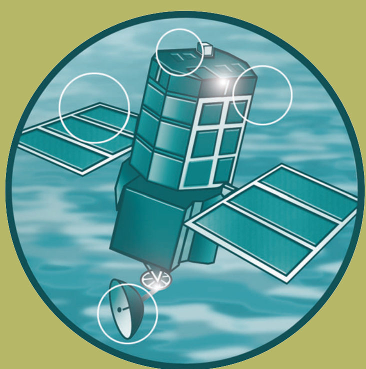
Joint Defra/EA Flood and Coastal Erosion
Risk Management R&D Programme

Development and Demonstration of Systems BasedEstuary Simulators (EstSim)

EstSim Mathematical Formalisation

R&D Project Record FD2117/PR3



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Development and Demonstration of Systems Based Estuary Simulators (EstSim)

Conceptualisation Report

Prepared by University of Plymouth for the Estuaries Research Programme (ERP Phase 2) within the Defra and Environment Agency Joint Broad Scale Modelling Theme

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Statement of use

This document provides information for the Department for Environment Food and Rural Affairs (Defra) and Environment Agency Staff (EA), researchers and consultants about the research project Development and Demonstration of Systems Based Estuary Simulators (EstSim) and constitutes an R&D output from the Joint Defra/Environment Agency Flood and Coastal Defence R&D Programme (Estuaries Research Programme Phase 2).

Contract Statement

This report describes work commissioned by Defra under Project FD2117 Development and Demonstration of Systems Based Estuary Simulators (EstSim). The Funders Nominated Project Officer was Kate Scott (Environment Agency: kate.scott@environment-agency.gov.uk). The ABPmer Project Number was R/3434 and the Project Manager at ABPmer was Alun Williams (Email: awilliams@abpmer.co.uk).

Collaboration Statement

This report was prepared by the EstSim Consortium comprising: ABP Marine Environmental Research Ltd (lead), University of Plymouth (School of Engineering), University College London (Coastal & Estuarine Research Unit), HR Wallingford, WL | Delft Hydraulics and Discovery Software.

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Keywords

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EXECUTIVE OVERVIEW OF FD2117: DEVELOPMENT AND DEMONSTRATION OF SYSTEMS BASED ESTUARY SIMULATORS (EstSim)

Project Inception Report, July 2004

Purpose

The Broad Scale Modelling Theme of the Defra/EA Joint Thematic R&D Programme for Flood & Coastal Defence has funded three contracts under the Estuaries Research Programme, Phase 2 (FD2107, FD2116 and FD2117). FD2117 (EstSim) started in April 2004 and has the following headline aims:

- To extend the ability to simulate estuary response to change.
- Facilitate knowledge exchange through accessibility of simulation results.

The Project

ABPmer, University College London, University of Plymouth, WL│Delft Hydraulics and Discovery Software are undertaking the project. The project was originally of 3 years duration (April 2004 – April 2007), but had an extension for completion in June 2007 and has nine Scientific Objectives as follows:

1. System Conceptualisation: Boundary setting and focusing of research effort.
2. Development of Management Questions: Rationalisation of management questions that can be informed through application of systems approach.
3. Development of Behavioural Statements: Formal definition of estuarine system in terms of systems approach and behavioural statements.
4. Mathematical Formalisation: Development of behavioural statements into a logically consistent mathematical framework.
5. Development of System Simulation: Development of architecture for estuary simulation based on the mathematical formulation of the system definition.
6. Manager System Interface: Explore the use of decision support systems and visualisation techniques for proof of concept testing.
7. Pilot Testing: Performance evaluation of estuary simulator.
8. Dissemination: Increase awareness of function and utility of research.
9. Peer Review: Ensure research lines deliver against Scientific Objectives.

This report follows on from the Development of Behavioural Statement (Objective 3) by taking the statements and them into a mathematical framework - the Mathematical Formalisation (Objective 4).

In developing mathematical formulation a number of stages have been undertaken, including:

- Review of approaches already in existence.
- Development of chosen formalisation - the Boolean Approach.
- Analysis of the system to describe generic estuary systems.
- Description of Boolean variables and states.
- Testing and validation of statements against a case study.

This report will be used to guide the development of the System Simulation using the statements defined in the Boolean approach which will be tested and validated in the next stage in the EstSim project (Scientific Objective 5).

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1. INTRODUCTION

1.1 Background

On 1st April 2004 ABP Marine Environmental Research Ltd (ABPmer) and its Project Partners were awarded research contract FD2117 (CSA 6064) within the Broad Scale Modelling Theme of the Defra/EA Joint Thematic R&D Programme for Flood & Coastal Defence.

The contract for FD2117 was awarded on the basis of a 'contract won in competition' after submission of a CSG7 (revised CSG7 submitted on 8th March 2004).

Entitled 'Development and Demonstration of Systems-Based Estuary Simulators' (hereafter EstSim), this research contract forms one of three contracts awarded under Phase 2 of the Estuary Research Programme (ERP). The two other contracts under the umbrella of ERP Phase 2 are (i) FD2107: Development of Estuary Morphological Models, and (ii) FD2116: Review and Formalisation of Geomorphological Concepts and Approaches.

The three phases of the Estuaries Research Programme seek to improve our understanding and prediction of estuarine morphological change over the medium to long-term, thereby facilitating strategic and sustainable decisions regarding flood and coastal defence.

The EMPHASYS Consortium undertook Phase 1 of this programme by evaluating existing morphological modelling approaches with the most promising of these approaches being developed within ERP Phase 2. It is anticipated that Phase 3 will seek to incorporate prior ERP research into an 'Integrated Estuary Management System'.

1.2 Project Aims

The overall aim of EstSim is to extend the ability to simulate estuarine response to change. This will be achieved through the delivery of research into the systems-based approach as an alternative yet complementary methodology to those research lines being undertaken within the other ERP Phase 2 projects (morphological concepts, bottom-up, top-down and hybrid methods). EstSim will also explore the simulation process in order to facilitate knowledge exchange between the systems-based tools and estuary managers. Integration of the systems based approach and existing methods is shown conceptually within Figure 1.

1.3 Project Structure

The project has been structured in to nine Scientific Objectives, covering the required lines of research and dissemination:

1. System Conceptualisation: Boundary setting and focusing of research effort.

2. Development of Management Questions: Rationalisation of management questions that can be informed through application of systems approach.

3. Development of Behavioural Statements: Formal definition of estuarine system in terms of systems approach and behavioural statements.

4. Mathematical Formalisation: Development of behavioural statements into a logically consistent mathematical framework.

5. Development of System Simulation: Development of architecture for estuary simulation based on the mathematical formulation of the system definition.

6. Manager System Interface: Explore the use of decision support systems and visualisation techniques for proof of concept testing.

7. Pilot Testing: Performance evaluation of estuary simulator.

8. Dissemination: Increase awareness of function and utility of research.

9. Peer Review: Ensure research lines deliver against Scientific Objectives.

1.4 Project Progress

Scientific Objective 2 was delivered through production of the EstSim Behavioural Statements Report (PR2). The Translation Workshop held on 14th December 2004 between EstSim Project Partners confirmed the content and way forward for Objective 4. PR2 presented the findings from that Workshop together with the research element of the Work package.

Following the completion of PR2, work was commenced on Objective 4.

1.5 Mathematical Formalisation (Objective 4)

The objective of this research element was to develop the behavioural statements (Objective 3) into a logically consistent mathematical framework that preserves the geomorphological characteristics.

Formalisation of behavioural statements required good knowledge of tools and techniques available for representing estuary processes and a review of what may need to be developed to provide simplified versions of existing numerical and expert modelling tools.

After delivery of the report on behavioural statements from Objective 3 the Mathematical Formalisation Objective began with an initial view presented on translation of the Objective 3 outputs.

In order to deliver this research a number of sub-tasks were defined, these are given in Table 1.

Delivery of this final Technical Report effectively completes all tasks required under Objective 4, as listed in Table 1. This document constitutes Project Record PR3 and as such is submitted in as Primary Milestone 04/02.

Table 1. Objective 4 tasks

Task	Description
4.1	Prepare the mathematical framework for describing the connectivity and flows of the system. This will include an investigation of different approaches such as linked differential equations, linked Boolean logic systems and statistical correlation models.
4.2	Review the proposed arrangement of objects and linkages within the system in relation to accepted means of describing the processes.
4.3	Parameterise these processes into a form that is amenable to the mathematical framework and which also preserves the required geomorphological characteristics.
4.4	Examine linear and non-linear stability issues and characteristics of the 'system' dynamics.
4.5	Simplification/parameterisation of existing geomorphological concepts and approaches (particularly those considered under FD2116).
4.6	Design, code and test the mathematical algorithms.
4.7	Hold team workshop to disseminate findings.
4.8	Produce Technical Report on findings.

1.6 Follow-on Research

The production of Boolean variables and states within the Mathematical Formalisation workpackage for estuarine geomorphological elements (Objective 3) provides the basis on which the System Simulation and Validation tasks could proceed, Objective 4.

1.7 Report Structure

The report has been structured to capture the main tasks but reordered for ease of presentation and review, as follows:

- Section 2: Estuary Description
- Section 3: System Based Approach
- Section 4: The Boolean Approach
- Section 5: Analysis of Generic Estuary Systems
- Section 6: Testing and Validation
- Section 7: Conclusions and Recommendations
- Section 8: The Way Forward (Future Objectives).
- Appendix A – Review of Approaches to Mathematical Formalization
- Appendix B – Boolean analysis of generic glacial valley estuary
- Appendix C – Boolean analysis of generic drowned river valley estuary.

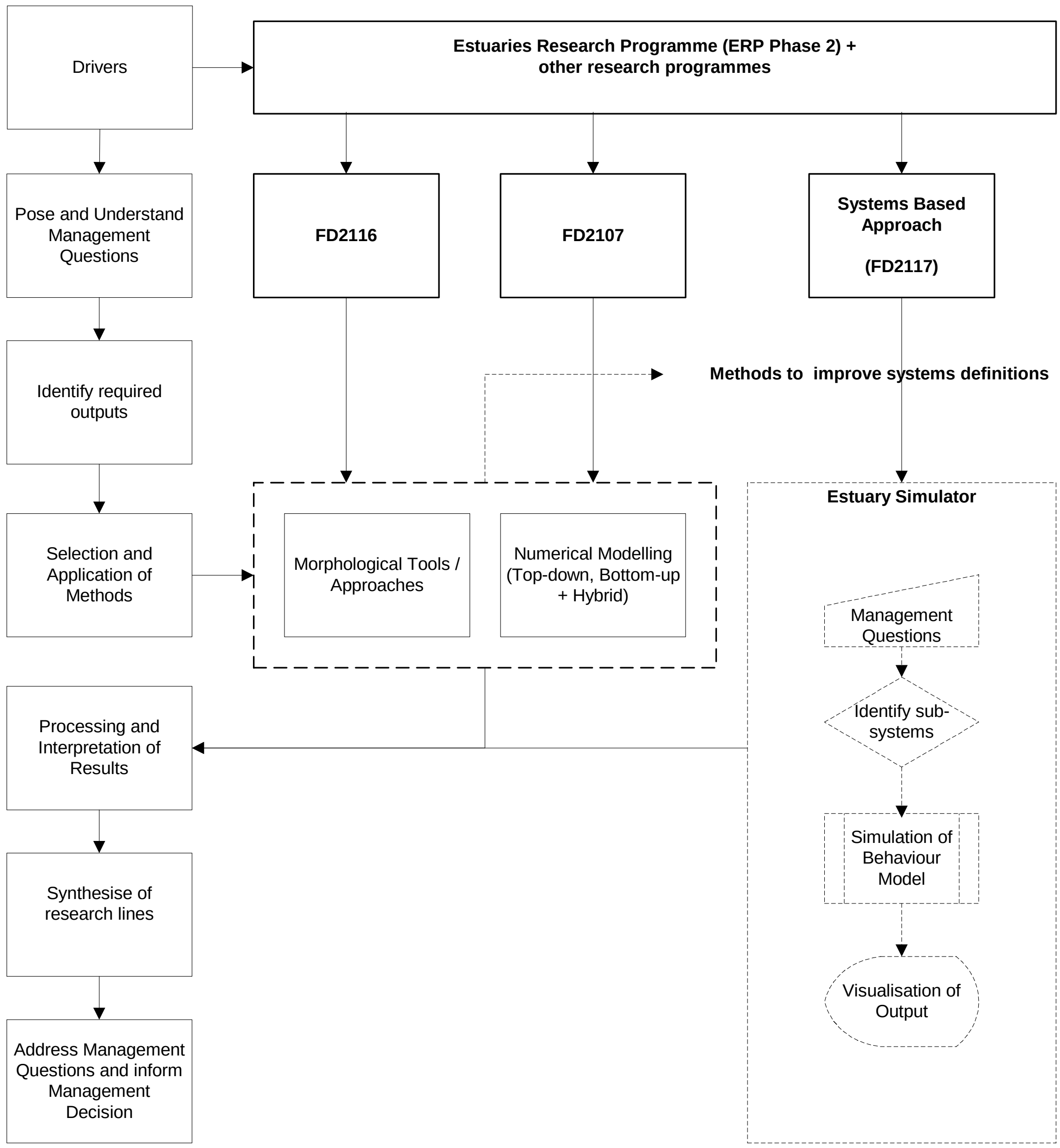

Figure 1. Integration of systems based approach

2. ESTUARY DESCRIPTION

2.1 Background

Estuaries are formed at the mouth of rivers, in the narrow boundary zone between the sea and the land. They are partially enclosed water bodies which are either periodically or permanently open to the sea within which water from the sea mixes with fresh water from rivers and streams. Estuaries are tidally driven, but partially sheltered from the full force of ocean waves.

Estuarine systems are highly complex morphological features. They are extremely dynamic land forms and continuously react to changes in marine and fluvial environmental forcing, such as rise in sea level, increase in storminess, etc. which may cause drastic changes in sediment and morphodynamics of estuaries. In addition, many estuaries are subject to morphological changes following human interference.

