Department for Education

NCETM
NATIONAL CENTRE FOR EXCELLENCE IN THE TEACHING OF MATHEMATICS

Sample Key Stage 3 Mathematics Curriculum Framework

Non-statutory guidance for the national curriculum in England

September 2021

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Contents

Summary

This publication provides non-statutory guidance from the Department for Education. It has been produced to help teachers and schools make effective use of the national curriculum to develop secondary school pupils' mastery of mathematics.

Who is this publication for?

This guidance is for:

- local authorities
- school leaders, school staff and governing bodies in all maintained schools, academies and free schools.

Aims

This publication aims to:

- Bring greater coherence to the national curriculum for mathematics by exemplifying the statutory guidance for Key Stage 3 (DfE, 2013) and giving schools, mathematics departments and teachers further guidance on how learning in mathematics develops across Key Stage 3.

Key considerations concerning how to distribute the national curriculum content across the key stage are discussed. A sample model of a curriculum framework is provided to help mathematics departments structure teaching and learning effectively. Guidance is given on how teachers within a school's mathematics department might collaborate to plan their long- and medium-term teaching.

Designing a coherent and connected curriculum

Purpose and rationale

High-quality teaching of mathematics in the classroom is, of course, what really makes a difference to students' learning. For maximum impact, all teachers need to work with an agreed school mathematics curriculum (or scheme of work) which:

- Offers a clear sequencing of mathematical ideas, concepts, knowledge, and techniques both within each year and across years so that new ideas are built on the firm foundations of existing ones.
- Gives a coherent view of mathematics that highlights important unifying ideas and links between them so that students experience mathematics not as a collection of disparate topics but as a connected whole.

The National Curriculum for mathematics sets out a broad statutory overview and curriculum content entitlement for all students. It is for individual schools to determine their own curriculum to meet these statutory requirements, to be implemented in their own classrooms with their own students.

This document gives guidance about what makes for a rigorous, coherent and connected Key Stage 3 mathematics curriculum and how this might be created. Alongside this is a sample Key Stage 3 curriculum framework, arranged by year group, which includes a detailed termly breakdown of the knowledge, skills and understanding required for Key Stage 3 mathematics. The examples and guidance offered within this document can be used regardless of whether or not a school chooses to teach mathematics in the order suggested in the sample curriculum framework.

This guidance is intended to help schools structure their Key Stage 3 curriculum so that students develop a deep and connected understanding of mathematics. The following principles are particularly important for coherent curriculum design:

- Certain images, techniques and concepts are **important precursors** to later ideas; sequencing these correctly is an important aspect of planning and teaching.
- When introducing new ideas, it is important to **make connections** with earlier ideas that are already well understood.
- When something has been deeply understood and mastered, it can and should be **used in the next steps of learning**.

In the short term, these materials can be used by secondary mathematics departments to review and develop the structure and focus of their curriculum in the context of the severe disruption to education caused by the Covid-19 pandemic. In the longer term, they will help mathematics departments to develop their curriculum to give it greater coherence.

Guidance

A fundamental principle of teaching effectively in mathematics is that key ideas need to be understood deeply before moving on. A curriculum which encourages teachers to move on to the next topic too quickly, before key ideas are deeply understood, results in superficial learning. While such an approach to 'covering' the curriculum at a rapid pace may seem to work in the short term, in the long term it is an inefficient use of precious curriculum time, because it leads to the same key ideas being retaught year after year.

Without a coherent, connected curriculum there is a danger that students will perceive the mathematics they learn as a bewilderingly large set of separate topics, each one with its own rules and techniques to remember. Students who have this view of mathematics often see it as a hard, impenetrable subject which they find difficult to learn. In contrast, students who experience the subject as a coherent set of connected ideas tend to find learning mathematics achievable, enjoyable, and stimulating.

This is outlined in the national curriculum mathematics programmes of study.

'Mathematics is an interconnected subject in which students need to be able to move fluently between representations of mathematical ideas. The programme of study for Key Stage 3 is organised into apparently distinct domains, but students should build on Key Stage 2 and connections across mathematical ideas to develop fluency, mathematical reasoning, and competence in solving increasingly sophisticated problems.'

A curriculum compatible with teaching for mastery rejects superficial short-term coverage in favour of developing deep, connected understanding of key ideas. This forms a secure foundation for future learning, so making more efficient use of teaching and learning time.

Discuss and agree the department's aims and values

The design of a curriculum should be based on a department's view of what constitutes good mathematics, good learning, and good teaching. If teachers do not already have an agreed view about this across the department, it is important to discuss shared aims and values before beginning to construct a curriculum.

