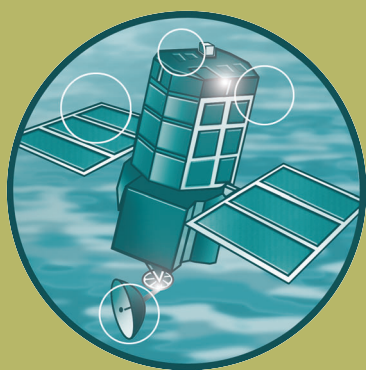# Development of estuary morphological models

Annex C: ASMITA Manual

R&D Project Record FD2107/PR

Environment Agency

defra
Department for Environment
Food and Rural Affairs

Environment Agency
Department for Environment, Food and
Rural Affairs

ASMITA Manual

Date: July 2007

Project Ref: R/3427/4

Report No: R.1373 (Version 1.0 July 2007)

ABP mer
marine environmental research

ASMITA Manual

Contents

Page

Appendix

Tables

Figures

1. Introduction

Definition: **A**ggregated **S**cale **M**orphological **I**nteraction between a **T**idal inlet and the **A**djacent coast (**ASMITA**).

This manual has been written as a guide to using the Matlab ASMITA model and illustrates the concepts and a methodology used in long-term morphological modelling.

1.1 Background Information

As part of the Estuaries Research Program Phase 2 (ERP2) funded by the Environment Agency and Department for Environment, Food and Rural Affairs (Defra) a long-term morphological prediction tool ASMITA has been developed. The contributions from the following project partners are gratefully acknowledged (Figure 1).

- ABP Marine Environmental Research Ltd (ABPmer);
- HR Wallingford;
- Delft Hydraulics;
- Delft Technical University.
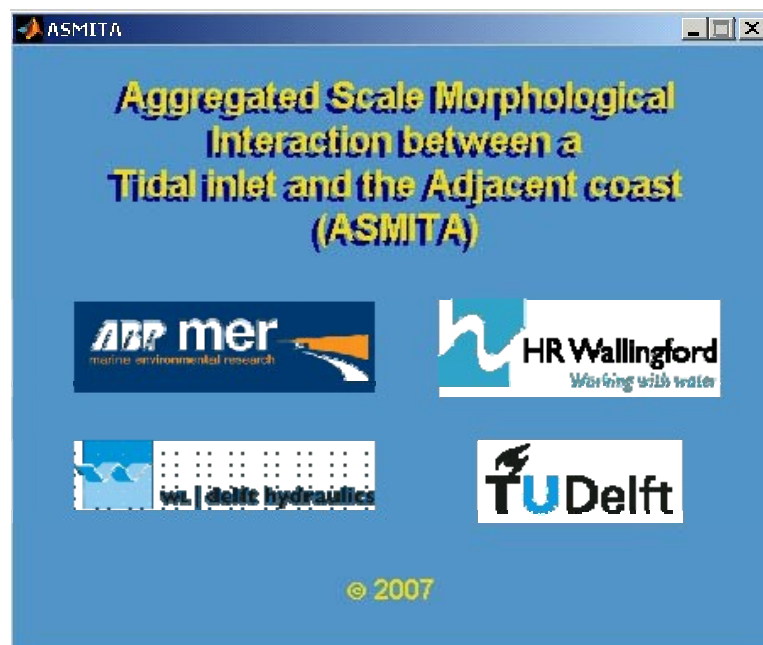
Figure 1.   Splash screen acknowledging the project partners who have contributed to the ASMITA program

1.2   What is ASMITA?
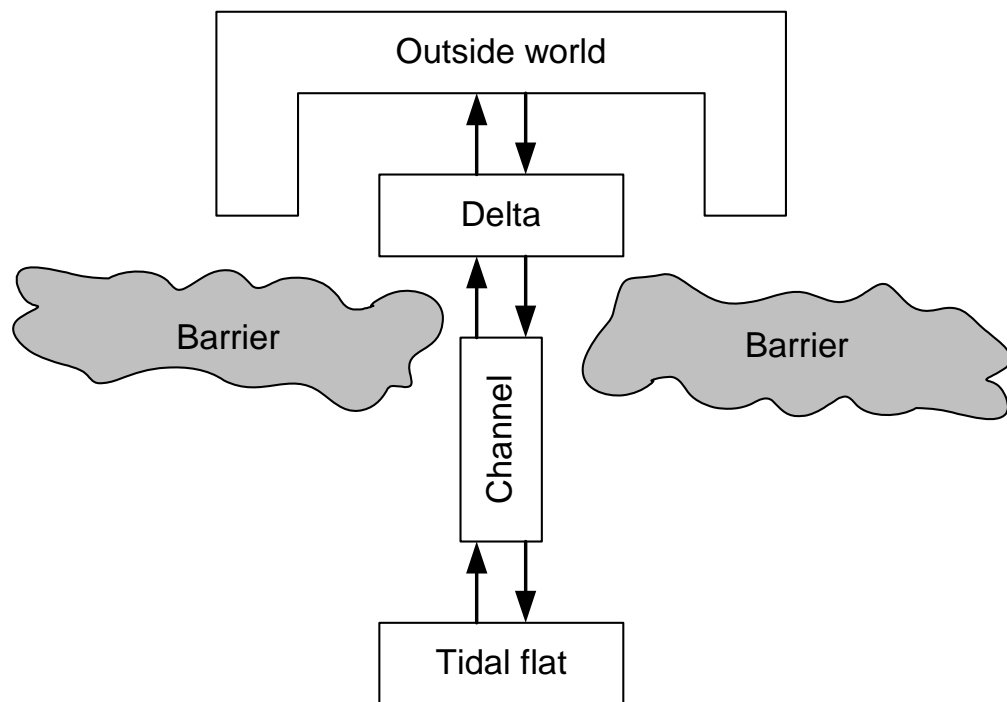
Figure 2.   A schematized representation of the concept of ASMITA

The idea behind ASMITA (Figure 2) is that a tidal basin, or any characteristic area (element) can reach an equilibrium volume relative to an invariable mean sea level. For example, under existing conditions the accommodation space (no net change in the change in the transport of sediment) is zero, with increased msl the accommodation space becomes positive and sediment may be imported, the opposite is true when sea level falls. The key concepts of the ASMITA model are:

- The estuary is schematized into a number of geomorphologic elements;
- The state of each element is described by its volume (water or sediment);
- Integrated parameters of hydrodynamics (tidal prism, or tidal range) are used;
- Empirical relationships define the morphological equilibrium for each element;
- Deviation from the morphological equilibrium causes sediment demand;
- A gradient in sediment demand drives sediment transport, and thereby morphological change.

Appendix A describes the theory and concepts used within the ASMITA model.

1.3   About ASMITA

The ASMITA software has been written using the software language Matlab, version R2007a (Figure 3). The software utilises only the functions present within Matlab and does not require additional toolboxes. Distributed alongside this manual is a word document that details the relationships between the various routines which makeup the ASMITA program.
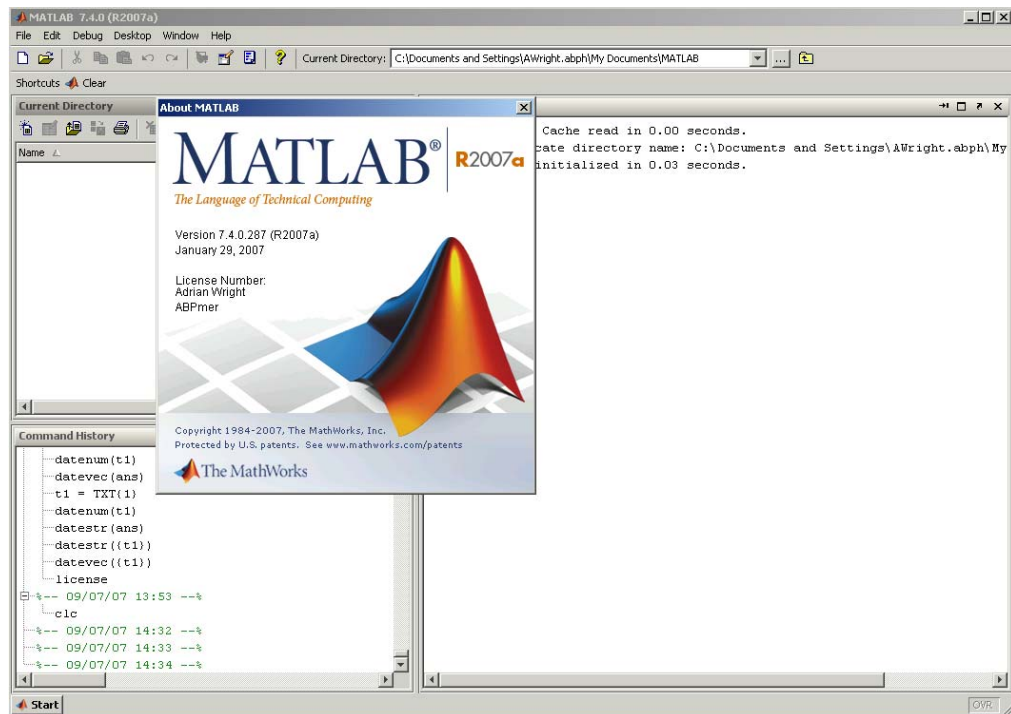
Figure 3.   The ASMITA program has been developed using the software Matlab R2007a from Mathworks

1.4 Opening ASMITA

Within Matlab navigate to the folder containing the Matlab ASMITA scripts. From the command line, type **asmui**; (Figure 4) this command will invoke the instillation and setup of the ASMITA graphical user interface.
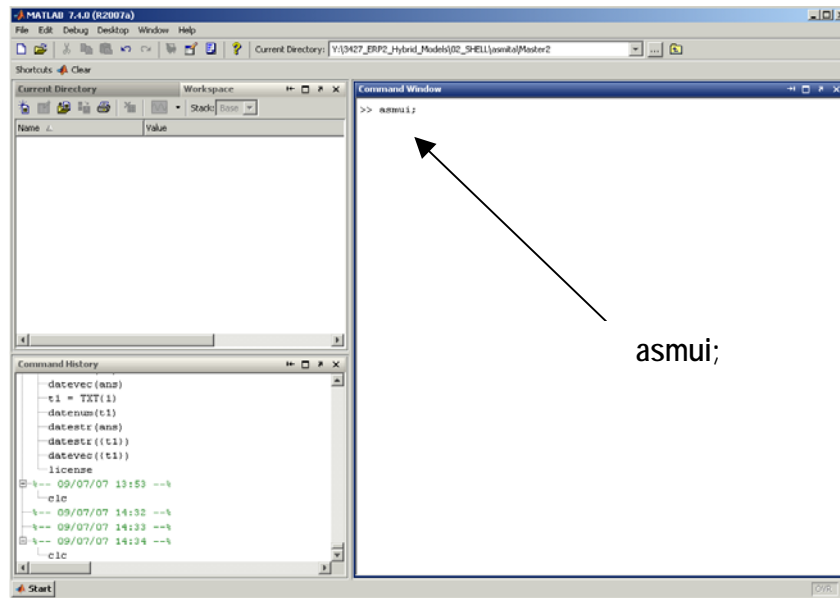
asmui;

Figure 4. To load and run ASMITA type 'asmui;' within the command widow

Global variables, these contain the information used for running ASMITA
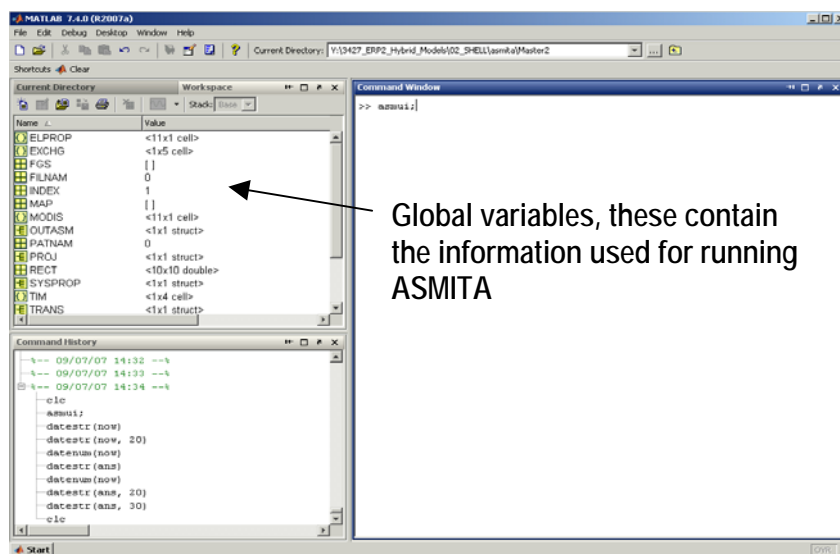
Figure 5. The Matlab environment after calling the ASMITA program

The information entered into the ASMITA model (type, properties, flow, diffusion rates and so on) is contained within global variables stored within the Matlab environment. This information is used throughout the many routines, therefore this information is stored in memory while the application is being run. The user can view this information either through the graphical interfaces provided in the software (discussed later) or by opening these global variables from the workspace window (Figure 5).

