1. Businesses would be more inclined to return capital to their owners and less inclined to invest. If expectations are more optimistic, the value would be greater than 1.

We do not have any reason to believe that economic agents' expectations would be misaligned with HS2 Ltd's forecasts. Nevertheless, this is a source of uncertainty within the model, and we have carried out sensitivity analysis - see Appendix B. We found that by increasing the parameter by 0.1%, there is a reduction in the amount of investment in the period prior to the operation of HS2. This could be interpreted as businesses returning capital to owners rather than reinvesting it. The test indicates that the level of capital stock is lower, leading to an estimated GDP impact that is 33% lower in 2051.

- **Steady state growth rate in the baseline.** We assume the UK real GDP would grow by 2.1% per year in the long run. This is based on the UK economy's historical growth rate in the ten years prior the COVID-19 pandemic, i.e., between 2009 and 2019. We recognise that this is higher than current DfT assumptions and OBR forecasts in the wake of the COVID-19 pandemic, which are between 1.5% and 1.7%. This assumption does not affect the model results in percentages difference, only the absolute estimate.

7.4. Benchmark comparisons for the model results

To contextualise the estimated outputs of our S-CGE model presented in chapter 8, the results are compared to the results of academic literature on government spending multipliers. Output multipliers are the change in economic output relative to the change in government investment. The multiplier effect implicitly captures the wider economic effects of government spending primarily through indirect (supply chain), induced (wages spent by supply chain workers) and productivity augmenting effects.

This evidence represents a useful benchmark to validate the results of our S-CGE model. Only a small number of studies focus on long-term multipliers, which is appropriate for HS2 as a transformative project. Table 13 lists the more robust and widely referenced papers on long term output multipliers in the academic literature. While methodological approaches vary, the range of estimates is relatively narrow (between 1 and 1.8).

Table 13: Comparison of multipliers from the literature

Approach	Context	Multiplier	Study
Intertemporal Spatial CGE model	An S-CGE model that estimates the net impact of HS2's Phases 1, 2a and 2b Western Leg	1.3	PwC (2022)
Structural vector autoregressions (SVAR) model	Effects of government spending in a wide range of circumstances.	1.6	Ilzetzki et al. (2013)
Panel Data	Study of multipliers for government investment and consumption spending in a panel of OECD countries	1.6	Boehm (2019)
Econometric	Effects of the building of the U.S. interstate highway system – includes anticipation effects and crowding-in of state and local spending on roads.	1.8	Yaffe (2020)
Econometric	Effects of U.S. Federal highway grants	1.3 – 1.7	Leduc and Wilson (2013)
Literature Review	Fiscal multipliers for government infrastructure spending in high income countries	1 – 1.5	Spillembergo (2011)

Our estimates of the impacts of HS2 operations on UK GDP from the S-CGE modelling are at the lower end of this range of multipliers from the literature. S-CGE multipliers tend to be lower than multipliers derived from econometric estimates for two reasons. Firstly, they are 'net' i.e., they account for displacement effects. Most studies attempt to account for it implicitly, but it is dealt with explicitly in an S-CGE model. This naturally induces downward bias as more emphasis is placed on the calibration of displacement effects. Secondly, our S-CGE model attaches adjustment costs to both capital and labour markets (described in the previous section). These costs are often absent from macroeconomic models and again have a downward effect on the estimates.

8. National results

8.1. Overview

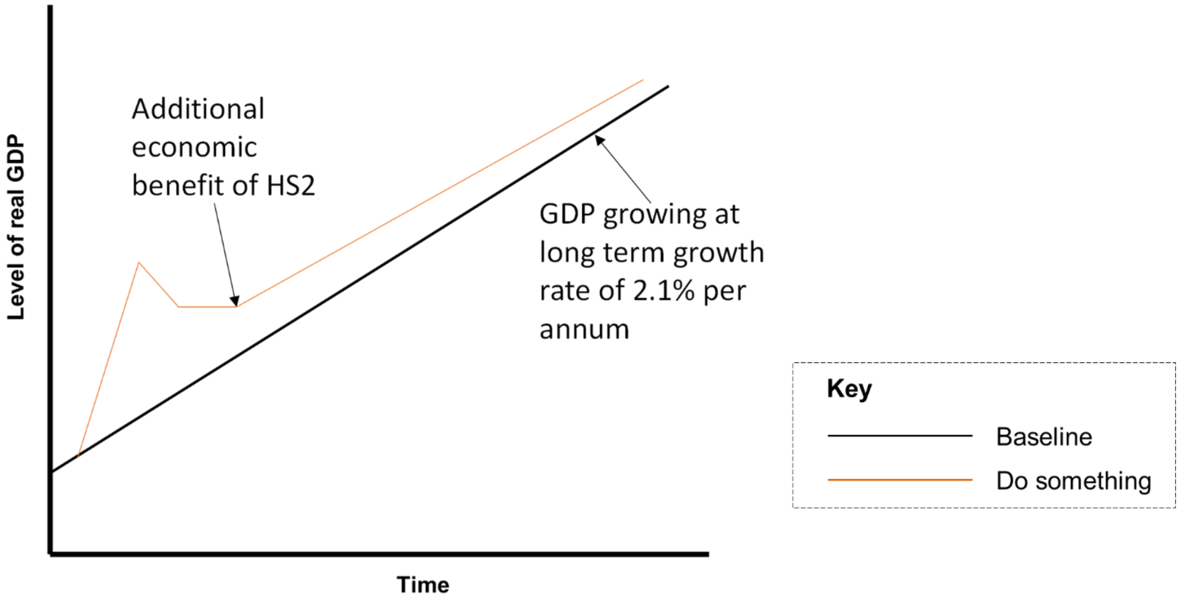
This chapter presents the results of the S-CGE modelling. The results for the impact of each of the three phases of the scheme assessed using the S-CGE model (Do Something) are compared to the baseline (Do Minimum) scenario in any year or period for the UK economy. The results are presented in level terms relative to an underlying baseline growth rate of real GDP. Figure 10 provides a guide to their interpretation.

The Do Minimum baseline (the black upward sloping line) is the UK long-term GDP growth rate without HS2. It is a scenario in which currently committed infrastructure projects and service level agreements are implemented, providing the reference case against which the HS2 scheme is to be compared. The Do Minimum baseline for all Train Operating Companies (TOCs) are based on assumptions instructed by the DfT and documented alongside the PFM v10a technical documentation.

The Do Something (the orange line) is the forecast future year scenario in which HS2 is operational, along with changes to conventional rail services due to the capacity that HS2 will release on the existing rail network. To isolate the specific effects of HS2, all results are presented relative to the baseline scenario.

The GDP results presented in the report are the sum of the area between the orange and black lines, i.e., the change in the level of real GDP relative to the baseline scenario in any year or period.

Figure 10: Interpreting the GDP impacts



The S-CGE model provides results at two-yearly intervals. We use linear interpolation to calculate results for the intermediate years. The S-CGE model provides results for a 40-year period from 2021 to 2060. We extrapolate these results from 2061 to 2100 by applying a constant assumed real GDP growth rate of 2.1% per annum. The S-CGE model excludes the economic impacts that will be realised as a result of the design and construction of HS2, such as spending and employment in the HS2 construction supply chain.

Our analysis focuses on the three phases of the HS2 Network:

- **Phases One + 2a** – This phase represents the potential impacts from the operation of HS2 between London and Crewe. We present total impact estimates for these Phases using a 60-year appraisal period from 2022 to 2081 (inclusive).
- **Phase 2b Western Leg increment** – This phase of the HS2 network is the additional section which extends from Crewe to Manchester and connects to the West Coast Mainline to allow HS2 services to continue to Scotland. For this phase we present total impact estimates using a 60-year appraisal period from 2026 to 2085 (inclusive).

- **Phase 2b Western Leg Full Network** – This represents the potential impacts from the sum of the above route sections of the HS2 network – London to Manchester, with a connection to the West Coast Mainline to allow HS2 services to continue to Scotland. We present total impact figures using a 64-year appraisal period from 2022 to 2085 (inclusive).

Our model results suggest that the biggest GDP impact may take place before HS2 services begin operating. Anticipatory investment is assumed to take place prior to the start of operational services as businesses look to capture the future productivity gains that they foresee will take place after HS2 services are operational.

The period in which businesses can adjust their investment behaviour in anticipation of future productivity gains when HS2 services are operational is constrained within the model. In the model, it is assumed for simplicity that businesses are unable to adjust their investment behaviours prior to the year in which the relevant phase of HS2 is assumed to secure parliamentary powers for its delivery, i.e., the Royal Assent date for the relevant hybrid Bill that secures those powers. Assumed dates for these 'trigger points' have been provided to us by HS2 Ltd and are outlined in Section 2.3.

The estimated GDP multipliers generated by the S-CGE model are provided in Table 14 below. For the Phase 2b Western Leg Full Network the estimated GDP multiplier is 1.32. This implies that for every £1 of static productivity gain (the input into the S-CGE model), UK GDP is estimated to increase by £1.32 over the 64-year appraisal period. The value of inputs into the CGE model (the denominator of the multiplier fraction) is defined as the sum of values of inputs over the appraisal period. This sum of inputs is obtained by applying the percentage impacts of productivity gains for all inputs to the baseline levels of sector outputs. This transforms the productivity inputs from percentage inputs into notional cash values comparable with our S-CGE model outputs, allowing us to calculate the multipliers.

We also built upon the benchmarking exercise in the previous Chapter to derive reasonable ranges around point estimates for these estimated GDP multipliers. As we set out in Section 7.4, we found that comparable studies suggest that a reasonable range of implied multipliers would be between 1.0 and 1.8 for all phases. These values have been adopted as lower and upper bounds of our reasonable ranges around point estimates for increases in the level of UK real GDP in this Chapter.

Table 14 shows that GDP multipliers are estimated to be larger for the Phase 2b Western Leg Increment (1.65) than for Phases One and 2a (1.21). However, these estimates lie within the reasonable range of implied multipliers from comparable studies (between 1.0 and 1.8) and hence the difference in estimated multipliers between phases may reflect uncertainties in the modelling rather than genuine differences in multiplier effects by phase.

Table 14: GDP multipliers by Phase

	Phases One + 2a	Phase 2b Western Leg Increment	Phase 2b Western Leg Full Network
Appraisal Period	31 Dec 2021 – 31 Dec 2081	31 Dec 2025 – 31 Dec 2085	31 Dec 2021 – 31 Dec 2085
Appraisal Length	60 years	60 years	64 years
GDP Multipliers	1.21	1.65	1.32

Source: PwC Analysis

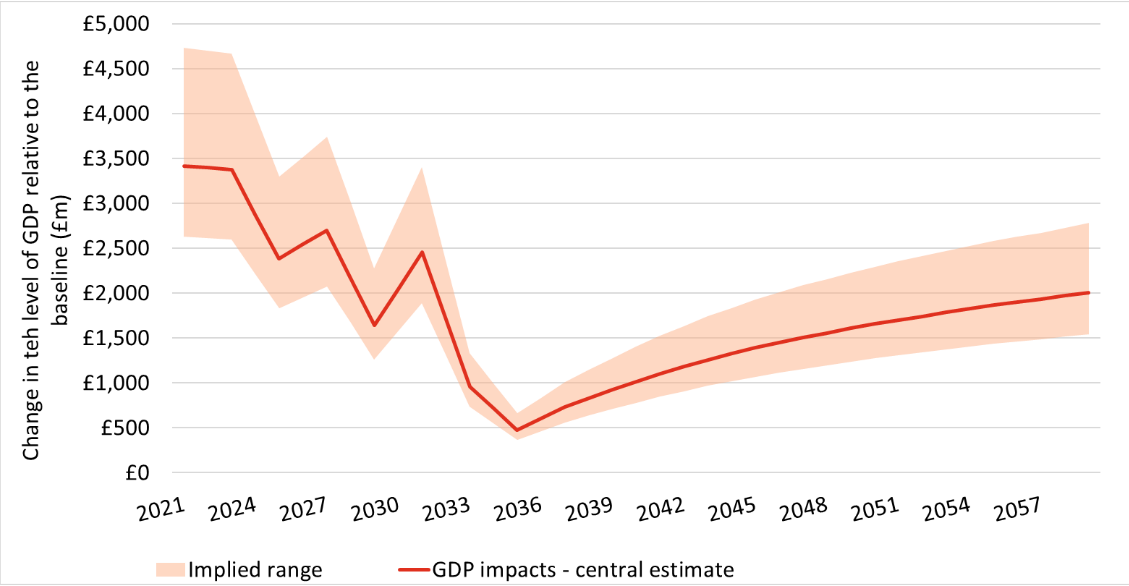
8.2. Absolute GDP impacts

In this section, we summarise our estimate of the national impacts of each of the three phases of the HS2 network. We present the total UK GDP impact associated with the static productivity gains from HS2 operations that are input into the S-CGE model.

Figure 11 illustrates the estimated impact of Phases One and 2a of HS2 on UK GDP. The S-CGE model estimates higher levels of UK GDP from 2021 onwards (the year in which the Phase 2a hybrid Bill received Royal Assent). This is driven by increased private sector investment in anticipation of future improvements in connectivity and associated productivity benefits.

Following the initial surge in private investment prior to the opening of HS2 services on Phases One and 2a, assumed in 2035, the increase in UK real GDP settles to a long-run steady state. As GDP grows over time in the baseline scenario without HS2 services, the level of GDP impacts of HS2 increases over time in absolute terms. In 2051, our model estimates the absolute increase in the level of UK real GDP to be between £1.4 billion and £2.5 billion (2015 prices, undiscounted) compared to the baseline. The point estimate is £1.7 billion.

Figure 11: UK real GDP impact, Phases One + 2a, £m (2015 prices, undiscounted)

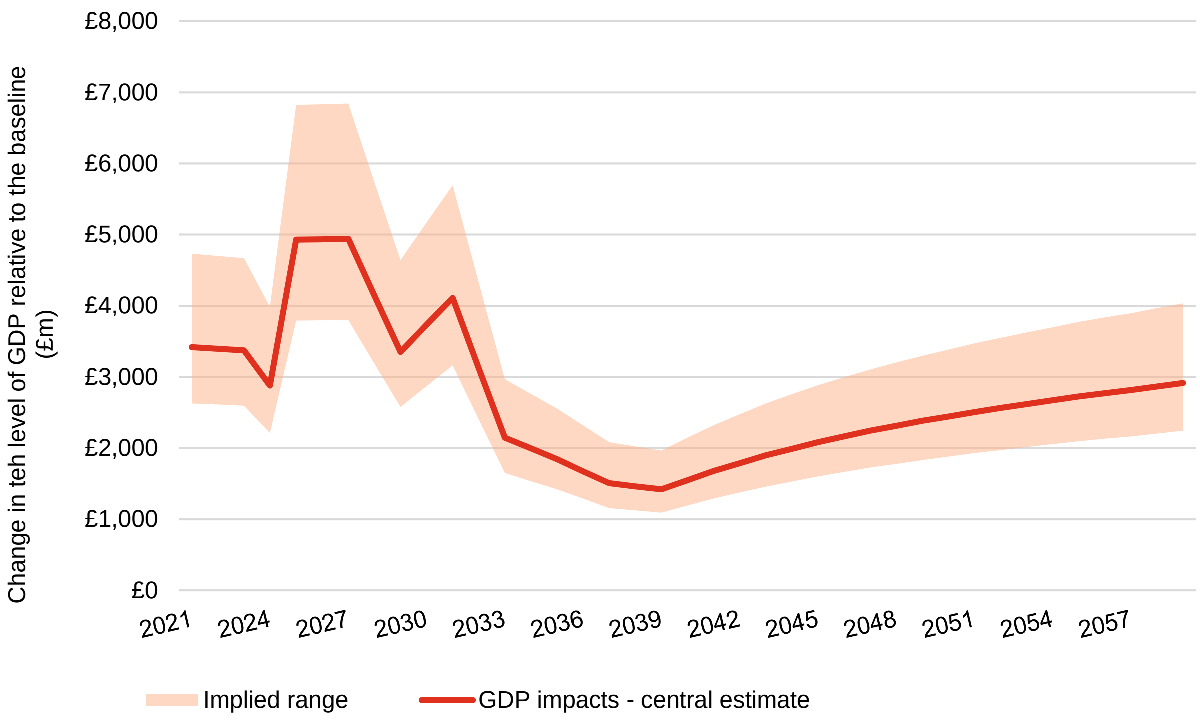


Source: PwC Analysis

Figure 12 illustrates the estimated impact of the Phase 2b Western Leg Full Network on UK GDP. The impact on the level of UK real GDP is estimated to peak between 2025 and 2027, following the assumed Royal Assent date for the Phase 2b Western Leg hybrid Bill in 2025. This stimulates additional private sector investment in anticipation of the productivity gains from completing the HS2 Western Leg network between Crewe and Manchester, in addition to the positive GDP impacts of Phases One and 2a of HS2 during the same period.

Following 2038, when the Phase 2b Western Leg is assumed to become operational, the impact of HS2 operations on the level of UK real GDP associated with the Phase 2b Western Leg Full Network increases over time in absolute terms as the increase in UK real GDP settles to a long-run steady state. In 2051, our model estimates the absolute increase in the level of UK real GDP to be between £1.9 billion and £3.5 billion (2015 prices, undiscounted) compared to the baseline. The point estimate is £2.5 billion.

Figure 12: UK real GDP impact, Phase 2b Western Leg Full Network, £m (2015 prices, undiscounted)



Source: PwC Analysis

8.3. Alignment of expectations and the time profile of impacts

In this section, we discuss how assumptions on the way in which economic agents (households and firms) form expectations impact on the level and profile of the estimated GDP impacts in Figure 11 and Figure 12.

Households and firms are assumed to form expectations of future economic variables (such as interest rates, the price level and output) and adjust their behaviour accordingly. The S-CGE model assumes that agents form 'rational expectations' (Muth, 1961)[47]. This is the standard approach used in contemporary macroeconomic models, such as CGE. Our central estimates of the economic impacts of HS2 assume that households and firms expect the same user benefits and agglomeration and labour supply impacts from HS2 operations as HS2 Ltd estimates using its transport models and will adjust their consumption and investment behaviour when powers are obtained for the construction of the scheme and not before.

As a corollary of this is the "alignment of expectations" assumption. In the S-CGE modelling additional investment is triggered by HS2 in the years prior to the opening of the scheme. Firms are assumed to position themselves to capture future productivity benefits from HS2. This anticipatory investment by firms is assumed to begin in the period immediately following Royal Assent for the hybrid Bill that secures powers for the construction of each phase of HS2. The consumption boost arises from households benefiting from incomes from this anticipatory investment and bringing forward future consumption in anticipation of higher incomes in future due to the productivity benefits of HS2.

As a corollary of this 'alignment of expectations' assumption, additional investment is triggered by HS2 in the years prior to the opening of the scheme and generates additional economic impacts. This anticipatory investment by firms in the period immediately following Royal Assent is due to businesses positioning themselves to capture future productivity benefits. The consumption boost relates to households benefiting from incomes from this anticipatory investment, as well as bringing forward consumption in anticipation of higher incomes in future due to the productivity benefits that HS2 will provide when operational.

The anticipatory increases in investment and consumption estimated by the S-CGE model reflect an assumption that economic agents are confident about the future economic benefits of HS2 services. The extent to which economic agents have confidence about future economic impacts and engage in anticipatory investment and consumption

[47] Muth, J.F., (1961). Rational Expectations and the Theory of Price Movements. Econometrica, Vol. 29, No. 3, pp. 315-335.

spending pre-opening is highly uncertain and is a judgement decision. The literature suggests that “perfect alignment of expectations” has proved to be a relatively good proxy for consumer and investor behaviour from a long-term macroeconomic perspective.[48]

We have seen upfront investment in advance of completion of large-scale rail upgrades in the case of investment in the King’s Cross area in London, prior to the completion of the upgraded rail stations and designation of St Pancras station as the UK terminus of Eurostar services. Investment has also taken place around Leeds station, in anticipation of the substantial station upgrade that began twenty years ago, in Birmingham to take advantage of the upgraded station and retail centre at New Street, and more recently, in anticipation of the future HS2 services to Birmingham Curzon Street station.

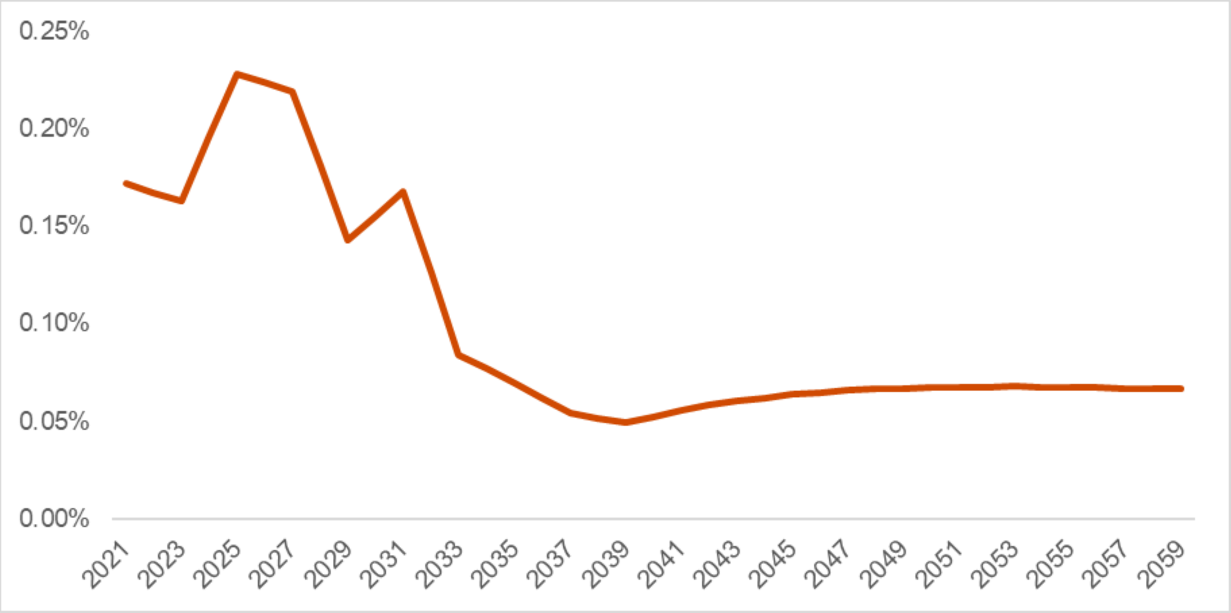
As a sensitivity test, we have tested how the model results change if economic agents are assumed to be less confident about the future benefits by making firms more risk averse in making investments in advance. The model has a variable to reflect businesses’ propensity to return capital to owners rather than retain to invest. The methods by which business can return capital to owners in the model include early repayment of debt, dividend payments, and/or share buybacks are not relevant within the model.