The national curriculum outlines general aims for the teaching of mathematics and the NCETM has published some related themes and key principles. Teachers may find these useful as starting points.

Identify key mathematical ideas which connect topics together

The NCETM secondary PD materials offer an example of a connected view of the Key Stage 3 mathematics curriculum through its structure of themes, core concepts and key ideas. Some examples are given below.

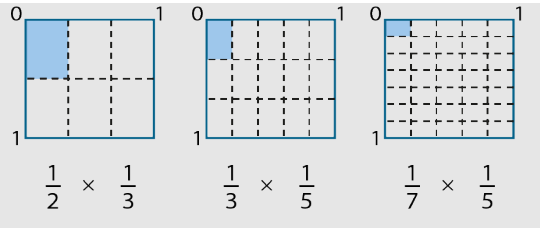
- The teaching of algebra is not treated as a separate, stand-alone idea but as a natural generalisation of the structures of number and number operations.
- Multiplicative reasoning is emphasised as a key idea. It is used to connect work in fractions, percentages, ratio, and proportion, and is linked to work on enlargement and scale.

The sample Key Stage 3 curriculum framework in this document illustrates how this can be achieved. It offers a clear structure in which over-arching themes provide the framework for a coherent and connected curriculum. Some key points to consider are given below.

Give sufficient time for learning and teaching the first time a new idea or concept is introduced. The sample Key Stage 3 curriculum framework offers an example of appropriate timings and the NCETM secondary assessment materials exemplify the important aspects of deep, embedded and sustainable understanding that are needed at each stage. Being clear about the important prerequisite knowledge from Key Stage 2 and allowing time to consolidate this and then build new Key Stage 3 ideas on these firm foundations is vital.

For example, for students to make sense of the structures underlying the multiplication and division of fractions and how these naturally build from ideas about multiplying integers, it will be important to make sure that:

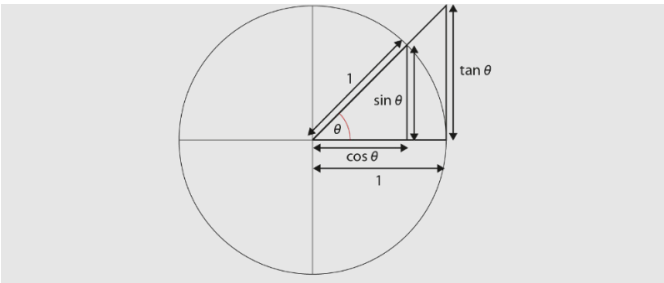
- They understand multiplication as scaling, as well as repeated addition.
- They understand division as grouping (quotative) as well as sharing (partitive).
- They are familiar with an area model for multiplication.



Ensure that, once new ideas have been mastered, they are used frequently, consolidated, and applied in future learning. For example, once the four operations with fractions have been introduced, fractions should feature regularly, such as:

- in topics on area, perimeter, and volume
- as coefficients when solving equations
- in statistics, where data in different forms are being analysed.

Identify how new ideas are connected with existing ones and how existing ideas can be used to make learning and teaching efficient. For example, how the teaching of trigonometry through the introduction of the unit circle uses existing ideas of similar triangles and scaling to introduce the use of the sine, cosine, and tangent ratios to solve problems involving right-angled triangles.



Ensure that students have opportunities to meet all three of the KS3 Mathematics National Curriculum aims: to develop fluency, to reason mathematically and to solve problems. Learning should incorporate a balance of these, with regular opportunities for students to apply their mathematics to non-routine questions. Such questions should not direct students towards a methodology for solution, but students should be aware that that they can answer them using the maths they have previously learnt. Reviewing and discussing approaches to such questions in class will enable students to develop their repertoire of strategies.

Ensure that assessment processes and procedures are integral to, and determined by, the curriculum. The assessments used should be formative. They should be an integral part of curriculum design and give teachers (and students) feedback on the extent to which they are developing the knowledge, skills and understanding embedded within that curriculum.

Teachers should not let the ‘assessment tail wag the curriculum dog’ by teaching to a test. Teaching to a well-designed curriculum and using assessments to give students the opportunity to demonstrate their understanding of the ideas and concepts taught is preferable. This will help to retain the coherence of the curriculum and give useful pointers to where it could be improved. If the Key Stage 3 curriculum is well designed and effectively taught, students will be prepared to perform to the best of their ability in GCSE Mathematics at the end of Key Stage 4.