Note: Within Matlab, information is stored as numbers, cell arrays and structured arrays.
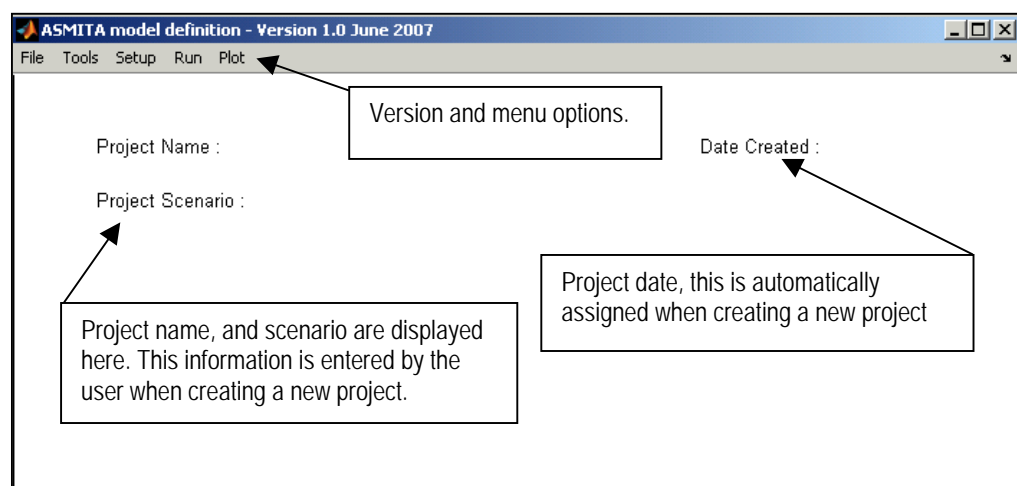
Figure 6.   The ASMITA interface

The user is able to control **all** aspects of the ASMITA model through the ASMITA interface (Figure 6), there is no requirement for the user to access this data through the Matlab command window.

Note: Users have full access to the code that will enable them to make fine adjustments to the controlling algorithms. For example, additional driving forces may be needed that are not described in the current setup. Having full access to the software code allows the user to add additional functionality.

2.   File Operations

From within the ASMITA interface (Figure 8) there are a number of menu options. The menu item **FILE** allows the user to open or create a new instance of ASMITA, exit the program or save the current model setup. The following section explains the options available to the user for each of these items under the FILE menu.

2.1 New (Create a New Model Setup)

After selecting **New** from the File menu option a series of popup widows will appear. The user is asked a number of questions regarding the project name and scenario (Figure 7). This information will be displayed in the ASMITA interface (Figure 6).
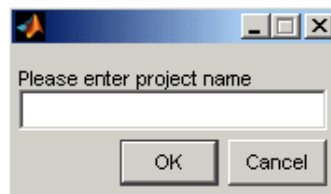
Figure 7.   A dialog windows that appears after selecting the New menu item

2.2 Open (Open an Existing Model)

Information regarding existing ASMITA models can be stored within a Matlab work file (*.mat). The *.mat file contains the information regarding the defined elements, their properties and system defined properties.
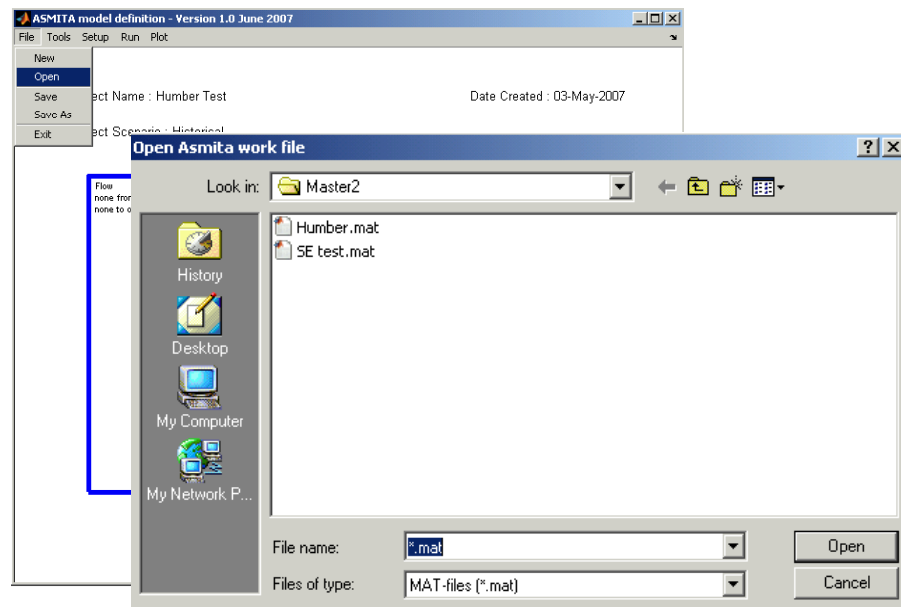
Figure 8.   Shows the dialog window that appears after selecting the Open menu item, only *.mat files should be selected

Select an existing ASMITA model setup file from the common dialog box. By default the *.mat file extension is visible (Figure 8). An example *.mat project is provided along with the model scripts.

2.3 Save and Save As

The user can save the current setup of the ASMITA model by selecting **Save** or **Save As** from the **File** menu item. If the user selects **Save** without first specifying a name then the program will by default request a name from the user.

2.4 Exit

The user can exit the ASMITA program at any point by going only to the **Exit** menu item in the **File** drop down list.

3. Tools

Within the ASMITA model software are a number of tools, these are designed to assist in the building and visualisation of the model elements. Also included are a number of tools that allow the user to clear unwanted figures or clear the whole ASMITA model setup. These tools are described in more detail below.

3.1 Load Image

In order to understand the complexity of the system an image can be loaded as a background map, showing the areas of intertidal, channel, delta and so on (Figure 9). This can help the user visualise the schematised approach.
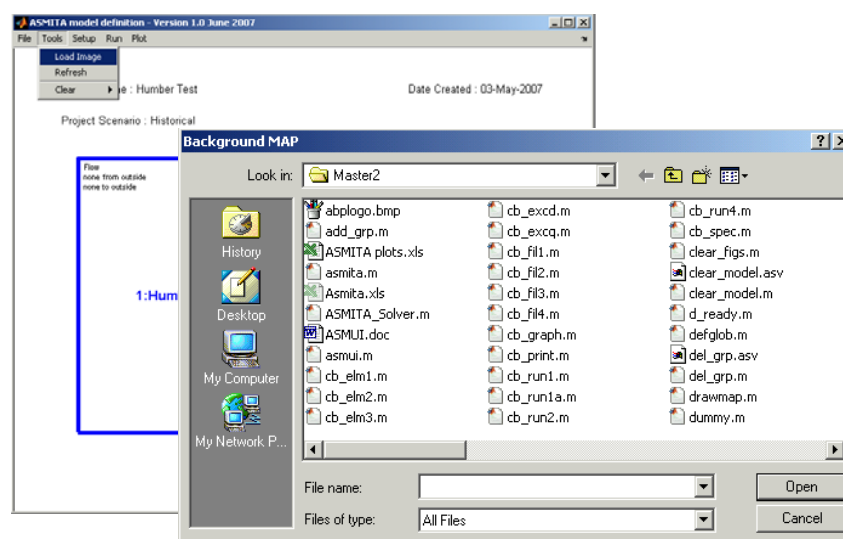
Figure 9.   The dialog box that appears after selecting the Load Image option from the Tools file menu

The user is able to select from the following image types:

ID Num	File Type
1	JPEG File Interchange Format (*.jpg, *.jpeg)
2	Tag Image File Format (*.tif)
3	Windows Bitmap (*.bmp)
4	Portable Network Graphics (*.png)
5	Hierarchical Data Format (*.hdf)
6	PC Paintbrush (*.pcx)
7	X Window Dump (*.xwd)

3.2 Refresh

The **Refresh** function resets the screen images. If an image appears incorrectly displayed then use the **Refresh** function to redraw the screen images.

3.3 Clear Model/Figures

The **Clear Model** function (Figure 10) resets all parameters in ASMITA to a blank (undefined) model. The **Clear Model** function allows the user to start the process of setting up an ASMITA model without the need to close the whole application.