The sensitivity test (which is described in more detail in Section B.2) indicates that by raising firms’ assumed propensity to return capital to owners, there would be a reduction in the level of capital investment in the years preceding the operation of HS2 services and substantially reducing the level of anticipatory investment and consumption. There would also be a smaller yet discernible reduction in the estimated GDP impacts after the commencement of HS2 operations. As an illustration, the model suggests that changing the assumed propensity of returning capital by 0.1 percentage point reduces the GDP impacts of the Phase 2b Western Leg Full Network by 87% in 2021 and 34% in 2051. This test suggests that this factor is a key variable in forecasting the GDP impacts of HS2 and is therefore one upon which further research might be undertaken in future.

8.4. Relative GDP Impacts

It is also helpful to present the estimated GDP impacts of HS2 operations as percentage changes to provide a sense of their scale. Figure 13 below shows the estimated GDP impact of the Phase 2b Western Leg Full Network as a percentage change compared to the baseline level of UK GDP. The initial impact is estimated to be around 0.2% of UK GDP, which then stabilises to around 0.07% in the long term. As explained in Section 8.3, the extent of the initial GDP impacts prior to the opening of HS2 services is highly uncertain.

Figure 13: UK real GDP impact, Phase 2b Western Leg Full Network, percentage change from baseline



[48] E.g., Svendsen, I., 1993, Empirical Tests of the Formation of Expectations, A Survey of Methods and Results, Issued in the series Social and Economic Studies (SES).

Source: PwC Analysis

8.5. Drivers of GDP impact

In the previous Section, we summarised the estimated economic impact of HS2 on UK GDP over time. In this Section, we examine the nature and drivers of these GDP impacts in greater detail. For the central point estimate impact of GDP, we present the S-CGE model's estimated impact on:

- UK GDP by expenditure component (i.e., consumption, investment, government spending, exports, and imports);
- UK GDP according to the transmission mechanisms through which HS2 generates static productivity gains (i.e., business user benefits, agglomeration, and labour supply impacts); and
- the output of different sectors of the UK economy.

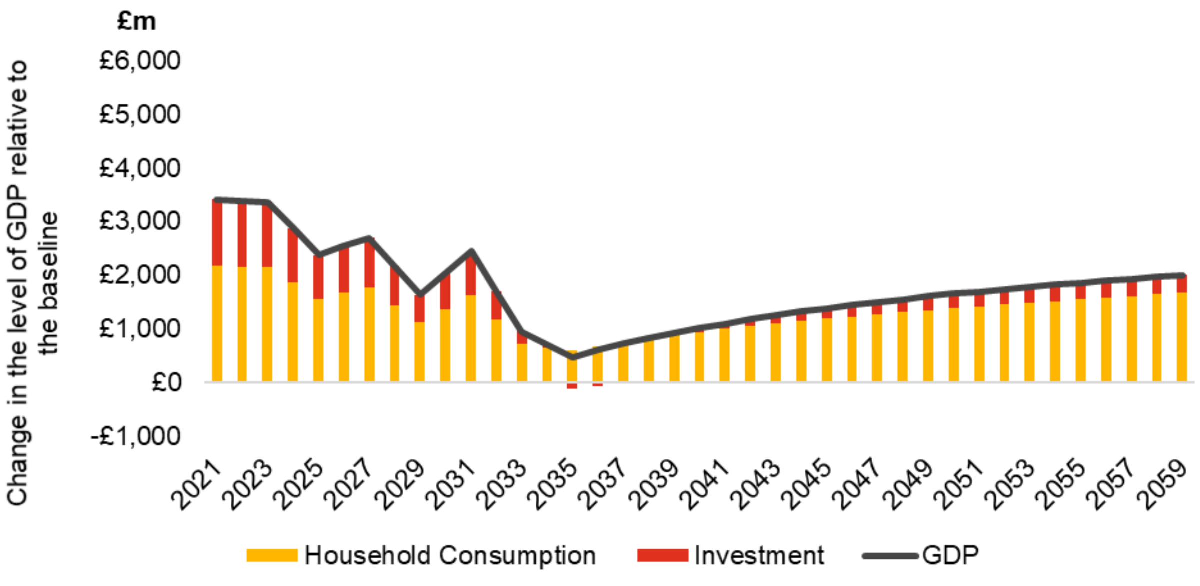
8.5.1. Impact by GDP expenditure component

One approach to measuring GDP is on an expenditure basis. It is calculated by summing the components of final demand by households, businesses and the government within the economy, plus net international trade.

By breaking down the estimated GDP impact of HS2 into these expenditure components, we can better understand which activities in the economy are driving the results. It is important to note that in the S-CGE modelling of HS2, we applied a Harberger closure rule in which it is assumed that net GDP impacts from government spending and trade is equal to zero.[49] This is a typical assumption in CGE models. The GDP impact by expenditure component analysis in this section therefore covers only the impacts of HS2 operations on household consumption and investment spending. These are provided in

Figure 14.

Figure 14: UK GDP impact by expenditure component, Phases 1+2a, £m undiscounted (2015 prices)



Source: PwC Analysis

The GDP impact of HS2 Phases One and 2a is estimated to be £0.7 billion once these phases are assumed to be operational in 2034 to grow thereafter in absolute terms.

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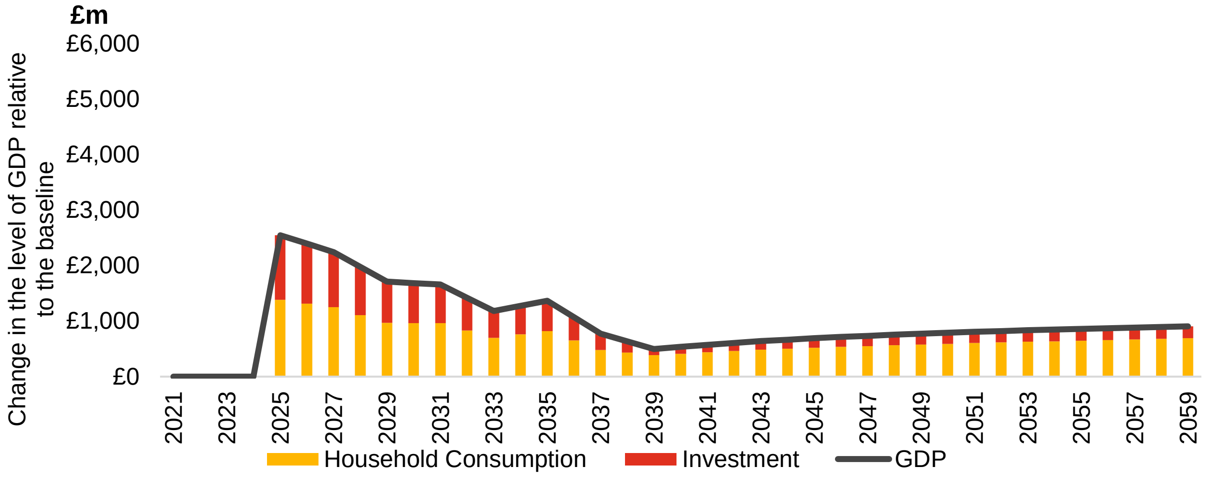
[49] Additional tax revenues associated with the higher GDP are assumed to be transferred back to households, so government spending does not increase in net terms. The international exchange rate largely neutralises international price differences, so changes in net international trade are not assessed.

Figure 15 shows the impacts by expenditure component of the Phase 2b Western Leg Increment. These GDP impacts are constrained in the model to start in 2025, when it is assumed for the purposes of this analysis that Parliamentary powers will be secured for the Phase 2b Western Leg.

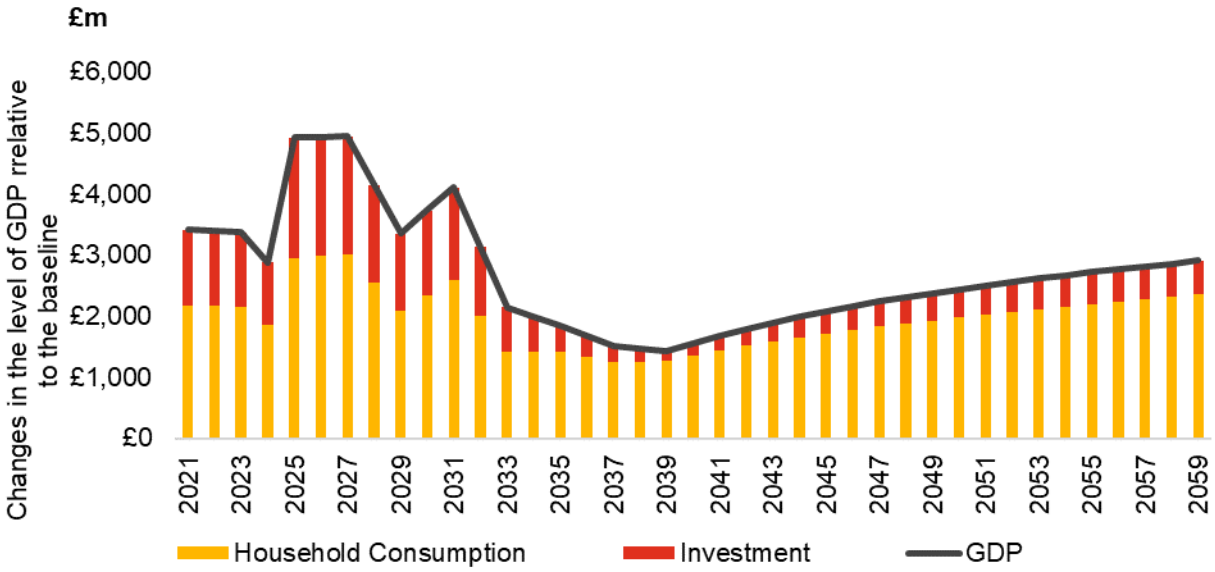
Figure 15: UK GDP impact by expenditure component, Phase 2b Western Leg Increment, £m undiscounted (2015 prices)



Source: PwC Analysis

Figure 16 shows the impacts by expenditure component of the Phase 2b Western Leg Full Network. The time profile of GDP impacts resembles that shown in Figure 13, Phases One and 2a deliver more than half of the total GDP impacts of the Full Network.

Figure 16: UK GDP impact by component, Phase 2b Western Leg Full Network, £m undiscounted (2015 prices)



Source: PwC Analysis

To explain the GDP impacts by expenditure component, we first explain the drivers of increased levels of investment:

- **An upfront investment effect:** businesses in the regions and sectors that stand to gain from the operation of HS2 will invest ahead of its opening to maximise its benefits. Private sector investment decisions are typically made with 20 to 30-year cycles in mind and are not instantaneous. For a business to capture the benefits of HS2, they must plan, as there are substantial adjustment costs (e.g., installation, learning how to utilise new capital) and investment cycles will not necessarily align with the specific opening dates of HS2 services. This leads to the initial 'hump shape' in the GDP profile. As noted earlier, the extent to which firms engage in this upfront investment is a product of the assumption in the S-CGE model of the "alignment of expectations" and is highly uncertain.

- **Supply businesses**: The increased economic activity and expected returns in associated sectors that supply businesses undertaking anticipatory investment leads to a further round of investment by businesses in these sectors. This contributes to the 'bumpy' time profile of investment, as these businesses are responding to investment by firms who directly benefit from the improved connectivity that HS2 will deliver when operational.
- **An ongoing investment effect:** once HS2 becomes operational there are continued productivity benefits to businesses due to the improved transport connectivity and they then invest on an ongoing basis. This effect is smaller but more sustained than the initial investment 'bump'.

Secondly, HS2 operations also increase levels of consumption spending by households. The increase in consumption makes a larger contribution to the GDP impacts of HS2 than the increase in investment. This is to be expected, as consumption accounts for substantially the largest proportion of UK GDP in the baseline. The drivers of the **consumption** increase are as follows:

- **Investment spill over effect (consumption):** as higher levels of investment by firms increase GDP, there will be a corresponding increase in consumption through the additional profits and wages that the investment creates, which accrue to households. The ratio of the final GDP increase, including consumption effects, to the increase in investment is substantial, as there is a relative shortage of capital in the UK economy – i.e., it is a scarce resource. Capital also needs to be combined with labour to be functional. The capital-labour ratio is assumed to be around 35:65, so for every 1 unit of capital generated via investment, around 2 additional units of labour will be utilised in the economy. The wages associated with this labour will translate into higher consumption.
- **Intertemporal consumption effect:** due to the forward-looking assumptions in the model, households will anticipate that they will become wealthier in the future due to HS2. They will bring forward some consumption from the future to derive higher utility over their lifetime. The magnitude of this effect is estimated to be marginal.

Marginal product of labour and capital effect: due to productivity increases associated with HS2, the marginal products of both labour and capital increase. This means that wages and returns to capital increase and consumption increase. Some firms that do not directly benefit from HS2 will raise wages and capital returns above the marginal products of labour and capital to retain workers and investment, and some firms may experience a loss in output due to restricted access to labour and/or capital. This latter effect will (partially) offset increased consumption due to increased productivity associated with HS2.

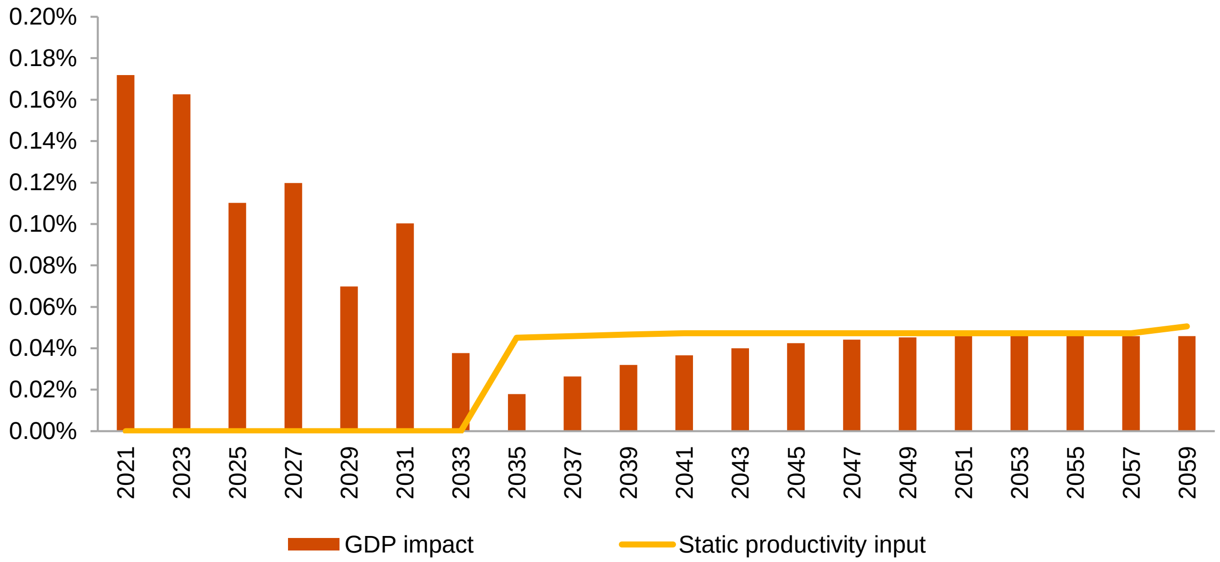
Figure 14 to Figure 16 show that a larger share of the GDP impacts are attributable to household consumption rather than to business investment. However, it is important to note that the GDP impact is driven by business investment: first, before the commencement of HS2 services, business investment activities enable higher wage income and consumption; secondly, after the commencement of HS2 services, the capital assets created by business investment enables the productivity gains to be captured by households through higher incomes.

We discuss this in more detail in Figure 17, Figure 18 and Figure 19 below, in which we set out the relationship between the static productivity gains that are inputted into the S-CGE model and our estimated GDP impacts. In all three Figures, we observe that the model suggests that HS2's GDP impact would materialise in three stages:

- In the first stage, before the commencement of HS2 services, businesses would invest in new capital in anticipation of resultant productivity gains. This is a combination of (i) additional capital investment, and (ii) capital investment that has been brought forward from periods further in the future. As noted earlier, the extent to which firms engage in this upfront investment is a product of the assumption in the S-CGE model of the "alignment of expectations" and is highly uncertain. The 'alignment of expectations" assumption is explored as a sensitivity test in Appendix B.
- In the second stage, in the few years immediately after the commencement of HS2 services, the model suggests that GDP impacts would be below the static productivity gains. In this intermediate stage, there could be lower levels of investment as some of the replacement capital expenditure in this period has been brought forward to the first stage by businesses.
- In the long run equilibrium, the GDP impacts converge towards the static productivity gains as the benefits of improved transport connectivity are being fully captured by the economy and as the increase in UK real GDP settles to a long-run steady state.

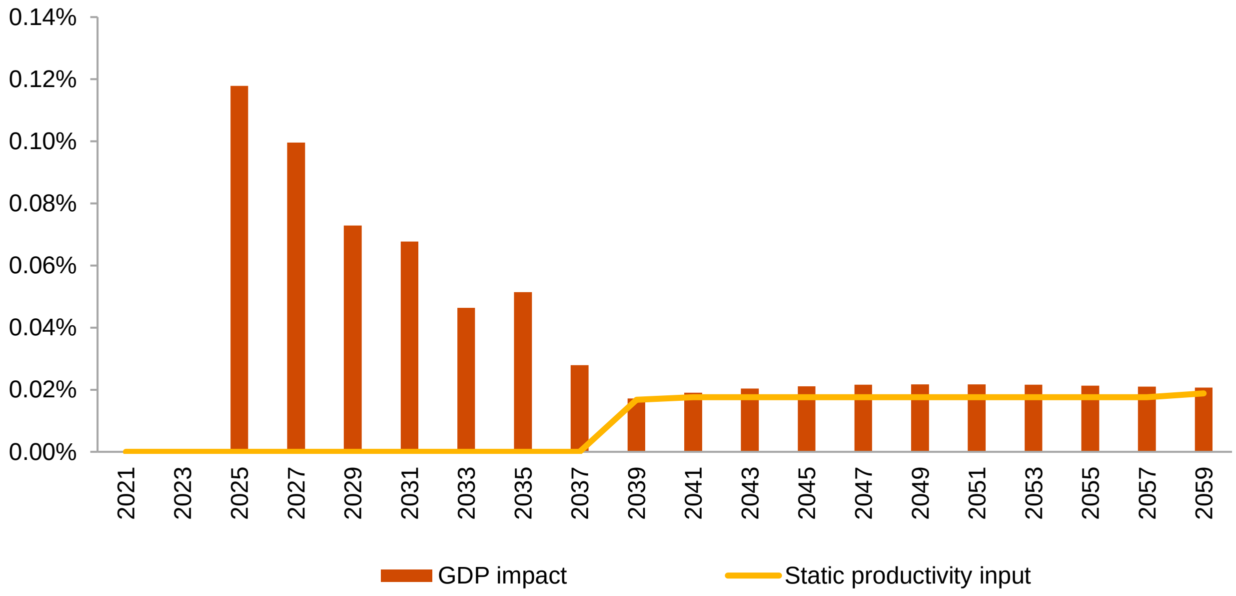
It is important to note that the GDP impact is larger than the inputs. One notable reason for this is the assumption of imperfect competition within the S-CGE model. As discussed in Section 3, in imperfectly competitive markets, the marginal benefits of increasing output (measured by price) are above marginal costs. An increase in output brought about by lower transport costs will therefore generate an additional benefit over and above that which would be experienced in perfectly competitive markets. Part of this accrues to firms in the form of higher profits and part accrues to workers in the form of higher wages, and hence, is reflected in the level of GDP

Figure 17: Total inputs and GDP impacts, Phases 1+2a (% of UK GDP in baseline)