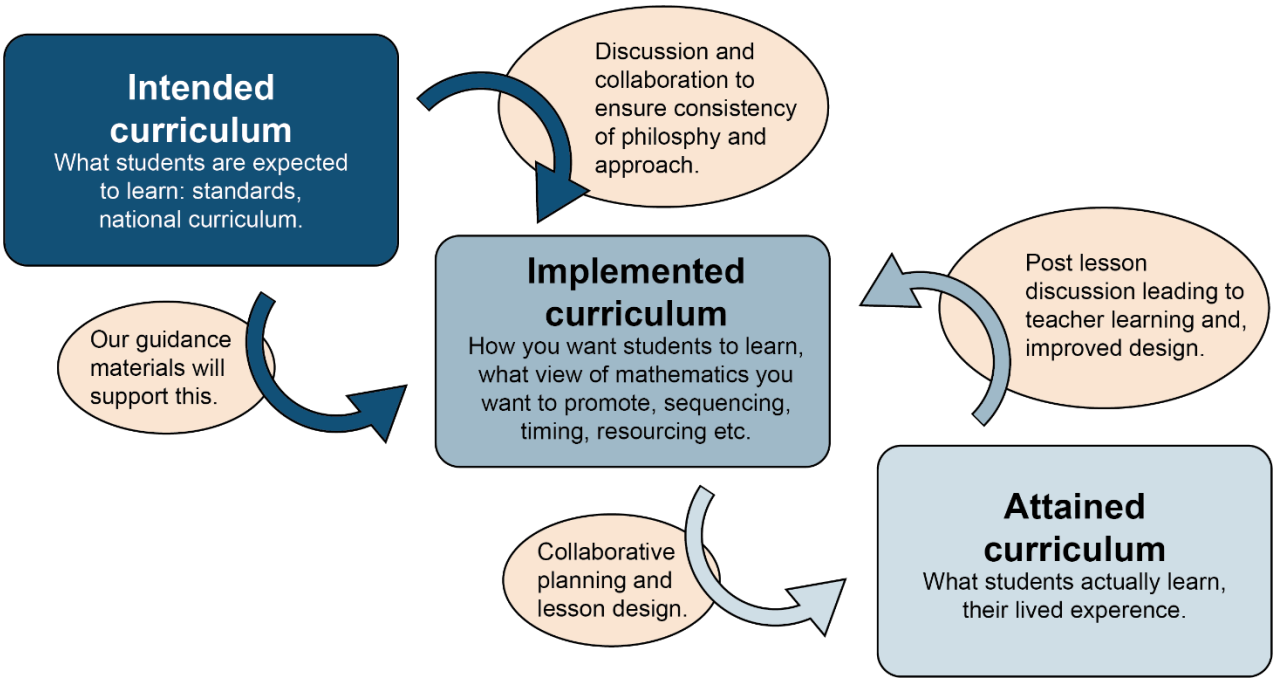
Work together – create, revise and refine

A school's curriculum or scheme of work should not be a static document which stays on the shelf for reference. It is a living document, encapsulating a department's philosophy and practice, and needs to be worked on continually.

As well as setting out what should be taught and when, curriculum outlines should also give a sense of *how* content should be taught and how teachers want their students to learn mathematics.

It is helpful to consider three different embodiments of a curriculum, the:

- intended curriculum
- implemented curriculum (the school curriculum or scheme of work)
- attained curriculum.



From intended, to implemented, to attained curriculum

This package of guidance and exemplars is intended to help secondary mathematics departments to construct their own curriculum framework and associated scheme of work. However, it is important that this is not the work of just one or two people; it should involve all those who teach mathematics working together, so that everyone understands the principles on which it is based.

Once a scheme of work is in place, all teachers can be working from the same set of principles and towards a common goal, using the shared document to support them in planning lessons. However, translating items in a scheme of work into the sort of lessons teachers want their students to experience is not a mechanical process but one that

requires discussion and debate. Establishing some form of collaborative planning where colleagues work together to plan lessons is very beneficial. These lessons can then become part of the department's shared resources and used as part of an ongoing cycle of improvement.

Well-constructed lesson designs do not guarantee rich learning experiences. They are always a hypothesis yet to be tested out in the classroom. The final piece of the collaborative planning jigsaw is regular post-lesson discussion of how successful the lesson was in terms of students' learning. Such discussions can lead to valuable professional development around subject knowledge, pedagogy and, where necessary, improved lesson design.