Clear Figures will delete any figure window open. Under the **Plot** menu item (discussed later) the user can plot selected information in numerous windows. To simply close all figure windows click the **Clear Figures** function from the drop down menu list (Figure 10).
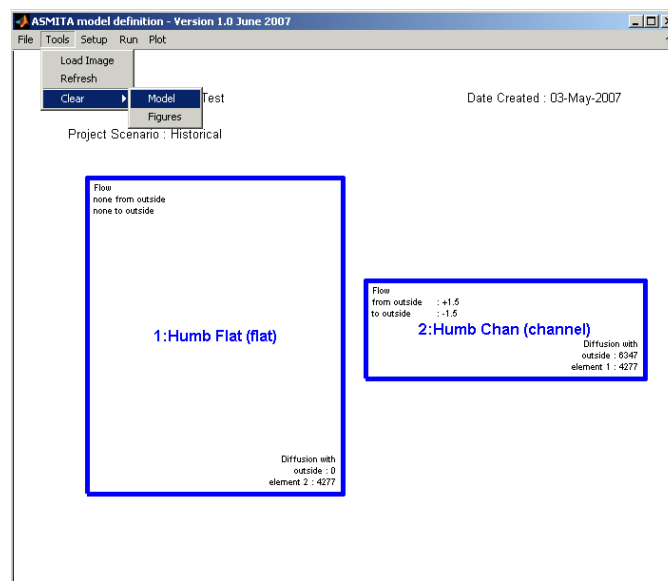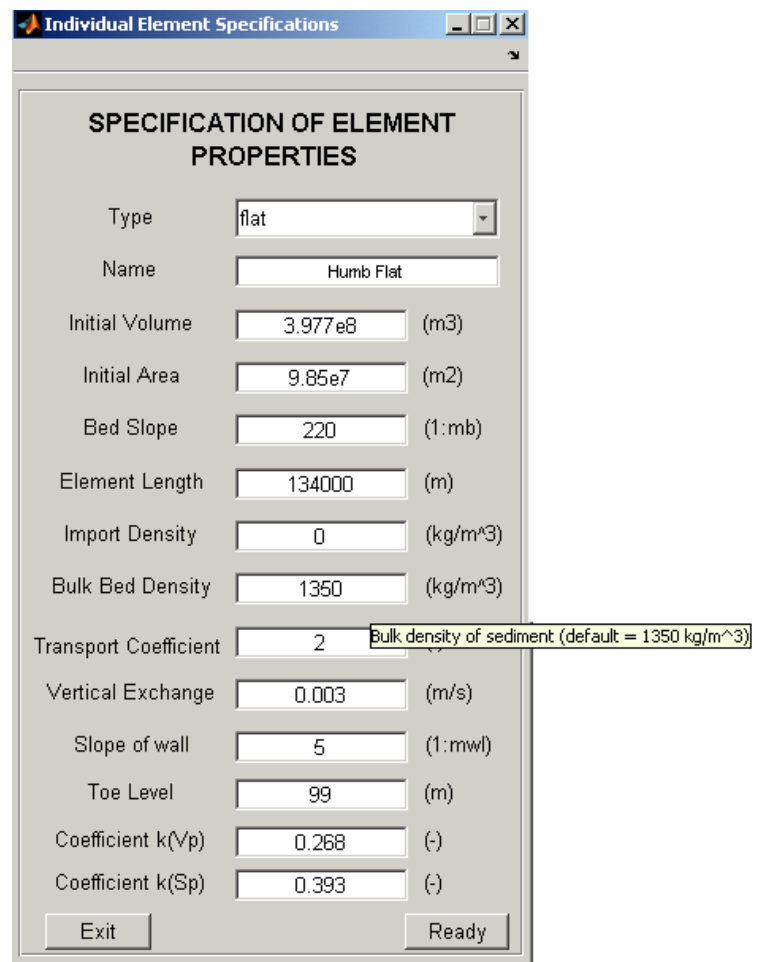
Figure 10. Clear model/figures can be accessed via the main interface

4. Element Setup

As discussed previously, the concept of ASMITA is a schematisation of the various parts of the environment under investigation. To define the individual environment elements the user must first know the following properties:

- Type;
- Initial Volume;
- Initial Area;
- Bed Slope;
- Element Length;
- Import Density;
- Bulk Bed Density;
- Transport Coefficient;
- Vertical Exchange;
- Slope of Wall;
- Toe Level;
- Regime coefficients.

Figure 11. ASMITA GUI that allows the user to enter element specific information

Since many of the items for the specified element may not be known tool tips are provided that gives the user additional information such as the default values. (Figure 11) shows the tool tip provided for bulk density.

4.1.1   Elements Units

Element definitions and units are shown in Figure 11, this information is presented in Table 1.

Table 1.   **Element unit definitions and associated default values where appropriate**

Parameter	Units
Initial Volume	m^3
Initial Area	m^2
Bed Slope	
Element Length	m
Import Density	kg/m^3
Bulk Bed Density	kg/m^3 (default value 1350)
Transport Coefficient	no units (default value 2)
Vertical Exchange	m/s
Slope of Wall	1:mean water level
Toe Level	m
Coefficient k (Vp)	no units
Coefficient k (Vp)	no units

Note: *A number of sensitivity runs should be performed by the user to determine how sensitive the system is to the information specified in the element and system specifications.*

4.2   Defining Elements

As discussed previously elements within the environment under investigation can be schematised into various parts depending on the individual characteristics, for example an element defined as a channel may have distinct properties to an element defined as intertidal and so on. To define an element the user must select the **Setup** – **Element** – **Define** item from the drop down menu item (Figure 12).

After selecting the **Setup** – **Element** – **Define** function, using the mouse draw a rectangle on the ASMITA interface. To draw (define) the element, select the position of the top right hand corner. Whilst holding down the mouse button, drag the mouse until you have reached the desired size for the element (rectangle). (Figures 12 and 13) illustrate this concept.

Note: *The size of the rectangle drawn plays no part in the calculation, the rectangle only represents the schematization of the environment under investigation.*
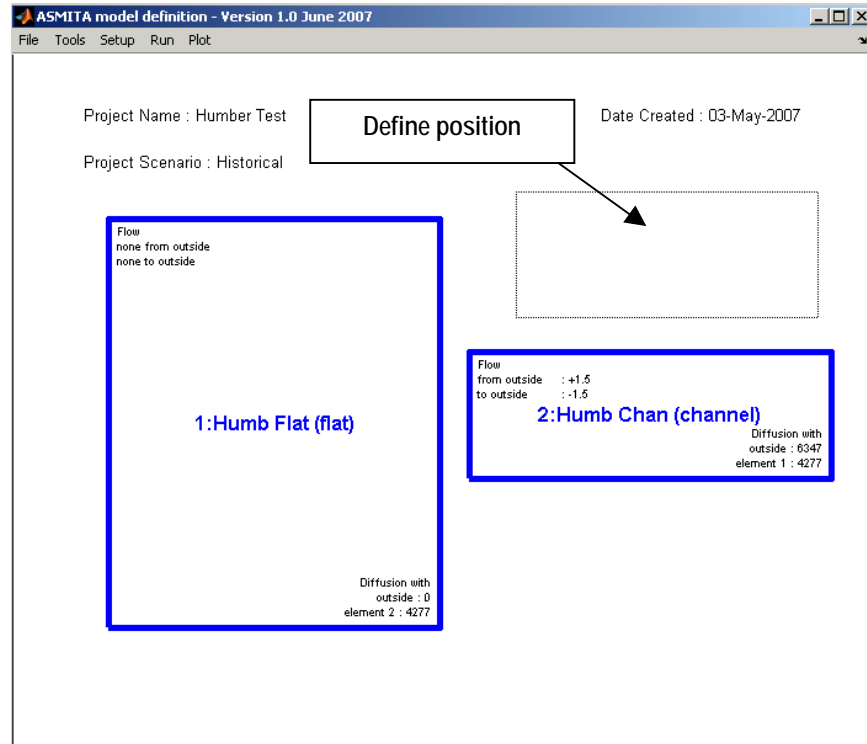
Figure 12.   Step one: define element position within the ASMITA interface
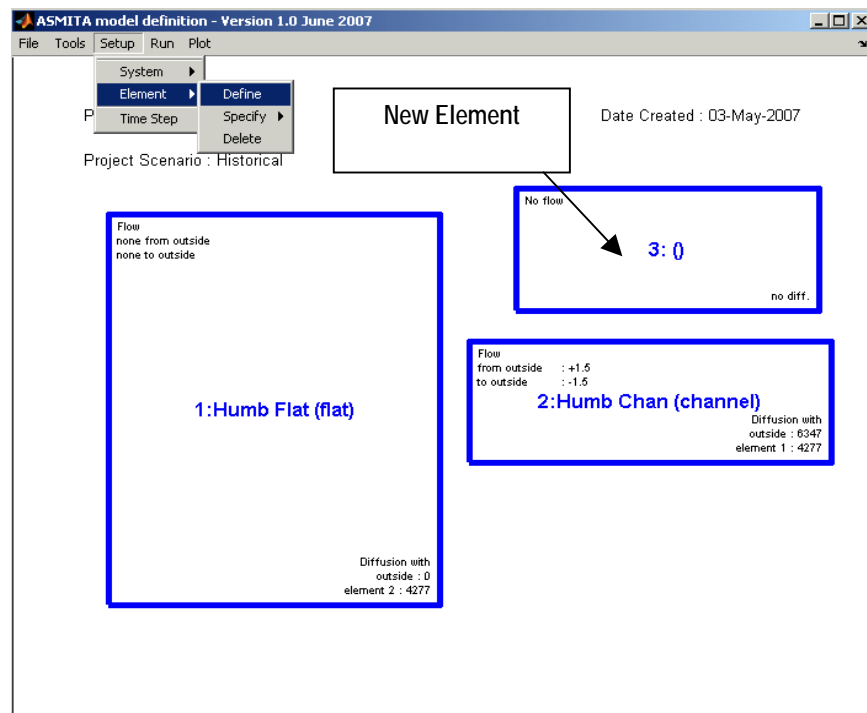
Figure 13.   Step two: new defined element within the ASMITA environment

No overlapping of elements is allowed, and no more than 10 elements can be specified in the current version. It is recommended that the user define at least 2 elements. It's also recommended that the user allow for a usable gap between elements, this ensures that when selecting elements the program is not confused by the close proximately of other elements.

Note: *The order of the elements specified by the user determines the order in which they are stored within the Matlab data structure.*

4.3   Element Properties

After defining a new element the user is asked to provide the information specific to this element type (Figure 14). Where possible, default information is provided; this information is displayed as tool tips (Figure 11) when the user hovers over the specific element property.

Note: *Default information is not element specific.*

To select the element property the user can right click on the specific element, and then select **Properties**. The selected element will be highlighted in red as shown in (Figure 14). Alternatively, the user can select **Setup – Element – Properties** from the drop down menu list. The last element selected will be chosen, if no elements have been selected in that session, then no element properties will be highlighted.

Note: *The right click method is the recommended approach.*
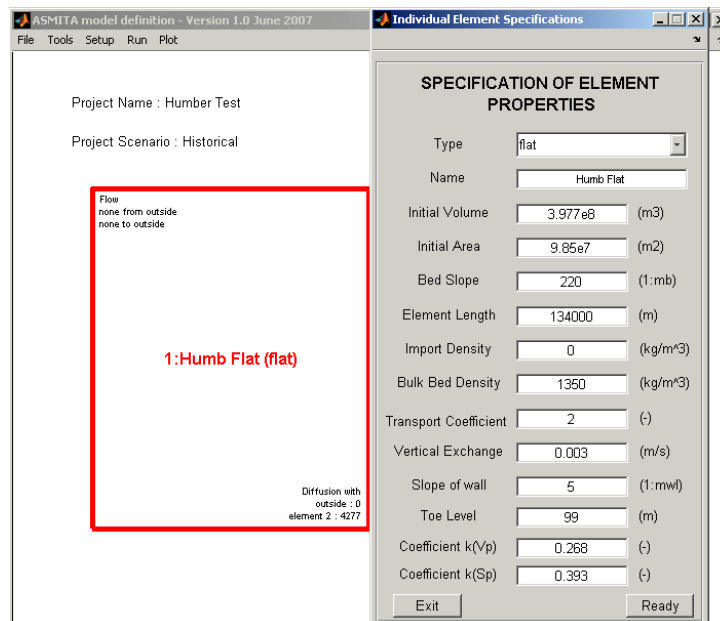
Figure 14.   Element specification window

4.3.1 Fixed Area Due to Sea Wall

The default values for the slope of a wall and the level of the toe of the wall are set to 3 and 99 respectively. Although this is set for all elements, only the values for the tidal flat have any effect. With the default values, high water would have to rise by >95m before this adjustment is called. By setting a value of 'Toe Level' greater than high water (half the tidal range) but less than the value at the end of the run (0.5*tidal range + slr) the slope used to calculate changes in the surface area switches from the high water slope specified for the flat element to the slope specified for the wall. This has the effect of constraining the increase in high water area and alters dynamical response of the flat and channel elements.