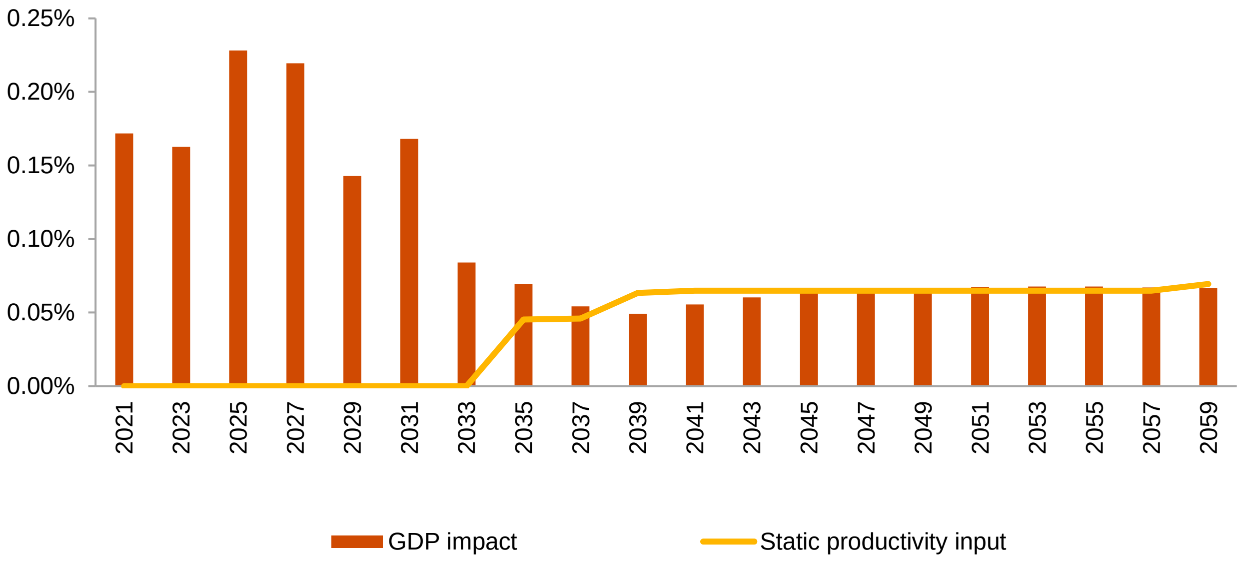
Source: PwC Analysis

Figure 18: Total inputs and GDP impact, Phase 2b Western Leg Increment (% of UK GDP)



Source: PwC Analysis

Figure 19: Total inputs and GDP impact, Phase 2b Western Leg Full Network (% of UK GDP in baseline)



Source: PwC Analysis

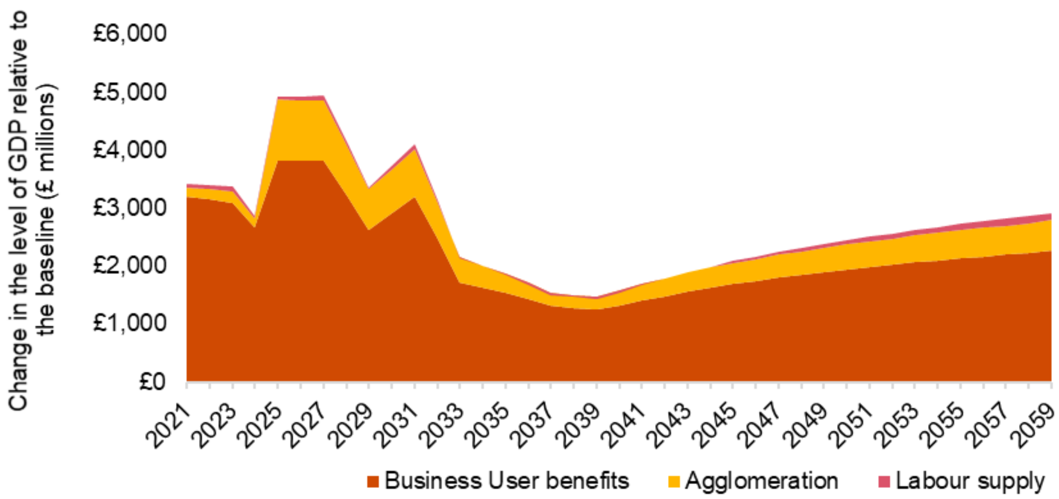
8.6. GDP impact by transmission mechanism

In this section we explore the impact of the three transmission mechanisms through which HS2 operations generate static productivity gains as inputs into the S-CGE model, as outlined in Chapter 3. These are:

1. **Business user benefits;**
2. **Agglomeration impacts; and**
3. **Labour supply effects.**

Figure 20 shows the estimated GDP impact of the Phase 2b Western Leg Full Network, broken down into these three transmission mechanisms.

Figure 20: GDP impact by transmission mechanism, Phase 2b Western Leg Full Network, stacked, £m undiscounted (2015 prices)



Source: PwC Analysis

During the operational phase, which is assumed to begin in 2038 for the Phase 2b Western Leg, the economic impact is dominated by the impact of business user benefits. This result is consistent with previous studies, which have found that ‘wider’ economic impacts, such as agglomeration, typically contribute substantially less than half of the overall impact.

It is important to note that the GDP impact is larger than the inputs. One notable reason for this is the assumption of imperfect competition within the S-CGE model. As discussed in Section 3, in imperfectly competitive markets, the marginal benefits of increasing output (measured by price) are above marginal costs. An increase in output brought about by lower transport costs will therefore generate an additional benefit over and above that which would be experienced in perfectly competitive markets. Part of this accrues to firms in the form of higher profits and part accrues to workers in the form of higher wages, and hence, is reflected in the level of GDP.

8.6.1. Business user benefits

Business user benefits are used to estimate the direct changes in productivity. A reduction in generalised travel costs for business users is equivalent to a reduction in the operating costs of those businesses. These productivity changes, which are specific to each sector and region, are then used as inputs to the S-CGE model. These direct changes in productivity then permeate throughout the economy, generating a wider GDP impact. These benefits are the net outcome of two economic effects: the multiplier effect and the diffusion effect, which act in opposite directions. Overall, the multiplier effect outweighs the diffusion effects and GDP increases, as resources are deployed more efficiently in the economy in response to higher levels of productivity generated by HS2 operations.

- **Multiplier effect** – firms expand output in response to improvements in rail connectivity and increases in productivity due to a reduction in costs. This positively affects the downstream supply chain for those firms, multiplying the positive aggregate demand effects. There will be a further knock-on impact on aggregate demand as workers employed may enjoy higher wages and shareholders enjoy higher dividends.
- **Diffusion effect** – to increase output within a given sector or region, firms need to use more labour and capital inputs. These inputs will need to be sourced from elsewhere in the economy, meaning that prices of scarce resources (such as wages) get ‘bid up’, resulting in higher costs elsewhere in the economy and reducing output. Similarly, to raise capital, firms will need to offer marginally higher interest rates than the market rate, raising the cost of borrowing for all other firms in the economy, again decreasing output overall.

8.6.2. Agglomeration and labour supply effects

The second effect captured within our model relates to the economic impact of additional agglomeration and labour supply effects. A new high speed rail link will increase the connectivity between existing economic centres (static clustering) and/or could cause individuals and firms to relocate across geographies, forming new clusters of activity (dynamic clustering). Better connectivity and greater clustering, whether static or dynamic, can then lead to productivity impacts through the mechanisms discussed below (and in more detail in Chapter 3).

The work of Marshall[50] is relevant when trying to understand the link between agglomeration and productivity in the context of transport investment. First, investment in transport infrastructure can result in knowledge spillovers as lower interaction costs enable firms to share knowledge, which facilitates productivity growth. Second, transport investments can reduce the costs associated with commuting to the workplace and provide firms with improved access to a larger pool of skilled workers. Third, transport infrastructure investment improves backward and forward linkages, thereby lowering costs associated with supplying goods and services to the market.

This increased connectivity enables firms to trade more frequently and more efficiently. We model static agglomeration effects (i.e., assuming no change in the location of individuals or firms) using static agglomeration and labour supply estimates provided by HS2 Ltd and generated using DfT’s WITA model, supplemented by our own analysis. We then input these estimates into the S-CGE model to analyse the general equilibrium effects of static clustering, as well as any dynamic clustering that may occur. Overall, the impacts of agglomeration and labour supply effects on GDP are modest relative to the impacts of business user benefits.

8.7. GDP impact by sector

In this section we explore the GDP impact of HS2 operations by sector. Figure 21 shows the overall GDP impact by sector for the Phase 2b Western Leg Full Network. For ease of reference, we note that the sector key follows the order of the sectors displayed in each bar and the sector description for each code is repeated below.

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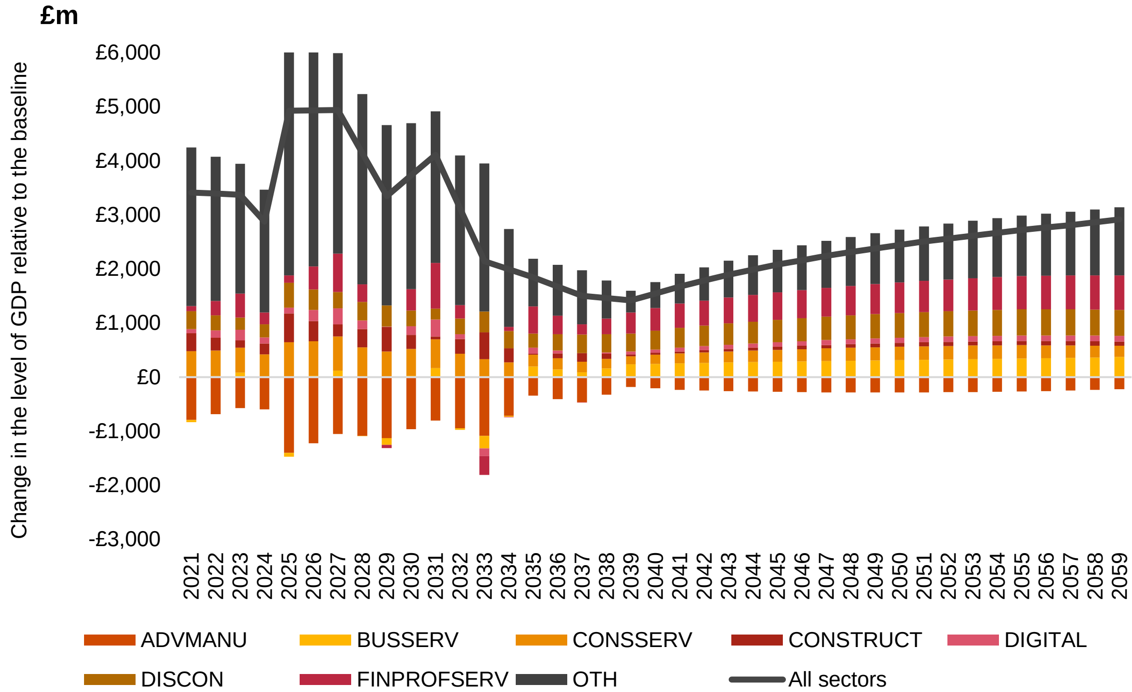
[50] Marshall, A. (1890). Principles of Economics. 8th edition. London, UK: Macmillan

Sector Grouping	Code (Abbreviation)
Advanced manufacturing	ADVMANU
Business services	BUSSERV
Construction	CONSTRUCT
Consumer services	CONSSERV
Digital	DIGITAL
Discretionary consumption	DISCON
Financial services & professional services	FINPROFSERV
Other	OTH

Figure 21: GDP impact by sector, Phase 2b Western Leg Full Network, £m undiscounted (2015 prices)



Source: PwC Analysis

It is important to note that these estimated GDP impacts by sector are strongly influenced by how the inputs to the S-CGE model are allocated by sector. As explained in Section 6, HS2 Ltd allocated business user benefits (which comprise the majority of the inputs into the S-CGE model) to each of the 10 sectors based on each sector's share of National Travel Survey (NTS) intercity (>50km) rail journeys between 2010 and 2019. The aim was to reflect the intensity of sectors using long distance rail. This approach to allocating the S-CGE model inputs by sector is subject to limitations in the NTS data.

Focusing on the period after HS2 services begin operations and when the UK real GDP impacts in the S-CGE model settle to a long-run steady state, the Business services sector it is estimated to account for a significant proportion of the overall positive GDP impact, stabilising at around 13% of the net UK GDP impact after 2046. This is significant for a sector that accounts for only 3% of UK GDP in our model baseline. Similarly, the Financial & Professional Services sector is estimated to account for around 23% of the net UK GDP impact after 2046, compared to an 8% share of UK GDP in our model baseline. This reflects the NTS data that was used to allocate business user benefits by sector in the inputs to the S-CGE model, which show that firms in these sectors are intensive users of long-distance rail.

9. Regional results

9.1. Overview

Using the S-CGE model, we have provided initial estimates of the impacts of HS2 operations on regional economies within the UK. We worked with HS2 Ltd to consider a variety of methods to estimate these regional impacts. Given the experimental nature of the regional analysis and limitations in the underlying data on UK regional economies (for example, the lack of regional I-O tables for England and Wales), the estimates produced for the regional economic impacts of HS2 are subject to a greater level of uncertainty than those produced for the national UK economy. As such, we only present the estimates of the regional analysis for the HS2 Phase 2b Full Network scheme.

To produce a productivity shock for each region to input into the S-CGE model, we worked together with HS2 Ltd to agree how to allocate Business User Benefits, Agglomeration and Labour Supply impacts to each of the ten regions and eight sectors within the spatial element of the S-CGE model. This is discussed in more detail in Chapter 6.

While we consider the approach to be appropriate for this study, it is worth noting that the productivity shocks were allocated to each region according to the regional distribution of agglomeration benefits derived from the WITA model.

We understand that the WITA tool has the advantage that agglomeration benefits are allocated based on regional weights. In the WITA model, static agglomeration benefits are predominately driven by improvements in transport connectivity over relatively short distance and hence are strongly influenced by improvements to local and regional rail services that use additional capacity created by HS2 on the existing rail network. This is a source of uncertainty when considering the regional estimates from a scheme such as HS2 that delivers improvements in long distance connectivity between regions.

Our discussion starts with an overview of the estimated GDP impacts by region generated by the HS2 Phase 2b Western Leg Full Network. We then discuss these impacts by the expenditure and output components of GDP.

While net exports at the national level are assumed to remain at the same level as in the baseline scenario, the model does allow for differences in the balance of exports between UK regions. This means that some UK regions may begin to export more within the UK, at the expense of other UK regions.

9.2. Regional impacts of HS2's Phase 2b Western Leg Full Network

In Table 15, we compare the long run (2051) GDP impact (column B) with the model inputs (i.e., the estimated static productivity gains) for each region (Column A). It suggests that all ten regions will experience positive GDP impacts from the Phase 2b Western Leg Full network (i.e., London to Manchester with HS2 services on the conventional rail network to Scotland), in 2051, relative to the “no HS2” baseline.

Table 15: Regional redistribution of estimated economic impacts, Phase 2b Western Leg Full Network (% of regional GDP, 2051, may not add up due to rounding)

Region	(A) Model inputs: Static productivity gain, 2051, as % of GDP	(B) Model outputs: Impact on regional GDP, 2051, (% of baseline regional GDP in 2051)
North West	0.19%	0.24%
West Midlands	0.09%	0.11%
East of England	0.08%	0.10%
South East	0.07%	0.07%
Scotland	0.04%	0.04%
East Midlands	0.03%	0.04%

Yorkshire and the Humber	0.03%	0.04%
Rest of the UK	0.03%	0.04%
London	0.05%	0.02%
North East	0.01%	0.01%
Total UK	**0.07%**	**0.07%**

Source: PwC Analysis

The North West and West Midlands are estimated to experience the largest GDP impacts relative to the size of their economies in the baseline without HS2 services in 2051 (0.24% and 0.11% respectively) (Column B). We find that the GDP impacts for these regions also materially exceed their estimated static productivity gains (Column A). This is driven by additional growth in household consumption and improvements in the regions' net regional and international export position, as well as higher levels of investment, due to a step change in journey times and connectivity and the associated productivity and income benefits.

It may appear counterintuitive that the GDP gain is smaller in percentage terms for the West Midlands compared to the North West, given the substantial improvement in connectivity that HS2 will provide to both regions. The relative size of the GDP impact is a function of how the static productivity gains are allocated by region in the S-CGE inputs. This follows the regional allocation in the WITA model, with the North West being allocated a substantially higher share of those static productivity gains than the West Midlands in the S-CGE model inputs.

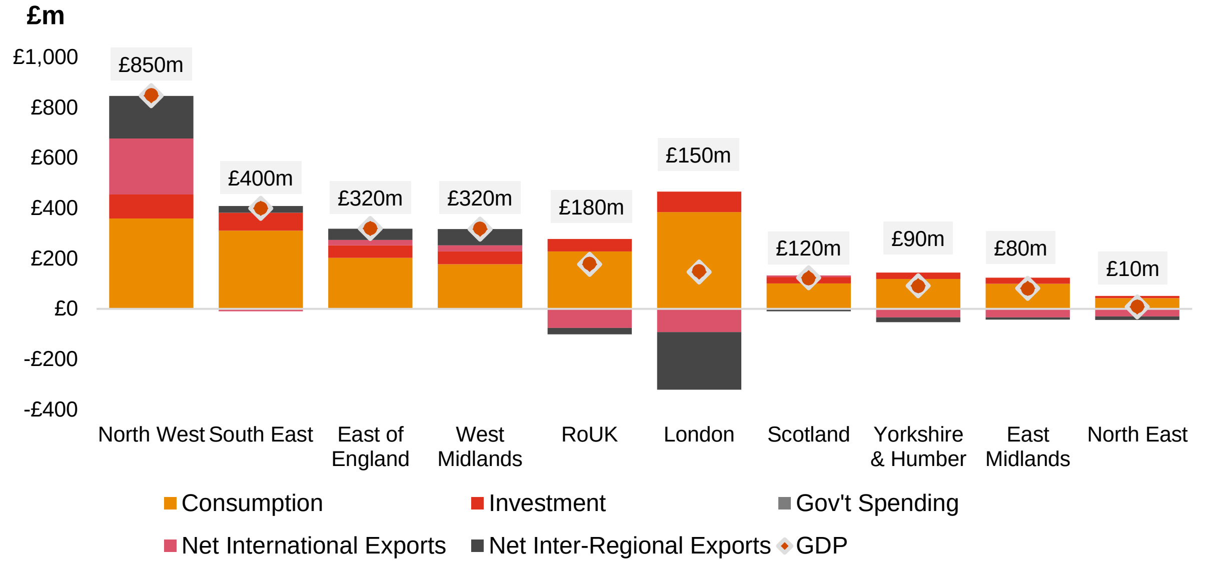
London is also projected to experience a net positive GDP impact in 2051, compared to the "no HS2" baseline. This implies that London's economy will be approximately 0.02% larger, compared to the baseline for this region. This is driven by higher levels of household consumption and, to a lesser extent, investment, which are partially offset by a reduction in net regional and international exports due to changes in the relative competitiveness of UK regions. This results in the GDP impact for London being less than its estimated static productivity gain.

Our S-CGE model also shows that HS2 Phase 2b Western Leg Full Network will positively impact the GDP of the South East and the East of England. While neither region has a station that will be served by HS2 services, both regions will benefit from increased demand for goods and services generated as second round effects through supply chain and labour market linkages.

Figure 23 shows GDP impacts in each region by sector in 2051 (Table 4 provides a key for the sector abbreviations). The GDP impacts are driven by increases in the output of the Financial & Professional Services, Business Services, and the Other Industries sectors. Much of the sectoral impacts, expressed here in absolute terms, is due to the size of these sectors in the "no HS2" baseline. In our model, the "Other" sector is the largest by output in the baseline.

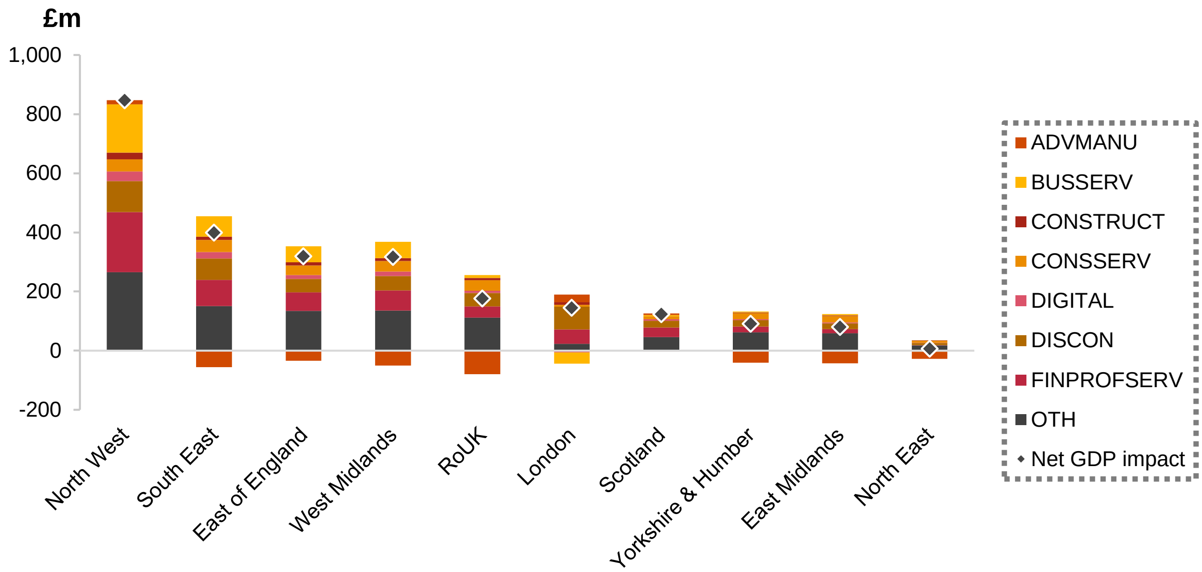
The Financial & Professional Services and the Business Services sectors are estimated to expand as the Business User Benefits that are input into the S-CGE model generate some of the largest static productivity gains in those sectors. This reflects National Travel Survey data which shows that these sectors account for a relatively large share of long-distance rail journeys in the UK, as explained in Chapter 6.