Understanding and predicting the morphodynamic behaviour of estuaries is still limited because of its complexity encompassing a large range of time and space scales. A central task faced by coastal and estuarine managers is to predict and manage the constantly moving coastal and estuarine environment in the medium to long term in the scale of 20-100 years.

Global climate change could significantly affect the morphological evolution of estuaries. In addition, a wide range of human activities such as port development, water extraction and discharge of waste and reclamation for agriculture, industry or places for dwelling already directly or indirectly influence the morphological behaviour of many estuaries in the UK. A basic requisite for making positive management decisions within the estuary environment in the future would be to have a sound understanding of estuary behaviour over both short (1-10 yrs) and long time scales and local and regional space scales.

A principal task of estuary managers, planners and regulators is to be able to predict, with reasonable confidence, the impacts on estuary attributes, the functioning of the system and on user interests of possible changes in either natural forcing factors or human activities. These changes in controlling factors may be extrinsic to the system (e.g. global sea level rise) or intrinsic (e.g. channel migration or reclamation for port development).

In the project ‘Review and formalisation of geomorphological concepts and approaches for estuaries (FD2116)’, a framework expert geomorphological assessment of estuaries is developed. Estuaries are classified according to the typography/geomorphology and various time scales and the underlying processes taking place within each time scale are discussed. In EstSim (FD2117), the classification given in FD2116 is amended and simplified to provide a working typology with which to progress the study for UK estuaries. The estuaries are classified to identify the range of geomorphological elements present in each type of estuary. The classification gives seven different types of estuaries (EstSim Behavioural Statement Report, 2005). Each generic estuary type incorporates a cluster of geomorphological elements. The understanding of the estuary behaviour combines those geomorphological elements, forcing parameters such as tidal and wave energy and the dynamics of the interactions between them. The time scales identified in FD 2116 are taken on board to develop system diagrams and behavioural statements for generic estuaries and estuarine geomorphological elements in the short/medium term and long term time scales.

2.2 Links to EstSim Behavioural Statement Report

The EstSim Behavioural Statement Report reviews the systems approach to provide background understanding for developing system diagrams for estuaries. Systems methods are often applied to provide insight into an understanding of the functioning of a complex system. The approach combines the physical elements of the system and the interactions between them and provides a model of how the different elements that make the system interact. The complexity of the system, the physical processes present and the spatial and temporal time scales involved determine how the system is represented. The changes in form and structure of the system through time and the flows governing the system are central to the understanding of the system behaviour. System diagrams provide a means of capturing the key attributes of a system by identifying the system elements and their interactions. A system diagram is a flow chart representation and its ability to capture the system behaviour depends on the fundamental knowledge of physical processes and the way in which these interact with morphology.

When applying this approach to natural systems such as estuaries and coasts it is necessary to consider a number of scales of behavioural sub-systems. For the purpose of understanding the behaviour of estuary systems, the time scales of hours to decades (meso-scale) is usually considered, but the system has to be set in the context in the time scale of decades to centuries (macro-scale) to identify the effects of regional and global changes. A starting point for a behavioural system of an estuary is the identification of the major energy and sediment pathways. The variations in forcing parameters and sediment supply should also be taken into account.

The EstSim Behavioural Statement Report provides a behavioural description for each generic estuary element. The report also gives a system definition and a system diagram indicating the interactions between geomorphological elements and external forcing for all generic estuaries found in the estuary classification. Figure 2 shows the system diagram for a tidal inlet.

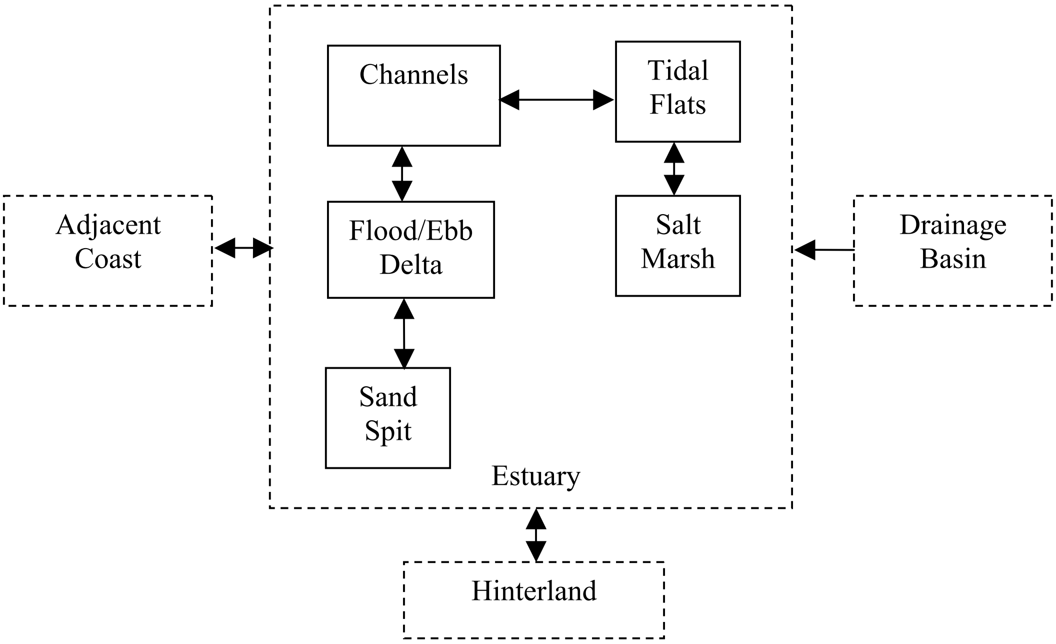

Figure 2. Systems diagram for generic tidal inlet (EstSim behavioural statement Report FD2117-PR2)

3. SYSTEM BASED APPROACH

3.1 Review of Approaches

Objective 4 of FD2117 is to develop a Mathematical Formalization to describe the connectivity and flow of complex estuarine systems using a systems based approach. Wang (2005, Appendix A) reviewed and compared three possible approaches namely:

- The Boolean Approach, developed by University of Plymouth (Harshinie Karunarathana & Dominic Reeve, 2005);
- The Network Dynamics Approach of UCL (Rob Seymour, 2005);
- The ASMITA Approach, developed by WL | Delft Hydraulics and Delft University of Technology (Stive *et al*, 1998, Stive and Wang, 2003, Kragtwijk *et al*, 2004, Van Goor *et al*, 2003).

The three approaches are almost totally different from each other. The only thing is common is the schematisation of an estuary into a “small” number of “large” elements. Nevertheless an attempt has been made to compare the three approaches in Table 2 by looking at the INPUT/OUTPUT of the approaches, by considering their behaviours, and by evaluating the known applications in the estuarine modelling so far.

As indicated in Table 2, the input and output of the Boolean approach are qualitative, the time step in the development is also qualitative. Therefore the interpretation of the output of the model is important in this approach. A whole story needs to be told on the basis of a series of Boolean vectors. The other two approaches use quantitative inputs and outputs, however the difference between the two methods is found in the physical meanings of the state variables. The state variables in the ASMITA model are in principle measurable physical quantities. The (uniform) way of treating all the node variables in the network dynamics implies that all these variables must have the same dimension. In practise this means that they should be dimensionless. Furthermore it is noted that the absolute values of the node variables are not important because of the (so called) scaling behaviour of the approach. By scaling behaviour it is meant that an equilibrium state (represented by a vector of non-negative real numbers) multiplied by any constant positive number remains an equilibrium state. It seems therefore that the physical meanings of the node variables and the way in which they should be made dimensionless are not clear. Another interpretation of this observation is that although the input and output of the network dynamics approach are quantitative in the mathematical sense, they are qualitative in the physical sense.

The qualitative character of an approach also makes it more flexible in applications. It is e.g. much easier to include an element or a process (as an example to extend an existing morphological model by including ecological influences) of different category in a Boolean approach than in ASMITA.

Table 2. Comparison between the three approaches

	Boolean Approach		Network Dynamics Approach		ASMITA Approach	
Input	Initial state, a vector of Boolean variables	Qualitative	▪ Initial state, a vector of non-negative real variables ▪ Parameters in response functions	Quantitative	▪ Initial state, a vector of non-negative real variables ▪ Parameters in the empirical relations ▪ (sediment transport) Parameters influencing time scales	Quantitative
Output	(a series of) vector (s) of Boolean variables	Qualitative	Evolution in time of the state vector of non-negative real variables	Quantitative	▪ Evolution in time of the volumes of the elements ▪ Sediment exchanges as function of time	Quantitative
Behaviour	▪ determined mainly by the Boolean functions ▪ discrete development (with qualitative time step) ▪ end state: fixed equilibrium or cyclic, never unstable due to fixed number of possible state ▪ complexity increases exponentially with number of elements		▪ determined mainly by the structure of the network ▪ discrete or continue development in time ▪ end state: stable equilibrium or cyclic, can be unstable ▪ possible equilibriums and their stability ▪ scaling ▪ complexity depending on the structure of the network		▪ The empirical relations determine the equilibrium states and the sediment transport parameters determine the evolution to the equilibrium state ▪ continue development in time ▪ end state: usually stable equilibrium, but can simulate unstable behaviour ▪ complexity depending on the structure of the network	
Application	▪ Example concerning estuarine morphology given in the documents ▪ The possible end states helpful for 'calibration'		Example concerning estuarine morphology not given in the document		▪ response to sea level rise, nodal tide, land subsidence ▪ response to engineering works, dredging / dumping prediction of dredging requirement ▪ need long-term historical data for calibration	

4. THE BOOLEAN APPROACH

4.1 Background

The Boolean network approach was described by Nicolis (1982) in a pioneering application of the technique to climate dynamics. The technique has since been developed with applications to different fields including seismology, climatology and meteorology (e.g. Ghil *et al* 1987, Wohlleben & Weaver 1995; Saunders & Ghil 2001; Zaliapin *et al* 2003). It is a modelling framework that is particularly suited to the mathematical formulation of conceptual models of systems that exhibit threshold behaviour, feedbacks and time delays. The approach is perhaps best considered to be an heuristic first step towards understanding problems currently too complex to model using systems of partial differential equations. Eventually it may well be possible to define and solve the exact equations that govern estuary morphology. However, in coastal morphodynamics and elsewhere in the natural sciences, much of the preliminary discourse is often conceptual. At the very least, Boolean expressions offer a formal mathematical language that may allow qualitative and quantitative approaches to be reconciled.

In this project, a Boolean approach has been used to develop mathematical formalisation of long-term morphodynamic evolution of complex estuary systems. This has involved development of Boolean networks combining geomorphological elements within the estuary system and external forcing which drive morphological evolution of it, and derivation of Boolean expressions that define the interactions between the network elements. The method provides a formal mathematical language that allows qualitative geomorphological ‘rules’ to be encapsulated and manipulated in a rigorous manner. The level of sophistication provided by the current formalism is relatively simplistic and survives on the basis of some generalising assumptions. The success of the modelling approach largely depends on the correct linkages between system elements. It should also be noted that this approach treats the system elements as homogeneous sedimentary deposits and assumes that all transfer of sediment contributes to any of the geomorphological elements. The approach does not currently take characteristic sedimentology of geomorphological elements in to account.

Nevertheless, this approach should be seen as an initial step in the development of more sophisticated Boolean networks that can address some of the acknowledged limitations. It should also be noted that in an hierarchical modelling framework, simple conceptual models are often employed to present hypotheses and identify mechanisms that are described in a qualitative or heuristic manner. In contrast, sophisticated process models are used to simulate the phenomena in more detail in order to compare results against observations in a quantitative manner. The Boolean approach provides the means to describe the dynamics of a complex system with a simple representation of the relevant geomorphological concepts.

The EstSim Behavioural Statement Report classifies estuaries into three categories (Glacial valley, Drowned river valley and Drowned coastal plain) depending on their origin and subdivided into seven types (Fjord, Fjard, Ria, spit enclosed drowned river valley, funnel shaped drowned river valley, embayment and tidal inlet) depending on their behaviour. The EstSim report gives a system diagram for each generic behavioural type, which incorporates system elements and their interactions. In addition, behavioural statements have been developed for each generic estuary at the whole estuary scale and for each individual geomorphological element at a generic level.

The mathematical formalisation of the estuary system behaviour has been developed using a Boolean approach by combining the estuary system diagrams and medium to long term behavioural response of geomorphological elements given in the EstSim Behavioural Statement Report.

4.2 Formulation of Problem

The formulation of the problem is described as follows:

Let x be a state variable. If the rate of change of x is considered a continuous process, then the rate equation could be written as

$$\frac{dx}{dt} = f(x,a) \qquad (1)$$

where f is a nonlinear function of x and a set of external parameters a. In most cases, f can be split into two parts: a highly non-linear term $X(x,a)$ describing the specific feedbacks inherent in the dynamics of the system and a quasi-linear decay term kx. k is an appropriate decay coefficient. Then, the equation (1) could be written as

$$\frac{dx}{dt} = X(x,a) - kx \qquad (2)$$

In the present context of estuary morphological evolution, the variable x represents the volume of a given geomorphological element within the estuary. The term a represents the set of external parameters which affect the rate of change of element volume includes waves, tides, sediment exchange rate between elements, and so on.

Now assume that X is quite small for $0 < x < x_0$, but becomes quite appreciable for $x > x_0$ and saturates to a threshold value of X_{max} shortly afterwards. If we idealise this situation by considering that both x and X are zero when $0 < x < x_0$ and x and X equal to one when $x > x_0$ then, x and X can be considered as a discontinuous Boolean variable and a discontinuous Boolean function respectively. We can then say that x is 'low' when $x < x_0$ and 'high' when $x > x_0$ respectively. Similar considerations apply for X. The term kx in Equation (2) characterises a delay in temporal evolution of the Boolean function with respect to the Boolean variable.