4.3.2 Equilibrium Volume and Area coefficients

The tidal flat and channel equilibrium conditions are specified by linear equations as a function of tidal prism of the form: Ve = kv*Vp and Se = ks*Sp, where Sp = Vp/tr, where Ve and Se are the equilibrium values, kv and ks are the scaling coefficients, Vp is the tidal prism and tr is the tidal range. A unique value of kv and ks can be specified for each element using the input boxes 'Coefficient k (Vp)' and 'Coefficient k (Sp)'. When a delta element is included, the delta equations are also multiplied by kv and ks, so they can be used to re-scale the delta equilibrium equations

If the values of 'Coefficient k (Vp)' are left at the default value of 1 for ALL elements, the model defaults to calculating the values of kv and ks internally. This is done using the initial values of element volume and tidal prism for kv and element surface area and prism area, Sp, for ks.

If the values of 'Coefficient k (Vp)' and set to 0 in ALL elements the code uses the hypsometry routine to calculate Ve and Se (see Appendix 1 for details).

If any of the values of 'Coefficient k (Vp)' differ from 1 or 0 the code uses the user-defined values in the linear equations noted above.

4.4 Deleting Elements

To delete an element from the ASMITA interface the user should right click on the element to be deleted. From the menu items, select **Delete**. A popup window will appear asking the user to confirm this action, click yes to permanently delete this element.

4.5 Defining Flow and Diffusion

One of the main drivers within the ASMITA model is the flow and diffusion of sediment into and out of the specified elements, either from one element to another or to and from the surrounding environment.

Summary of defined **flow** and pathways:

- Element to Element Flow;
- Environment to Element Flow;
- Element to Environment Flow.

In order to specify the flow from **Element to Element** the user should right click on the specified element in which they want the flow to originate, and select Flow from the popup menu. A window will appear showing the specified element flow arrangement (Figure 15) the user should then click on the element in which they wish the flow to enter. After specifying the second element a textbox will appear on the flow specification window, the user should define the rate here.

To specify flow from **Element to Environment** the user should follow the method stated above, however, rather than selecting a second element the user should click outside any elements within the ASMITA interface (Outside environment).

To select the **Environment to Element** the user has to access the Flow method from the main menu drop down list. The user should select **Setup** – **Element** – **Flow**, using the mouse click onto the **Outside Environment**. The user can then select the element in which flow will enter from the outside world.
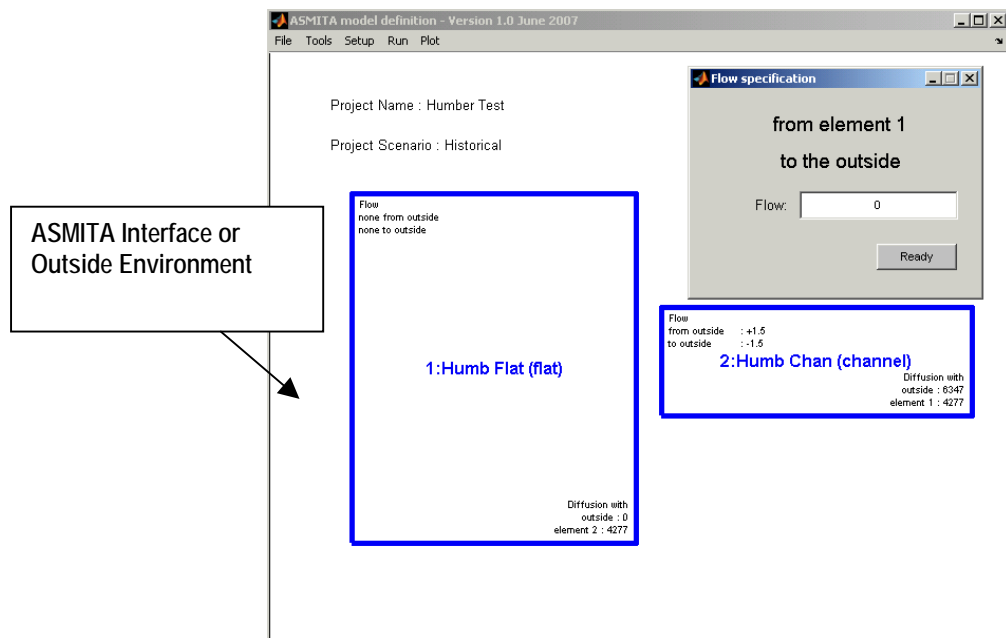
Figure 15. Definition of the flow/diffusion through the various pathways

The process is similar for defining the diffusion between elements. However, diffusion is not-directional and so one can either select an element and select Diffusion from the popup window, or access the same utility using **Setup** – **Element** – **Diffusion**. Popup windows guide the user to select elements and enter diffusion rate. The order in which elements or the Outside Environment are selected does not matter for diffusion.

4.6   Defining Change

Changes can be defined as one or more adjustments to the volume and/or area of individual elements to represent alterations to the system   The user can specify specific periods where a known change in an element may have occurred, for example a capital dredge or some engineering works, or a repeating cycle such as may be needed to represent maintenance dredging.

To specify a change in the element the user should select **Setup** – **Element** – **Change** or alternatively, right click on the element and select **Change**. In the element change window (Figure 16) the user can enter the period and associated changes in volume and area. The user can enter a cyclic period by using Matlab notation. For example the following notations can be used in the '**Years**' box:

[1:10:100] - specifies a change at 10 year intervals from year 1 to 100
86 99 – specifies changes in years 86 and 99
[1:10:100] **86 99** – combines the two.

The '**Change in Volume**' and '**Change in Area**' boxes than require values to be specified for every entry in the '**Years**' box, so the above entries would require:

V1 and **A1** – to specify the change to be applied at 10-year intervals
V2 V3 and **A2 A3** – to specify the changes to be applied in years 86 and 89
V1 V2 V3 and **A1 A2 A3** – to specify both.

This short hand notation significantly reduces the effort needed to specify dates and volume/area changes.
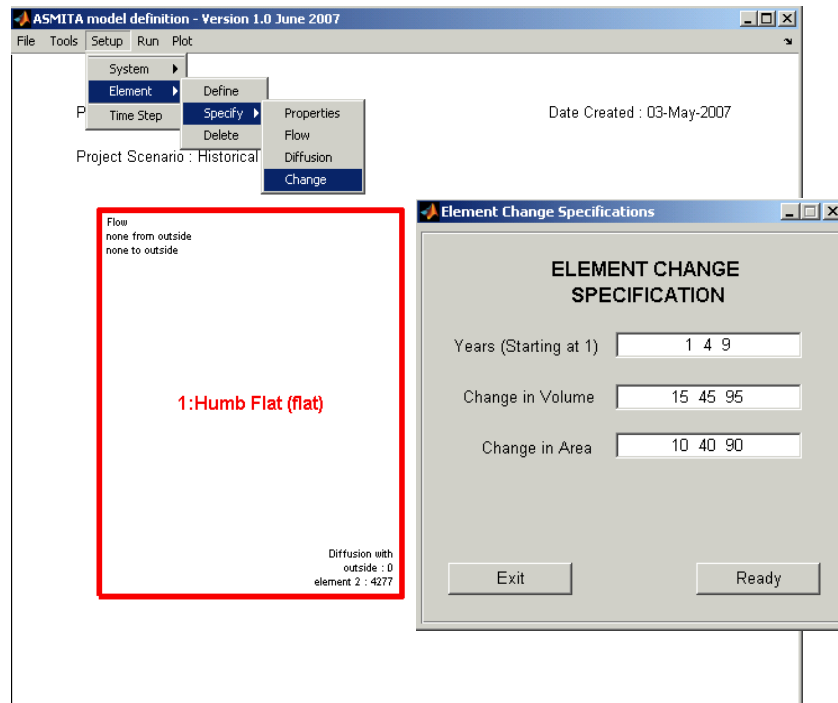
Figure 16.   Popup change window for selected element

5.   System Setup

5.1   Properties

System properties are not element dependent, that is to say the information defined within the system properties represents the whole environment. The system properties include:

- Tidal Range;
- Global Sediment Equilibrium Concentration;
- Water Density;
- Sediment Density;
- Sea Level Rise (SLR);
- Cyclic information:
  - Number of cycles;
  - Phase;
  - Amplitude;
  - Period;
- Project Details:
  - Name;
  - Scenario;
  - Date.

The user can access the system properties by selecting **Setup – System – Properties** from the menu drop down list (Figure 17)
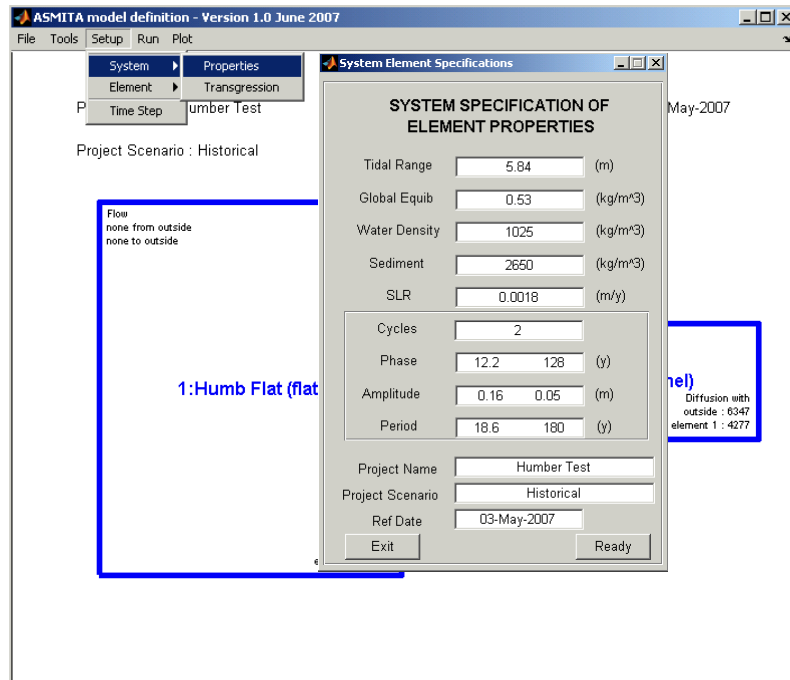
Figure 17.   Element specification popup window

6.   Configure Time Step

The number of time steps represents the length of time the simulation will be run. For example, (Figure 19) shows 500 time steps at an interval of 0.5 starting from year 1800. In this example, the model will run for the period 1800 to 2300 and will calculate the change in element areas and volumes at an interval of 0.5years. The fixed surface check box is used to calculate the volume and areas based on a fixed surface area. Leaving this unchecked means that the surface area is allowed to vary, as well as the volume, in response to perturbations.

Summary of time step configuration parameters:

- Size of time step (years);
- Number of time steps (years);
- Start year (year);
- Fixed surface calculation (optional).

To access the time step configuration window select **Setup – Time Step** from the main menu drop down list (Figure 19).
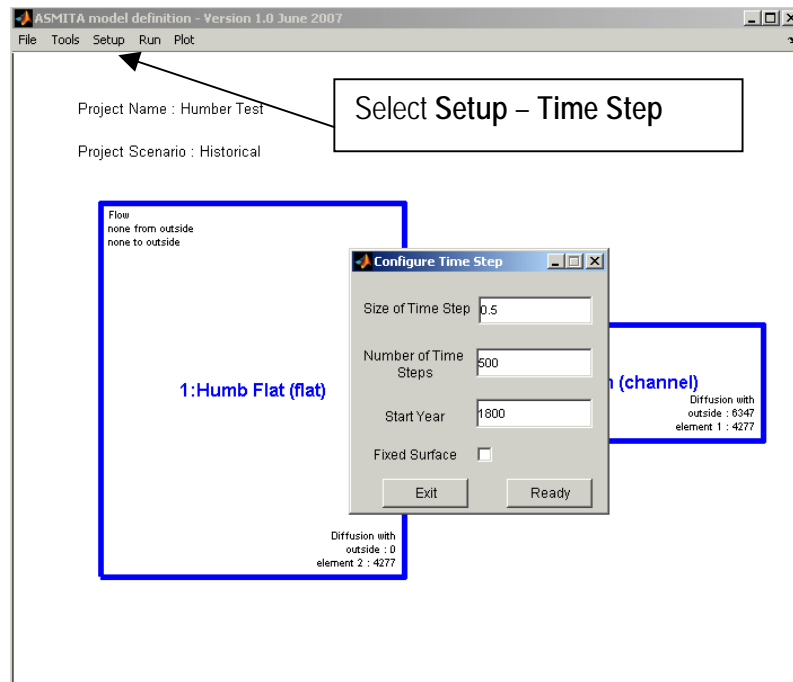
Figure 18.   Configuration Time Step window

7.   Running ASMITA

7.1   Check Input

After defining the individual elements and assigning the element and system properties the user has the option to check the input data and run the ASMITA model. The **Check Input** function checks all data entered to ensure consistency. The diagnostic information is reported back to the Matlab command window (Figure 20). Note the Check Input function is called when the user chooses the **Run Model** option. However, it might be more convenient to establish the model setup without actually running ASMITA, in this circumstance the user should select the **Check Input** function.