Figure 22: Phase 2b Western Leg Full Network GDP impact by expenditure component in 2051, undiscounted (£m, 2015 prices)



Source: PwC analysis

Figure 23: Phase 2b Western Leg Full Network GDP impact by region & sector in 2051, undiscounted (£m, 2015 prices)



Source: PwC analysis

10. Welfare impact

10.1. Overview

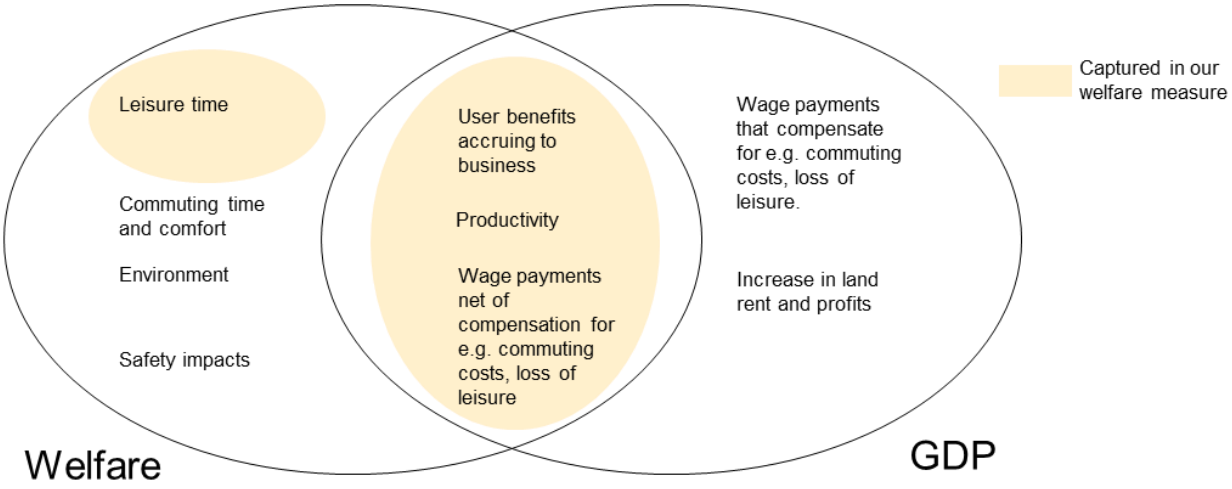
Venables et al. (2014) emphasise that any approach to assessing the effects of transport on the economy requires a framework that can ascribe 'social value' to economic changes. The two main measures of social value used in economic appraisal are economic welfare (the sum of consumer and producer surpluses plus government receipts) and GDP (an aggregation of the value of production in an economy). Whilst these two measures share similarities, they are neither the same nor additive.

Some of the welfare gains associated with transport improvements do not directly affect GDP. For example, time savings to leisure and commuter travellers are valued by individuals and are regarded as a key component of a measure of economic welfare. These are not usually thought to have a direct impact on economic output (GDP).

Other effects influence GDP, but do not have the same impact on welfare. For instance, a person joining the labour market will produce output and therefore increase GDP, but the welfare impact to the individual may be lower than the GDP impact if that person experiences disutility from working.

Our S-CGE model only captures and reports HS2's impact on the part of welfare that can be valued in market prices, which constitutes of (i) household consumption and (ii) workers' leisure time, valued at market wages. We also provide a reconciliation between GDP and the S-CGE welfare measure in Section 10.2. Unless otherwise specified, this is the definition of welfare impacts we report in throughout this chapter and in particular in Sections 10.3, 10.4, and 10.5. It may also be referred to as "S-CGE welfare" or "Reported Welfare" when there is a need to distinguish it from other measures of welfare. The scope of this "S-CGE welfare" measure is set out in the context of Venables (2014)'s theoretical framework in Figure 24 below.

Figure 24: The overlapping welfare and GDP measures & scope of the welfare measure in our S-CGE model



Source: Venables et al. (2014), adapted and annotated by PwC

In this chapter, we also used the term "welfare" in two other meanings:

1. In Section 10.2, the **"general equilibrium welfare"** impact includes both (i) S-CGE reported welfare, and (ii) the user benefits for non-business travellers – whose value of time is not included in our reported welfare measure.

2. In Section 10.6, the term "welfare" has been used in a more general manner as analyses have been carried out to decompose the Reported Welfare into Levels 1, 2 and 3 impacts under TAG, as well as to test its sensitivity to certain appraisal assumptions. We will discuss this in more detail in the rest of this section.

As we discussed in Section 3.3, S-CGE is one of the "Supplementary Economic Models" (SEMs) in the current TAG, which – as TAG's Unit M5.3 sets out – is to be used to *'supplement rather than replace conventional appraisal methods set out in TAG Units A1 and A2'*. This principle applies to both GDP and welfare impacts.

We observe that TAG's conventional appraisal methods – especially in Level 1 – tend to emphasise a micro user perspective. On the other hand, the S-CGE model – being a Level 3 SEM in TAG – captures a more macro economy-wide perspective. There are overlaps in approaches. For example, both the TAG's conventional appraisal methods and the S-CGE model encapsulate business user benefits in their measure of the economic benefits.

Both approaches have their merits – conventional appraisal methods are effective under a well-defined set of circumstances where the economy is assumed to operate effectively, while S-CGE models are effective where there are feedback effects because of land use changes or market failures that mean that the economy is not functioning efficiently.

Differences in modelling approach, methods and assumptions between TAG's conventional appraisal methods and S-CGE modelling approaches make a full and accurate reconciliation between GDP estimates and welfare difficult and inherently uncertain. In the final section of this chapter, we have attempted to provide a calculation which reconciles estimates from the TAG's conventional appraisal methods and S-CGE approaches so that they can be compared. This is illustrative, as our final economic welfare estimate derived from S-CGE modelling is not directly comparable to the estimated level 1 and level 2 welfare benefits of HS2 in the HS2 business case.

10.2. From S-CGE model results to TAG welfare estimates

Figure 25 illustrates each of the steps in our reconciliation of the GDP and welfare measures for the Phase 2b Western Leg Full Network. The welfare results presented in this Section are all undiscounted estimates in 2051, shown in 2015 prices and presented for illustrative purposes.

Figure 25 presents the scale of the business user benefits, agglomeration benefits and labour supply inputs into the CGE model. These inputs (£2,407 million in Step 1) are less than the GDP impacts estimated for the year 2051 (£2,507 million in Step 2), providing a multiplier of 1.042 for Phase 2b Western Leg Full Network in that year. The General Equilibrium welfare estimate of £4.0bn in 2051 (Step 3e) is higher than the GDP impact of around £2,507 million estimated in that year (Step 2), due to the following steps:

- **Investment (Step 3a)**: This general equilibrium GDP impact in Step 2 is made up of consumption and investment. While consumption can be thought of as a component of welfare, investment is not directly a welfare impact. The returns to investment will accrue to households in future years, leading to increased levels of consumption by households and welfare in those years. Investment must therefore be subtracted when transitioning to a welfare-based estimate.
- **Leisure (Step 3b)**: Leisure is a component of social welfare but does not directly contribute to economic output and as such is not included in a GDP estimate. In the S-CGE model, we assume that the relationship between income and leisure is positive, i.e., as incomes rise due to higher levels of productivity and hence wages, individuals demand more leisure. The value of this increased leisure time is added to the GDP estimate in Step 3b. Section 10.3 discusses the modelling of households' choice between working (and hence generating income for consumption) and leisure and our approach to valuing leisure time in monetary terms.

  There is uncertainty as to the strength of these income and substitution effects between work and leisure, and on the appropriate method to value the increase or decrease in leisure time resulting from a change in hours worked. We have not reviewed the empirical literature on labour supply responses to increases in household incomes, but there is the potential for households to choose to maintain or increase their number of hours worked, rather than decrease them, when HS2 increases productivity and wages.

  It is important to note that changes to how the labour supply response is modelled would have a less significant impact on the headline welfare estimate than on the headline GDP estimates, providing that leisure is valued using the marginal wage rate. This implies that if there was an increase in working hours, opposed to a decrease, the net additional GDP from the increase in hours worked would outweigh the reduction in welfare from leisure. The valuation of leisure is discussed in Section 10.3.
- **User benefits for non-business travellers (Step 3d)**: Finally, Step 3d adds user benefits that accrue to leisure travellers and commuters as a result of the introduction of HS2 services. These are a component of the welfare benefits of HS2 and were not included in the inputs to the S-CGE model to generate GDP impacts.

Figure 25: Linking inputs, GDP outputs and welfare estimates in 2051, Phase 2b Western Leg Full Network, undiscounted (£m, 2015 prices)

Source: PwC and HS2 Ltd

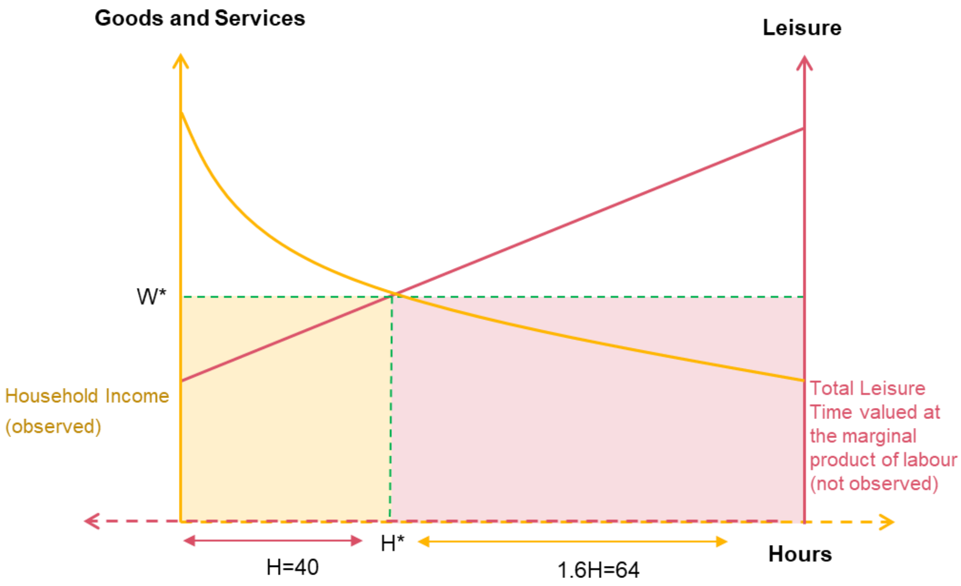
10.3. Consumption-leisure substitution in our S-CGE model

The difference between our estimated GDP and welfare impacts is due in part to households' decision to maximise their utility through the substitution between consumption of goods and services and the consumption of leisure. This is often referred to as the 'time allocation' function in the literature. On the one hand, a household could choose to increase consumption of goods and services by working more, but only at the expense of having less leisure. On the other hand, having more leisure implies a lower income that could be spent on the consumption of goods and services.

In constructing our welfare estimates, we further assume that a household has a fixed number of hours per week to be allocated between work and leisure. The representative household's marginal utility of consumable goods and services (i.e., consumption) diminishes with quantity, which is shown by the yellow line and left-hand axis in Figure 26. Similarly, the representative household's marginal utility of leisure also diminishes with quantity, which is shown by the pink line and right-hand axis in Figure 26.

The fixed number of hours is represented by the distance between the two vertical axes in Figure 26. Household utility maximisation occurs at the point where the marginal utility of consumption and leisure curves intersect, shown by H* hours of work for W* wages per hour.

Figure 26: Illustrative depiction of household choice between consumption and leisure



Source: PwC analysis

Interactions with business production functions require W* to be both the representative household's marginal utility and the marginal product of labour per hour. The total value of household income valued at the marginal product of labour W* – the yellow shaded area – is a known quantity from national statistics.

The total value of leisure is not an observable quantity. For our modelling purposes, we have assumed that in the baseline, before any shocks are applied, the number of hours spent in leisure to be 1.6 times that spent working. If we assume that the value of leisure is set at the marginal product of labour W*, the total value represented by the pink area is also 1.6 times that of the value of household income.

The impacts of HS2 operations on the 'value of leisure' (i.e., step 3b in Figure 10.2) are the result of an increase in the number of leisure hours spent, which are in turn valued at the marginal productivity of labour, W*, as estimated by the wage rate.

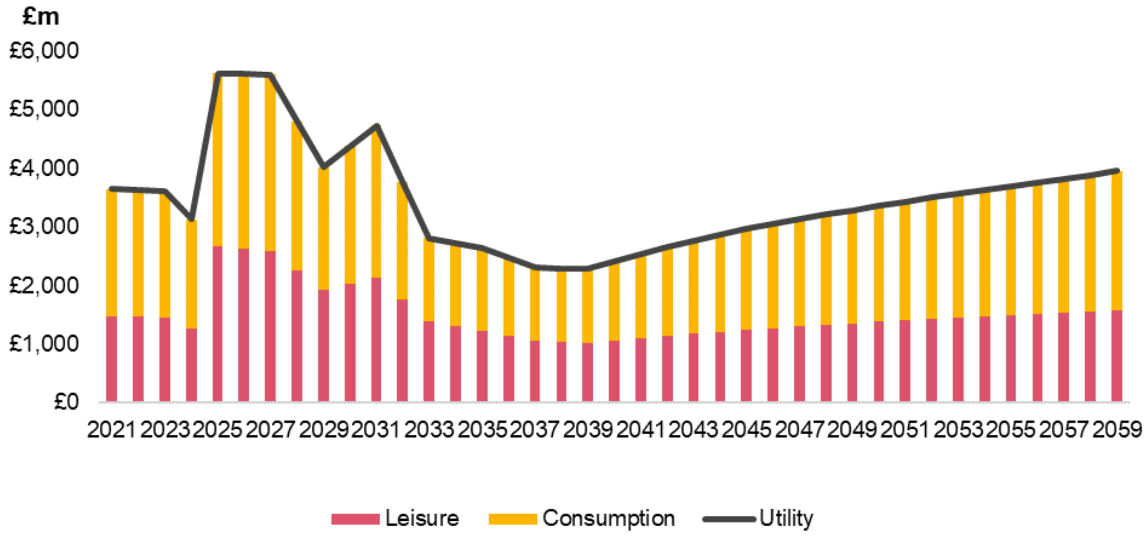
The marginal wage rate W* represents a reasonable proxy for the value that households attach to marginal changes in the number of hours spent in leisure. However, the S-CGE modelling of HS2 suggests a substantial increase in the number of hours spent in leisure, which may be non-marginal changes. It is not clear what alternatives to the marginal wage rate might be used to value non-marginal changes in leisure. One possibility would be to employ the value of time for leisure travellers that is set out in the Department for Transport's Transport Analysis Guidance

(TAG), which is around one third of the marginal wage rate W* used in the S-CGE modelling. Employing this lower value of time to value increases in the number of hours spent in leisure would therefore have the effect of reducing the welfare value of leisure by around two thirds, with a corresponding reduction in the overall welfare estimate (see step G in Table 17).

10.4. Welfare impact results (undiscounted)

Figure 27 shows the undiscounted national welfare impact results for the Phase 2b Western Leg Full Network. In 2051, the net additional welfare impact from the Phase 2b Western Leg Full Network is estimated to be around £3.4 billion. Approximately 60% of the welfare impact derives from increased consumption and 40% from increased leisure.

Figure 27: Phase 2b Western Leg Full Network welfare impact by component, undiscounted (£m, 2015 prices)

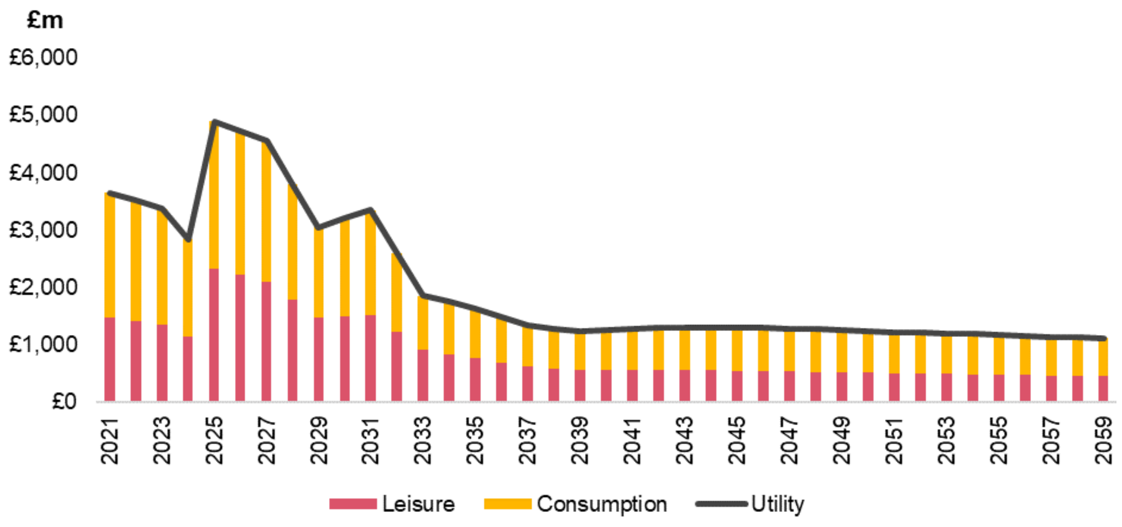


Source: PwC analysis

10.5. Welfare impact results (discounted)

Figure 28 shows national welfare estimates over time for the HS2 Phase 2b Western Leg Full Network in discounted terms. In the long run (2051), the discounted net additional welfare is estimated to be around £1.2 billion. Close to 60% of the discounted welfare impact derives from consumption and around 40% from leisure.

Figure 28: Phase 2b Western Leg Full Network welfare impact on baseline by component, present value, discounted to 2021, (£m, 2015 prices)



Source: PwC analysis

10.6. Welfare Impacts over the appraisal periods

This section describes the final step in reconciling the welfare and GDP estimates.

The Department for Transport's TAG does not specify the appropriate length of an appraisal period for supplementary economic modelling (SEM). The default appraisal period in TAG is 60 years. Table 16 sets outs the appraisal periods we have used to convert annual welfare estimates into net present values (NPVs).

Table 16: Appraisal periods for discounting welfare impacts by Phase

	Appraisal period	Total number of years included in appraisal period
Phases One and 2a	31 Dec 2021 – 31 Dec 2081	60
Phase 2b Western Leg Increment	31 Dec 2025 – 31 Dec 2085	60
Phase 2b Western Leg Full Network	31 Dec 2021 – 31 Dec 2085	64

Based on these appraisal periods, Table 17 sets out the national economic welfare impacts by phase in net present value (NPV) terms and steps through how these welfare estimates would change if Level 1 and 2 welfare benefits are subtracted and sensitivity tests are applied to account for uncertainties regarding the extent of benefits that are realised prior to scheme opening and the value applied to an hour of leisure time.

Table 17: National economic welfare impacts by phase, net present value, discounted to 2021, Q1 2015 prices, £m

		Phases One and 2a	Phase 2b Western Leg Increment	Phase 2b Western Leg Full Network
A	Total S-CGE reported welfare	61,200	34,600	100,800
B	Level 1 - Total User Benefits (incl. Other User Benefits)	29,000	10,100	40,200
C	Level 2 - Agglomeration + Labour Supply Benefits + Imperfect Competition	8,800	3,700	12,800
D	Level 1 + Level 2 Benefits	37,700	13,800	53,000
E	Level 3 - Dynamic Benefits (A-B-C)	23,400	20,800	47,700
F	Level 3 - Dynamic Benefits that occur after HS2 opens	13,200	10,400	27,100
G	Level 3 - Dynamic Benefits after Value of Leisure Adjustment	7,600	9,100	18,600
H	Level 3 - Dynamic Benefits after HS2 opens & after Value of Leisure Adjustment	4,300	4,700	10,800

Source: Calculation by HS2 Ltd
Note: Numbers may not add up due to rounding to the nearest £0.1bn.

Focusing on the Phase 2b Western Leg Increment and stepping through these welfare estimates in Table 10.2:

- **Total S-CGE Welfare (Step A)**: Applying the methodology set out in Section 10.2, the general equilibrium economic welfare for the Phase 2b Western Leg Increment is estimated to be approximately £34.6 billion (2015 prices).

- **Adjusting for Level 1 and 2 Welfare Benefits (Step D)**: The total welfare estimate in Step A includes Level 1 benefits (i.e., benefits to transport users in Step B) and Level 2 benefits (static wider economic impacts in Step C), both of which are accounted for in the standard appraisal of value for money in the HS2 business case. To isolate the transformational (level 3 benefits) generated by HS2 in Step D, we subtract Level 1 and 2 benefits of around £13.8 billion (2015 prices) for the Phase 2b Western Leg Increment from Total General Equilibrium Welfare to generate estimated Level 3 benefits of £20.8 billion (2015 prices) in step E.

- **Sensitivity test – Extent of benefits prior to scheme opening (Step F)**: As discussed in Section 8.3, there is uncertainty as to the extent to which benefits will be realised prior to the operation of HS2 services. We therefore provide a sensitivity test that excludes all welfare benefits that are realised prior to scheme opening, which reduces the estimate of Level 3 benefits of the Phase 2b Western Leg Increment to 10.4 billion (2015 prices) in Step F, while noting that we would expect some benefits to be realised prior to scheme opening as firms increase investment in anticipation of future productivity gains.