5. ANALYSIS OF GENERIC ESTUARY SYSTEMS

5.1 Methodology

Boolean networks are constructed for each type of generic estuary. Each element in a Boolean network has two states, ‘high’ or ‘low’ (also called ‘on’ or ‘off’, ‘true’ or ‘false’). To indicate its state, each element has an associated value 1 for ‘high’ and 0 for ‘low’. The future state of one element in the network depends on the states of the other elements in the network which are designated as that element’s inputs. The element may feedback its own state as a self-input. The state of an element in a Boolean network at a future time is governed by a logical rule or Boolean function, which operates on the element’s inputs. Each geomorphological element and the external forcing parameters which drive morphological changes in the estuary are represented by an individual element in the network. The network is formed by combining the estuary system diagram with medium to long term behavioural response of geomorphological elements given in the EstSim Behavioural Statement Report. Once the nodal points of the Boolean network are finalised based on the estuary system diagram, all possible feedback between geomorphological elements and external forcing which drives the morphological evolution of the estuary in the medium to long term are derived from the behavioural description and the system diagram of the geomorphological elements given in the report. The effects of change in environmental forcing parameters on the morphological evolution of the estuary are incorporated through waves and tides. Human interference is modelled through feedback from control structures (e.g. training wall, jetties) and dredging. The feedback from the sub systems are represented by the sediment flow.

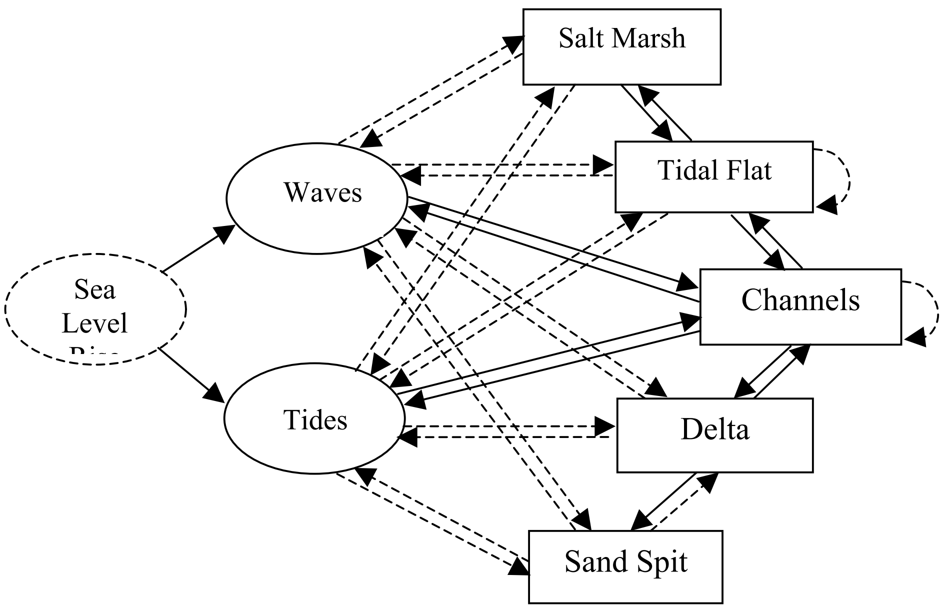

Figure 3. Boolean network for a generic tidal inlet (little or no sediment flow from outside). Dark arrows and broken arrows in the network represent positive and negative feedback respectively

The Boolean network for a generic Drowned coastal plain estuary (tidal inlet) tidal inlet experiencing sea level rise with constrained (little or no) sediment flow is shown in Figure 3. The system diagram for a generic tidal inlet (Figure 2) together with the behavioural descriptions and medium to long term system diagrams for geomorphological element present in a tidal inlet (EstSim Behavioural Statement Report FD2117-PR2) are used to form the Boolean network.

5.2 Boolean Variables

Once the network is completed, a Boolean variable is assigned to each element in the network. Then, a Boolean function for each variable is derived by combining Boolean variables within a logical framework. The logical framework operates on the feedback from designated 'input' elements in the network. The truth table is then developed by solving the logical expressions for Boolean functions. The truth table gives Boolean states corresponding to various combinations of Boolean variables and resulting Boolean functions. The logical framework for the estuary system shown in Figure 2 is shown below.

$$\left.\begin{array}{ll}
W = sm' \vee tf' \vee cc \vee dd' \vee ss' & \text{(a)} \\
T = sm' \vee tf' \vee cc \vee dd' \vee ss' & \text{(b)} \\
SM = (w' \wedge t \wedge tf) & \text{(c)} \\
TF = ((sm \vee cc) \wedge t) \vee (tf' \wedge w') & \text{(d)} \\
CC = (w \vee t) \wedge (tf \vee dd) \vee cc' & \text{(e)} \\
DD = (w' \vee t') \wedge ss' \vee (t \wedge cc) & \text{(f)} \\
SS = (w' \vee t') \wedge dd & \text{(g)}
\end{array}\right\} \quad (3)$$

Following notations stand for the variables used in Equation 3.

Network element	Boolean variable	Boolean function
waves	*w*	*W*
tides	*t*	*T*
salt marsh	*sm*	*SM*
tidal flats	*tf*	*TF*
channels	*cc*	*CC*
delta	*dd*	*DD*
sand spit	*ss*	*SS*

Following convention is used to form the logical expressions.

Convention	Description
a'	not *a*
a v *b*	*a* or *b*
a ^ *b*	*a* and *b*

The mathematical framework is a set of linear equations which can be easily coded using any programming language (MATLAB was used for the examples presented here). Validity of the mathematical framework can be verified by the outcome of the truth table where an inconsistent logical framework leads to unrealistic Boolean output states.

Equations 3(a) and 3(b), express that little or no existence of salt marsh, tidal flat, delta and sand spit individually enforce negative feedback on wave and tidal forcing respectively as the presence of each of these geomorphological elements causes dissipation of wave and tidal energy. According to Equation 3(c), tidal forcing enforces a positive feedback on the salt marsh by supplying sediment from tidal flats but wave forcing should be low at the same time to reduce marsh erosion.

For each variable, the corresponding Boolean function is deduced from the logical expressions given in Equation (3) above, considering all possible combinations of Boolean variables then forms the Boolean matrix. The system has $2^7 = 128$ states. Table 3 shows some selected states from the Boolean matrix. These states represent initial states of the estuary which can realistically exist in nature.

Table 3. Some selected states from the Boolean matrix for tidal inlet with constrained sediment inflow

	w	t	sm	tf	cc	dd	ss	W	T	SM	TF	CC	DD	SS
1	1	1	0	1	1	1	0	1	1	0	1	1	1	0
2	1	0	1	1	1	1	1	1	1	0	0	1	0	1
3	1	0	0	1	1	1	1	1	1	0	0	1	0	1
4	1	1	0	0	1	0	1	1	1	0	1	0	1	0
5	1	1	0	1	0	1	0	1	1	0	0	1	0	0
6	1	1	0	0	1	0	0	1	1	0	1	0	1	0
7	0	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	0	1	1	1	0
9	0	1	0	1	1	1	1	1	1	1	1	1	1	1

5.3 Boolean States

A Boolean state in which all the Boolean variables and corresponding functions take the same value indicates a stable state where the system is bound to no further changes (Nicolis, 1987). When a stable state is reached, neither the Boolean variables nor the Boolean functions are bound to change further. In certain situations, the truth table indicates oscillatory system behaviour where the system evolves between two or more states and completes a cyclic evolutionary path. If the evolution of the system initiates from a state other than the stable state or the system does not follow an oscillatory evolutionary pattern, then, for certain initial states, the system follows one or more evolutionary stages to reach the stable state.

In the present case, the truth table corresponding to the logical framework given in equation (3) indicates one stable state, which is shown by the encircled row in Table 3 (1101110). This state corresponds to a generic tidal flat with high wave and tidal forcing. At this state, the estuary possesses little or no salt marshes and no sand spit due to lack of sediment flow from outside to maintain these geomorphological elements against sea level rise.

Now, consider the initial state where the generic tidal inlet is wave dominated and contains salt marshes, tidal flats, a delta and a sand spit (row 2 of Table 3). Sea level rise increases wave and tidal forcing within the estuary system. According to the logical matrix, salt marshes, tidal flats and the delta structure begin to recede as the initial response to sea level rise. But, the estuary evolves further by exchanging sediment between estuary elements. Tidal flats accrete but channels become shallower. The delta is bound to change again and the sand spit recedes. The logical matrix shows that the estuary is bound to change once again to a state with deep channels, little or no tidal flats. Once the estuary reaches this state, the logical matrix indicates a reversal of its state in to the previous state thereby following a cyclic evolutionary pattern as shown below.

$$1011111 \rightarrow 1100101 \rightarrow 1101010 \leftrightarrow 1100100$$

Under constrained sediment supply, tidal flats in general recede before that of salt marshes in the event of sea level rise. In this situation, the logical matrix shows that the estuary reaches a state where tidal flats are replenished by the sediment from receding salt marshes before it reaches the cyclic evolutionary pattern as shown below.

$$1011111 \rightarrow 1110101 \rightarrow 1101010 \leftrightarrow 1100100$$

Let us next consider a tide dominated generic tidal inlet which contains salt marshes, tidal flats, a sand spit and a delta (row 7 of Table 3). Both tidal and wave forcing in the estuary increase as a result of sea level rise. For this initial state, the logical matrix indicates the following morphological evolutionary path against sea level rise:

$$0111111 \rightarrow 1111111 \rightarrow 1101110$$

The salt marshes and the sand spit recede as the estuary evolves due to lack of sediment inflow to maintain them against sea level rise. The estuary reaches a stable state with little or no salt marshes and a sand spit. However, the decay rates of salt marshes and the sand spit are likely to be different, and therefore the estuary will have the following alternative evolutionary paths:

$$0111111 \rightarrow 1111111 \begin{cases} \rightarrow 1101111 \rightarrow 1101110 & \text{(Path 1)} \\ \rightarrow 1111110 \rightarrow 1101110 & \text{(Path 2)} \end{cases}$$

Irrespective of the difference in the decay rates of salt marshes and the sand spit, the estuary finally reaches the same stable state.

The selection among different evolutionary pathways depends on various factors. For example, geometry of the estuary, channel geometry, extent of salt marshes and tidal flats, sediment composition and concentrations, flood and ebb tidal current intensity and patterns, biological aspects related to sediment flocculation and deposition are some of the factors that affect the receding or accretion rate of geomorphological elements of estuaries.

In the event of anthropogenic influences such as dredging, the system is forced to a situation similar to a sudden removal of sediment. For example, if dredging of tidal flats is started at a state where tidal flats hold a state ‘high’ (associates value ‘1’), then before evolving to the next state, this value is artificially changed to ‘0’ (‘low’).

5.4 Illustration of the Methodology

Let us consider a wave dominated tidal inlet with unconstrained sediment flow and have well developed tidal flats and shallow channels (initial state before dredging is 1011011). If dredging is done to remove sediment from tidal flats and channels, the system is forced to the state 1010111 (low tidal flats and deep channels). After this stage, the system evolves according to the logical matrix and the full evolutionary path before reaching a stable state is shown below:

1011011 → 1010111 → 1101111 → 1111111

After dredging is carried out to remove material from channels and tidal flats, sediment exchange takes place between tidal flats and salt marshes resulting tidal flat accretion and salt marsh recession. With further increase in sea level rise and unconstrained sediment inflow, the estuary reaches a stable state with fully developed tidal flats and salt marshes.

Similar analyses were done for generic glacial valley estuary and drowned river valley estuary. The Boolean networks and selected cases from the Boolean matrices for generic glacial valley and drowned river valley estuaries are given in Appendix B and C respectively.

It should be noted that considering fluvial discharge and its change associated with global climate change is significantly less than the tidal flows in most of the UK estuaries, river flow is not taken as an environmental forcing in the formation of Boolean networks at the initial stage. But, at a later stage, the generic estuaries were re-analysed taking fluvial discharge as an additional environmental forcing in Boolean networks. It was found that it did not affect the stable state or other evolutionary states of the estuary leading to the stable state or the cyclic evolutionary nature.

6. TESTING AND VALIDATION

6.1 Background to Test Case

Historic data from the Ribble estuary, UK, was used for testing and validation of the approach. The estuary is located in the north-west of England between Southport and Litham. An extensive study of morphological changes in the Ribble estuary was reported by Van der Wal, *et al*. (2002), and this work forms the basis of data used for this case study. The estuary is a funnel shaped drowned river valley. The mouth of the estuary is approximately 15 km wide and the main channel is about 28 km long. The estuary occupies 12 000 ha of inter-tidal area. It is macrotidal and partially mixed. The mean spring and mean neap tidal ranges are 8.0 m and 4.4 m respectively. The tides are semi-diurnal. Mean freshwater flow (44 m^3/s) into the estuary is very small compared to the average tidal inflow (12 000 m^3/s). The middle and outer estuary is subjected to moderate wave energy.

The long term morphological evolution of the Ribble estuary is analysed in detail by Van der Wal *et al*. (2002) using Admiralty charts and charts published by the Port of Preston Authority since 1847. In addition, spot depth and height measurements carried out during a survey on sand banks (1850), height information from a salt marsh fringing from the southern shore of the estuary (1824), spot survey data on sand banks (1997) and LIDAR survey data (1999) have been used to supplement the information from the charts. Further information has been collected by topographic maps published by the Ordnance survey, plans published by the Port of Preston Authority, cross shore profiles based on annual ground surveys and aerial photographs.

In 1847, the main channel of the Ribble estuary was trained between Preston and The Naze. The channel was separated from the southern shoreline by inter-tidal sand flats. In the outer estuary, the channel split into two: the northern channel which was -13 m Ordnance Datum Newlyn (ODN) and ran northwards close to Lytham and the south channel, which was approximately 14 m deep and ran in the south westerly direction. The outer estuary is dominated by inter-tidal sand banks. The navigation channel had been extended through the sand banks in the outer estuary in 1904 and training walls were constructed along almost 20 km length of it. The trained channel had a depth of up to -7 m ODN. The channel was further deepened in 1951 to a depth of -10 m ODN at its deepest point.

Following the construction of the training wall along the main navigation channel, the northern channel in-filled. The south channel also was partially in-filled. The infilling of the main channels was attributed to the erosion of inter-tidal sand banks. There were both net erosion zones and accretion zones in the outer estuary between 1847 and 1904. The centre of the outer estuary was eroded while higher inter-tidal areas accreted during this period.