To access the **Check Input** function the user should select **Run – Check Input** from the main menu drop down items (Figure 21)
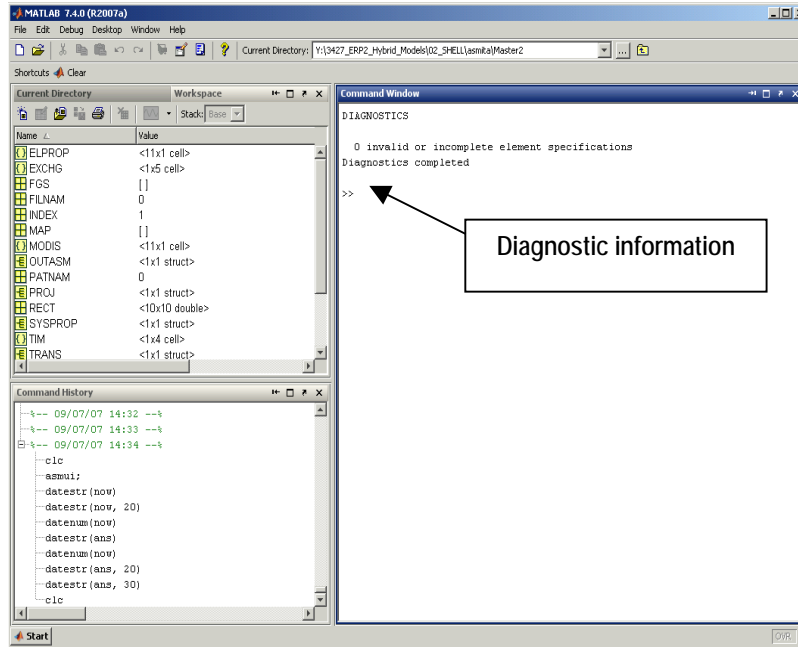
Figure 19. The Matlab command window where the diagnostic information is reported back to the user

7.2 Run Model

To run the ASMITA model the user should select **Run – Run Model** from the main menu drop down list (Figure 21)
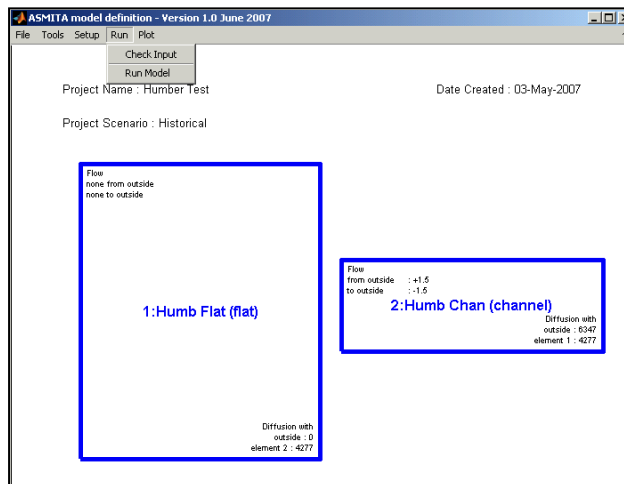
Figure 20. The Check Input and Run Model menu items

After selecting the **Run Model** option the ASMITA program will evaluate the data entered by the user. Before the model is initiated the ASMITA program asks for a file name (Figure 22) with **NO** extension. After running ASMITA there will be 2 output files. One will be a text file with the setup information and a second Microsoft Excel file. The excel file will contain results from the model simulation as well as the setup information. The name of both the text and Excel files are identical, except for the file extension.
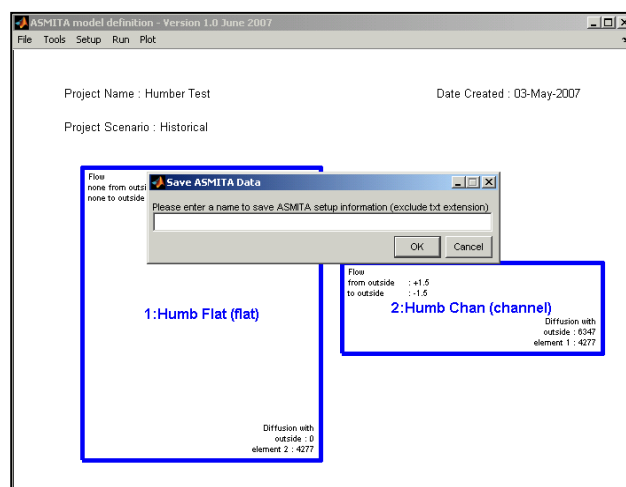
Figure 21. Dialog box asking for user defined name

8. Results

The results that can be obtained from ASMITA to inform the user of the consequences of an engineering project, dredge, climate change and so on, are the following:

Volumes/Areas:
- Moving Surface Volume/Area (values relative to any changed water level);
- Equilibrium Volume/Area (values that determine the equilibrium condition);
- Fixed Surface Volume/Area (values relative to a fixed (initial) water level).

9. Plotting Routines

In order to visualise the results generated by the ASMITA model a plotting routine has been developed (Figure 23). To access the plotting routine select **Plot – Plot Results** from the drop down menu item.

Choose between elements defined in ASMITA model

Option to display area and volume information

The user can create a new plot or add to an existing figure. The user can delete data from a current plot figure
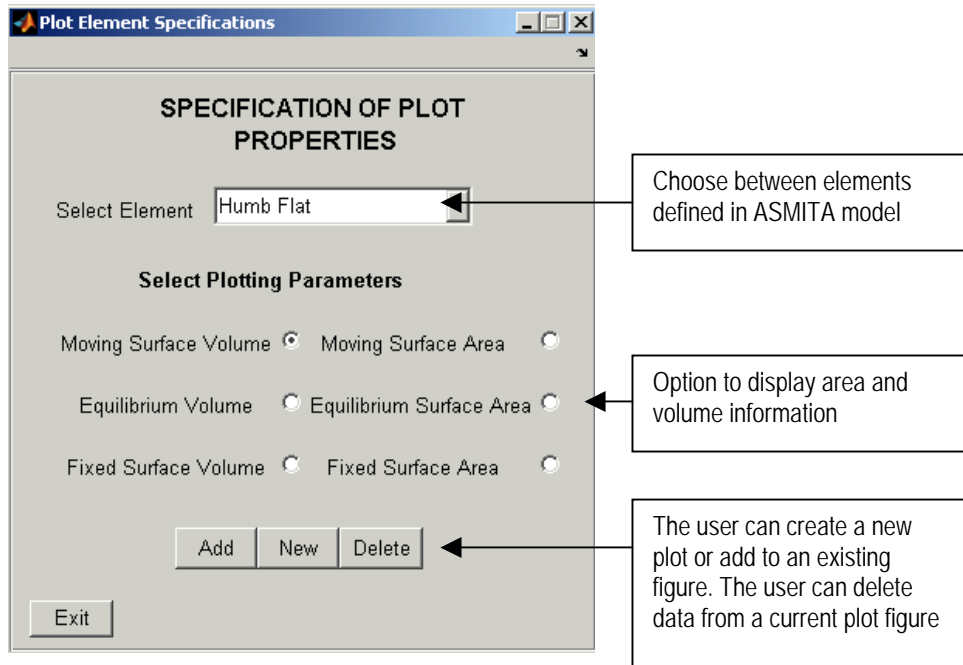
Figure 22. Plot specification window, allows the visualisation of ASMITA results

A typical example of the output from ASMITA is displayed in Figure 24. The user can access this data either through Matlab or from the stored data in the excel sheet

Results from selected element. Each result is represented by a different colour
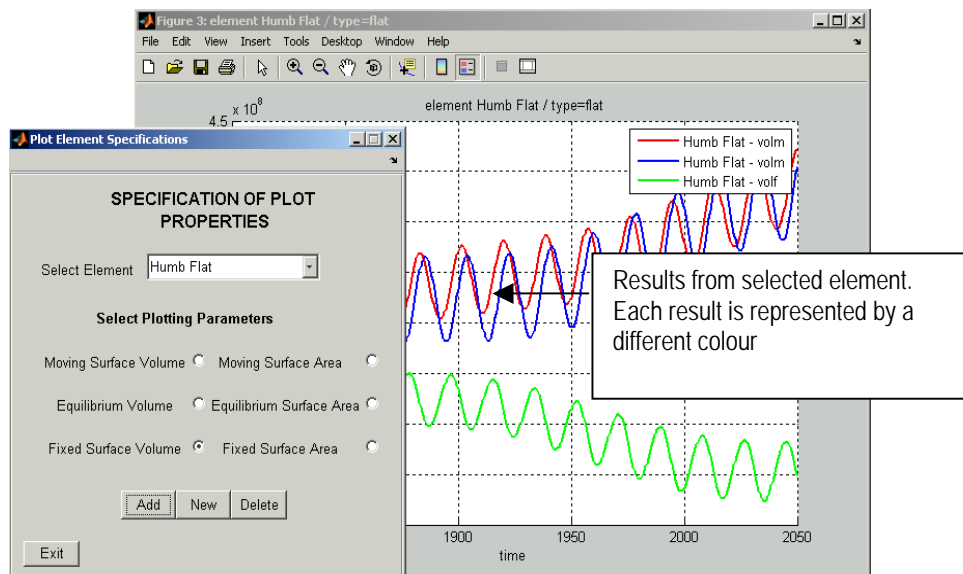
Figure 23. Example of results generated using ASMITA

10. Program Structure

vardef	definition of variables
defglob	definition of global variables
moddef_fig	model definition (figure)
cb_fil1	menu item 'file, open'
cb_fil2	menu item 'file, save'
cb_fil3	menu item 'file, save as'
cb_fil4	menu item 'map'
cb_elm1	menu item 'element, define'
cb_elm2	menu item 'element, specify'
elspec_fig	element specification form (figure)
cb_spec	button 'ready' in this form
cb_elm3	menu item 'element, delete'
cb_excq	discharge related exchange between elements
excq_fig	discharge specification form (figure)
q_ready	button 'ready' in this form
cb_excd	diffusion related exchange between elements
excd_fig	diffusion specification form (figure)
d_ready	button 'ready' in this form
cb_run1	menu item 'run, set time step'
timest_fig	time step specification form (figure)
cb_run1a	button 'ready' in this form
cb_run2	menu item 'run, check input'
cb_print	write input data to file
cb_run3	menu item 'run, run model'
asmita	model
eq_vol	model
cb_graph	graphs of computational results
exit_fig	menu item 'exit'
drawmap	draw worksheet and background map, if applicable
getelem	select element by clicking on it
getindex	find or set reference to information on exchange between elements

Appendix A

ASMITA Background Understanding

Area and volume changes in an estuary

Introduction

ASMITA (Aggregated Scale Morphological Interaction between a Tidal Basin and the Adjacent coast, Stive *et al*. 1998) is an approach developed for the study of estuary response to sea level rise. The model is based on an aggregated model that considers the estuary as a box (or set of boxes) that fills and empties under tidal action. The methodology was developed to look at the combined response of tidal delta, channel and tidal flats and has been applied to the impact of sea level rise on tidal inlets (van Goor *et al*. 2003) and the response of an estuary to the nodal tidal cycle (Jeuken *et al*. 2003).