- **Sensitivity test – Value of leisure time (Step G)**: As discussed in Section 10.3, there is also uncertainty regarding the value of additional leisure time, which is a component of the estimated welfare benefits generated by HS2. The welfare estimates generated by the S-CGE model assume that households value leisure time at the marginal wage rate. The sensitivity test adopts the value that leisure travellers place on their time, as set out in DfT's Transport Analysis Guidance (TAG), as an alternative estimate for the value of increased leisure time. This value of time for leisure travellers is around one third of the marginal wage rate used in the S-CGE model. As a result, this sensitivity test reduces estimated Level 3 benefits of the Phase 2b Western Leg Increment to around £9.1 billion in Step G. We note that in reality, the value that households place on increased leisure time may lie somewhere between their wage rate and the value of time estimate in DfT's TAG.

- **Sensitivity test - Extent of benefits prior to scheme opening and value of leisure time (Step H)**: This sensitivity test estimates the combined impact of excluding benefits that are realised prior to scheme opening and of adopting the lower value of leisure time from DfT's TAG. This sensitivity test reduces estimated Level 3 benefits of the Phase 2b Western Leg Increment to around £4.7 billion in Step H. We note that this sensitivity test adopts conservative assumptions and should be viewed as a lower bound estimate, rather than a central estimate, for Level 3 welfare benefits.

Appendix A: Structure of our S-CGE model

In this Appendix we:

- Outline the structure of the S-CGE model used in this analysis. We begin by explaining how the S-CGE model maps the circular flow of income and then we go on to outline the two main components, or ‘blocks’, in the model – the consumption block and the production block;
- Discuss the role played by the government, specifically, how government influences consumption and production; and
- Identify the nuances of the model, including the labour market, the dynamic nature of the model, imperfect competition among firms, inter-regional trade, non-resident household expenditure and international trade.

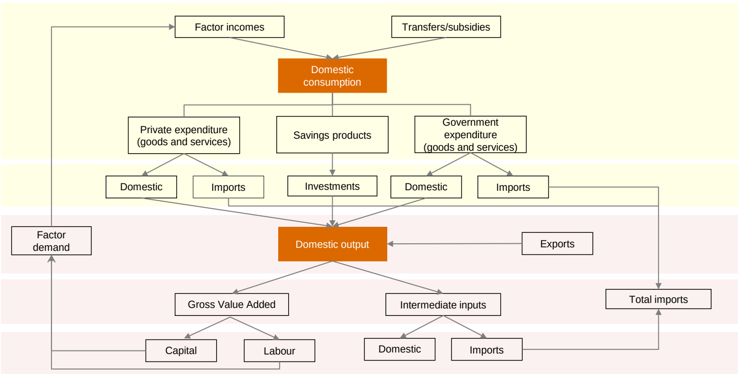
A.1. Model structure

S-CGE models are based on the circular flow of income model which illustrates how economic agents receive and spend income in the economy. In Figure 29, we provide a summary of the circular flow in the context of the S-CGE model. We have split the economy into two main components, or ‘blocks’ as they are often referred to when describing S-CGE models. The consumption block outlines the structure of consumption within the economy and identifies sources of income and how that income can be spent. The production block explains the organisation of the productive side of the economy and how domestic output is determined. In Figure 29, imports and exports refer to both international and inter-regional imports and exports.

Each block contains equations and data that correspond to a key feature of the model. For instance, in the consumption block, the private expenditure feature contains mathematical equations and data that determine private consumption, investment and transfer payments. The diagram we have used to present these blocks does not capture every single economic linkage in the S-CGE model. However, it summarises the most important economic interactions in an intuitive way.

In the figures in this appendix, the rectangular shapes represent sets of model equations and data within the two blocks. The solid arrows represent two-way direct, indirect, and induced linkages in economic activity. The dotted arrows used in later figures represent elasticity parameters and their associated functional forms that govern the interactions of these relationships. The direction of the arrows denotes the flow of money (e.g., payment for intermediate inputs or final goods). The income and price elasticities in our model, where not specified, are derived from the GTAP database[51] in line with the HMRC CGE modelling[52] methodology. We have used these pictorial definitions throughout this chapter.

Figure 29: The circular flow of income



Source: PwC

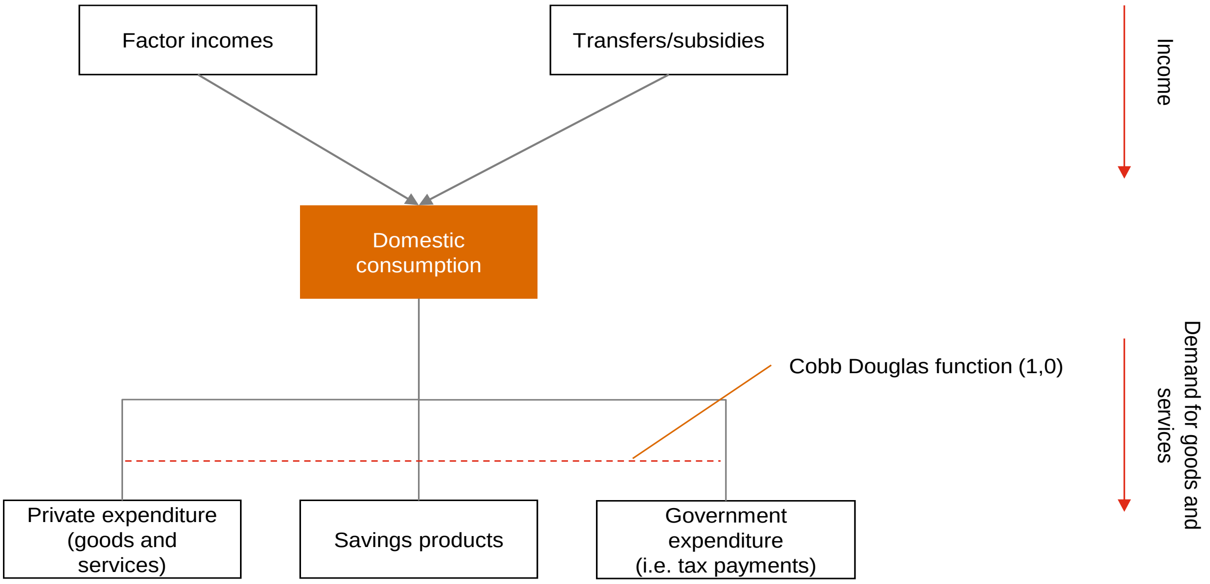
[51] Hertel et al, Behavioural Parameters, https://www.gtap.agecon.purdue.edu/resources/download/4184.pdf

[52] HMRC’s CGE model documentation (December 2013). https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/263652/CGE_model_doc_131204_new.pdf.

A.2. The consumption block

The consumption block outlines the organisation of consumption within the S-CGE model. In our model there are two sources of income: factor incomes, such as wages and gross operating surplus, and government transfers/subsidies from the redistribution of taxes collected. This income can then be spent in three ways: government expenditure, private expenditure, and savings products – otherwise known as national savings. Changes in the relative price of consumption are governed by a Cobb-Douglas function[53] with unit elasticity. This means that a tax rise which increases the relative price of private expenditure by 1% will initially lead to a 1% reduction in the relative quantity of private expenditure. However, in the S-CGE model, ensuing behavioural effects and relative price changes are likely to reduce this effect. Private and government expenditure lead to demand for domestic and imported goods, while savings products drive investments in the economy

Figure 30: The domestic consumption block



Source: PwC

In this S-CGE model, we assume that consumers and users view domestic and imported goods of the same product category as imperfect substitutes under an Armington preference system. This is discussed in more detail in Section A.8 below.

The Linear Expenditure System (LES) governs spending by households. The functional form of the LES requires households to have a minimum level of consumption for subsistence (e.g., housing and food). The scale of these minimum requirements is linked to the income elasticity of demand for these goods and typically subsistence goods have an income elasticity of less than 1. This means that as income falls, say by 20%, demand will fall by a figure less than 20%. The minimum requirements for subsistence are calibrated to achieve certain income elasticities of demand for goods as specified by the UK estimates generated by the GTAP (Hertel, 1997)[54] database.

Investments in the economy are driven by purchases of savings or national savings. Government spending is ruled by a Cobb-Douglas function with unit elasticity to represent spending in each sector being a constant proportion of total government spending. The S-CGE model accounts for international capital mobility through the balance of payments, i.e., a current account deficit must be matched by a capital account surplus.

Households maximise inter-temporal utility subject to:

- Expected prices of goods and services; and
- Factor earnings.

---

[53] We specify utility functions of the Cobb–Douglas form. The Cobb-Douglas function is a common functional form used in economic theory (the most well-known use being in production).

[54] Hertel, T.W. (1997). Global Trade Analysis: Modelling and Applications. New York, NY: Cambridge University Press.

The model does not account for wealth distributions explicitly as there is no data available, but the model accounts for regional variation in households through regional household labour and non-labour income and consumption distributions. Demand for domestically produced goods and return on investments drives domestic production, which in turn drives the demand for factors of production.

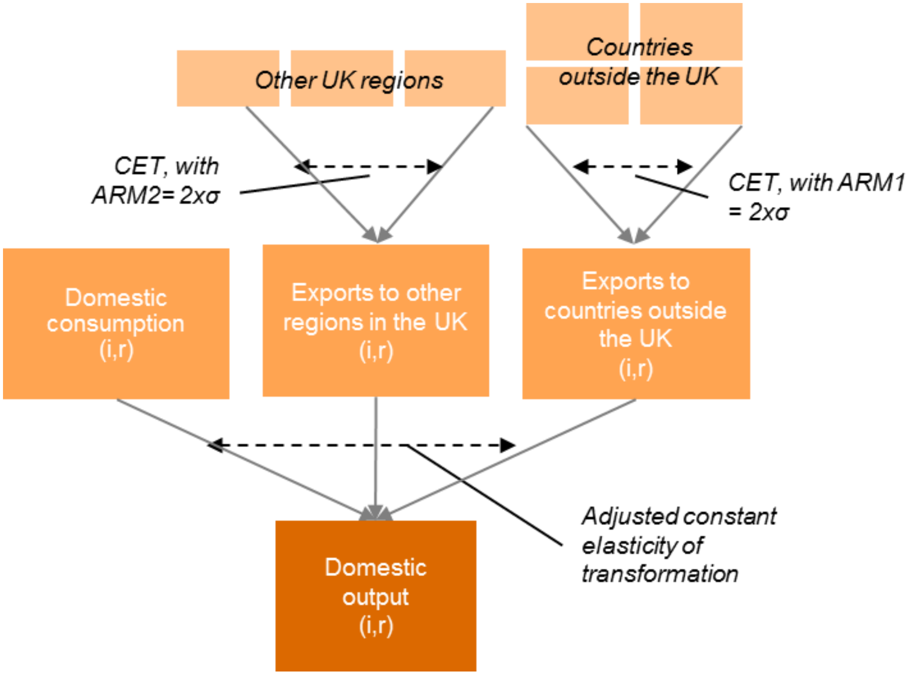
The factor incomes received by households and firms support domestic consumption in a circular loop.

A.3. The production block

The production block contains the structure of the productive side of the economy within the S-CGE model. Demand from domestic (in S-CGE model, domestic would mean a 'region') and foreign consumers results in output being sold in both markets; firms decide on the amount they want to supply to each market recognising that there is a cost involved in changing markets.

Figure 31 illustrates these relationships.

Figure 31: The domestic output nest



Source: PwC

In the model, output produced in a sector, i, and a region, r, can be exported overseas or to other UK regions, or be consumed by households in region r. When an economic scenario is imposed on the model, the proportions that are exported and consumed adjust according to changes in relative export and domestic prices that are determined endogenously within the model. The rate at which these proportions change is governed by a Constant Elasticity of Transformation (CET)[55] function. For example, if the CET equals 3, a 1% rise in the relative price of exports outside the UK causes firms to increase the relative quantity of exports to countries outside the UK by 3%.

Domestic output is produced from intermediate inputs used in the production process, both imported and domestically produced, and Gross Value Added (GVA). In the S-CGE model, the Leontief production function[56] is used to represent the technological relationship between the amount of inputs (GVA and intermediate inputs) used and the

---

[55] The CET models producers' decisions about how they allocate production to the domestic and export markets. It is the corollary of the constant elasticity of substitution, CES, function. For a more detailed description see Powell & Gruen (1968). Powell, A.A., & Gruen, F. (1968). The constant elasticity of transformation production frontier and linear supply system. International Economic Review, 9(3), 315-328.

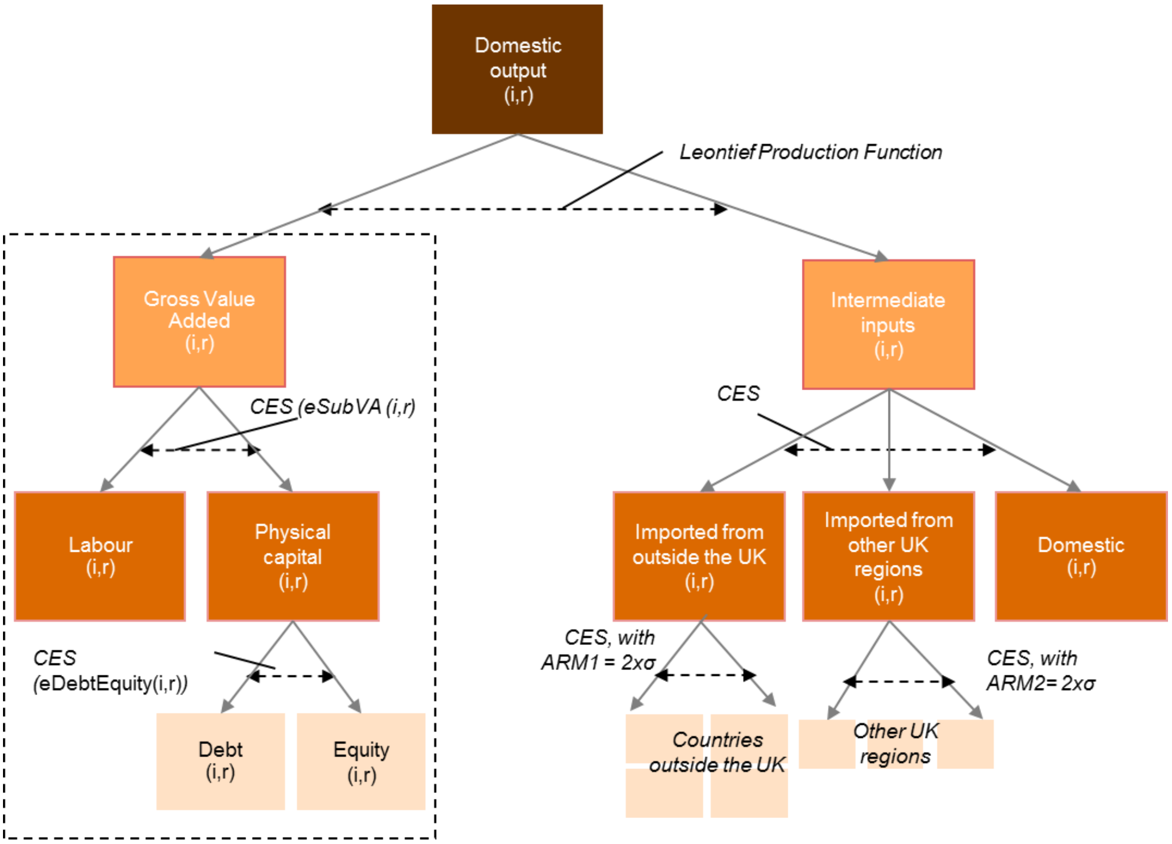
[56] The Leontief production function or fixed proportions production function is a production function that implies the factors of production will be used in fixed (technologically pre-determined) proportions, as there is no substitutability between factors. For a more detailed discussion see Allen, R.G.D. (1968). Macro-economic Theory: A Mathematical Treatment. London: Macmillan. Our approach is consistent with the HMRC CGE modelling methodology.

amount of output that can be produced. Using this function ensures that the proportion of inputs (GVA and intermediate inputs) is fixed. These relationships are illustrated in the domestic production nest in Figure 32 below.

The domestic production nest lays out the transmission mechanism that when an economic scenario is imposed on the CGE model then:

- GVA will vary across sector i and region r.
- Use of intermediate inputs will vary across sector i, region r and product j. They are sourced from the use matrix and are subject to taxes on different production processes.

Figure 32: The domestic production nest



Source: PwC

Producers can choose between using physical capital and labour for production, and the decision depends on the marginal productivity of these inputs and their relative prices. The size of the Constant Elasticity of Substitution (CES)[57] governs the rate at which the proportions of labour and physical capital change. For example, if CES = 3, then a rise in the relative price of labour of 1% leads to a 3% increase in the relative quantity of physical capital used in production as firms substitute away from more expensive labour.

A wide range of intermediate inputs are used in the production of domestic output. In the model, these inputs are purchased from the 8 different sectors. For example, if a manufacturing company in region r purchases business services in region r, this will be recorded as expenditure by the manufacturing company on domestic intermediate inputs.

Furthermore, in the S-CGE model, there is a regional element of cross-border intermediate purchases which is captured in the inter-regional trade flows, although the data are limited in distinguishing explicitly between final demand and intermediates imported. Inputs are either sourced from domestic producers, or they are imported from

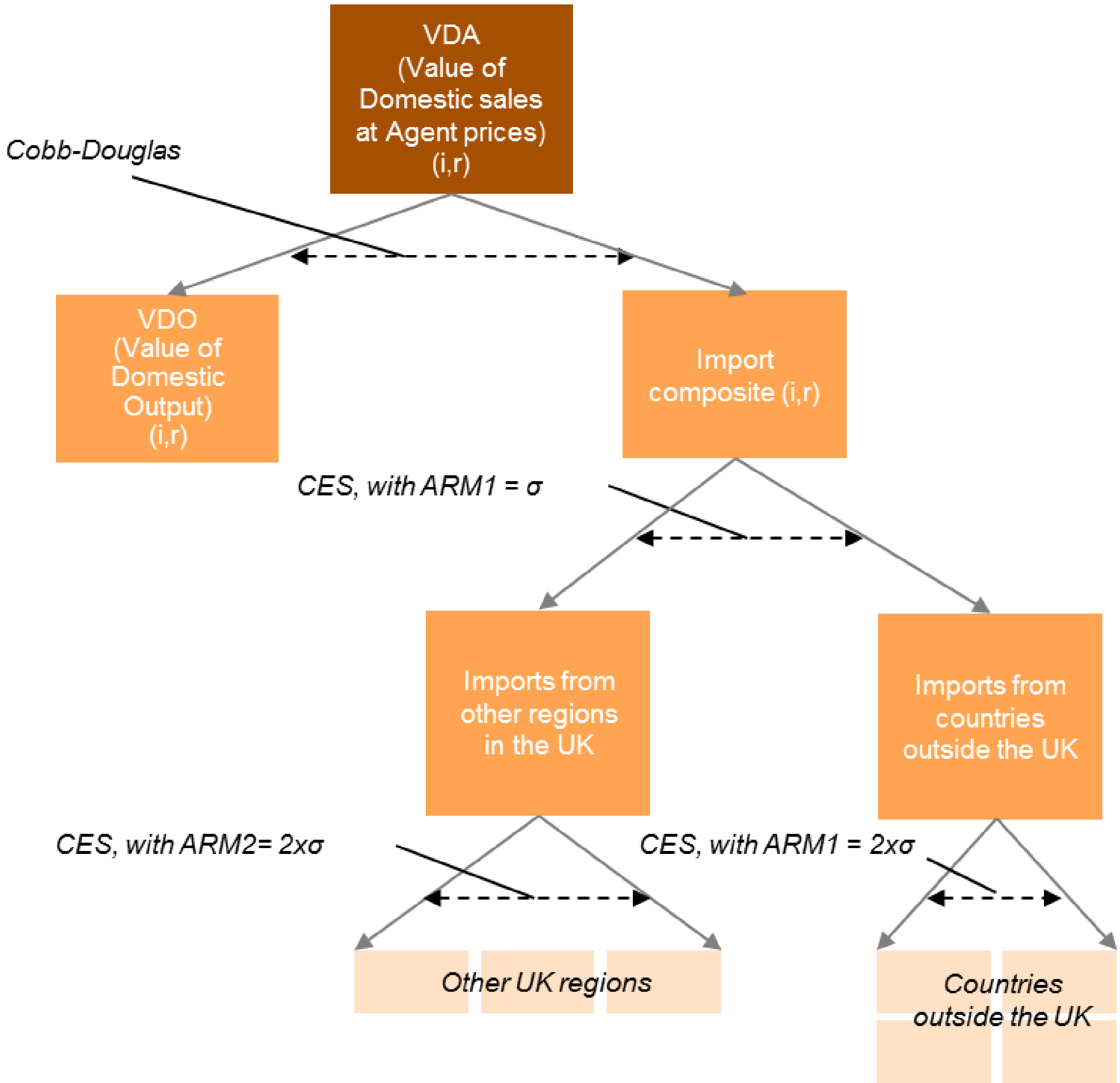
[57] The two factor (Capital, Labour) CES production function introduced by Solow, R.M. (1956). A contribution to the theory of economic growth. The quarterly journal of economics, 65-94.

other regions in the UK, or from outside the UK. The choice between domestic inputs and imported inputs is also determined by a CES function under an Armington preference system.

A Cobb-Douglas relationship is used to represent the relationship between domestic output and exports. A Cobb-Douglas function with unit elasticity of substitution means that a 1% rise in the relative domestic price of a product will result in a 1% increase in the relative quantity of exports of the same product.

Armington preferences[58] govern the choice between importing from other regions in the UK and from outside the UK. This takes into account consumers' preference for variety, in terms of the source of their purchases. The choice of different UK regions for imports from other UK regions, and the choice of different regions for imports from the rest of the world are also based on Armington preferences, with the elasticity of substitution being twice that of the higher nest as seen in Figure A.5 on the following page.