Following the deepening of the navigation channel in 1951, accretion was dominant in the main inter-tidal sand banks and salt marshes and also in the outer estuary to the west. But, seaward margins of the inter-tidal sand banks were subjected to net erosion. In 1994, the navigation channel dominated, but had experienced considerable infilling. It had been partially blocked by a growth of sand banks towards the south. Accretion

had taken place both in inter-tidal and sub-tidal areas. Recent LIDAR surveys, ground surveys and aerial photographs show accretion and progradation of salt marshes in the recent decades with the highest sedimentation in the upper vegetated salt marsh areas.
Sediment Budget of Ribble Estuary

The sediment budget of the Ribble estuary is found to be positive for the last 150 years and a net accretional trend is observed. The sediment influx into the estuary is found to be mainly from the Irish sea where ebb tidal currents bring sandy sediment from the sea floor. Littoral drift, although to a lesser extent, is also found to be contributing to the sedimentation of the estuary. It has also been found that the relative sea level in the area of the Ribble estuary is rising at a rate of 1 to 2 mm/yr since the late nineteenth century (Woodworth *et al*., 1999).

A logical analysis is performed by using the Boolean approach to predict morphological evolution of the Ribble estuary against rising sea level and the construction of a training wall along the main channel. The sediment flux into the estuary is considered as continuous throughout. Figure 4 shows the Boolean network for the Ribble estuary.

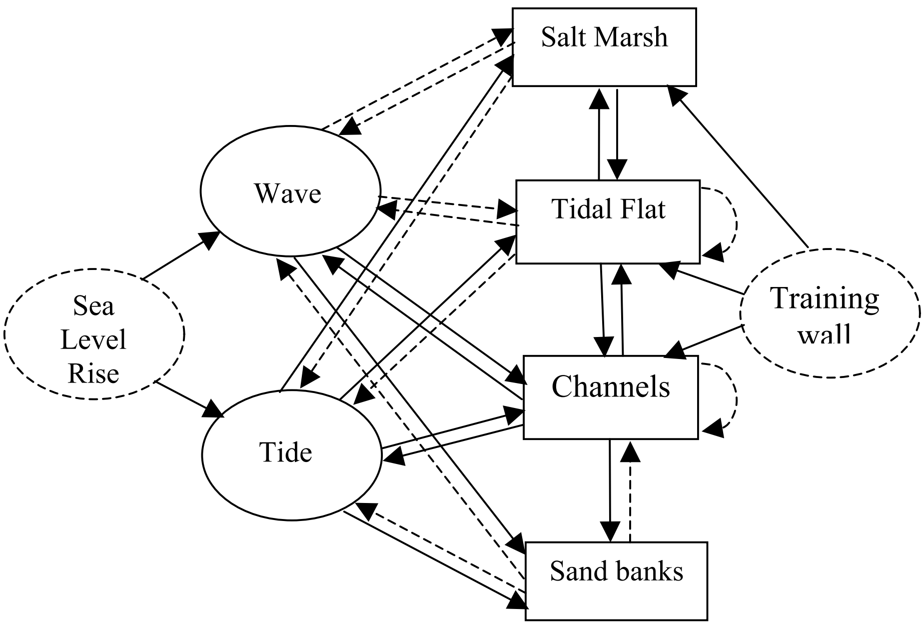

Figure 4. Boolean network for the Ribble Estuary system

Increase in wave energy in the estuary due to sea level rise will impose a negative feedback on salt marshes and tidal flats by eroding them. But, tidal currents tend to supply sediment into the system from outside when there are sediment available. Salt marshes and tidal flats force tidal currents to slow down thereby allowing sediment to settle down. Simultaneously, tidal flats and salt marshes impose negative feedback on tidal and wave energy by forcing them to dissipate. Channel deepening and widening take place due to the passage of high tidal and wave energy. On the other hand, deep and wide channels allows tidal and wave energy to be carried into the inner estuary. Therefore, they exert positive feedback on each other.

Sand banks are fed by waves and currents by bringing sediment into them from the sea. But sand banks exert negative feedback on both wave and tidal energy through dissipation. Salt marshes and tidal flats exert positive feedback on each other by acting as sediment sources. Tidal flats and channels also exert positive feedback on each other as accretion of tidal flats is associated with deep channels. Sand banks supply sediment into channels which will result channel infilling. But, channels bring sediment into sand banks and therefore exert positive feedback on them.

Tidal flats and channels exert negative feedback on themselves. Shallow flats are more prone to erosion by waves and currents. Deep channels attract more sediment from outside and tend to infill. Construction of training walls in the estuary to maintain the navigation channel is expected to enforce a positive feedback on the system, any evolutionary process of the channel is controlled by the training walls. Also, the training wall controls the sediment outflow from the marshes and the tidal flats.
Application of Boolean Approach

The Boolean expressions corresponding to the Boolean network shown in Figure 4 are given below.

$$\left.\begin{array}{ll} W = sm' \vee tf' \vee cc \vee sb' & \text{(a)} \\ T = sm' \vee \mathrm{tf}' \vee cc \vee sb' & \text{(b)} \\ SM = (w' \wedge t \wedge tf) \vee (t \wedge st) & \text{(c)} \\ TF = (sm \vee cc) \wedge t \vee (tf' \wedge w') \vee (t \wedge st) & \text{(d)} \\ CC = (w \vee t) \wedge (tf \vee sb') \vee cc' \vee st & \text{(e)} \\ SB = w \vee (t \wedge tf) & \text{(f)} \\ ST = st & \text{(g)} \end{array}\right\} \quad (4)$$

sb and SB refer to Boolean variable and Boolean function for sand banks respectively.

The Boolean matrix resulting from the above logical framework defines the evolutionary processes shown in Table 4.

Table 4. Some selected options from the Boolean matrix corresponding to the logical framework for Ribble estuary against sea level rise and training wall construction given by Equation 4

	w	t	sm	tf	cc	sb	st	W	T	SM	TF	CC	SB	ST
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	1	1	1	1	1	1	1	1
3	1	1	0	0	1	1	1	1	1	1	1	1	1	1
4	1	1	0	0	1	0	1	1	1	1	1	1	1	1
5	1	1	0	1	1	0	1	1	1	1	1	1	1	1
6	1	0	1	1	1	1	1	1	1	0	0	1	1	1
7	1	0	0	1	1	1	1	1	1	0	0	1	1	1
8	1	0	0	0	1	1	1	1	1	0	0	1	1	1
9	1	0	0	0	1	0	1	1	1	0	0	1	1	1
10	1	0	1	1	1	0	1	1	1	0	0	1	1	1
11	0	1	1	1	1	1	1	1	1	1	1	1	1	1
12	0	1	0	1	1	1	1	1	1	1	1	1	1	1
13	0	1	0	0	1	1	1	1	1	1	1	1	1	1
14	0	1	0	0	1	0	1	1	1	1	1	1	1	1

One stable state was found which corresponds to the estuary with well developed salt marshes, tidal flats and sand banks (row 1, Table 4). With continuous influx of sediment, the estuary is stable with all geomorphological elements remaining stable against sea level rise. This is very much in line with the findings of O'Connor (1987) who states that the estuary at present is in a dynamic equilibrium state.

The Ribble estuary is found to be macro-tidal which experiences moderate wave energy. Prior to the construction of the training wall, the estuary had very little salt marsh and a few sand banks at the outer estuary (Van de Wal, 2002). If we consider this situation as the initial state of the estuary (1101101), the logical matrix shows that the estuary will reach the stable state by developing salt marshes and accreting sand banks when the sediment supply continues (row 5, Table 4). Depending on the relative accretion rates of salt marshes and sand banks, the estuary evolves along the following pathways according to the logical matrix:

1101101 → 1101111 → 1111111 (Path 1)
1101101 → 1111101 → 1111111 (Path 2)

If the accretion rate of sand bank is higher than that of salt marshes, the estuary will evolve along path (1) to reach the stable state. If salt marshes accrete faster than sand banks, path (2) would be followed.

The quantitative analysis of the evolution of the Ribble estuary by Van de Wal (2002) has shown that the estuary had extensive inter-tidal flats at the southern side of the estuary, prior to the construction of the training wall. The two main channels of the estuary were deep and extensive. Following the construction of the training wall along the main navigation channel in 1904, inter-tidal flat and sand banks have shown accretion. Following further deepening of the navigation channel in 1951, existing sand banks have further accreted, new sand banks have appeared and salt marshes were developed. Channel infilling had also taken place during this period but, channel depths had been maintained through frequent dredging operations. During the period after 1994, further accretion of sand banks and tidal flats has taken place together with the development of salt marshes. However, there were areas of net erosion and accretion in the outer estuary. The recent LIDAR surveys, ground surveys and aerial photographs have shown that highest sedimentation occurred in the upper salt marshes in the recent years.

The outcome of the logical analysis on the morphological evolution of the Ribble estuary is very much in qualitative agreement with the observations of Van der Wal (2002). Accumulation of salt marshes and sand banks has taken place simultaneously prior to 1994 and therefore, it is not possible to distinguish the exact evolutionary pathway followed by the estuary. However, after 1994, salt marshes appeared to be accumulating faster and therefore, the estuary is expected to follow the Path 2 above.

7. CONCLUSIONS AND RECOMMENDATIONS

A Boolean approach has been used for the mathematical formulation of the estuary simulator. Analysis of Boolean networks for various types of estuaries has shown that this is a feasible approach to model of complex systems. But, it should be noted that it is a complementary approach rather than a substitute for other form of quantitative estuary morphology modelling.

Boolean analysis was performed to investigate the response of different types of generic estuaries to sea level rise and construction of control structures, which are the most prominent causes of long-term morphological changes in estuaries. First, a Boolean network is formed to interpret the complex feedbacks involved between various elements in the estuary system. Then, a logical framework is developed by considering the types of feedback between the physical elements of the estuary and external forcing. The logical framework is then transformed into a Boolean matrix and different potential behavioural pathways of estuary morphology are identified and discussed.

According to the above analysis, some generic estuaries indicate a stable state against changes in external forcing due to sea level rise or human intervention. If such a situation is reached, no further morphological changes take place. Under most circumstances, the estuary evolved through several states before it reaches a stable state. In some situations, the estuary continues to evolve in a cyclic pattern where the state of the estuary changes between two or more different states.

The mathematical framework developed using the Boolean approach is a set of simple logical expressions which could be solved using any programming language. MatLab programming and numerical analysis software has been used to obtain the truth table from the logical expressions in the present study. Once the truth table is derived, the output Boolean states could be picked up for any given input Boolean state.

The simulator should include Boolean networks for different types of estuaries under various sediment budget and human intervention scenarios. Then, the logical framework for each combination of conditions should be incorporated. Once the type of the estuary is identified and the sediment budget is known, a system could be developed to select the appropriate Boolean network and the corresponding logical framework when the external interference to the estuary (dredging, construction, etc.) apart from natural physical forcing is known.

8. THE WAY FORWARD (FUTURE OBJECTIVES)

Objective 4, Mathematical Formalisation, has provided the system approach to be used in the System Simulation based on the outcome of the Behavioural Statements Report.

In order to facilitate this a workshop on EstSim was held at university of Plymouth in December 2005. The objective of the workshop was to present the development of the mathematical formalisation of the estuary simulator to the project partners and open it for comments and discussion and to discuss the way forward to system simulation using the mathematical formalisation. A draft report on the Mathematical Formalisation was distributed among the project partners prior to the meeting. The workshop was attended by all partners of the EstSim project. Wang (TU Delft) presented a review and comparison of the three approaches Boolean approach, network analysis and ASMITA model) put forward for the development of mathematical formalization of the estuary simulator. UoP presented development and application of Boolean approach for the mathematical formalization for a generic tidal inlet, the results were discussed and the implementation of the Boolean approach agreed.

8.1 Objective 5: System Simulation and Validation

The objective of this research element is to set-up the architecture and methodology for estuary simulation based on the system definition comprising the behavioural statements its mathematical formulation. The simulation phase thereby provides the system approach output to inform answers to the specific management questions. This work is being based on the outcomes for Objective 4 namely the use of the Boolean approach to describe the estuary types.

9. REFERENCES

ABP Marine Environmental Research Limited, UK, 2005, EstSim Behavioural Statements Report FD2117, Project Record PR2.

Carter, R.W.G., 1988, Coastal Environments, Academic Press, London.

Dyer, K., 1997, Estuaries A Physical Introduction, John Wiley and Sons.

Ghil, M., Mullhaupt, A. M. and Pestiaux, P., 1987. Deep water formation and Quaternary glaciations. Climate Dynamics, 2, pp. 1-10.

HR Wallingford 1997, Estuaries. The Case for research into Morphology and Processes, Report SR478

HR Wallingford 2000a, Modelling estuary morphology and process: Final Report, Report TR111.

HR Wallingford 2000b, A guide to prediction of morphological change within estuarine systems, Report TR114.

HR Wallingford 2000c, Recommendations for Phase 2 of the Estuaries Research Programme, Report TR113.

HR Wallingford, ABPmer and Pethick, J., 2006, Review and formalisation of geomorphological concepts and approaches to estuaries, Final Report FD2116/TR2.

Nicolis, C., 1982, A Boolean approach to climate dynamics. Quarterly Journal of Royal Meteorological Society, No.108, pp. 707-715.

O'Connor, B.A., 1987, Short and long term changes in estuary capacity. Journal of Geological Society, London, No.144, pp.187-195.

Pethick, J.S., 1984, An introduction to coastal geomorphology, Edward Arnold, London.

Pethick, J.S., 1992, Saltmarsh geomorphology, In Saltmarshes, Morphodynamics, Conservation and Engineering Significance. Eds: Allen J.R.J and Pye, K., Cambridge University Press.

Pethick, J.S., 1994, Estuaries and wetlands: function and form., In: Wetland Management, Thomas Telford, London, pp. 75-87.

Saunders, A., and Ghil, M., 2001, A Boolean delay equation model of ENSO variability, Physica D, 160, pp. 54-78.