This note summaries the basis of the ASMITA model, combining multiple elements with marine and fluvial sources. In addition the existing ASMITA, which only considers variations in estuary volume, is extended to represent time varying surface area. This allows perturbations to be introduced in the form of changes to the volumes and areas due to interventions such as dredging and reclamation, as well as sea level rise and cyclic variations in the tidal range. The basis of the empirical relationship used to define the equilibrium relationships is also given some further consideration.

Single element volume model

For a single element model, comprising just an estuary channel, the equilibrium state is derived from the equilibrium relationship assumed between the channel volume and the tidal prism:

$$V_{ce} = f(P) \tag{1}$$

Where V_{ce} is the equilibrium volume of the channel and P is the tidal prism (Eysink, 1990).

The other assumption made is that the ratio of the actual flow velocity to the equilibrium condition is proportional to the ratio of the equilibrium volume and actual volume. The local equilibrium concentration can therefore be written in terms of the actual volume, V, and the equilibrium volume, V_{ce}:

$$c_{ce} = c_E \left(\frac{V_{ce}}{V} \right)^n \tag{2}$$

Here c_{ce} is the local equilibrium concentration, n is the concentration transport exponent and c_E is the equilibrium concentration for the system as a whole (usually taken as the open boundary value). The difference between the actual concentration and the local equilibrium value induces morphological change governed by the exchange between the water column and the bed:

$$\frac{dV}{dt} = wS(c_{ce} - c) \tag{3}$$

Where w is the vertical exchange coefficient, S is the surface or plan area of the channel and c is the actual concentration. If we now consider a sediment balance where the (long-term residual) sediment transport between elements is assumed to be simply diffusive (in this case the channel and the external environment):

$$wS(c_{ce} - c) = \delta_{co}(c - c_E) \tag{4}$$

The parameter δ_{co} is the horizontal exchange coefficient and the r.h.s. of equation (4) represents the exchange with the external environment.

Combining equations (2)-(4) and adopting a simple linear relationship for equation (1) of the form $V_e = \alpha P$, where α is an empirical coefficient, the moving surface volume, V_m, is given by:

$$\frac{dV_m}{dt} = \frac{w\delta c_E S}{\delta + wS}\left[\left(\frac{V_e(t)}{V_m(t)}\right)^n - 1\right] + \frac{d\varsigma}{dt} \cdot S + i\omega\frac{\eta}{2}e^{i\omega t} \cdot S \qquad (5)$$

Which includes the variation in volume due to sea level rise and the some cyclic variation in tidal range, such as the nodal tidal cycle.

In this form, the equilibrium volume, V_e, and the fixed surface volume, V_f can be written:

$$V_e(t) = \alpha \cdot P = V_o + \alpha \cdot \Delta P$$
$$\Delta P = \eta \cdot e^{i\omega t} \cdot S \qquad (6)$$
$$V_f(t) = V_m(t) - \frac{d\varsigma}{dt} \cdot S \cdot t - \frac{\eta}{2}e^{i\omega t} \cdot S$$

Where the variables are defined as follows:

V_e – equilibrium volume
V_o – initial volume at time t=0
V_m – moving surface volume
V_f – fixed surface volume
P – tidal prism

ς - vertical sea level movement
ω - angular frequency of nodal tide
η - amplitude of nodal tide
S – surface area of basin
n – concentration transport exponent, usually taken as ~2.

w – vertical exchange rate
δ - horizontal exchange rate
c_E – global equilibrium concentration

In this simple model the plan area is treated as constant. On the basis that $V_e \sim fn(P)$ any hydraulic changes that result in a change in tidal prism are represented in V_e but those that change the volume of the system (such as sea level rise, dredging or reclamation) are represented in V_m.

The variation in equilibrium, moving and fixed estuary volumes, due to a linear rise in sea level and the nodal tidal cycle, are illustrated in Figure 1. The lag and damping of the response, relative to the equilibrium volume, is due to the dynamics associated with the morphological response time of the estuary (Jeuken *et al*. 2003).

The reduction of the volume relative to a fixed surface, shown in Figure 1, reflects the infilling of the basin that takes place, in order for the morphology to warp up vertically to keep pace with sea level rise (and the superimposed nodal cycle). Thus, vertical translation of the system is incorporated in this model. If, however, we wish to include the possibility of horizontal translation, as well as the vertical response, it is necessary to adjust the plan area rather than treat it as constant.

Multi-element volume model

The basic concept of the multi-element model is to subdivide the estuary into a number of elements and define the exchanges between elements and the equilibrium conditions for each element. As already noted, this has been presented in the literature as the ASMITA model (Aggregated Scale Morphological Interaction between a Tidal basin the Adjacent coast) and was introduced by Stive et al (1998; see also van Goor *et al*. 2003; and Kragtwijk *et al*. 2004). The system can be schematised into any number of discrete elements, which might be sections along the channel as used in ESTMORF (Wang *et al*. 1998), or geomorphological components, such as the channel and tidal flats as typically used in ASMITA. This is illustrated in Figure 2, which shows the linkage between tidal flat, channel and tidal delta through to the open sea, referred to as the outside world. For each

component the volume can be defined in terms of the sediment or water volume. In the derivation presented here, only water volumes are used and the equations presented reflect this (for the more general case see the papers noted above). So, for example, the scheme shown in Figure 2 would be represented by:

Tidal delta -   total water volume over delta (where the delta has a volume relative to undisturbed coastal bed)
Channel   -   total water volume below MLW
Tidal flats -   total water volume between MLW and MHW over flats (ie not including the prism over the channel)

The variation in these volumes depends on the transport of sediment in and out of the elements and any changes to the water volume itself. The latter may be due to sea level rise, subsidence of the bed, or any form of progressive change in the basin volume. Hence, over the long-term (time scales much longer than a tidal cycle) the rate of change of the element volume depends on the residual flux, the change in sea level as follows and any change in tidal range:

$$\frac{dV_i}{dt} = \sum_j J_{i,j} + S_i \frac{d\zeta}{dt} + S_i \frac{d(tr)}{dt} \qquad (7)$$

where J is the sediment volume flux between elements i and j, S is the plan area of the element and ζ is the elevation of mean sea level. The residual sediment flux between two elements is assumed to have advective and diffusive components:

$$J_{i,j} = q_{i,j} \cdot c_i - D_{i,j} A_{i,j} \frac{\partial c}{\partial x} \qquad (8)$$

where q (m^{3}s^{-1}) is the residual discharge rate, D (m^{2}s^{-1}) is the diffusion coefficient between the two elements and A (m^2) is the vertical area of the interface between the two elements. This can be written in a form similar to the r.h.s. of equation (4):

$$J_{i,j} = q_{i,j} \cdot c_i - \delta_{i,j} \cdot \left(c_i - c_j \right) \qquad (9)$$

In equations (8) and (9) the subscripts i and j refer to the elements the transport is from and to respectively. The advective component assumes that the concentration of the flux is that of the element supplying the sediment, which is important for external inputs such as from the rivers. In contrast the diffusive component simply uses the gradient between the elements.

The equations presented for the single element model can be conveniently written in matrix form to represent a multi-element model. Here we follow the naming convention of Kragtwijk et al (2004) for volumes and add some additional terms to make the model more general with respect to diffusion, advection and the sources of perturbations in the volume.

Vectors are defined for a number of the variables as follows:

$\underline{V}$	element volumes	$\underline{\gamma}$	volume ratios $(V_{ke}/V_k)^n$
$\underline{S}$	element surface areas	$\underline{\sigma}$	surface area ratios $(S_{ke}/S_k)^n$
$\underline{c_e}$	local equilibrium concentrations	$\underline{\delta_E}$	horizontal exchange coefficients with outside world
$\underline{c}$	element concentrations	$\underline{n}$	concentration transport exponent, taken as positive for wet volumes and negative for sediment volumes

In addition use is made of the following diagonal matrices:

W   vertical exchange coefficient w;
S   surface areas;
I   unit or identity matrix;
M   unit matrix with sign of n.
$\mathbf{C}_{ext}$ matrix of concentrations for fluxes into the system from the environment

The matrix **D** reflects the structure of the horizontal exchange between elements and with the outside world. For an n-element system with all elements linked and exchanging sediment with external environments, this would take the following form:

$$\mathbf{D} = \begin{pmatrix} \sum \delta_{1,n} + \delta_{1,E} & -\delta_{1,2} & \dots & -\delta_{1,n} \\ -\delta_{2,1} & \sum \delta_{2,n} + \delta_{2,E} & \dots & -\delta_{2,n} \\ \dots & \dots & \dots & \dots \\ -\delta_{n,1} & -\delta_{n,2} & \dots & \sum \delta_{n,n} + \delta_{n,E} \end{pmatrix} \tag{10}$$

For a 3-element system in which only element 3 is connected to the external environment (eg a tidal flat, channel and delta) this would take the form:

$$\mathbf{D} = \begin{pmatrix} \delta_{1,2} & -\delta_{1,2} & 0 \\ -\delta_{2,1} & \delta_{2,1} + \delta_{2,3} & -\delta_{2,3} \\ 0 & -\delta_{3,2} & \delta_{3,2} + \delta_{3,E} \end{pmatrix} \tag{11}$$

For diffusion the matrix is symmetric because $\delta_{i,j} = \delta_{j,i}$ and the direction of transport depends on the concentration gradient. However, for residual discharge the transport has a specified direction and as already noted we adopt the convention of transport from element i to element j. The matrix for an n-element system is then given by:

$$\mathbf{Q} = \begin{pmatrix} -\sum q_{1,n} + q_{1,E} & q_{1,2} & \dots & q_{1,n} \\ q_{2,1} & -\sum q_{2,n} + q_{2,E} & \dots & q_{2,n} \\ \dots & \dots & \dots & \dots \\ q_{n,1} & q_{n,2} & \dots & -\sum q_{n,n} + q_{n,E} \end{pmatrix} \tag{12}$$

Where $q_{n,E}$ all relate to fluxes out of the system. In order to ensure continuity of water mass we also require that, for each element, the sum of the discharges in and out of the element is zero, ie $\Sigma q_i = 0$.[1]

---

[1] However, provision is made for the sediment concentrations associated with these discharges to vary.