Figure 33: Structure of inter-regional output



Source: PwC

[58] An Armington elasticity represents the elasticity of substitution between products of different countries, and is based on the assumption that products traded internationally are differentiated by country of origin. For a more detailed discussion see: Armington, P.S. (1969). A Theory of Demand for Products Distinguished by Place of Production. Staff Papers-International Monetary Fund, 159-178.

A.4. The Government sector

Government performs two roles in the S-CGE model: collecting taxes and spending money. Government expenditure is split into the two main functions used in government accounting: Departmental Expenditure Limits (DEL)[59] and Annually Managed Expenditure (AME)[60].

There is a specific category in the S-CGE which is a catch-all for the non-capital elements of government department spending (R-DEL). The model also captures capital (DEL or C-DEL) which is government expenditure on capital investment projects such as infrastructure spending. The model estimates the government's capital stock using the assumption that it depreciates at a rate of 5% per annum, so some government investment is necessary to preserve the level of capital.

AME data is available at the household and regional level from the Department of Work and Pensions (DWP)[61], and is split into tax credits/benefits linked explicitly to hours worked, state pensions, disability benefits and other benefits.

The model was constructed using assumptions about the burden of taxation across households and regions in line with published HMRC statistics[62] about regional tax payments. The S-CGE model captures approximately 95% of all tax payments to the UK exchequer –taxes paid on a realisation basis, such as stamp duty and capital gains tax, are not modelled.

The government budget balance is dictated by what is known in CGE modelling terms as a closure rule, i.e. the gap between government spending and receipts must be 'closed' within the model[63]. Suppose government DEL spending increases – then there are four main ways in which it can be closed in the model:

- **Harberger closure rule** – Through an ad-hoc lump-sum tax on households;
- **Tax closure rule** – Specific tax rates (VAT, corporation tax, income tax etc.) can increase to finance the additional spending;
- **Debt closure rule** – The extra spending can be funded by increasing the fiscal deficit; and
- **Household transfer closure** – Benefits can be cut.

Correspondingly, if taxes were cut, then in addition to the debt, or household transfer closure rules being invoked, government spending could be cut to restore balance to the public sector balance sheet (government closure).

The tax closure rule allows the model to investigate revenue neutral tax reform options. For instance, if the government chose to increase income tax to finance a cut in VAT, then this could be specified as a separate closure rule in the model. This allows the economic efficiency of different tax options to be compared. For instance, if a cut in VAT which is financed by an increase in income tax led to a reduction in GDP, this would imply that income tax is less economically efficient tax than VAT. The outcome of this experiment would be dependent on a range of factors, e.g., which bands of income tax were cut, which elements of VAT were increased (the standard or reduced rate), the initial rates of taxation, the surrounding assumptions about the economic environment (e.g. strong consumption outlook or weak labour market outlook). The full range of revenue neutral options are not specified in the basic version of the model but are specified separately depending on the scenario modelled.

Rather than specifying any taxes to offset each other, or to avoid using a specific tax category to close the model when investigating changes in spending or government debt, an alternative closure rule is specified to proxy the

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[59] Spending which is planned and controlled on a three year basis in Spending Reviews. The DEL is the annual spending limit imposed on a government department arising from its agreed, longer-term financial settlement with DFP.

[60] Expenditure that is relatively volatile and largely demand-led that cannot reasonably be given firm, multi-year limits in the same way as DEL. AME includes social security benefits, local authority self-financed expenditure, debt interest, and payments to EU institutions.

[61] Department for Work and Pensions (2010). Regional Benefit Expenditure. Available at: http://data.gov.uk/dataset/gb-regional-benefit-expenditure

[62] HM Revenue & Customs (2013). A disaggregation of HMRC tax receipts between England, Wales, Scotland & Northern Ireland. Methodology Note. Available at: http://webarchive.nationalarchives.gov.uk/20140109143644/http://www.hmrc.gov.uk/statistics/receipts/disagg-method.pdf

[63] The description here refers to the various closure rule options. However, the construction 'shock' was designed such that the increased government spending due to the LTC is matched by cuts in other areas.

effects of an assumed ‘lump-sum’ tax. This tax assumption is taken from Harberger (1962)[64] and is the most common approach used in CGE models as a neutral closure assumption. The lump sum tax levies an equal tax on each household in response to a change in spending, government debt or other taxes.

The debt closure rule assumes that changes in the budget position are not offset by other policies but affect the level of national debt. For example, an increase in government debt levels leads to a transfer from the corporate sector which buys gilt-edged bonds to finance the government debt. The model includes an assumption whereby the level of national debt affects the risk premium faced by investors in the economy. Increasing levels of national debt raise the risk premium thereby resulting in lower returns to investors. The model allows this relationship to be strengthened or weakened in sensitivity testing.

The S-CGE model has built in ‘closure rules’ to maintain fiscal balances. For instance, if the government chose to cut the corporate tax rate, then this would need to be financed from government spending, transfer payments, debt, or increases in other taxes. Furthermore, if the corporate tax cut increased the level of activity in the UK, then tax receipts in the Rest of the UK would increase. The model would automatically invoke the Harberger closure rule to bring fiscal positions back into balance.

In this modelling exercise, we used the Harberger closure throughout unless otherwise specified.

A.5. The labour market and migration flows

A dynamic labour market function underpins the S-CGE model. It incorporates a direct relationship between employment, wages and levels of economic activity. Its core properties are as follows:

In any given region, changes in wages can lead to workers entering or exiting the labour market. This can take place in the three forms below:

- Migration
- Inter-regional commuting, and
- Changes in labour participation rate and intensity within each region.

In this modelling exercise, labour supply decisions under different wage levels are governed by a single labour-leisure substitution elasticity. As such, labour supply responds to changes in wages in each region. However, it does not distinguish between three different channels through which labour supply could change when wages change set out above.

Data on employee compensation are taken from ONS GVA figures, with gross wages being a subcomponent of employee compensation, the other component being benefits in kind (BIKs). BIKs consist of a range of financial and non-financial employee remuneration such as company mobile phones, vehicles, accommodation allowances etc. Employee compensation data are broken down by region and by sector in the S-CGE model.

A.6. Model dynamics

The S-CGE model is dynamic. This means that it makes a forward-looking projection of the economy over time. The model assumes full alignment of expectations between the economic agents and HS2 Ltd’s estimates. It simulates a total of 25 time periods.

The length of time over which the model can simulate the economy is dependent on two main factors:

- **The complexity of the scenario and the magnitude of changes:** More complex scenarios or scenarios with large changes to the economy use more computing power and make the model harder to solve, thus necessitating the need to reduce the number of time periods; and
- **The overall size of the model:** the additional equations relating to the dynamic labour market and imperfect competition increase the size of the model considerably and therefore the required computing power.

Time periods are linked through savings, household utility, and capital accumulation. In each time period capital adjustment is governed by a standard depreciation plus investment function. The model is calibrated so that each

[64] Ibid.

time period is equal to 2 years. However, this is approximate and where possible the adjustment processes in the model need to be compared directly to econometric evidence about adjustment speeds to policy changes to refine the model's accuracy.

Investment in each sector, and by type of capital, is subject to installation costs (Uzawa 1961,[65] Markusen et al. 2000)[66] whereby the cost of investment is related to the amount of installed capital. The equations are set-up so that more rapid capital accumulation therefore becomes increasingly costly.

A.7. Imperfect competition

In the model, sectors are imperfectly competitive with increasing returns to scale. Imperfect competition is based on the Dixit-Stiglitz large-firm Cournot structure[67]. In particular, the **profit function**, Π_(i,r), of a representative firm in sector (i) and region (r) is:

$$L = \Pi_{i,r} = p(d_{i,r})d_{i,r} + p(e_i)e_i + p(m_{i,r})m_{i,r} - c_i\left(d_{i,r} + e_i + m_{i,r}\right) - f_i$$

The demand for the firms' products in the domestic region is represented by $d_{i,r}$ and its price is given by $p(d_{i,r})$. Demand in the export market is e_i and its price is given by $p(e_i)$. Demand in a particular region r is given by $m_{i,r}$ with the price being represented by $p(m_{i,r})$. Marginal cost is a function of total demand, $c_i\left(d_{i,r} + e_i + m_{i,r}\right)$, and fixed costs are given by f_i.

Using the profit function as the Lagrangian and solving for the profit maximisation problem, the first order conditions (FOC) below show the relationship between mark-up and price elasticity of demand. Similar to other CGE modelling literature using models with imperfect competition and increasing returns at the firm level, this model employs the Lerner formula to endogenously set the price mark-up.

$$\frac{p(d_{i,r}) - c_i}{p(d_{i,r})} = \frac{1}{|\tau_{i,r}|}, \ \tau_{i,r} < -1$$

$$\frac{p(e_i) - c_i}{p(we_i)} = \frac{1}{|\delta_i|}, \delta_i < -1$$

$$\frac{p(m_{i,r}) - c_i}{p(m_{i,r})} = \frac{1}{|\eta_{i,r}|}, \eta_i < -1$$

The price elasticity of domestic demand for a product in sector i and in region r is given by $\tau_{i,r}$. The price elasticity of export demand is represented by δ_i, and price elasticity of a particular region's demand is given by η_i.

The demand elasticity that the representative firm faces is then calibrated with our estimates of the Herfindahl-Hirschman Index (HHI) in each sector.

The HHI captures the degree of competition and is calculated by summing the squares of the market shares of the firms within a given sector, where the market shares are expressed as fractions. This gives a figure between 0 and 1, which is proportional to the weighted average market share. For instance, if there were only one firm in a given sector, that firm would have 100% market share and the HHI would equal 1. Alternatively, if there were hundreds of firms competing, each would have a market share closer to 1%, and the HHI would be close to 0.01, indicating nearly perfect competition.

The HHI values we have used in our model were computed by the Office of National Statistics in 2010 using data from the Business Register and Employment Survey (BRES).

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[65] Uzawa, H. (1961). Neutral inventions and the stability of growth equilibrium. The Review of Economic Studies, 28(2), 117-124.

[66] Markusen, J., Rutherford, T.F., & David T. (2000). Foreign Direct Investments in Services and the Domestic Market for Expertise. NBER Working Papers 7700. National Bureau of Economic Research.

[67] Dixit, A.K., & Stiglitz, J.E. (1977). Monopolistic competition and optimum product diversity. The American Economic Review, 297-308.

A.8. Inter-regional trade

The way in which the foreign sector is modelled in our S-CGE model is largely governed by the Armington (1969)[68] assumption, whereby domestically produced and imported goods are treated as imperfect substitutes. The elasticity of substitution between domestic (or local) and imported goods is then known as the Armington elasticity.

This captures a key characteristic of real-world trading patterns, in which countries and regions often simultaneously import and export goods in the same product category.

In this modelling exercise, the elasticities are sourced from the GTAP 10 database.

A.9. Inter-regional trade data

Usually when CGE models are built regional trade data are available: examples include Spain (Gillham, 2004)[69] and Australia (Regional version of the ORANI model, which is a general equilibrium model of the Australian economy). There are other models which partially cover the regional trade data; the general equilibrium model for the US, USAGE-51, is built on commodity flow data, but ignores transaction data.

However, readily available, relatively up-to-date sectoral inter-regional trade flow data for the UK do not exist, and therefore we are required to use the existing data available from different sources and estimate regional export and import data for the UK. We have already set this out in Chapter 4 in the body of the report.

There are examples of models where regional I-O tables are constructed. Most of these come from the US such as IMPLAN (Robinson and Liu, 2006)[70] and Regional Breakdown of GTAP (USAGE-51) Dixon, Rimmer and Wittwer (2012)[71].

Consumers and users within each region are also assumed to view in-region products as imperfect substitutes as inter-regionally imported goods, which we modelled also with an Armington preference system.

A.10. Conjectural Variation

Conjectural variation is the view that a firm takes on how its competitors may react if it varies its price or output. The collection of views in the market then defines how firms interact and this determines the number of firms in the sector. To deter new entrants, existing firms have two options:

- **To gain market share incumbent firms could improve their product to differentiate their goods.** In this instance the consumer's price elasticity of demand falls, i.e., consumers are willing to pay more for the differentiated good offered by the incumbents, and new entrants cannot compete; and
- **Incumbent firms could price out new entrants.** By increasing output existing firms can lower prices[72]. New entrants would not be able to compete as consumers would buy the incumbent firms' lower priced products. No new firms enter and so there is no increase in the number of varieties.

Conjectural variation is therefore an important factor in determining how many new firms enter the market, which in turn affects the scale of the agglomeration effect. Interactions between domestic, regional, and foreign firms are captured by the dashed arrows in Figure 34.

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[68] Armington, P.S. (1969). A Theory of Demand for Products Distinguished by Place of Production. Staff Papers-International Monetary Fund, 159-178.

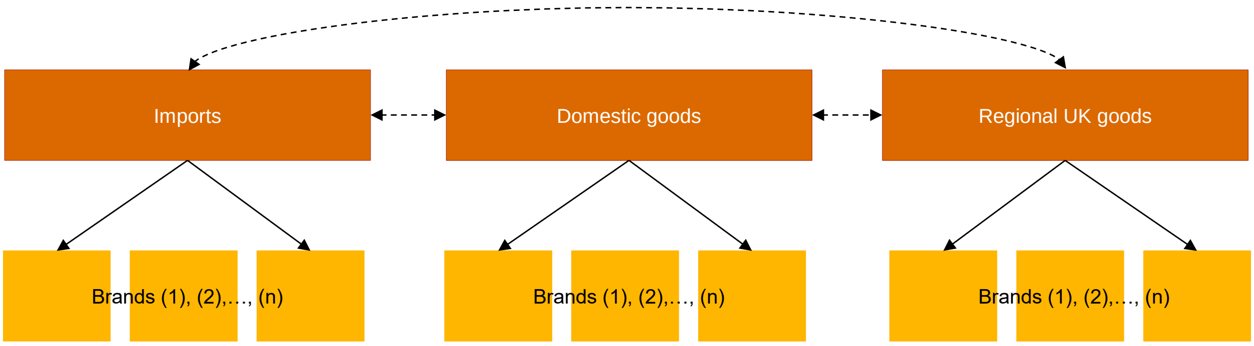
[69] Gillham, J. (2004). The economic inter-relationships of tourism: A Computable General Equilibrium Analysis. PhD Thesis, University of Nottingham. Available at: http://etheses.nottingham.ac.uk/1330/1/436728_VOL1.pdf

[70] Robinson, D.P., & Liu, Z. (2006). The effects of interregional trade flow estimating procedures on multiregional social accounting matrix multipliers. Journal of Regional Analysis & Policy, 36(1), 94-114.

[71] Dixon, P.B., Rimmer, M.T., & Wittwer, G. (2012). USAGE-R51, a state-level multi-regional CGE model of the US economy. GTAP Working Paper. Available at: https://www.gtap.agecon.purdue.edu/resources/download/5933.pdf

[72] The concept of predation has been discussed extensively in the academic literature. Kreps and Wilson (1982) and Milgrom and Roberts (1982) have shown that predation occurs when firms with a reputation for being predatory exist. Fudenberg and Tirole (1986) have shown that predation can exist even without reputation effects.

Figure 34: Interaction between firms



Source: PwC

The conjectural variation parameters are endogenously calibrated and are a function of the consumer price elasticity of demand. When consumer demand is more price elastic, more firms will enter the market, thereby inducing a greater agglomeration effect.

A.11. Sector definition for this S-GCE modelling exercise

In this section, we set out the definition of the sectors we used in this S-CGE modelling exercise. This has been agreed with HS2 Ltd and a wider set of stakeholders.

Table 18: Interaction between firms

Section (SIC07, Alphabetic)	Sector (SIC07, 2-digit)	Sector description	Sectoral allocation
A	1	Agriculture and hunting	OTH
A	2	Forestry and logging	OTH
A	3	Fishing and aquaculture	OTH
B	5-8	Mining and quarrying, excluding support activities	OTH
B	9	Mining support service activities	OTH
C	10	Manufacture of food products	OTH
C	11-12	Manufacture of beverages and tobacco products	OTH
C	13	Manufacture of textiles	OTH
C	14	Manufacture of wearing apparel	OTH
C	15	Manufacture of leather products	OTH
C	16	Manufacture of wood products, except furniture	OTH
C	17	Manufacture of paper products	OTH
C	18	Printing and reproduction of recorded media	OTH
C	19-20	Manufacture of coke, refined petroleum and chemicals	OTH
C	21	Manufacture of pharmaceutical products	ADVMANU
C	22	Manufacture of rubber and plastic products	ADVMANU
C	23	Manufacture of other non-metallic mineral products	ADVMANU
C	24	Manufacture of basic metals	ADVMANU

Section (SIC07, Alphabetic)	Sector (SIC07, 2-digit)	Sector description	Sectoral allocation
C	25	Manufacture of fabricated metal products	ADVMANU
C	26	Manufacture of computer, electronic and optical products	ADVMANU
C	27	Manufacture of electrical equipment	ADVMANU
C	28	Manufacture of machinery and equipment	ADVMANU
C	29	Manufacture of motor vehicles	ADVMANU
C	30	Manufacture of other transport equipment	ADVMANU
C	31	Manufacture of furniture	OTH
C	32	Other manufacturing	OTH
C	33	Repair and installation of machinery and equipment	ADVMANU
D	35	Electricity, gas, steam and air conditioning supply	OTH
E	36-37	Water supply and sewerage	OTH
E	38	Waste collection, treatment and disposal activities	OTH
E	39	Remediation and other waste management services	OTH
F	41	Construction of buildings	CONSTRUCT
F	42	Civil engineering	CONSTRUCT
F	43	Specialised construction activities	CONSTRUCT
G	45	Motor trades	CONSSERV
G	46	Wholesale trade	CONSSERV
G	47	Retail trade	CONSSERV
H	49	Land transport	OTH
H	50	Water transport	OTH
H	51	Air transport	OTH
H	52	Warehousing and transport support activities	OTH
H	53	Postal and courier activities	OTH
I	55	Accommodation	DISCON
I	56	Food and beverage service activities	DISCON
J	58	Publishing activities	DIGITAL
J	59	Motion picture, video and TV programme production	DIGITAL
J	60	Programming and broadcasting activities	DIGITAL
J	61	Telecommunications	DIGITAL
J	62	Computer programming and consultancy	DIGITAL
J	63	Information service activities	DIGITAL
K	64	Financial service activities	FINPROFSERV

Section (SIC07, Alphabetic)	Sector (SIC07, 2-digit)	Sector description	Sectoral allocation
K	65	Insurance and pension funding	FINPROFSERV
K	66	Activities auxiliary to finance and insurance	FINPROFSERV
L	68	Real estate activities, excluding imputed rental	OTH
L	68IMP	Owner-occupiers' imputed rental	OTH
M	69	Legal and accounting activities	FINPROFSERV
M	70	Head offices and management consultancy	BUSSERV
M	71	Architectural and engineering activities	BUSSERV
M	72	Scientific research and development	BUSSERV
M	73	Advertising and market research	BUSSERV
M	74	Other professional, scientific and technical activities	BUSSERV
M	75	Veterinary activities	BUSSERV
N	77	Rental and leasing activities	BUSSERV
N	78	Employment activities	BUSSERV
N	79	Travel agency and tour operator activities	DISCON
N	80	Security and investigation activities	BUSSERV
N	81	Services to buildings and landscape activities	BUSSERV
N	82	Office administration and business support activities	BUSSERV
O	84	Public administration and defence	OTH
P	85	Education	OTH
Q	86	Human health activities	OTH
Q	87	Residential care activities	OTH
Q	88	Social work activities	OTH
R	90	Creative, arts and entertainment activities	DISCON
R	91	Libraries, archives, museums and other cultural activities	DISCON
R	92	Gambling and betting activities	CONSSERV
R	93	Sports, amusement and recreation activities	DISCON
S	94	Activities of membership organisations	CONSSERV
S	95	Repair of computers, personal and household goods	CONSSERV
S	96	Other personal service activities	CONSSERV
T	97-98	Households as employers and own use production	OTH

Note: ADVMANU = Advanced Manufacturing; CONSTRUCT = Construction, CONSSERV = Consumer Services, BUSSERV = Business Services, DIGITAL = Digital sectors; DISCON = Discretionary Consumption, FINPROFSERV = Financial and Professional Services, OTH = Others.

Appendix B: Sensitivity tests

We have run various sensitivities as part of the modelling exercise to better understand the impact of certain parameters on our modelling outputs. We examined three sets of sensitivity tests, which consist of:

- Scenarios in which inter-regional trade is affected by distances between regions
- Scenarios in which economic agents' confidence of HS2's benefits do not fully align to HS2 Ltd's estimates; and
- Scenarios in which the labour-leisure elasticity parameter is varied.

These sensitivities have been run for both Phase One+2a and Phase 2b WL increment. We set out and discuss the results below.