Townend, I.H., 2003, Coast and estuary behaviour systems, Coastal Sediment '03, ASCE.

Van der Wal, D., Pye, K., and Neil, A., 2002, Long-term morphological change in the Ribble Estuary, northwest England., Marine Geology, No.189, pp.249-266.

Woodworth, P.L., Tsimplis, M.N., Flather, R.A. and Shennan, I., 1999, A review of the trends observed in British Isles mean sea level data measured by tide gauges, Geophy. J. Int., Vol. 136, pp. 651-670.

Van Goor, M.A., Stive, M.J.F., Wang, Z.B. and Zitman, T.J., 2001, Influence of relative sea level rise on coastal inlets and tidal basins, Coastal Dynamics 2001, ASCE, Lund, Sweden.

Wohlleben, T. M. H. and Weaver, A. J., 1995, Interdecadal climate variability in the subpolar North Atlantic, Climate Dynamics, 11(8), pp. 459 – 467.

Zaliapin, I., Keilis-Borok, V., and Ghil, M., 2003. A Boolean delay equation model of colliding cascades. Part I: Multiple seismic regimes. J. Stat. Phys., 111, pp.815-837.

APPENDICES A-C

Appendix A. Review of Approaches to Mathematical Formalization

Report by Zheng Bing Wang, December 2005

Prepared for:

Review of Approaches to Mathematical Formalization

EstSim

Report

December, 2005

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Prepared for:

Review of Approaches to Mathematical Formalization

EstSim

Zheng Bing Wang

Report

December, 2005

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1 Introduction

At the EstSim workshop on 4th October 2005, University of Plymouth, it has been discussed how to take the mathematical formalisation task forward into the system simulation task. To inform these discussions a review of the various approaches available for mathematical formalisation has been carried out. The following three approaches have been considered in the review:

1. The Boolean Approach, developed by University of Plymouth (Harshinie Karunarathana & Dominic Reeve, 2005);
2. The Network Dynamics Approach of UCL (Rob Seymour, 2005);
3. The ASMITA Approach, developed by WL | Delft Hydraulics and Delft University of Technology (Stive et al, 1998, Stive and Wang, 2003, Kragtwijk et al, 2004, Van Goor et al, 2003).

This document summarises the review. First the three approaches have been briefly described (chapter 2) and then a comparing discussion has been given (chapter 3).

2 Brief description of the approaches

2.1 Boolean Approach

An estuary is schematised into a number of geomorphologic elements. The state of each element is described by a Boolean variable, low=0, high=1. The same description for the state of hydrodynamics (tide, wave). The state of an estuary at a certain time is thus described by a vector of Boolean variables.

A Boolean function for each variable is derived by combining Boolean variables within a logical framework, for the simulation of the evolution of the estuary.

The approach can best be illustrated by the example given in the document of Karunarathana & Reeve (2005). The schematisation of the system is shown in Fig.1. For each of the 5 morphological elements (marshes, flats, channels, delta and sand spit) and the 2 hydrodynamic forcing (waves and tides) a Boolean variable and a corresponding Boolean function is defined (see table 1).

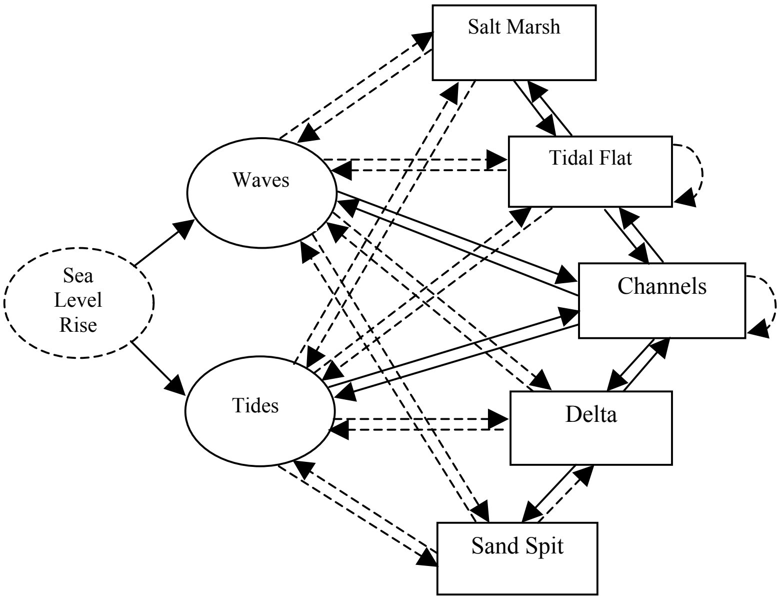


Fig.1

Table 1 Boolean variables describing the elements and their corresponding Boolean function

Network element	Boolean variable	Boolean function
waves	*w*	*W*
tides	*t*	*T*
salt marsh	*sm*	*SM*
tidal flats	*tf*	*TF*
channels	*cc*	*CC*
delta	*dd*	*DD*
sand spit	*ss*	*SS*

The Boolean functions describe how the Boolean variable evolves depending on the other variables (and possibly itself) as indicated in Fig.1.

$$\left.\begin{aligned}
W &= sm' \vee tf' \vee cc \vee dd' \vee ss' && \text{(a)}\\
T &= sm' \vee tf' \vee cc \vee dd' \vee ss' && \text{(b)}\\
SM &= (w' \wedge t \wedge tf) && \text{(c)}\\
TF &= (\,(sm \vee cc) \wedge t\,) \vee (tf' \wedge w') && \text{(d)}\\
CC &= (w \vee t) \wedge (\,tf \vee dd) \vee cc' && \text{(e)}\\
DD &= (w' \vee t') \wedge ss' \vee (t \wedge cc) && \text{(f)}\\
SS &= (w' \vee t') \wedge dd && \text{(g)}
\end{aligned}\right\} \qquad (1)$$

The following table shows some examples of the evaluation of the Boolean functions according to Eq.(1). The framed row indicates an equilibrium state as the output is exactly equal to the input according to the Boolean functions.

Table 2 – Some selected states from the Boolean matrix for tidal inlet with constrained sediment inflow.

	w	t	sm	tf	cc	dd	ss	W	T	SM	TF	CC	DD	SS
1	1	1	0	1	1	1	0	1	1	0	1	1	1	0
2	1	0	1	1	1	1	1	1	1	0	0	1	0	1
3	1	0	0	1	1	1	1	1	1	0	0	1	0	1
4	1	1	0	0	1	0	1	1	1	0	1	0	1	0
5	1	1	0	1	0	1	0	1	1	0	0	1	0	0
6	1	1	0	0	1	0	0	1	1	0	1	0	1	0
7	0	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	0	1	1	1	0
9	0	1	0	1	1	1	1	1	1	1	1	1	1	1

2.2 Network Dynamics

The state of an estuary is described by a vector of state variables. Each of the state variable xi ≥0 describes the state of a node in the network. The links between nodes indicate influences between the nodes: structure of the network. Mathematically the structure of a network (of n nodes) is expressed in a (n x n) matrix W of connection weight (-1, 0, or 1).

The net influence of the network on the node is defined as:

$$w_i(x) = (xW)_i = \sum_j x_j w_{ji} \qquad (2)$$

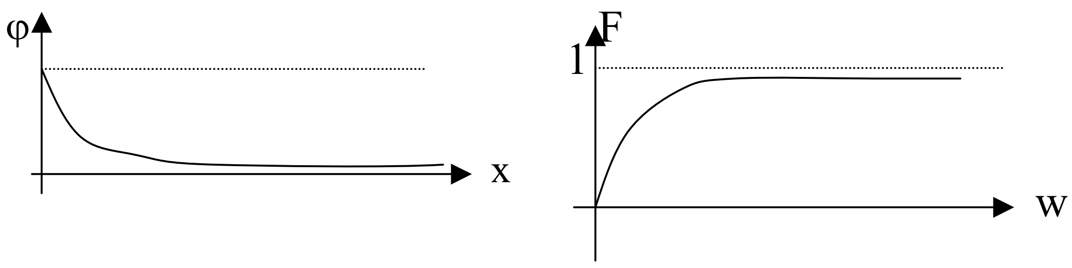
Evolution of the state of a node x_i in time is determined by x_i it self and by the net influence of the network on the node:

$$\dot{x}_i = R_i\left(x_i, w_i(x)\right) \tag{3}$$

The response function R is calculated as follows:

$$R(x,w) = \varphi(x)F(w) \text{ if } x > 0,\ w \geq 0, \tag{4a}$$
$$R(x,w) = -x\varphi(x)F(-w/x) \text{ if } x > 0,\ w < 0. \tag{4b}$$

The two functions φ and F can be chosen but the need to satisfy a number of requirements. Both functions cannot take negative values. φ is a monotonously non-increasing function and F is an monotonously increasing function with F(0)=0 and F(∞)=1. Typical examples of the two functions are shown in Fig.2.



F.g.2

Figure 3 shows an example of network, given in Seymour (2004).

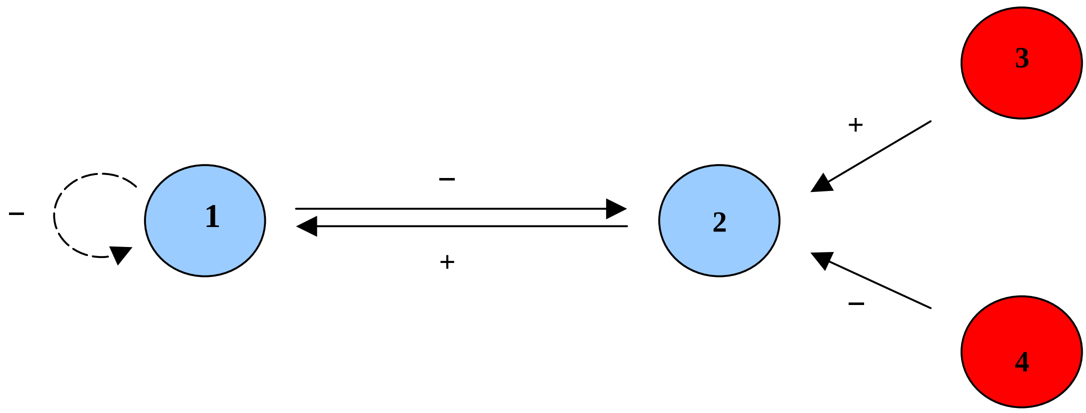


Fig.3

The structure of this network is mathematically represented by the following matrix:

$$W = \begin{pmatrix} -\varepsilon & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{5}$$

So the net influence vector can be calculated as:

$$(w_1, w_2, w_3, w_4) = (x_1, x_2, x_3, x_4)W = (-\varepsilon x_1 + x_2, -x_1 + x_3 - x_4, 0, 0) \tag{6}$$

In this example the nodes 3 and 4 are external nodes and their net influence by the network is zero, so they do not change in time (as F(0)=0). Figure 4 shows examples of the possible evolutions of the two internal nodes 1 and 2.

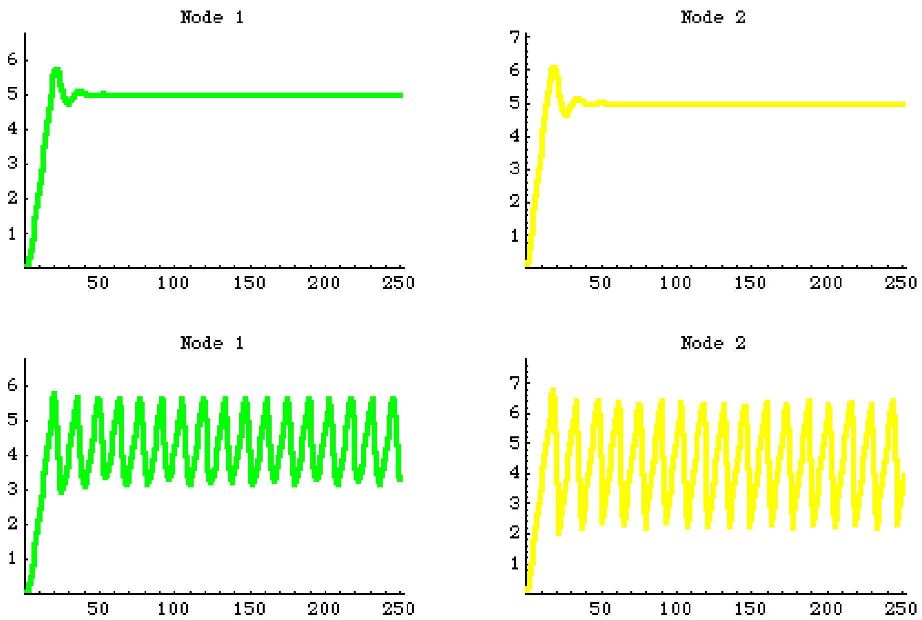


Fig.4

2.3 ASMITA

ASMITA is a semi-empirical model for the long-term morphological development of tidal inlet systems (Stive et al, 1998, Stive and Wang, 2003, Van Goor et al, 2003, Kragtwijk et al, 2004). A more detailed description of this model is given in Annex A.

2.4 Common example

2.4.1 The problem

In order to better illustrate the various approaches all three approaches are applied to a common problem. Consider the morphological development of an estuary / tidal inlet under influence of tide and (accelerated) sea-level rise. For simplicity reasons the morphological state is schematised with a single variable, e.g. the averaged depth.

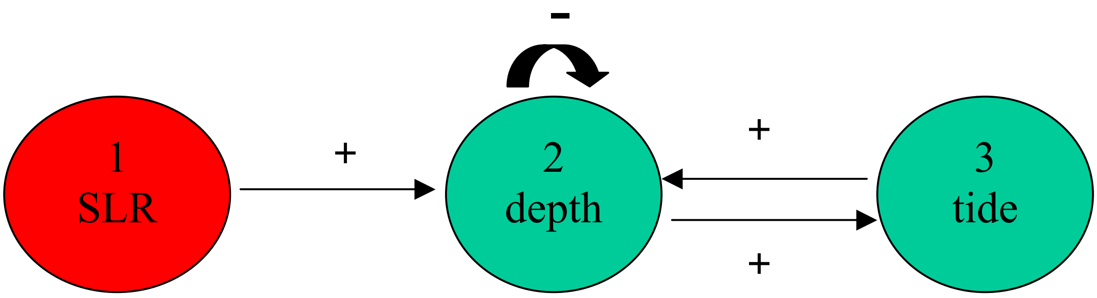


Fig.5

Figure 5 shows the network representation of the problem. Sea-level rise causes a direct increase of the depth. An increase of depth can cause more tidal intrusion in the system, which in turn can cause increase of the water depth. When the water depth is too large it can cause deposition again, therefore there is a negative feedback of the depth to itself.