Sample Key Stage 3 curriculum framework

Teaching that aims for deep and sustainable learning is rooted in an appreciation of the connectedness of mathematical ideas and an understanding of the underlying structures. It emphasises the need to go beyond being able to memorise facts and practise procedures and routines. This requires teachers to 'look through' the national curriculum statements of content and descriptions of what students need to be able to do, to discern what students need to *be aware of and understand* in order to do those things fluently.

The sample curriculum framework below is based on the NCETM secondary mastery PD materials for Key Stage 3. These offer a 'fine-grained' description of the key themes and big ideas of the curriculum, detailing:

- six broad mathematical 'themes'
- a number of 'core concepts' within each theme
- a set of 'knowledge, skills and understanding' statements within each core concept
- a collection of focused 'key ideas' within each of those statements.

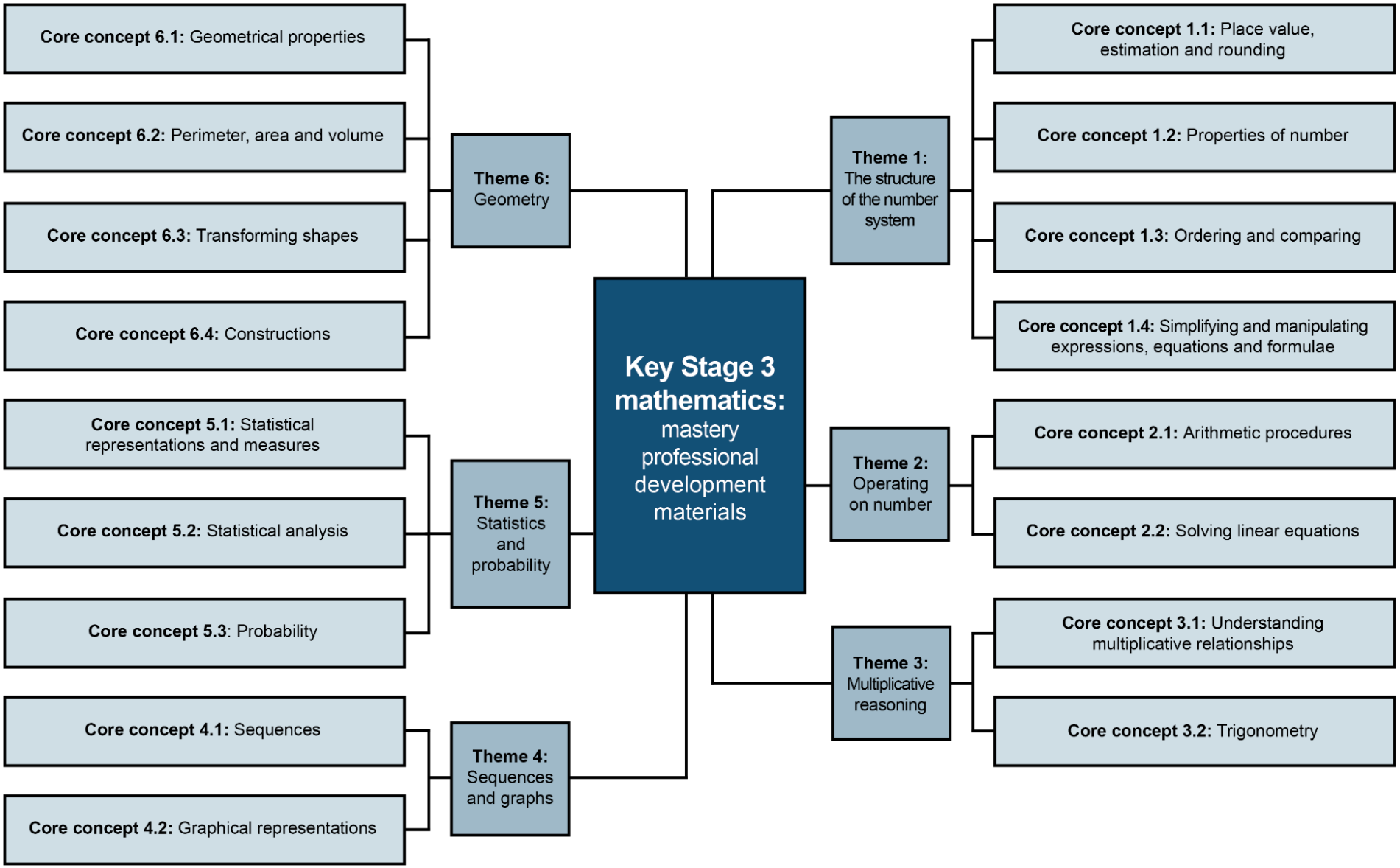
There are many ways to organise the curriculum, and individual schools will make their own decisions. This sample curriculum framework is designed to support schools in their decision-making processes by offering an example of how teaching of the 'knowledge, skills and understanding' statements could be distributed.