B.1. Inter-regional trade's sensitivity to distance

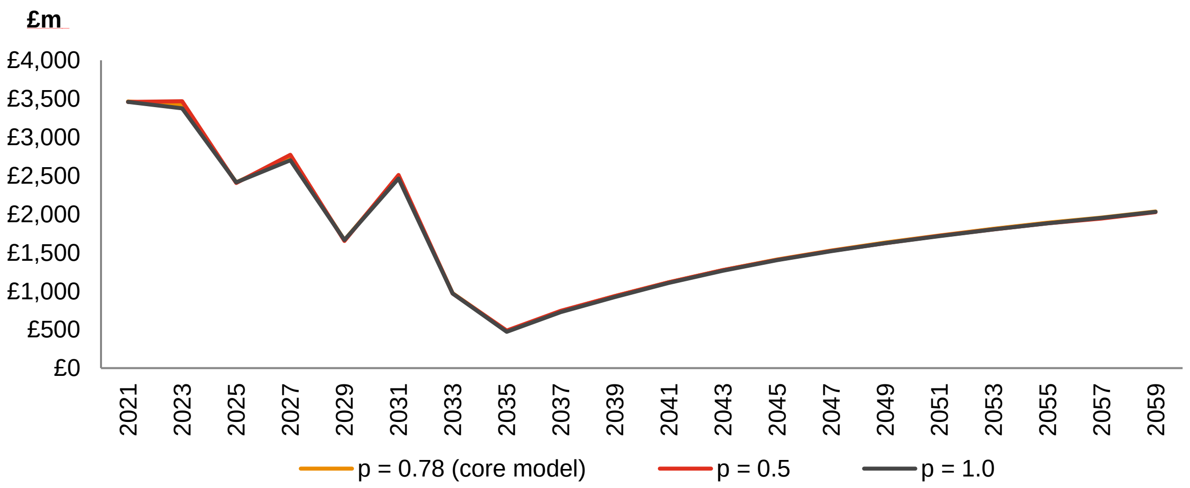
In constructing an inter-regional set of input-output tables we assumed a distance decay factor of 0.78, as set out in Section 4.2. This is based on a meta-analysis by Gert-Jan M. Linders (2005)[1] who found that the average distance elasticity (p) across a sample of studies to be 0.78, with estimates ranging from 0.2 to 1.21.

The purpose of this sensitivity test is to look at the various effects of distance on trade and the implications this could have on the results. To do this, we adjusted the distance decay factor to 0.50 and 1.00 in the model for both Ph One+2a and Ph2b WL incremental.

Our results from this exercise show that changing the distance decay factor would not have a material impact on the results for national GDP. For Phase One+2a changing the parameter has almost no impact, whilst for Phase 2b WL Increment the sensitivity has only a small effect on the level of change, with the profile of the shocks remaining largely unchanged. In summary, how inter-regional trade is allocated in the base data has little effect on the results at a national level.

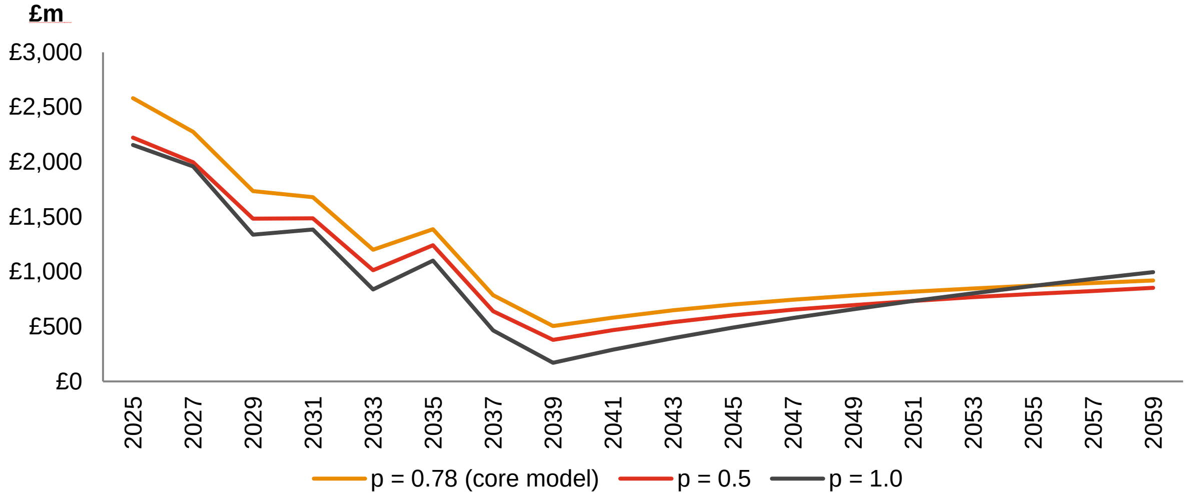
The results of this sensitivity in B.1 show that for both Phases One+2a and the Phase 2b Western Leg Increment, the profile of the results in each sensitivity align with the profile of the central case modelled. Moreover, the divergence from the central case is not large. For Phases One+2a the maximum divergence of either the low or high distance decay scenario in any one year is 2.62%, with results for most years being within 1% of the GDP impact in the central case.

Figure 35: GDP impact, Phase One and 2a, Undiscounted, Distance Decay Sensitivity



Meanwhile, for the WL Increment, the maximum difference from the central case for any one year is 66% (in 2039) however most outputs in the sensitivities are within 15% of the central case, a modest divergence given the substantial change in the distance decay factor (28% and 64% above and below the central distance decay parameter value). This suggests that the nature of regional trade flows assumed in our model does not have a significant impact on our estimated national impacts.

Figure 36: GDP impact, Phase 2b WL Increment, Undiscounted, Distance Decay Sensitivity



B.2. Alignment between HS2 Ltd's estimates and economic agents' expectations

In the main scenario of the model, we have assumed that agents' expectations of HS2's benefits are fully aligned with HS2 Ltd's estimates from the point of announcement. In this section, we test how the model results would react if economic agents were less confident about the future benefits before HS2's opening and are therefore more risk-averse in making investments in advance.

Specifically, this test is implemented by changing businesses' propensity to return capital to owners in the period prior to the operation of HS2; this can take the form of early repayment of debt, dividend payments, or share buybacks – the model is agnostic in this regard.

In the charts below, we present the impacts on GDP under different assumptions for the propensity to return parameter for both Ph One and 2a and the Ph2b WL Increment. The model suggests that for both phases, when businesses are more inclined to return capital (i.e., they have lower confidence in HS2's benefits than HS2 Ltd estimates), less investment would be made in the pre-HS2 period. For example, by adjusting the businesses propensity to return capital by + 0.1%, the GDP impact for Phase One and 2a, in 2051, would be 23% lower; For Ph2b WL Increment, the GDP impact in 2051 would be 53% lower.

For Phase One and 2a, the GDP impact's upward trajectory remains unchanged for both scenarios. The results for the Ph2b WL Increment also show a level shift in the results, but it may be more affected by the fact that Ph2b WL Increment's benefits are more unbalanced regionally.

Figure 37: GDP impact, Phase One and 2a, Undiscounted, different levels of confidence

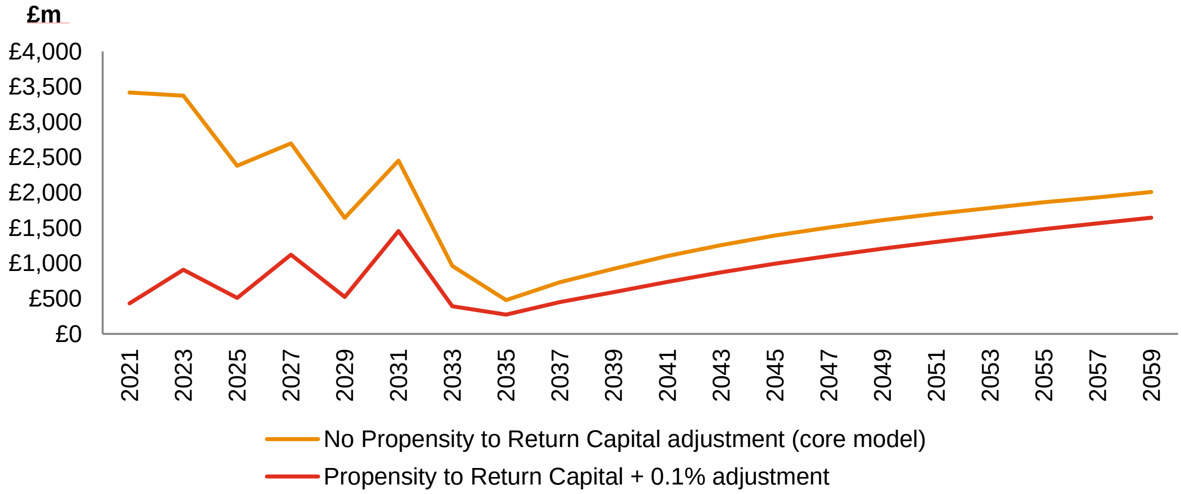
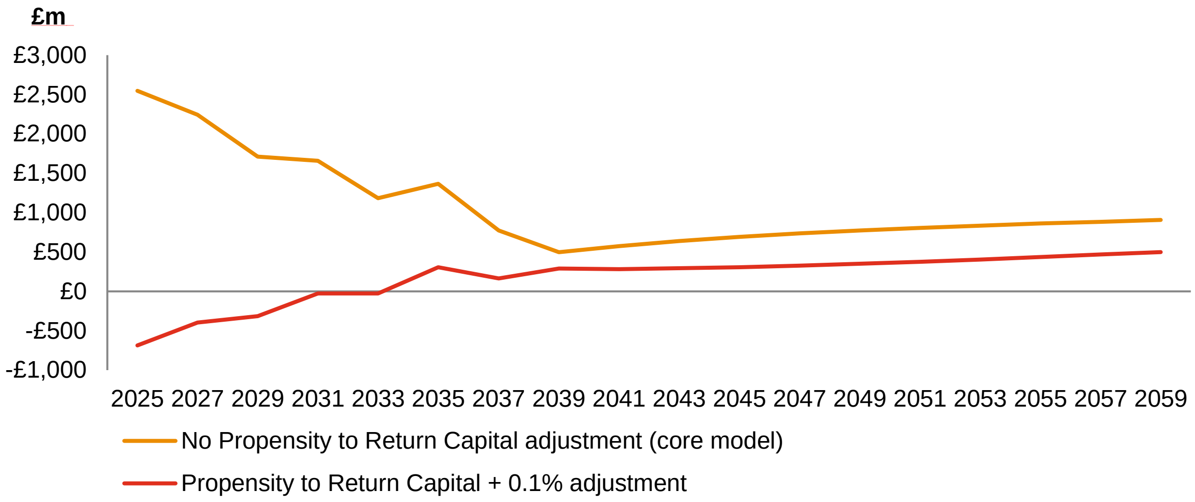


Figure 38: GDP impact, Phase 2b WL Increment, Undiscounted, different levels of confidence



B.3. Labour-leisure substitution elasticity

To test the results' sensitivity to labour supply elasticities, we have carried out sensitivity analysis on consumption-leisure substitution effect. The current model assumes the elasticity to be 0.3 under a Constant Elasticity of Substitution (CES) utility function. The charts below show the net additional GDP impact obtained from adjusting the consumption-leisure elasticity to 1.0 and 0.2. In summary we found that:

At a higher elasticity of substitution such as 1.0, households choose to have even more leisure in the pre-HS2 period as they smooth their overall utility over time. This results in a lower GDP impact. For Phase One and 2a, the GDP impact would be 48% lower. For Phase 2b WL Increment, the GDP impact would be 26% lower.

At a lower elasticity of substitution such as 0.2, households choose to have less leisure in the pre-HS2 period as they choose to consume more in the short-term. This results in a higher GDP impact. For Phase One and 2a, the GDP impact would be 68% higher in 2051. For Phase 2b WL Increment, the GDP impact would be 22% higher in 2051.

In the latter part of the forecasted period, net additional GDP impact across both assumptions starts to converge, showing the economy reaching its steady state equilibrium in the long run.

Figure 39: GDP impact, Phase One and 2a, Undiscounted, Labour-Leisure Sensitivity

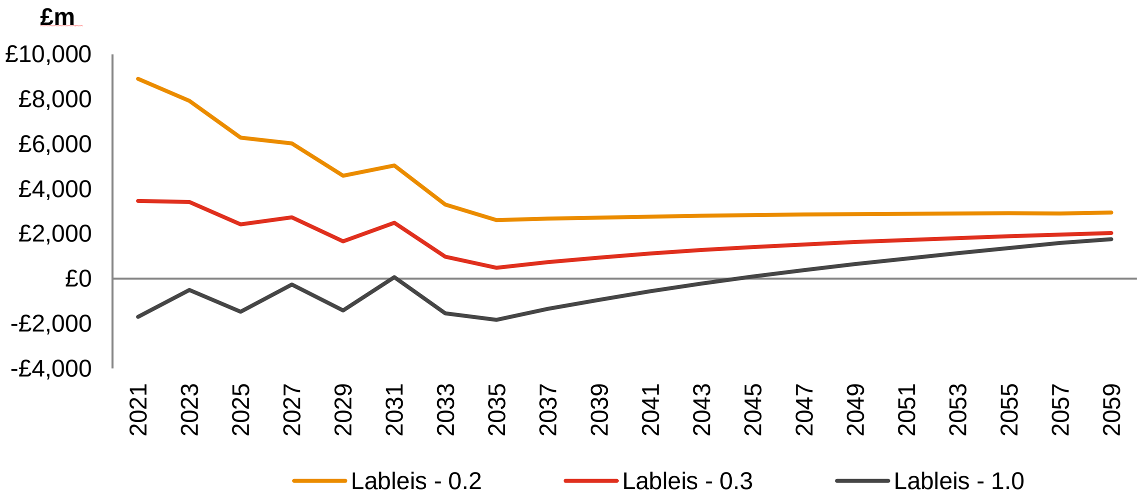
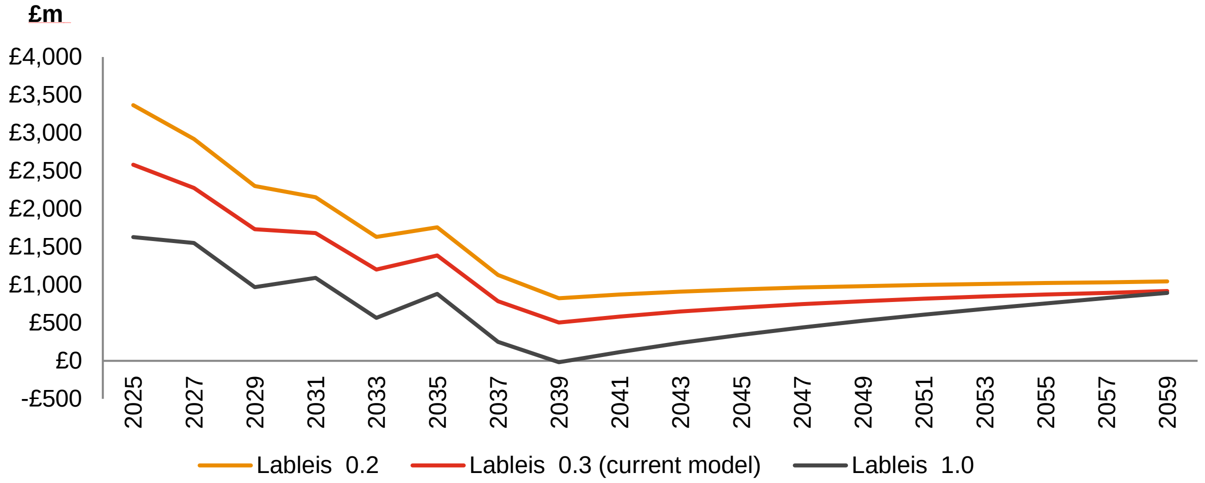


Figure 40: Phase 2b WL Increment, Undiscounted, Labour-Leisure Sensitivity



Appendix C: Assumptions in the model

We first set out a review of the key assumptions in this study (Section C.1). We then discuss in more detail four topics that stakeholders have shown particular interests; namely the derivation of the Supply Table from public sources (Section C.2), the choice of the capital adjustment cost parameter (Section C.3); the derivation of a UK-wide inter-regional input-output table (section C.4); and the allocation of BUBs to sectors and regions (Section C.5).

C.1 Review of assumptions

It is important to understand the possible sources of uncertainty and assumptions associated with our inputs and findings. Table 19 sets out an overview of the assumptions made with the inputs used in S-CGE model and their rationale. Some key considerations with regards to the model input uncertainty are as follows:

- **Measurability:** Can the input be readily observed and measured? The ability to measure a variable with confidence – even if only ex-post – is important in determining the certainty associated with an input. If the variable being used as an input can be easily measured, historic experience can be used to guide forecasts for the future and ex-post data can be used to assess whether the predicted impacts have been realised. On the other hand, some parameters are grounded in economic theory and could only be estimated indirectly.
- **Modelling approach:** What is the level of certainty surrounding the modelling approach used to generate the inputs? The more robust the modelling approach used to generate the inputs; the greater confidence can be attached to the resulting outputs emerging from the framework.
- **Model incorporation:** Can the inputs be incorporated into the model directly and with relatively few additional assumptions? Some of the effects can be readily and directly incorporated into the S-CGE framework. Others, on the other hand, have to be integrated into the model by making certain assumptions, which reduces confidence in their use as inputs to the framework.

Table 19: Summary of assumptions made for the inputs in the S-CGE model

Effect/parameters	Measurability	Modelling approach	Model incorporation
I-O inputs			
Inter-regional trade	Our analysis used the 2017 UK I-O table published by the Office of National Statistics and the 2017 Scottish I-O table published by the Scottish Government.	We combined the UK and Scottish I-O tables for 2017 together to create a bespoke table. We used data from the Scottish I-O table as a basis on which to derive trade between Scotland and the Rest of the UK. We derived the inter-regional trade estimates by adjusting for the GVA share, each sector's location quotients and regional distance weightings.	These estimates were not used as shock inputs into the S-CGE model. They form the baseline economic condition for the model with respect to trade across the 8 sectors and 10 regions. We also assessed the multipliers produced as an intermediate output for sense-checking.
Non-I-O inputs			
Use table	We used the 2017 Use Table published by the Office of National Statistics	A range of components were derived to be used in creating the Use table. Specifically, for the Intermediate consumption we used the inter-regional	It forms the baseline economic condition for the model with respect to inter-regional trade across the 8 sectors and 10 regions.

Effect/parameters	Measurability	Modelling approach	Model incorporation
		trade estimates derived from the I-O table with the addition of imports and product tax (e.g. VAT) included in the estimates. Similarly, for household expenditure, government expenditure and international exports this was taken from the I-O table with adjustments made to account for product taxes and import use. We also included gross fixed capital formation, capital allowance and household incomes in the Use table, our approach is outlined below.	
Supply table	The Use, Supply and Input-Output tables are linked with each other by a mathematical relationship. With knowledge of two of them, the third can be derived algebraically (refer to the appendix section C.2 for more detail on approach).	We derived the UK and regional Supply Tables with the Use Tables	It forms the baseline economic condition for the model with respect to inter-regional trade across the 8 sectors and 10 regions.
Value Added Tax ('VAT')	We derived two components of VAT: 1) VAT on current use ('VATC') and; 2) VAT on capital ('VATK'). The estimates for VATC data were derived by apportioning product tax estimates (taken from the ONS) using the total use proportion for each region and sector. VATK was estimated by configuring a weight for investment tax by using GFCF estimates and applying an effective tax rate for each sector and region. This weight was then used to apportion product tax for investment by region and sector.	Due to the lack of granular data available by sector and region for VAT. We have apportioned the product tax values provided by the ONS to derive estimated figures for VAT across the 8 sectors and 10 regions.	We used the VAT estimates as a supplementary variable that is inputted into the S-CGE framework to form our baseline equilibrium market condition.
Gross fixed capital formation ('GFCF')	Our 2017 GFCF estimates were based on a combination of data from the ONS and Scottish Government.	We disaggregate the sectoral GFCF estimates by region by using the proportion of use and GVA for each of the 10 regions. We remove any negative GFCF estimates as we deem a negative value to be inappropriate for investment.	These estimates are incorporated into the S-CGE model as an input (i.e. investment) to derive the baseline market economy before any productivity shocks are applied.
Capital Allowance	To derive the capital allowance estimate we used	We apportion the regional capital allowance by sector	Similarly, the capital allowance estimates were

Effect/parameters	Measurability	Modelling approach	Model incorporation
	the total value of corporate subsidy for 2017-18 sourced from the ONS.	using the share of GFCF for each specific sector within a given region (each of the 10 regional groups).	used as inputs to derive the balanced baseline equilibrium.
Household incomes	Each of the components of household incomes, e.g. Employee's National Insurance Contribution, state pension etc, were sourced from the ONS. We multiply the average household incomes per individual with the equivalised number of individuals per household to give us the total estimates.	In our model we assume that household disposable income can only be used for consumption and saving purposes. As such the summation of our consumption and savings total is equivalent to our Household income estimates.	Each of the parameters in the household incomes are inputted into the S-CGE framework to generate an initial equilibrium prior to adjustments from the productivity shocks.
Elasticity of substitution between domestic and imported goods	The elasticity of substitution parameters used are sourced from Global Trade Analysis Project (GTAP) database.	Regions can import and export goods and services. For each type of good or service, economic agents (consumers, producers and government) may choose between domestic and imported products. They are assumed to be imperfect substitutes with each other and are joined together with a CES function. In line with HMRC's inter-regional model, we assumed the Armington elasticity between regions to be half of the values in the GTAP database. We then followed GTAP's example and assume the elasticity of substitution between imported products from different regions to be twice the Armington elasticity (a.k.a. the 'rule of two').	We apply the 'Armington elasticity' to the S-CGE model to measure the responsiveness to economic changes given the different elasticity parameters in the model.
Adjustment cost of capital assumptions	In our model we have assumed an adjustment cost of capital assumptions (referred to as φ) of 2. This assumption is based on the literature review we conducted which is described in detail in the appendix section C.3.	Our model assumes a forward-looking investment rule with rational expectations. It assumes businesses make investment decisions based on their expectations. Capital stock is accumulated over time by investment, whereby investment is subject to 'installation cost' or adjustment costs.	This assumption on adjustment cost of capital assumptions is incorporated throughout the S-CGE model as an input to derive the baseline and new market economy before and after productivity shocks are applied.
Business User Benefit productivity shock			
Railway and Highways user benefits	The Business User Benefits are allocated to each sector by their share in National	As benefits were not available at the level of sector-region pairs, they	The process of incorporating Business User Benefits into the S-CGE model requires

Effect/parameters	Measurability	Modelling approach	Model incorporation
	Travel Survey (NTS) intercity (>50km) rail journeys, with adjustments by HS2 Ltd. Separately, HS2 Ltd allocates them across regions in line with agglomeration impacts from WITA. This was measured using the value of journey time savings, demand measured by the number of passenger trips in 24 hours and the general journey times for each origin-destination pair across Britain.	must be estimated. This by nature requires making assumptions about the future level of traffic, the make-up of users, their values of time and so forth. Using the Business User Benefits estimates provided PwC then applies a cross-entropy method to calculate a set of inputs for each sector in each region that both (i) adds up to the regional and sectoral sums as calculated by HS2, and (ii) aligns with the contours of sectoral output in each of them.	that we assume that user benefits to firms manifest themselves as increased productivity in the rail transport sector. Whilst this assumption seems reasonable, it does introduce an additional layer of uncertainty. We have sought to deal with this by making conservative assumptions about which user benefits manifest themselves as real economic effects.
Non-labour Agglomeration productivity shock			
Static Agglomeration	The Agglomeration effects are derived from the WITA model combined with our in-house analysis. This looks at the changes in productivity as a result of firms or individuals being located more closely together. Standard TAG guidance is to use estimates of agglomeration elasticities and decay parameters provided by Graham et al. (2010).	We assumed that agglomeration benefits grow linearly between 2034 and 2041 which thereafter becomes constant (flat line growth) in Scenario 2. For Scenario 3, growth follows the same trajectory as Scenario 2 up to 2038, and then when Phase 2b (WL) opens in 2038 there will be accelerated growth up to 2041, followed by constant growth. We only considered agglomeration impacts as provided by HS2.	Static agglomeration (within-region and between-region) is incorporated via a general productivity shock into the S-CGE framework. The benefit from greater agglomeration arises from firms using resources more productively. To value this input as a productivity shock we therefore compare this benefit as a percentage of total use of all resources employed ('Total output'). As such, a relatively direct mechanism is available for the incorporation of S-CGE model inputs.
Labour Agglomeration productivity shock			
Labour supply impacts	The labour supply effect was estimated from the WITA model that HS2 provided to us. It was then, as HS2 suggested, adjusted for by dividing each estimate by the 40% tax wedge to derive the labour supply benefits. The labour supply effect is a measure of the welfare impact that is additional transport user benefits from greater employment.	To disaggregate the labour supply effect by sector, we assume a similar sectoral agglomeration ratio. We apply these ratios to the given regional labour supply effect (split by the 10-regional allocation) from the WITA model. A similar time profile for Scenario 2 and 3 in the non-labour agglomeration, as mentioned above, was applied.	The labour supply benefits are incorporated into the S-CGE model in the form of a productivity shock. Like the agglomeration productivity shock we value this input as a productivity shock by comparing the benefits as a percentage of total use of all resources employed.