Note that Fig.5 can be used as basis for the Boolean approach as well as for the network dynamics approach. However, there is an essential difference in using such network schemes in the two approaches. In the network dynamics approach the positive and negative influences as indicated in the figure is interpreted into the influence matrix in a fixed way. In the Boolean approach, on the other hand, these influences are represented in the Boolean function not according to a single way. In other words, the figure is sufficient to determine the behaviour of the system in the network dynamics approach, but is not sufficient in the Boolean approach.

2.4.2 Boolean Approach

The sea-level rise is an external node and is considered as given. The Boolean variables for the two internal node and the corresponding Boolean functions are defined as follows:

Table 3.

Network element	Boolean variable	Boolean function
depth	d	D
tides	t	T

$$
\begin{aligned}
D &= d' \vee t \\
T &= d
\end{aligned}
\tag{7}
$$

As there are only two Boolean variables, there in total 4 (=2^2) possible state of the system. These 4 state and the responses according tot the Boolean functions (7) are given in the following table.

Table 4

state	d	t	D	T
1	0	0	1	0
2	0	1	1	0
3	1	0	0	1
4	1	1	1	1

It is interesting to notice that state 4 is a steady equilibrium and states 2 and 3 form a cyclic equilibrium. State 4 is the end state only if it is the initial state. In all other cases the system ends with the cyclic development 2⇔ 3.

2.4.3 Network dynamics

The network as show in Fig.5 is represented by the following matrix:

$$W = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{8}$$

So the net influence vector becomes:

$$(w_1, w_2, w_3) = (x_1, x_2, x_3)W = (0, x_1 - x_2 + x_3, x_2) \tag{9}$$

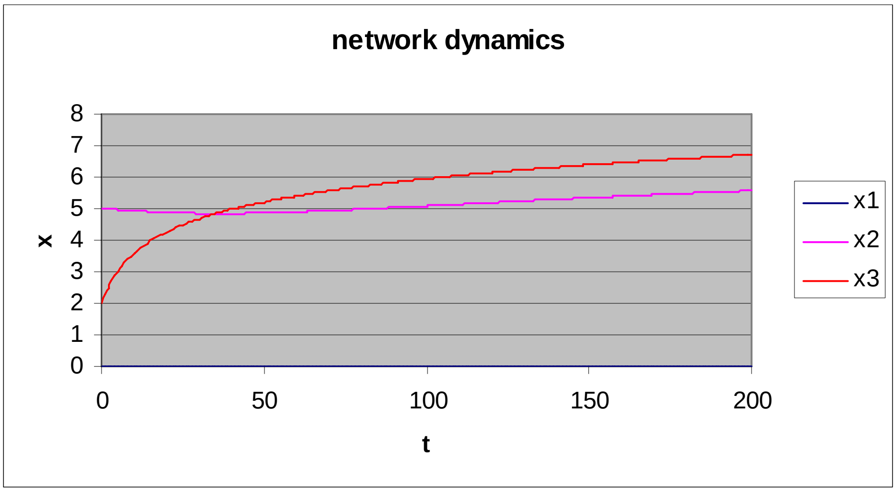


Fig.6

Using the following choices for F and φ and with the initial state (x_2=5, x_3=2) the model results as shown in Fig.6 are obtained.

$$\begin{aligned} F(w) &= \tanh(w) \\ \varphi(x) &= \exp(-x) \end{aligned} \tag{10}$$

The results in Fig.6 suggest that both tide and water depth in the estuary is increasing in time.

2.4.4 ASMITA

The results of the single element ASMITA model is shown in Fig.7. The simulation starts with an estuary in equilibrium, before the sea-level rise starts at t=0. The volume under moving sea-level increases to a higher new (dynamic) equilibrium value. At the end state the sedimentation (decrease of the volume under fixed level) exactly balances the sea-level rise.

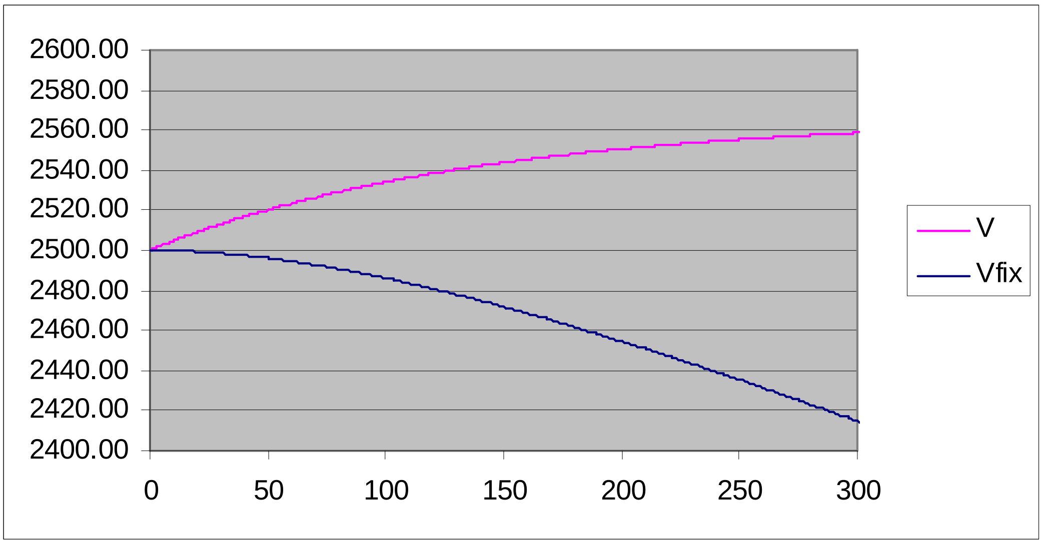


Fig.7 Development of the volume of the estuary under moving sea level (V) and under (fixed) initial sea level (Vfix) simulated by the single element ASMITA model.

3 Concluding discussions

The three approaches under consideration are in fact almost totally different from each other. The only thing is common is the schematisation of an estuary into a “small” number of “large” elements. Nevertheless an attempt is made to compare the three approaches in table 5 by looking at the INPUT/OUTPUT of the approaches, by considering their behaviours, and by evaluating the known applications in the estuarine modelling so far.

As indicated in table 5, the input and output of the Boolean approach are qualitative. Also the time step in the development is qualitative. Therefore the interpretation of the output of the model is very important in this approach. A whole story needs to be told on the basis of a series of Boolean vectors. The other two approaches use quantitative input and output. However there is a difference between the two methods concerning the physical meanings of the state variables. The state variables in the ASMITA model are in principle measurable physical quantities. The (uniform) way of treating all the node variables in the network dynamics implies that all these variables must have the same dimension. In practise this means that they should be dimensionless. Furthermore it is noted that the absolute values of the node variables are not important because of the (so called) scaling behaviour of the approach. By scaling behaviour it is meant that an equilibrium state (represented by a vector of non-negative real numbers) multiplied by any constant positive number remains an equilibrium state. It seems therefore that the physical meanings of the node variables and the way in which they should be made dimensionless are not clear. Another interpretation of this observation is that although the input and output of the network dynamics approach are quantitative in the mathematical sense, they are qualitative in the physical sense.

The qualitative character of an approach also makes it more flexible in applications. It is e.g. much easier to include an element or a process (as an example to extend an existing morphological model by including ecological influences) of different category in a Boolean approach than in ASMITA.

The real estuarine system is too complex to be fully described by any of the considered approaches. Due to the differences between the methods and due to our restricted understanding of the system we the three approaches should be considered as complementary rather than competitive to each other. The recommendation is thus to remain applying all the approaches, at least for the time being.

Table 5 Comparison between the three approaches

	Boolean Approach		Network Dynamics Approach		ASMITA Approach	
Input	Initial state, a vector of Boolean variables	Qualitative	• Initial state, a vector of non-negative real variables • Parameters in response functions	Quantitative	• Initial state, a vector of non-negative real variables • Parameters in the empirical relations • (sediment transport) Parameters influencing time scales	Quantitative
Output	(a series of) vector (s) of Boolean variables	Qualitative	Evolution in time of the state vector of non-negative real variables	Quantitative	• Evolution in time of the volumes of the elements • Sediment exchanges as function of time	Quantitative
Behaviour	• determined mainly by the Boolean functions • discrete development (with qualitative time step) • end state: fixed equilibrium or cyclic, never unstable due to fixed number of possible state • complexity increases exponentially with number of elements		• determined mainly by the structure of the network • discrete or continue development in time • end state: stable equilibrium or cyclic, can be unstable • possible equilibriums and their stability • scaling • complexity depending on the structure of the network		• The empirical relations determine the equilibrium states and the sediment transport parameters determine the evolution to the equilibrium state • continue development in time • end state: usually stable equilibrium, but can simulate unstable behaviour • complexity depending on the structure of the network	
Application	• Example concerning estuarine morphology given in the documents • The possible end states helpful for 'calibration'		• Example concerning estuarine morphology not given in the document		• response to sea level rise, nodal tide, land subsidence • response to engineering works, dredging / dumping • prediction of dredging requirement • need long-term historical data for calibration	

4 References

Harshinie Karunarathana & Dominic Reeve, 2005, Mathematical formulation of estuary simulator, University of Plymouth.

Rob Seymour, 2004, Notes on Network Dynamics, UCL.

N.G. KRAGTWIJK, M. J.F. STIVE, Z.B. WANG and T. J. ZITMAN, 2004, Morphological response of tidal basin to human interventions, Coastal Engineering, 51 (2004), 207-221.

Stive, M.J.F., Capobianco, M., Wang, Z.B., Ruol, P. and Buijsman, M.C., 1998. Morphodynamics of a tidal lagoon and the adjacent Coast. In: J. Dronkers and M. Scheffers (eds.), *Physics of estuaries and coastal seas*, Balkema, Rotterdam, pp 397-407.

M.J.F. Stive and Z.B. Wang, 2003, Morphodynamic modelling of tidal basins and coastal inlets, In C. Lakkhan (Ed.) *advances in coastal modelling,* Elsvier Sciences, pp. 367-392.

M.A. Van Goor, M.J.F. Stive, Z.B. Wang, T.J. Zitman, 2003, Impact of sea level rise on the morphological stability of tidal inlets, Marine Geology, Volume 202, issues 3-4, pp.211-227.

A Description of ASMITA

A.1 Introduction

The ASMITA (Aggregated Scale Morphological Interaction between a Tidal basin and the Adjacent coast) model, has recently been introduced by Stive et al (1998, see also Wang and Stive 2003, Van Goor et al, 2003 and Kragtwijk et al, 2004). The approach heavily depends on the concept of the existence of morphodynamic equilibrium in case of constant forcing based on the above-mentioned and to a certain degree explained empirical observations. These findings are used to study morphodynamic evolution of near-equilibrium basins when forced out-of-equilibrium.

A.2 Model concept

The basic idea of the approach is that a tidal inlet can be schematised into a number of morphological elements. For each element a volume can be defined acting as integral state variable. The level of schematisation is similar to that of the ebb-tidal delta by Dean and Walton (1975). A tidal inlet is thus schematised into (see Fig.A.1):

- the ebb-tidal delta (state variable = integral excess sediment volume relative to an undisturbed coastal bed profile, V_d);
- the inter-tidal flat area in the tidal basin (state variable = integral sediment volume between MLW and MHW, V_f);
- the total channel volume in the tidal basin (state variable = integral water volume below MLW, V_c);

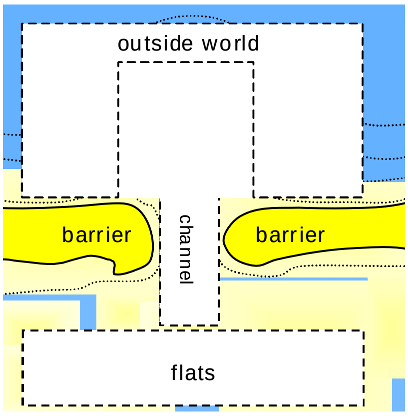


Figure A.1. Basic macro-scale elements of a tidal basin.

Following this schematisation the adjacent coastal stretches are considered as an external boundary - "the outside world" - which can exchange sediment with the considered inlet system.

As mentioned before, the important hypothesis used in the model concept is that a morphological equilibrium can be defined for each element depending on the hydrodynamic conditions (e.g. tidal prism, tidal range) and morphometric conditions (e.g. basin area). Theoretical arguments for the existence of such equilibrium were given by Dronkers (1998), but is also supported by various field investigations, which have resulted in empirical relations between state variables and parameters of the governing hydrodynamic and morphometric conditions (cf. Eysink 1990). In general, the (dry or wet) volume V_{ne} [m^3] of an arbitrary element (n) in a state of morphodynamic equilibrium, has appeared to be highly correlated to the tidal range H [m], the tidal prism P [m^3] and the basin area A_b [m^2]:

$$V_{ne} = V_{ne}(P, H, A_b) \qquad \text{(A.1)}$$

According to this hypothesis no morphological change takes place when all elements in the system are in equilibrium. When one or more elements are out of equilibrium morphological changes will take place tending to restore the system to (a possibly new) equilibrium.