This curriculum framework outlines the skills, knowledge and understanding to be developed in each term; it does not specify particular resources or activities. When putting together a curriculum framework it is important to consider the order of development of learning so that content is covered in a coherent way, and structures and connections within the mathematics are emphasised. This will help to ensure that students' learning is sustainable over time. When developing a scheme of work from a curriculum framework, time needs to be built in to ensure that students have the prerequisite knowledge and skills for the forthcoming modules of work, and time for both formative and summative assessments will need to be included. Schools will need to keep this in mind when using this framework to inform their planning.

The essential features of a teaching for mastery approach in maths: working to develop a deep and connected understanding, developing procedural fluency and conceptual understanding in tandem, developing fluent knowledge of key facts and techniques, keeping the class together working on the same content, and believing that every child can succeed, can be applied in either setted, streamed or mixed attainment classes. This sample curriculum framework can be used regardless of the choice made by the individual secondary school.

Within this sample curriculum framework, the 'knowledge, skills and understanding' statements are not of equal size and do not require the same amount of curriculum time; some will require more than others. Schools should make their own decisions about

timings based on their knowledge of their students. They will take into consideration that a teaching for mastery approach includes significant time spent developing a deep understanding of the key ideas and concepts that are needed to underpin future learning. The model exemplifies a *three-year* Key Stage 3; it is not recommended that the content is condensed into two years as the necessary depth of understanding is unlikely to be attained within a shorter time frame.


Key Stage 3 mathematics: mastery professional development materials
Theme 1: The structure of the number system
Core concept 1.1: Place value, estimation and rounding
Core concept 1.2: Properties of number
Core concept 1.3: Ordering and comparing
Core concept 1.4: Simplifying and manipulating expressions, equations and formulae
Theme 2: Operating on number
Core concept 2.1: Arithmetic procedures
Core concept 2.2: Solving linear equations
Theme 3: Multiplicative reasoning
Core concept 3.1: Understanding multiplicative relationships
Core concept 3.2: Trigonometry
Theme 4: Sequences and graphs
Core concept 4.1: Sequences
Core concept 4.2: Graphical representations
Theme 5: Statistics and probability
Core concept 5.1: Statistical representations and measures
Core concept 5.2: Statistical analysis
Core concept 5.3: Probability
Theme 6: Geometry
Core concept 6.1: Geometrical properties
Core concept 6.2: Perimeter, area and volume
Core concept 6.3: Transforming shapes
Core concept 6.4: Constructions

Year 7 sample curriculum framework

Autumn term

Place value

Understand the value of digits in decimals, measure and integers

Properties of number: factors, multiples, squares and cubes

Understand multiples

Understand integer exponents and roots

Understand and use the unique prime factorisation of a number

Arithmetic procedures with integers and decimals

Understand and use the structures that underpin addition and subtraction strategies

Understand and use the structures that underpin multiplication and division strategies

Use the laws and conventions of arithmetic to calculate efficiently

Expressions and equations

Understand and use the conventions and vocabulary of algebra including forming and interpreting algebraic expressions and equations

Simplify algebraic expressions by collecting like terms to maintain equivalence

Manipulate algebraic expressions using the distributive law to maintain equivalence

Spring term

Plotting coordinates

Connect coordinates, equations and graphs[$]

Perimeter and area

Understand the concept of perimeter and use it in a range of problem-solving situations[$]

Understand the concept of area and use it in a range of problem-solving situations[$]

Arithmetic procedures including fractions

Work interchangeably with terminating decimals and their corresponding fractions

Compare and order positive and negative integers, decimals and fractions

Know, understand and use fluently a range of calculation strategies for addition and subtraction of fractions

Know, understand and use fluently a range of calculation strategies for multiplication and division of fractions

Summer term

Understanding multiplicative relationships: fractions and ratio

Understand the concept of multiplicative relationships

Understand that multiplicative relationships can be represented in a number of ways and connect and move between those different representations[$]

Understand that fractions are an example of a multiplicative relationship and apply this understanding to a range of contexts

Understand that ratios are an example of a multiplicative relationship and apply this understanding to a range of contexts