The basic equations can now be written:

$$\frac{dV}{dt} = \mathbf{MW}(\underline{c_e} - \underline{c})$$

$$(\mathbf{D} + \mathbf{Q}^T)\underline{c} = \mathbf{W}(\underline{c_e} - \underline{c}) + \mathbf{C}_{ext}\left(\underline{\delta_{ext}} - \underline{q_{ext}}\right)$$

$$\underline{c_e} = c_E \underline{\gamma}$$

(13)

From which the following expression can be derived for the rate of change of volume (note the vector $\underline{d}$ has the opposite sign to that given in equation (17) of the paper by Kragtwijk et al):

$$\frac{dV}{dt} = \mathbf{B}\underline{\gamma} - \underline{d} + \underline{S}\frac{d\zeta}{dt} + \underline{S}. \times \underline{trs}\frac{d(tr)}{dt} + \underline{\Delta V}$$

where

$$\mathbf{B} = c_E \mathbf{MWS}(\mathbf{I} - (\mathbf{D} + \mathbf{Q}^T + \mathbf{WS})^{-1}\mathbf{WS})$$

$$\underline{d} = \mathbf{C}_{ext}\mathbf{MWS}(\mathbf{D} + \mathbf{Q}^T + \mathbf{WS})^{-1}(\underline{\delta_{ext}} - \underline{q_{ext}})$$

(14)

Here *tr* refers to the tidal range and the expression $.\times \underline{trs}$ refers to element by element multiplication by ½ with a sign that reflects whether the element is influenced by high or low water. The term $\underline{\Delta V}$ refers to any other changes in volume, introduced at any given time, within each element, eg due to reclamation or dredging.

Equilibrium and fixed volumes then have a similar form to equation (6), where δtr is the variation in the tidal range from its mean value:

$$\underline{V_e} = \underline{\alpha} \cdot P = \underline{V_o} + \underline{\alpha} \cdot \Delta P$$

$$\underline{V_f} = \underline{V_m} - \underline{S} \cdot \zeta - \underline{S}. \times \underline{trs} \cdot \delta tr$$

(15)

The relationship between estuary volume and surface area

As estuaries vary throughout their length the change in depth is generally much smaller than the change in width. This obvious statement induces a corollary that the tidal, wave, fluvial and geotechnical effects that define an estuary geometry induce much larger changes to the width of a cross-section than to the depth. In addition, the downstream part of an estuary, which is predominantly tidal, tends to experience less proportional change in depth with distance than the head of an estuary which is much more affected by fluvial discharges. Since the volume and surface area within an estuary tends to be dominated by the much wider and deeper downstream reaches, where cross-section area correlates highly with width, estuary volume also tends to correlate highly with surface area.

Figure 3 indicates how in simple terms a change (δA) in channel cross-section area can be related to the change in width (δW), ie. δA= h.δW. Because of the above arguments it is possible to extend this idea to surface area (S) and volume (V), i.e. δV= h.δS . Moreover the same argument can be made for change in tidal flat volume and surface area.

Variable Surface Area

For a single element (such as a channel) linked to the open sea, the above arguments lead to the following expression (in the absence of sea level rise and tidal node variation:

$$\frac{dS}{dt} = \frac{1}{h} \cdot \frac{dV_m}{dt} = \frac{1}{h} \cdot \frac{w\delta c_E S}{\delta + wS}\left[\left(\frac{V_e(t)}{V_m(t)}\right)^n - 1\right] = \frac{\hat{w}\hat{\delta} c_E S}{\hat{\delta} + \hat{w}S}\left[\left(\frac{S_e(t)}{S_m(t)}\right)^n - 1\right] \qquad (16)$$

where:

$$\hat{q} = \frac{q}{h}; \quad \hat{\delta} = \frac{\delta}{h} \quad \text{and} \quad \hat{w} = \frac{w}{h} \qquad (17)$$

The effect of sea level rise and nodal tide variation is included by considering the change in surface area that arises from a change in mean sea level and a change in tidal range. The non-linear equations (for the single element model) describing the variation in surface area are therefore similar to equation (5) and (6) for volumes, viz:

$$S_e(t) = \hat{\alpha} \cdot \frac{P}{tr} = S_o + \hat{\alpha} \cdot \frac{\Delta P}{tr}$$

$$\frac{dS_m}{dt} = \frac{\hat{w}\hat{\delta} c_E S}{\hat{\delta} + \hat{w}S}\left[\left(\frac{S_e(t)}{S_m(t)}\right)^2 - 1\right] + \frac{d\varsigma}{dt} \cdot R + i\omega\frac{\eta}{2}e^{i\omega t} \cdot R \qquad (18)$$

$$S_f(t) = S_m(t) - \frac{d\varsigma}{dt} \cdot R \cdot t - \frac{\eta}{2}e^{i\omega t} \cdot R$$

The parameter R represents the change in area for a unit change in water level and is given by $R = n \cdot L m_b$, where n is the number of bed surfaces in the element (generally 2 for the two sides of the estuary), m_b is the transverse bed slope (ie, slope is given by the ratio $1:m_b$) and L is the length of the element.

As for volumes, this can be applied to multiple elements and the rate of change for the surface area **S**, is thus given by:

$$\frac{d\underline{S}}{dt} = \mathbf{F}\underline{\sigma} - \underline{e} + \underline{R}\frac{d\zeta}{dt} + \underline{R}.\times\underline{trs}\frac{d(tr)}{dt} + \underline{\Delta S}$$

where

$$\mathbf{F} = c_E \mathbf{M}\hat{\mathbf{W}}\mathbf{S}(\mathbf{I} - (\hat{\mathbf{D}} + \hat{\mathbf{Q}}^T + \hat{\mathbf{W}}\mathbf{S})^{-1}\hat{\mathbf{W}}\mathbf{S}) \qquad (19)$$

$$\underline{e} = \mathbf{C}_{ext}\mathbf{M}\hat{\mathbf{W}}\mathbf{S}(\hat{\mathbf{D}} + \hat{\mathbf{Q}}^T + \hat{\mathbf{W}}\mathbf{S})^{-1} \cdot (\hat{\underline{\delta}}_{ext} - \hat{\underline{q}}_{ext})$$

where $\hat{\mathbf{D}}$, $\hat{\underline{\delta}}_E$ and $\hat{\mathbf{W}}$ are derived from **D**, $\underline{\delta}_E$ and **W** in accordance with equation (17). The equilibrium and fixed surface areas then have a similar form to equation (15):

$$\underline{S}_e = \hat{\underline{\alpha}} \cdot P = \underline{S}_o + \hat{\underline{\alpha}} \cdot \Delta P$$

$$\underline{S}_f = \underline{S}_m - \underline{R} \cdot \zeta - \underline{R}.\times\underline{trs} \cdot \delta tr \qquad (20)$$

Tidal Prism Relationships

Equation 1 postulates a link between the equilibrium volume and the tidal prism. Similarly the value of the equilibrium surface area S_e in Equation 17 is likely to vary with the the tidal prism. Townend (2005) showed that the tidal prism exhibits a strong correlation with the plan area and volume of the estuary. From a sample of 66 UK estuaries he found that the total surface area, total volume and tidal prism exhibited the following relationships:

$$S_{mtl} = 0.42P^{0.96}, \qquad R^2 = 0.92;$$
$$V_{mtl} = 0.073P^{1.13}, \qquad R^2 = 0.92. \qquad\qquad (21)$$

The elements used within ASMITA comprise deltas, channels and tidal flats. The UK estuaries database suggests that the channels and flats (when represented as a single element) can be adequately represented by simple linear relationships with exponents of one:

Table 1 Linear form-prism ratios

	Volume	R^2	Surface Area	R^2
Channel	$V_c = 0.418P$	0.93	$S_c = 0.076P$	0.95
Tidal flats	$V_f = 0.163P$	0.74	$S_f = 0.077P$	0.47
			$S_f = 28.9P^{0.75}$	0.82

However, individual estuaries can have values significantly different from the above. The following are the average values for the Humber over the period 1851-2000 (based on 22 bathymetric data sets) with standard deviations indicating a small variation over time:

Table 2 – Linear form-prism ratios for the Humber Estuary

	Volume	St.dev	Surface Area	St.dev
Channel	$V_c = 0.738P$	0.021	$S_c = 0.130P$	0.003
Tidal flats	$V_f = 0.249P$	0.011	$S_f = 0.067P$	0.003

One of the strongest correlations revealed by the UK data is between the surface area at mean tide level, prism and tidal range, which takes the form:

$$S_{mtl} = 1.07\frac{P}{tr} \quad (R^2 = 0.996) \qquad\qquad (22)$$

This implies that the tidal prism is essentially the plan area at mean tide multiplied by the tidal range (particularly for the larger estuaries with $P > 10^7 m^3$) and that the cross-shore shape of the intertidal is of secondary importance. Although the regression is strong it must be recognised that the error in predicting the plan area of an individual estuary can still be ±30% based on this data set, with most of the data points falling below this forced regression line (ie exponent=1 and intercept=0). The surface area at low water is also reasonably well represented by this form of relationship but the area of the tidal flats shows a poorer correlation, Table 3. The coefficients specific to the Humber are similar for the mean tide and channel (mlw) relationships but differs significantly for the tidal flats.

Table 3 Linear form-prism ratios for surface area as a function of P/tr

	UK estuaries	R^2	Humber estuary
Channel	$S_c = 0.775P/tr$	0.97	$S_c = 0.733P/tr$
Tidal flats	$S_f = 0.945P/tr$	0.77	$S_f = 0.386P/tr$
Mean tide level	$S_{mtl} = 1.07P/tr$	0.99	$S_f = 0.96P/tr$

Hypsometry Relationships

An alternative approach is to use the equilibrium hypsometry relationships for channels {Cao, 1997 236 /id} and tidal flats {Friedrichs, 1996 359 /id} to define the form of the estuary cross-section. The resulting relationships can then be written in terms of volume and surface area by considering a representative length. Some trials for a number of estuaries suggest that this form description provides a reasonable characterisation of estuary cross-sections.

Friedrichs and Aubrey (1996) show that the lower intertidal width is a function of the equilibrium flow velocity and the angular tidal frequency: *(w_o-w_{lw}) = u_{eq}/ω.* This implies that the lower intertidal width reflects flow field, the erodibility of the sediments and the rate of convergence of the estuary. We therefore assume that the slope of the lower intertidal is a characteristic value, which in terms of surface area is given by:

$$m_{eq} = \frac{S_o - S_{lw}}{2\eta} \tag{23}$$

Using the initial estuary surface areas at mtl and lw, with the tidal range, the equilibrium slope can be calculated using equation (23). Similarly, the d-n ratio = *(d-η)/(n+1)* can be obtained as the ratio of the volume and area at low water. If the slope and d-n ratio are treated as being constant, equations (24) can be used to derive updated equilibrium volumes and areas for the channels and flats given a new value of the tidal prism.

$$S_{lw} = \frac{1}{2\eta}\left(V_p - (1+\pi)\cdot\eta^2 m_{eq}\right) \qquad V_{lw} = \frac{d-\eta}{n+1}\cdot S_{lw}$$

$$S_{fl} = \left(1 + \frac{\pi}{2}\right)\cdot 2\eta\cdot m_{eq} \qquad V_{fl} = (1+\pi)\eta^2 m_{eq} \tag{24}$$

Fitting (24) to the UK estuaries database produces reasonable fits to the data. In particular, V_{lw} is well represented and the fit lines go through the centre of the data, whereas for V_{fl}, and S_{fl} the fitted lines represent lower bounds.

Basis of equilibrium volumes and areas in ASMITA

The ASMITA model, as presented, makes use of the general nature of the results described above and provides the user a number of options.

(i)   Equilibrium based on linear tidal prism functions

This method assumes that the governing equation for equilibrium volume is a linear function of the tidal prism and for surface area is a linear function of prism/tidal range. The user is able to set the values of the proportionality coefficients separately for each element. In the code the default values for these coefficients are 1 and if these are used, the routine calculates the coefficients from the initial values of volume, surface area, tidal prism and tidal range for each element. Thereafter, the coefficients are treated as constants. (NB: This uses the individual element volume or surface area as a proportion of the total tidal prism. This implicitly scales the coefficient to the size of the element relative to the total tidal prism. When considering internal perturbations to the system (eg due to reclamation or dredging) it may be more appropriate to use the prism from each element to the tidal limit. This option has not however been implemented.)