Obviously, sediment transport must accompany morphological changes. It is assumed that suspended load is representative for the transport mode. The sediment transport formulation is basically the same as for any other suspended sediment transport model. However, unlike process-based models describing flow and sediment transport within tidal cycles residual sediment transport T is directly modelled here. This means that the long-term (time scale much larger than tidal period) mass-balance is considered for every morphological element:

$$\pm \frac{\mathrm{d}V_n}{\mathrm{d}t} = \sum_i T_{ni} \qquad \text{(A.2)}$$

The left-hand side of this equation represents the erosion rate within the element. Its sign is positive for a wet volume and negative for a dry volume. The right-hand side represents the sum of the transports leaving the element via all connections to other elements including the outside world. The erosion rate is assumed to be proportional to the difference between the local equilibrium concentration and the actual concentration like the depth-averaged model for suspended sediment transport of Galappatti and Vreugdenhil (1985):

$$\pm \frac{dV_n}{dt} = w_s \cdot A_n \cdot \left(c_{ne} - c_n\right) \qquad \text{(A.3)}$$

Herein w_s [m/s] is the vertical exchange rate and A_n [m^2] is the horizontal area of the element. Erosion occurs when the actual sediment concentration c_n is smaller than the equilibrium concentration c_{ne}, sedimentation occurs when the actual sediment concentration is larger than the equilibrium concentration. Also like any suspended sediment transport model the (long-term residual) sediment transport between two elements is assumed to be of the advective-diffusive type:

$$T_{ni} = Q_{ni}\left(c_n + c_i\right) + \delta_{ni}\left(c_n - c_i\right) \qquad \text{(A.4)}$$

Herein Q [m^3/s] is the residual flow rate, δ [m^3/s] the diffusion exchange rate between the two elements and c_i the sediment concentration in the adjacent element.

Substituting (A.3) and (A.4) into (A.2) yields an equation for the sediment concentration for each element. In this way a system of coupled equations for the sediment concentrations in all elements is established. It can readily be solved if the local equilibrium concentration is known. Equilibrium sediment concentration according to most sediment transport theories can be considered as proportional to a certain power of the flow velocity. In an aggregated scale model as considered here, flow velocity is not an available hydrodynamic parameter. However, the ratio between the equilibrium volume and the actual volume of e.g. the channel can be considered as the ratio between the flow velocity and that under equilibrium condition. Therefore the following formulation is used:

$$c_{ne} = c_E \cdot \left(\frac{V_{ne}}{V_n} \right)^r \tag{A.5}$$

The magnitude of power r is larger than one, commonly taken as 2 in compliance with a third power for the sediment transport as a non-linear function of the mean flow velocity. Its sign depends on the definition of the element volume, V_n, positive for wet volume and negative for dry volume.

The parameter c_E has the dimension of sediment concentration. When the whole system is in equilibrium the sediment concentration in all elements will be the same and equal to c_E. Therefore it is called the overall sediment concentration. It is usually prescribed at the outside world as boundary condition if the outside world can be considered as in equilibrium, which physically means that there is no limitation for supply or accommodation of sediment adjacent to the system under consideration. In the cases that we describe this is proven to be valid. However, in case the adjacent coast is protected, constrained by headlands, or in general insufficiently dynamic this assumption may not hold. In this case the adjacent coast needs to be introduced as an intrinsic morphological element to the system.

Morphological time scales

In order to obtain insight into the behaviour of the model we first demonstrate how the system reacts to a disturbance of the morphological equilibrium. To do this we first introduce a model with only one morphological element, e.g. the channel volume within the basin. In this case there is only one connection for diffusive sediment transport, i.e. the connection to the outside world. From the formulation described in the previous section the following equation can be derived for the channel volume:

$$\frac{\mathrm{d}V_c}{\mathrm{d}t} = \frac{\delta_{co} \cdot w_s \cdot A_c \cdot c_E}{\delta_{co} + w_s \cdot A_c} \left[\left(\frac{V_{ce}}{V_c} \right)^r - 1 \right] \tag{A.6}$$

Obviously, V_c will increase if it is smaller than its equilibrium value and vice versa. In other words, a disturbance always tends to be damped out and the equilibrium can be restored. For small disturbances Eq.(15) can be linearised:

$$\frac{dV_c'}{dt} = -\frac{w_s \cdot A_c \cdot \delta_{co} \cdot c_E \cdot r}{(\delta_{co} + w_s \cdot A_c) \cdot V_{ce}} \cdot V_c' \tag{A.7}$$

with $V_c' = V_c - V_{ce}$ expressing the disturbance. The solution of this differential equation is an exponential function:

$$V_c'(t) = V_{c0}' \cdot \exp(\frac{-t}{\tau}) \tag{A.8}$$

where V_{c0}' is the initial disturbance V_{c0}-V_{ce}, and

$$\tau = \frac{1}{c_E \cdot r}\left(\frac{V_{ce}}{w_s \cdot A_c} + \frac{V_{ce}}{\delta_{co}}\right) \tag{A.9}$$

The exponential decay of a disturbance from morphological equilibrium has been suggested before (see e.g. Eysink, 1990). However, there are a number of differences between the model described here and the earlier suggestions. First, according to the present model the exponential decay only applies to small disturbances whereas Eysink (1990) applies the exponential decay function to an arbitrary disturbance. Second, the time scale for the decay in the model of Eysink is an empirical input parameter, whereas equation (18) relates this time scale to a number of physical characteristics of the system. Equation (18) actually shows that the time scale consists of two parts, one for the vertical sediment exchange (first term within the bracket) and the other for the horizontal sediment exchange (second term within the bracket). Furthermore, the time scale is found to be inversely proportional to the overall equilibrium concentration and the power *r,* in agreement with the process-based models.

The most important difference with the classic empirical models can readily be made clear when two morphological elements are considered in the model, take e.g. the inter tidal flat volume, V_f, and the channel volume, V_c, in a tidal basin as state variables. Without going into the mathematical details the equations for the morphological evolution can be written in the following general form:

$$\begin{aligned} \frac{\mathrm{d}V_f}{\mathrm{d}t} &= \mathrm{F}_1\left(V_f, V_c\right) \\ \frac{\mathrm{d}V_c}{\mathrm{d}t} &= \mathrm{F}_2\left(V_f, V_c\right) \end{aligned} \tag{A.10}$$

This is a system of coupled first order non-linear differential equations. Linearisation of the system yields:

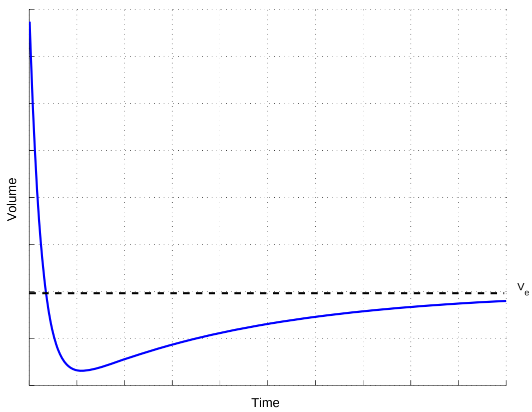
$$\frac{d}{dt}\begin{pmatrix} V_f' \\ V_c' \end{pmatrix} = \left.\begin{pmatrix} \frac{\partial F_1}{\partial V_f} & \frac{\partial F_1}{\partial V_c} \\ \frac{\partial F_2}{\partial V_f} & \frac{\partial F_2}{\partial V_c} \end{pmatrix}\right|_E \begin{pmatrix} V_f' \\ V_c' \end{pmatrix} \tag{A.11}$$

The solution of this system is:

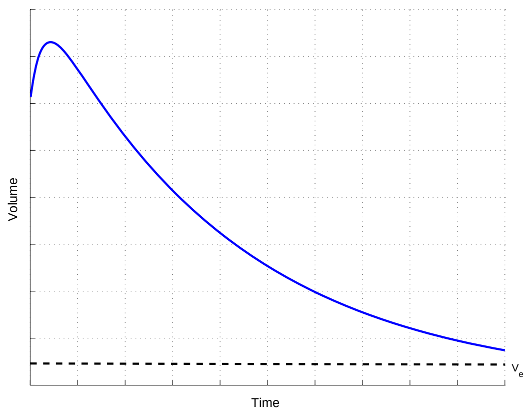
$$\begin{pmatrix} V_f'(t) \\ V_c'(t) \end{pmatrix} = C_1 \cdot \begin{pmatrix} Q_{11} \\ Q_{12} \end{pmatrix} e^{\lambda_1 \cdot t} + C_2 \cdot \begin{pmatrix} Q_{21} \\ Q_{22} \end{pmatrix} e^{\lambda_2 \cdot t} \tag{A.12}$$

Herein λ_1 and λ_2 are the eigenvalues of the Jacobian matrix in equation (A.11) with the corresponding eigenvectors Q_1 and Q_2. The constants C_1 and C_2 depend on the imposed disturbances (initial condition). The result is that the morphological behaviour is determined by two morphological time scales, which are equal to the inverse of the eigenvalues ($T_1 = -1/\lambda_1$ and $T_2 = -1/\lambda_2$). These time-scales are time-scales not of the individual elements, but of *the system* and they depend on geometric and exchange parameters (input parameters ASMITA). The larger time scale is related to the situation when both elements have a sediment demand or sediment surplus. The smaller time scale is related to the situation when one element has a sediment surplus while the other one has a sediment demand. An arbitrary disturbance is damped out by a combination of two exponential functions with two time-scales.

If the system is given sufficient time to respond to a disturbance, it will always evolve towards an equilibrium state. This evolution is not necessarily monotonous. Depending on the combination of the disturbances, the initial response of an element may be away from its equilibrium (this type of behaviour is referred as ‘bump’ behaviour). It is also possible that an element overshoots its equilibrium. These two contrasting situations are illustrated in Figure A.2. Physically it means that sedimentation does not necessarily occur in an element when it has a sediment demand, but that this depends on the situation in the surrounding elements. In fact sediment transport within the system is in the direction of the gradient of sediment demand.



a



b

Figure A.2. Characteristic morphological development, a: overshoot, b: bump (after Kragtwijk et al, 2002)

For the general case, if N morphological elements are included in the model there will be N morphological time scales. All the time scales are system time scales related to different type of disturbances represented by the corresponding eigenvectors. None of the time scales can be related to an individual element in the system.

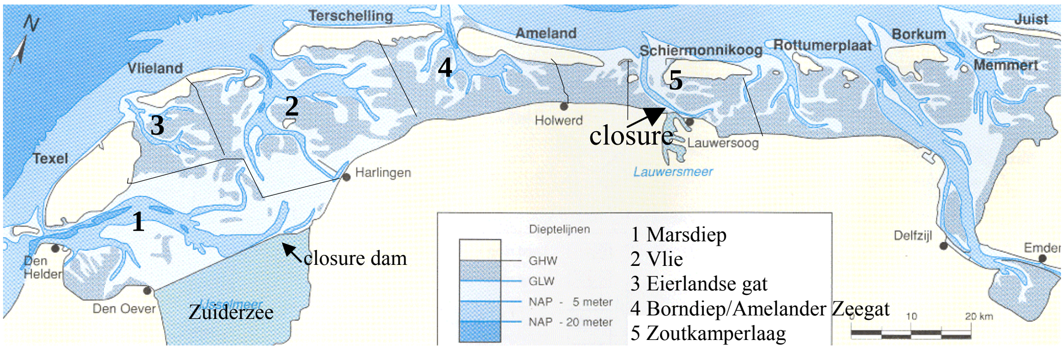


Figure A.3. Overview tidal inlets Western Wadden Sea (with approximate tidal divides)

Kragtwijk et al (2002) analysed the morphological time scales of five tidal inlets in the Dutch Wadden Sea (Figure A.3). In this analysis each tidal inlet is schematised into three morphological elements, i.e. the ebb-tidal delta, the channel in the basin and the intertidal flat in the basin. So three morphological time scales are found for each inlet as presented in Table A.1.

Inlet	**T_1 (years)**	**T_2 (years)**	**T_3 (years)**
Marsdiep	3	12	198
Eierlandse gat	3	7	38
Vlie	8	11	130
Borndiep	6	8	69
Zoutkamperlaag	5	8	48

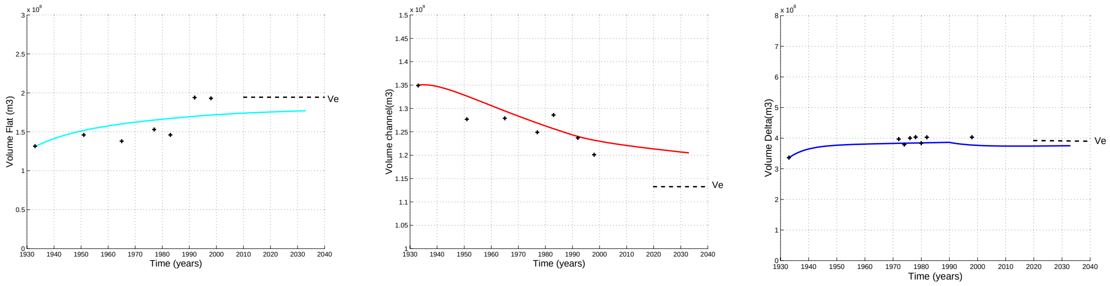
Table A.1. System time-scales for various Wadden inlets (after Kragtwijk et al, 2002)

From this table, we can conclude that the Wadden inlets can be characterised by two comparatively small time-scales and one larger time-scale. The main differences between the inlets are present in the largest time-scales, which characterise the overall development of the tidal inlet. For a relatively large tidal inlet like the Marsdiep system time-scale T_3 (see Table 1) is considerably larger than for a small inlet like the Eierlandse Gat. This means that a tidal inlet consisting of large morphological elements, has a comparatively large adaptation time.

Since the derived time-scales are system time scales and not interaction related, we emphasise that these findings are generic for tidal basins of similar size.