Transformations

Understand and use translations

Understand and use rotations

Understand and use reflections

Understand and use enlargements

Year 8 sample curriculum framework

Autumn term

Estimation and rounding

Round numbers to a required number of decimal places

Round numbers to a required number of significant figures

Estimate calculations by rounding

Sequences

Understand the features of a sequence

Recognise and describe arithmetic sequences

Graphical representations of linear relationships

Connect coordinates, equations and graphs[$]

Explore linear relationships

Solving linear equations

Understand what is meant by finding a solution to a linear equation with one unknown

Solve a linear equation with a single unknown on one side where obtaining the solution requires one step

Solve a linear equation with a single unknown where obtaining the solution requires two or more steps (no brackets)

Solve efficiently a linear equation with a single unknown involving brackets

Spring term

Understanding multiplicative relationships: percentages and proportionality

Understand that multiplicative relationships can be represented in a number of ways and connect and move between those different representations[$]

Understand that percentages are an example of a multiplicative relationship and apply this understanding to a range of contexts

Understand proportionality

Statistical representations, measures and analysis

Understand and calculate accurately measures of central tendency and spread

Construct accurately statistical representations

Interpret reasonably statistical measures and representations

Choose appropriately statistical measures and representations

Summer term

Perimeter, area and volume

Understand the concept of perimeter and use it in a range of problem-solving situations[$]

Understand the concept of area and use it in a range of problem-solving situations[$]

Understand the concept of volume and use it in a range of problem-solving situations

Geometrical properties: polygons

Understand and use angle properties

Constructions

Use the properties of a circle in constructions

Use the properties of a rhombus in constructions

Year 9 sample curriculum framework

Autumn term

Geometrical properties: similarity and Pythagoras' theorem

Understand and use similarity and congruence

Understand and use Pythagoras' theorem

Probability

Explore, describe and analyse the frequency of outcomes in a range of situations

Systematically record outcomes to find theoretical probabilities

Calculate and use probabilities of single and combined events

Spring term

Non-linear relationships

Recognise and describe other types of sequences (non-arithmetic)

Expressions and formulae

Find products of binomials

Rearrange formulae to change the subject

Trigonometry

Understand the trigonometric functions

Use trigonometry to solve problems in a range of contexts

Summer term

Standard form

Interpret and compare numbers in standard form $A \times 10^n$, $1 \leq A < 10$

Graphical representations

Model and interpret a range of situations graphically

The notation '$' indicates where key ideas within the 'knowledge, skills and understanding' statements have been split in order to sequence learning more effectively.

Split statements of knowledge, skills and understanding

In some cases key content has been split between episodes of learning and hence the same, or a very similar, statement will appear in multiple locations. This is indicated by the notation $. It is intended that the following key ideas should be covered in the terms specified.

Understand that multiplicative relationships can be represented in a number of ways and connect and move between those different representations

Use a double number line to represent a multiplicative relationship and connect to other known representations (Year 7 summer term)

Understand the language and notation of ratio and use a ratio table to represent a multiplicative relationship and connect to other known representations (Year 7 summer term)

Use a graph to represent a multiplicative relationship and connect to other known representations (Year 8 spring term)

Use a scaling diagram to represent a multiplicative relationship and connect to other known representations (Year 8 spring term)

Connect coordinates, equations and graphs

Describe and plot coordinates, including non-integer values, in all four quadrants (Year 7 spring term)

Solve a range of problems involving coordinates (Year 7 spring term)

Know that a set of coordinates, constructed according to a mathematical rule, can be represented algebraically and graphically (Year 8 autumn term)

Understand that a graphical representation shows all of the points (within a range) that satisfy a relationship (Year 8 autumn term)

Understand the concept of perimeter and use it in a range of problem-solving situations

Use the properties of a range of polygons to deduce their perimeters (Year 7 spring term)

Recognise that there is constant multiplicative relationship (π) between the diameter and circumference of a circle (Year 8 summer term)

Use the relationship $C = \pi d$ to calculate unknown lengths in contexts involving the circumference of circles (Year 8 summer term)

Understand the concept of area and use it in a range of problem-solving situations

Derive and use the formula for the area of a trapezium (Year 7 spring term)

Understand that the areas of composite shapes can be found in different ways (Year 7 spring term)

Understand the derivation of, and use the formula for, the area of a circle (Year 8 summer term)

Solve area problems of composite shapes involving whole and/or part circles, including finding the radius or diameter given the area (Year 8 summer term)

Understand the concept of surface area and find the surface area of 3D shapes in an efficient way (Year 8 summer term)

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