(ii)   Equilibrium based on hypsometry functions
 This method uses the initial values of volume, surface area, tidal prism and tidal range to calculate the equilibrium slope and the d-n ratio, as defined above. These values are then treated as constants and the equations for the equilibrium hypsometry (24) are used to compute volumes and areas for the channels and flats, taking account of change in tidal range and tidal prism. The method is activated by setting the volume equilibrium coefficients for every element to zero.

References

Cao S, Knight D, 1997, Entropy-based design approach of threshold alluvial channels, Journal of Hydraulic Research, 35(4), 505-524.

Eysink WD, 1990, Morphological response of tidal basins to change, In: 22nd International Conference on Coastal Engineering, ASCE, Washington, 2, pp. 1948-1961.

Friedrichs CT, Aubrey DG, 1996, Uniform bottom shear stress and equilibrium hypsometry of intertidal flats, In: Pattiaratchi C (Ed.), Mixing in estuaries and coastal seas, American Geophysical Union, Washington, pp. 405-429.

Hughes SA, 2002, Equilibrium cross sectional area at tidal inlets, Journal of Coastal Research, 18(1), 160-174.

Jeuken MCJL, Wang ZB, Keiller D, Townend IH, Liek GA, 2003, Morphological response of estuaries to nodal tide variation, In: International Conference on Estuaries & Coasts (ICEC-2003), pp. 167-173.

Kragtwijk NG, Stive MJF, Wang ZB, Zitman TJ, 2004, Morphological response of tidal basins to human interventions, Coastal Engineering, 51, 207-221.

Kraus NC, 1998, Inlet cross-sectional area calculated by process-based model, In: 26th International Coastal Engineering Conference, American Society of Civil Engineers, 3, pp. 3265-3278.

O'Brien MP, 1931, Estuary tidal prism related to entrance areas, Civil Engineering, 1(8), 738-739.

Prandle D, 2006, Dynamic controls on estuarine bathymetry: assessment against UK database, Estuarine Coastal and Shelf Science, 1-7.

Spearman JR, 1995, Estuary Regimes and Methods For Prediction of Long-Term Changes, Oxford Brooks University, PhD Thesis.

Stive MJF, Capobianco M, Wang ZB, Ruol P, Buijsman MC, 1998, Morphodynamics of a tidal lagoon and adjacent coast, In: Physics of Estuaries and Coastal Seas: 8th International Biennial Conference on Physics of Estuaries and Coastal Seas, 1996, A A Balkema, Rotterdam, pp. 397-407.

Townend IH, 2005, An examination of empirical stability relationships for UK estuaries, Journal of Coastal Research, 21(5), 1042-1053.

van Goor MA, Zitman TJ, Wang ZB, Stive MJF, 2003, Impact of sea-level rise on the morphological equilibrium state of tidal inlets, Marine Geology, 202, 211-227.

Wang ZB, Jeuken MCJL, Gerritsen H, de Vriend HJ, Kornman BA, 2002, Morphology and asymmetry of the vertical tide in the Westerschelde estuary, Continental Shelf Research, 22, 2599-2609.

Wang ZB, Karssen B, Fokkink RJ, Langerak A, 1998, A dynamic-empirical model for estuarine morphology, In: Dronkers J, Scheffers MBAM (Eds.), Physics of Estuaries and Coastal Seas, Balkema, Rotterdam, pp. 279-286.

Figure 1 – Variation in estuary volume for rising sea level and nodal tidal cycle as predicted by the single element model
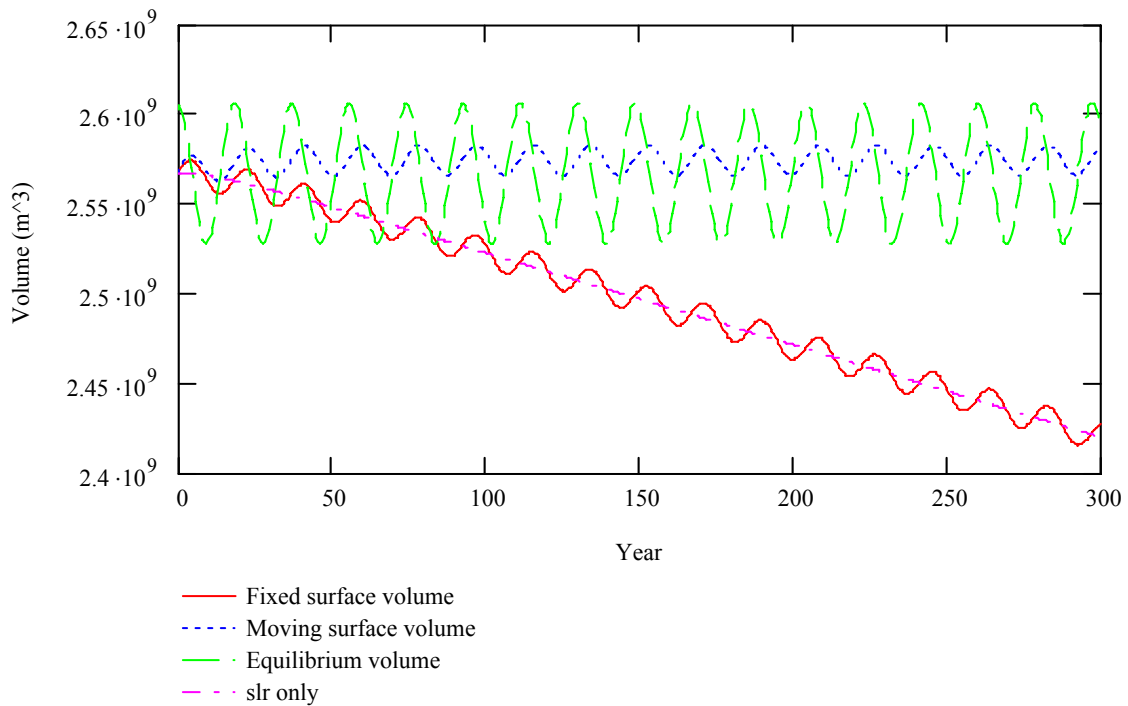

Figure 2 – Schematic of elements for a tidal inlet as used in the ASMITA model
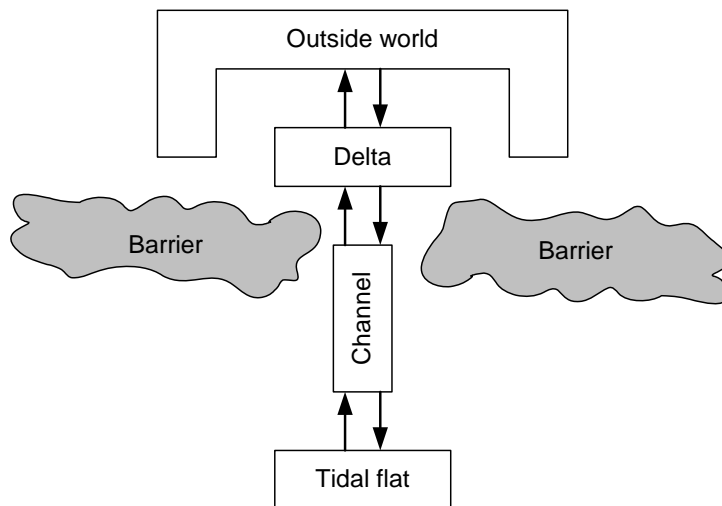
Figure 3 – Sketch to illustrate relationship between changes in width and cross-sectional area
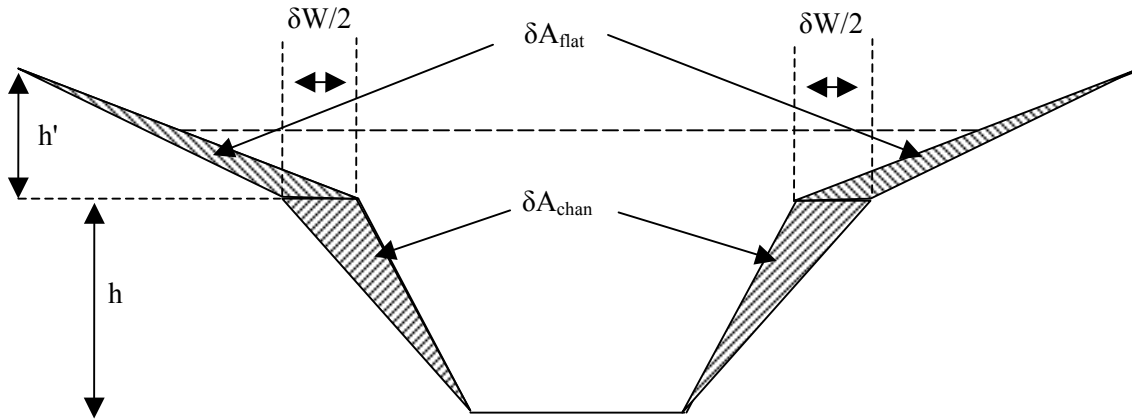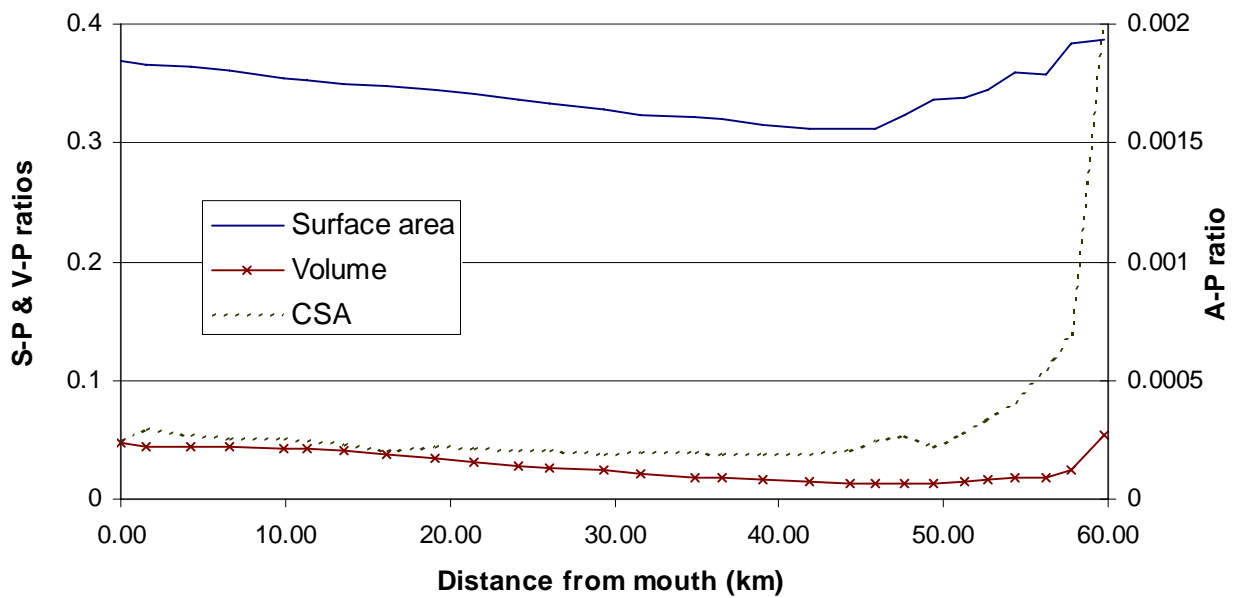

Figure 4 – Tidal prism ratios for Cross-section area, Surface area and volume along the Humber estuary

SGS   UKAS   INVESTOR IN PEOPLE

Ergon House
Horseferry Road
London SW1P 2AL

www.defra.gov.uk