Source: PwC

C.2 Deriving the supply table mathematically

The Combined Use Table, U_C, sets out the value of each product category – both domestic and imported – that is used by each industry-sector to produce its outputs. It is sector-by-product: each column of U_C represents a product-using sector, and each row represents a product that could be used. The Domestic Use Table U_D and the Imported Use Table U_1 respectively set out only domestic and imported products in each of the rows.

The Supply Table, S, sets out the value of each *product* category that is *produced* by each *industry sector*. In general, the great majority of the value of each sector's output could be found in its own product category, i.e., on the top-left to bottom-right diagonal of the Supply Table.

If we assume that each product has its own specific market shares (deliveries to industry sectors) independent of the industry sector where it is produced – this is known as the *fixed product sales structure assumption*, then we could assume that for each product, the share produced by each industry sector is a constant. We can derive from the Supply Table a matrix of product factors, S_t, by dividing each cell by their respective row sum (i.e. total value of each product's supply).

The industry-by-industry IOT, IO_{IxI} could then be derived from the Domestic Use table by allocating the value of products used by each industry sector by the producing domestic industry sector, i.e.:

$$IO_{IxI} = (S_t)'\, U_D \quad (1)$$

While the ONS no longer publishes the full Supply Table, it still publishes the UK industry-by-industry IOT, $IO_{IxI}$and the domestic use table, U_D. Following Equation (1), it is possible to derive the UK Supply Table by the following equation:

$$S_t = IO_{IxI}\, (U_D)^{-1} \quad (2)$$

Since two of the UK Input-Output sectors (Imputed Rent of Owner-Occupiers and Services of Households as Employers) does not supply any other sectors – all products are used by final users – they have to be removed from the IOT and Domestic Use Table for the purpose of the calculation in Equation (2). Otherwise, U_D will not be invertible.

C.3: Detail of assumption on adjustment cost of capital

Our model assumes a forward-looking investment rule with rational expectations. It assumes businesses make investment decisions based on their expectations. Capital stock is accumulated over time by investment, whereby investment is subject to "installation cost" or adjustment cost of capital. The formula below demonstrates the relationship between gross investment with regards to the net investment, capital stock and adjustment cost of capital[73]

$$I_{j,t} = J_{j,t}\left(1+\phi\frac{J_{j,t}}{2K_{j,t}}\right)$$

where ø = adjustment cost of capital, *I* is gross investment and *J is net investment.*

As mentioned above, the adjustment cost of capital assumption of 2 incorporated in the S-CGE model is based on our literature review of academic studies looking at the assumptions that other CGE model practitioners used in their work. Table 20 lists the number of studies and their adjustment cost of capital estimates:

Table 20: Summary of assumptions made on adjustment cost of capital

Study	Beta	Adjustment cost of capital estimate
Richard Blundell, Stephen Bond, Michael Devereux, and Fabio Schiantarelli. Investment and Tobin's q: Evidence from company panel data. Journal of Econometrics, 51(1/2):233-257, 1992.	<0.01	>50
Cesar Alonso-Borrega and Samuel Bentolila. Investment and q in Spanish manufacturing firms. Oxford Bulletin of Economics and Statistics, 56(1):49-65, 1994.	0.021 to 0.027	18-23
Fumio Hayashi and Tohru Inoue. The relation between firm growth and q with multiple capital goods: Theory and evidence from panel data on Japanese firms. Econometrica, 59(3):731-753, 1991. p.748.	0.029	17
Summers, L.H., Bosworth, B.P., Tobin, J. and White, P.M., 1981. Taxation and corporate investment: A q-theory approach. Brookings Papers on Economic Activity, 1981(1), pp.67-140.	0.031	16
Cummins, J.G., Hassett, K.A., Hubbard, R.G., Hall, R.E. and Caballero, R.J., 1994. A reconsideration of investment behavior using tax reforms as natural experiments. Brookings papers on economic activity, 1994(2), pp.1-74.	0.5	1
Cooper, R.W. and Haltiwanger, J.C., 2006. On the nature of capital adjustment costs. The Review of Economic Studies, 73(3), pp.611-633.	N/A	0.049
Groth, C. (2005). Estimating UK capital adjustment costs. Bank of England Working Paper no. 258	N/A	0.40-1.20
GROTH, C., & KHAN, H. (2010). Investment Adjustment Costs: An Empirical Assessment. Journal of Money, Credit and Banking.	N/A	0.001

[73] HM Revenue & Customs (2013), 'HMRC's CGE model documentation', pg.22

Study	Beta	Adjustment cost of capital estimate
Paoli, B.D., Scott, A.M., & Weeken, O. (2007). Asset pricing implications of a New Keynesian model. Journal of Economic Dynamics and Control, 34, 2056-2073.	N/A	0.30
Mun 2002 Computer Adjustment Costs: Is Quality Improvement Important?	N/A	0.60-1.60 (computer investment)
Torres, J. L. (2020). Introduction to dynamic macroeconomic general equilibrium models. Vernon Press.	0.97	6.00

From our review, it is worth noting that there is a significant wide range of estimates on the adjustment cost of capital ranging from 0.001 to greater than 50. However, we also note that most CGE models choose a value around the 1.0-2.0 range. Therefore, we have chosen a φ of 2.0.

C.4. Constructing an interregional I-O table

This section of the Appendix adds in additional information with regard to Step 3 and 4 of the four-step approach used to build the inter-regional I-O table. For reference, these steps are outlined below:

- **Step 1:** Data collection
- **Step 2:** Amalgamation of the UK and Scottish tables
- **Step 3:** Apportioning the aggregated I-O table to the regions
- **Step 4:** Applying entropy algorithm to balance the inter-regional I-O table

C.4.1. Apportioning the aggregated I-O tables to regions

This section reviews our adjustment to the Amalgamated UK and Scottish input-output table. Three main adjustments were made to apportion the aggregated I-O tables into an extensive inter-regional table that reflects the trade between the 10 regions: (1) Gross Value-Added weights; (2) industry sector location quotients; and (3) region-to-region inverse distance weightings. These three adjustments are detailed below:

Adjustment 1: Gross Value-Added weights

Gross Value-Added ('GVA') weights were applied to the relevant components in order to apportion the estimates for non-Scottish parts of the UK into the 10 regions. To calculate the regional GVA share, we compared the regional specific GVA for each sector against the overall level, i.e. the GVA of the UK economy excluding Scotland, using the ONS data. The formula below illustrates an example of the regional GVA share calculation that was used in deriving the GVA share of the Digital sector in the North West of the UK:

$$GVA\ share_{Digital,North\ West} = \frac{GVA_{Digital,North\ West}}{GVA_{Digital,UK\ excluding\ Scotland}}$$

Adjustment 2: Industry sector location quotients ('LQ')

We adjusted the intermediate consumption/demand estimates with the industry sector location quotients ('LQs') to account for users' tendency to use products from the local area than from the rest of the country, i.e. the 'home bias' they demonstrate, as well as to account for different industry sector mixes in each region. To this end, we applied the location quotient approach developed by Flegg et al. (1995)[74] where the Simple Location Quotient ('SLQ') was firstly calculated. The SLQ is a ratio which tells us the concentration of a sector in the specified region (e.g. North West) in comparison to a larger reference region, in this case the UK. This is calculated for each sector as follows:

[74] Flegg A. T., Webber C. D. and Elliott M. V. (1995) 'On the appropriate use of location quotients in generating regional input−output tables', Regional Studies 29, 547−61.

$$SLQ_i = \frac{\frac{GVA_{i,North\ West}}{GVA_{\ North\ West}}}{\frac{GVA_{i,UK}}{GVA_{\ UK}}}$$

In this case, if sector i had a value of 2, this would mean that sector i in North West is twice as large as a share of the UK's economy, so sector i is effectively self-sufficient. We cap these Simple LQs at 1, so that we only adjust sectors which are relatively underrepresented in the selected regions compared to the UK and use these to scale final demand by rows. Note that instead of using employment shares, as the original papers do, we use GVA shares instead. We then calculated two adjustment factors based on Flegg et al. (1995) and Flegg et al. (1997)[75] respectively:

$$\lambda_{1995} = \frac{GVA_{North\ West}}{GVA_{UK\ excluding\ Scotland}} / \left[log_2\left(1 + \frac{GVA_{North\ West}}{GVA_{UK\ excluding\ Scotland}}\right)\right]^{\beta}$$

$$\lambda_{1997} = \left[log_2\left(1 + \frac{GVA_{North\ West}}{GVA_{UK\ excluding\ Scotland}}\right)\right]^{\delta}$$

Our chosen adjustment factor $\underline{\lambda} *$ is the average of λ_{1995} and λ_{1997} above, a minor but positive factor, where $\beta = 5$ as in Flegg *et al* (1995) and $\delta = 0.25$ as in Flegg *et al* (1997). Again, note that instead of using employment shares, as the original papers do, we used GVA shares instead.

Using these adjustment factors, we then calculate LQ_{ij} as follows:

If i = j: $LQ_{ij} = \underline{\lambda} * SLQ_i$

If i ≠ j: $LQ_{ij} = \underline{\lambda} * \frac{SLQ_i}{SLQj}$

where *i* = buying (column) IOT sector and *j* = supplying (row) IOT sector.

Like above with the SLQs, we cap these pairwise Location Quotients at 1, so that we only adjust sectors which are relatively underrepresented in the selected regions versus the UK as a whole. These LQs are used to scale intermediate demand.

Adjustments 3: Region-to-region inverse distance weightings ('IDW')

We applied region-to-region inverse distance weighting to our estimates to account for the effects of distance on trade. A distance matrix was constructed using pairwise distance estimates for the 10 regions. The formula used to derive the region-to-region IDW is as follows:

$$Region - to - region\ inverse\ distance\ weighting = \frac{1}{Distance_{a,b}^{p}}$$

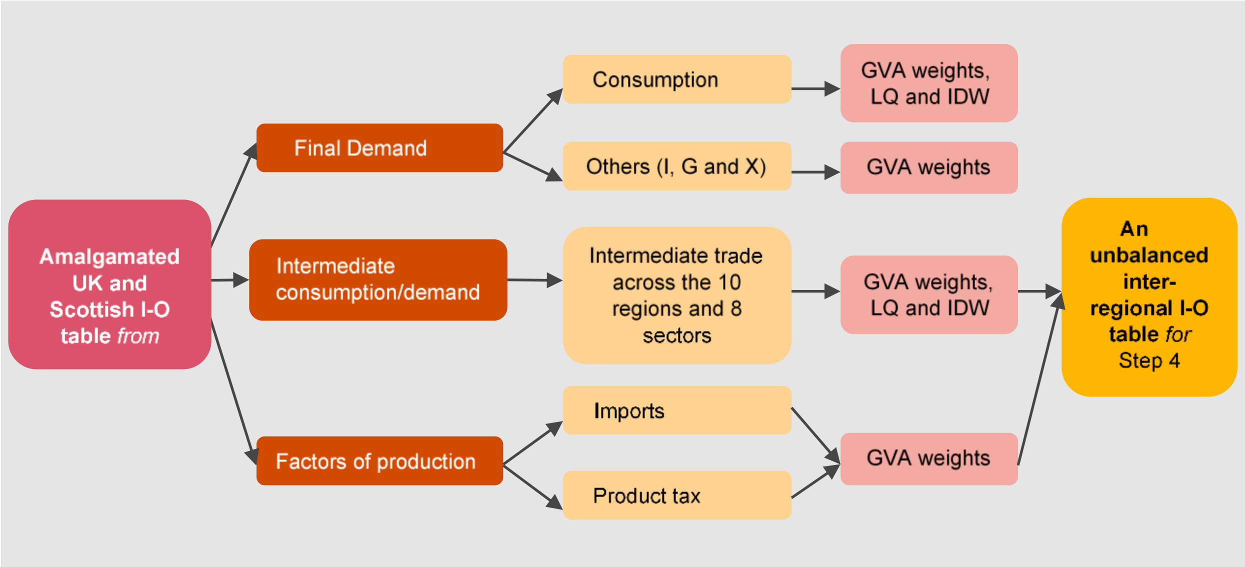
We set our distance elasticity (the rate at which the weights change with respect to changes in distance) equal to 0.78 when estimating the IDW. This is based on a meta-analysis by Gert-Jan M. Linders (2005)[76] which found that the average distance elasticity, p, across a sample of studies is c.0.78, with estimates ranging from 0.2 to 1.21.

The flow chart in Figure 41 below summarises the adjustments made to the Amalgamated table with respect to final demand, intermediate trade and factors of production components.

[75] Flegg A. T. and C.D. Webber. (1997) On the Appropriate Use of Location Quotients in Generating Regional Input-Output Tables: Reply, Regional Studies, 1997, vol. 31, issue 8, 795-805.

[76] Gert-Jan M. Linders, 2005. 'Distance Decay in International Trade Patterns – a Meta-analysis,' ERSA conference papers ersa05p679, European Regional Science Association.

Figure 41: Flow chart of our approach in apportioning the amalgamated I-O table[77]



Source: PwC

C.4.2 Applying entropy algorithm to balance the inter-regional I-O table

We used a Cross-Entropy methodology to balance the I-O table, the details of which are included below.

The cross-entropy method was developed by Shannon (1948)[78] and was first applied to balancing I-O by Sherman Robinson and others between 1998-2000.[79] To balance the I-O table, we looked to find a new I-O, X^1, that is close to the existing SAM, X^0, by minimising the cross-entropy distance between them whilst respecting all constraints.
For this procedure, we started with the estimates from the unbalanced adjusted I-O table cells and assumed the column sums for X^0 were equivalent to the column totals before the IDW adjustment whilst setting a constraint on the Scottish estimates so that it remained unchanged. This ensured that the relationship within the trade estimates between each region and sector remained intact. This approach follows the probability formula mentioned in a study by Lemelin, Fofana and Cockburn (2005)[80] with:

$$H_{t^1} = \sum_i \sum_j t^1_{ij} \, ln \, ln \, \frac{t^1_{ij}}{t^0_{ij}} = \sum_i \sum_j t^1_{ij} \, ln \, ln \, t^1_{ij} - \sum_i \sum_j t^1_{ij} \, ln \, ln \, t^0_{ij}$$

$$H = \sum_j \sum_i t^1_{ij} \, ln \, ln \left(\frac{t^1_{ij}}{t^0_{ij}} \right)$$

Where:

t^0_{ij} = The original estimates of ij from the adjusted I-O table

t^0_{ij} = The new estimates of ij and is between 0 and 1

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[77] Others' refers to the rest of the Final Demand components which are Investment (I), Government expenditure (G) and Exports (X).

[78] Shannon, C.E. (1948). A mathematical Theory of Communication, Reprinted with corrections from The Bell System Technical Journal, Vol. 27, pp. 379–423, 623–656, July, October, 1948. Accessible at: https://people.math.harvard.edu/~ctm/home/text/others/shannon/entropy/entropy.pdf

[79] For instance, see: Robinson, S., Cattaneo, A. and El-Said, M., Updating and Estimating a Social Accounting Matrix Using Cross Entropy Methods, August 200. Accessible at: https://www.un.org/en/development/desa/policy/mdg_workshops/training_material/robinson_cattaneo_and_el-said_2001.pdf

[80] Lemelin, A., Fofana, I., & Cockburn, J. (2005). Balancing a Social Accounting Matrix: Theory and Application (Revised Edition).

Whilst subject to: $\sum_j t^1_{ij} X_j = X_i$ and $\sum_j t^1_{ij} = 1$

Alternatively, when substitution proportions, t_{ij}, in the probability formula by transaction flows, x_{ij}, the following formula is derived:

$$H = \sum_j \sum_i \frac{x_{ij}}{x_{\circ\circ}} ln\, ln \left[\frac{\left(\frac{x_{ij}}{x_{\circ\circ}}\right)}{\left(\frac{x^0_{ij}}{x^0_{\circ\circ}}\right)} \right] \text{with } x_{\circ\circ} = \sum_i \sum_j x_{ij} \text{ and } x^0_{\circ\circ} = \sum_i \sum_j x^0_{ij}$$

C.5. Applying entropy algorithm to allocate Business User Benefits (BUBs)

This sub-section describes in more detail the cross-entropy method to calculate a set of inputs for each sector in each region. There were five steps in this method, outlined in Section 6.3 of the report. Here, we provide more detail on the fourth step of this process.

To balance the user benefit matrices, we looked to find a new matrix, X^1, that is close to the existing user benefits matrix, X^o, by minimising the cross-entropy distance between them whilst respecting all constraints. For this procedure, we started with the estimates from the adjusted user benefit matrices (accounting for the adjusted sectoral NTS and regional WITA weights combined with regional re-allocation in line with industry sector outputs) and assumed each sectoral (column) sums and regional (row) sums are equivalent to the column and row totals before the GVA apportioning procedure was undertaken.

This approach follows the probability formula mentioned in a study by Lemelin, Fofana and Cockburn (2005)[81] with:

$$\min_{\{t^1\}} H = \sum_i \sum_j t^1_{ij} \ln \frac{t^1_{ij}}{t^0_{ij}} = \sum_i \sum_j t^1_{ij} \ln t^1_{ij} - \sum_i \sum_j t^1_{ij} \ln t^0_{ij} \quad \text{or} \quad H = \sum_j \sum_i t^1_{ij} \ln\left(\frac{t^1_{ij}}{t^0_{ij}}\right)$$

Where t^0_{ij} = original estimates of ij from the matrix and t^1_{ij} = new estimates of ij and is between 0 and 1.

Whilst subject to: $\sum_j t^1_{ij} X_j = X_i \qquad \sum_j t^1_{ij} = 1$

When substituting proportions t_{ij} in the probability formula by transaction flows, x_{ij}, the following formula is derived:

$$H = \sum_j \sum_i \frac{x_{ij}}{x_{\bullet\bullet}} \ln \left[\frac{\left(x_{ij} / x_{\bullet\bullet} \right)}{\left(x^0_{ij} / x^0_{\bullet\bullet} \right)} \right] \qquad \text{with } x_{\bullet\bullet} = \sum_i \sum_j x_{ij} \text{ and } x^0_{\bullet\bullet} = \sum_i \sum_j x^0_{ij}$$

In calibrating the benefits matrices for Ph2b(WL) full network, we have also imposed a condition that the BUB allocated to each cell is no less than the corresponding BUB value Phases One and 2a of HS2.

[81] Fofana, I., Lemelin, A and Cockburn, J., Balancing a Social Accounting Matrix: Theory and application, August 2005. Accessible at: https://www.un.org/en/development/desa/policy/mdg_workshops/eclac_training_mdgs/fofana_lemelin_cockburn_2005.pdf