A.3 Response to human interference

There have been two major human interferences in the Dutch Wadden Sea in the last century, the closure of the Zuiderzee (IJssel Lake presently) in 1932 and the closure of the Lauwerszee (Lauwers Lake presently) in 1969. The former closure has impacted the morphological development of the Marsdiep and the Vlie (Figure A.4) and the latter has impacted the development of the Zoutkamperlaag (Figure A.5). The effects of both closures have been monitored, although the early years data concerning the first closure are less extensive and less accurate. The effect of the closure of Zuiderzee has been simulated by Kragtwijk et al (2002) using the ASMITA model with three elements as described above. Figure A.4 shows the computed and observed development of the Vlie. Figure A.5 shows the hindcast simulation of the development of the Zoutkamperlaag after the closure of the Lauwerszee by Van Goor et al (2002).



a Flat Vlie   b Channel Vlie   c Delta Vlie

Figure A.4. Model results for Vlie inlet (after Kragtwijk et al, 2002)

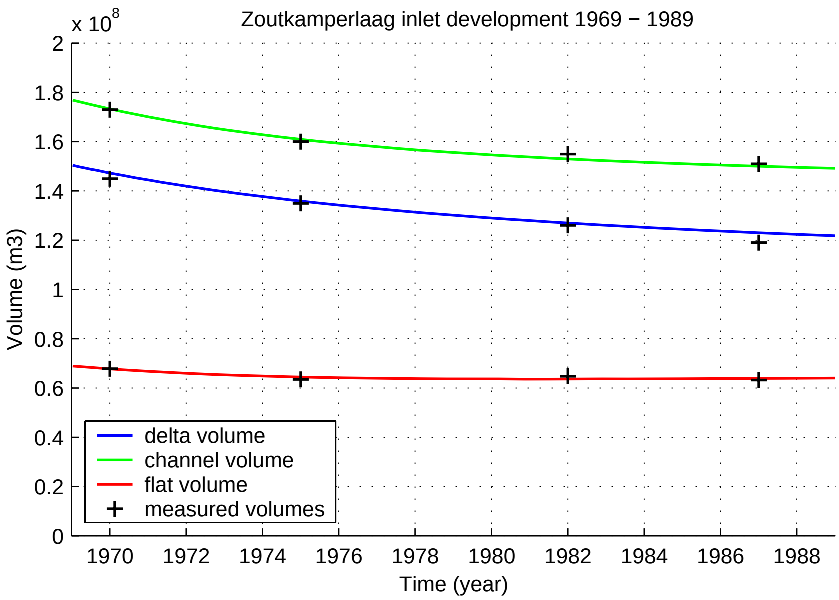


Figure A.5. Simulation of volume development of Zoutkamperlaag inlet after closure Lauwerszee in 1969 (After Van Goor et al, 2002).

Both cases show a satisfactory agreement between the computed development and the observations. More important, the experiences with the parameter setting of the model can be used for other applications of the model, e.g. for studying the impact of sea level rise.

A.4 Response to sea level rise

Sea level rise induces a special kind of disturbance or forcing for the morphological development. The volume of a morphological element will then not only change due to sedimentation or erosion but also due to the change of the mean sea level. This can be implemented into the model by adding a term to Eq.(A.3)

$$\pm\frac{dV_n}{dt} = w_s \cdot A_n \cdot \left(c_{ne} - c_n\right) + A_n \frac{d\zeta}{dt} \tag{A.13}$$

Herein ζ is the mean sea level. For the model with a single element the equation for the morphological evolution (Eq.A.6) becomes:

$$\frac{dV_c}{dt} = \frac{w_s \cdot A_c \cdot \delta_{oc} \cdot c_E}{\delta_{oc} + w_s \cdot A_c}\left(\left(\frac{V_{ce}}{V_c}\right)^r - 1\right) + A_c \cdot \frac{d\zeta}{dt} \tag{A.14}$$

The volume of the channel V_c will now still be changing in time if it is equal to its equilibrium value. When sea-level rise rate is constant a dynamic equilibrium can be established and the equation becomes:

$$V_{ce}^* = V_c\Big|_{\frac{dV_c}{dt}=0} = \frac{V_{ce}}{\left(1 - \frac{d\zeta}{dt}\cdot\frac{\delta_{oc} + w_s \cdot A_c}{w_s \cdot \delta_{oc} \cdot c_E}\right)^{\frac{1}{r}}} \tag{A.15}$$

Equation (A.14) gives the channel volume in case of a dynamic equilibrium under external forcing of a constant sea level rise. In case of no sea level rise ($d\zeta/dt = 0$) equation (A.15) reduces to $V_{ce}^*=V_{ce}$. However, in case of a constant rate sea-level rise ($d\zeta/dt > 0$) equation (A.15) gives a 'new' dynamic equilibrium volume which is larger than the original equilibrium volume ($V_{ce}^*>V_{ce}$). Apparently there is a permanent difference between the equilibrium volume with *SLR* (V_{ce}^*) and the equilibrium volume without *SLR* (V_{ce}). This difference in equilibrium volume is necessary to maintain the demand of sediment that drives sediment imports into the system to such extent that the system does not drown. Equation (A.15) also shows that the equilibrium volume V_{ce}^* becomes infinitely large when:

$$\frac{d\zeta}{dt} = \frac{w_s \cdot \delta_{oc} \cdot c_E}{\delta_{oc} + w_s \cdot A_c} \tag{A.16}$$

Apparently, there is a maximum *SLR* rate an inlet can keep pace with. It is referred to as critical sea level rise rate as it indicates the transition between preservation and degeneration of the inlet. The dynamic equilibrium volume (relative to the equilibrium value) as a function of the sea level rise rate (relative to the critical value) is shown in Figure A.6.

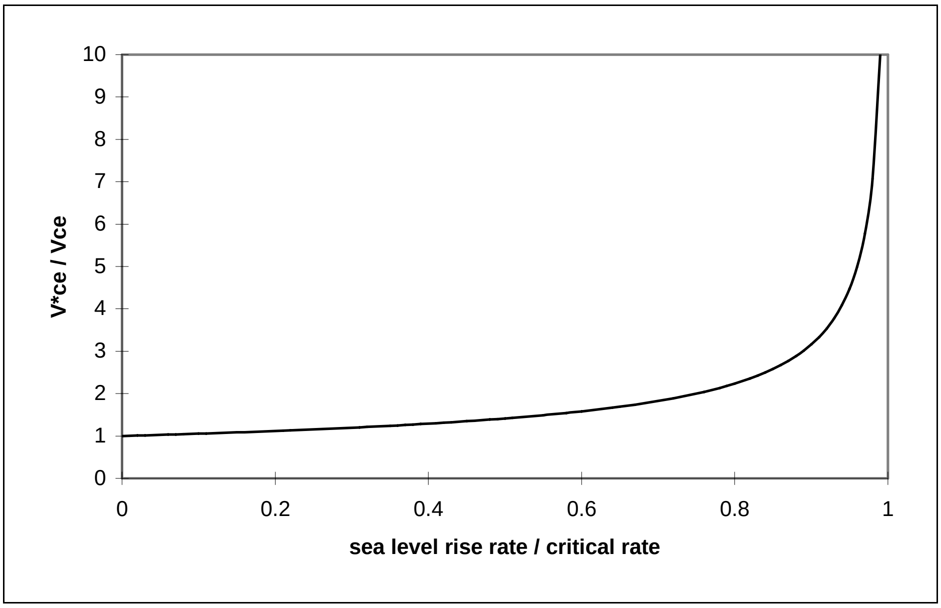


Figure A.6. Dynamic equilibrium volume under influence of sea level rise (r = 2).

That there is a critical value of the sea level rise rate at which a tidal inlet will drown can also be found if the system is divided into more morphological elements. Van Goor et al (2002) applied the three-elements model as described above to investigate the impact of sea level rise on two tidal inlets in the Dutch Wadden Sea, the 'Amelander Zeegat' and the ‘Eierlandse Gat’ (see Figure A.7). In this figure the dynamic equilibrium volumes of the inlet elements are plotted as a function of sea level rise rates. The vertical line represents the current state of dynamic equilibrium under the present rate of *SLR*. With a larger rate of *SLR* (moving to the right in the figures) we see that the dynamic equilibrium volume of the channel element increases (wet volume) and that the dynamic equilibrium volumes of the flat and ebb-tidal delta element decrease (sediment volume).

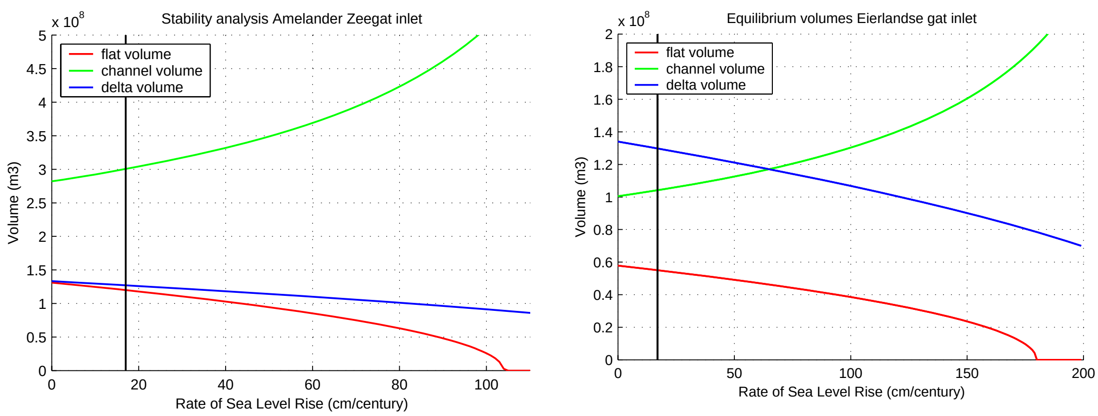


Figure A.7 Dynamic equilibrium volumes and stability limit of the Amelander Zeegat and Eierlandse Gat (after Van Goor et al, 2002).

As shown, the faster the sea level rises, the more the element volumes have to deviate from the equilibrium volumes (V_{ne}) belonging to a constant sea level ($d\zeta/dt = 0$). This deepening of the basin stimulates the system to follow the rising sea level. For the Amelander Zeegat case Figure A.7 shows that the critical sea-level rise rate is 105 cm per century. For the Eierlandse Gat the critical sea level rise rate is 180 cm per century. As the sediment demand of a smaller basin is less, the latter inlet finds it easier to adapt to a higher rate of sea level rise.

In order to cope with the uncertainties in the used input parameters for calculating the critical sea level rise rate Van Goor et al (2002) employed a probabilistic approach. The ASMITA model is especially suited for such an approach because of its simplicity. The end result is the probability that a tidal inlet can no longer follow the rise of sea level as function of the sea level rise rate, as shown in Figure A.8. The three different lines in the figure indicate three different methods for determining the overall equilibrium sediment concentration.

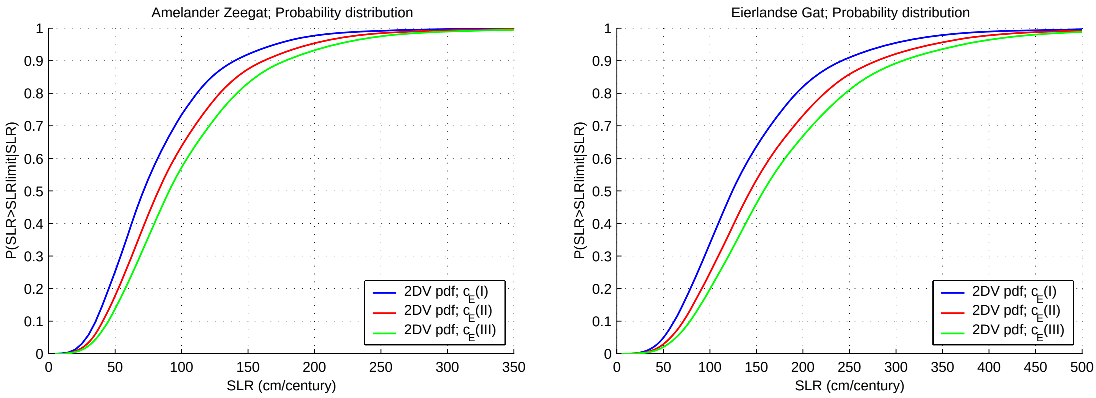


Figure A.8. Probability distribution (2D) of Amelander Zeegat and Eierlandse Gat for present and changed wave climate scenarios (after Van Goor et al, 2002).

Validation of these findings is difficult since morphological observations of tidal basins under increasing rates of sea level rise are – to the authors' knowledge - non-existing. From geological reconstruction of the Holland coast (Beets et al., 1992), however, validation in a reverse manner is to a certain extent confirmed. When sea level rise rates decreased to less than about 1 m/century the central Holland tidal basins changed from drowned systems to well-developed intertidal basins.

A.5 References

Beets, D.J., Van der Valk, L., and Stive, M.J.F., 1992. Holocene evolution of the coast of Holland. *Marine Geology*, 103, pp. 423-443.

Di Silvio, G., 1989: Modelling the morphological evolution of tidal lagoons and their equilibrium configurations. XXII Congress of the IAHR, Ottawa, Canada, 21-25 August 1989.

Eysink, W.D., 1990. Morphological response of tidal basins to change. *Proc. 22nd International Conference on Coastal Engineering*, ASCE, Delft, July 2-6, Vol. 2, pp. 1948-1961.

Gallappatti, R. and C.B. Vreugdenhil, 1985. A depth-integrated model for suspended sediment transport. *Journal of Hydraulic Research*, Vol. 23, no. 4, p. 359-377.

Kragtwijk, N.G., 2001. Aggregated scale modelling of tidal inlets of the Wadden Sea, *WL | Delft Hydraulics/Delft Cluster*, Report Z2822/DC03.01.03b.

O'Brien, M.P., 1969. Equilibrium flow areas of inlets on sandy coasts. *Journal of the Waterways and Harbour Div.*, ASCE, 95(WW1), pp. 43-52.

Stive, M.J.F., Capobianco, M., Wang, Z.B., Ruol, P. and Buijsman, M.C., 1998. Morphodynamics of a tidal lagoon and the adjacent Coast. In: J. Dronkers and M. Scheffers (eds.), *Physics of estuaries and coastal seas*, Balkema, Rotterdam, pp 397-407.

Van Goor, M.A., 2001. Influence of Relative Sea Level Rise on Coastal Inlets and Tidal Basins, *WL | Delft Hydraulics/Delft Cluster*, Report Z2822/DC03.01.03a.

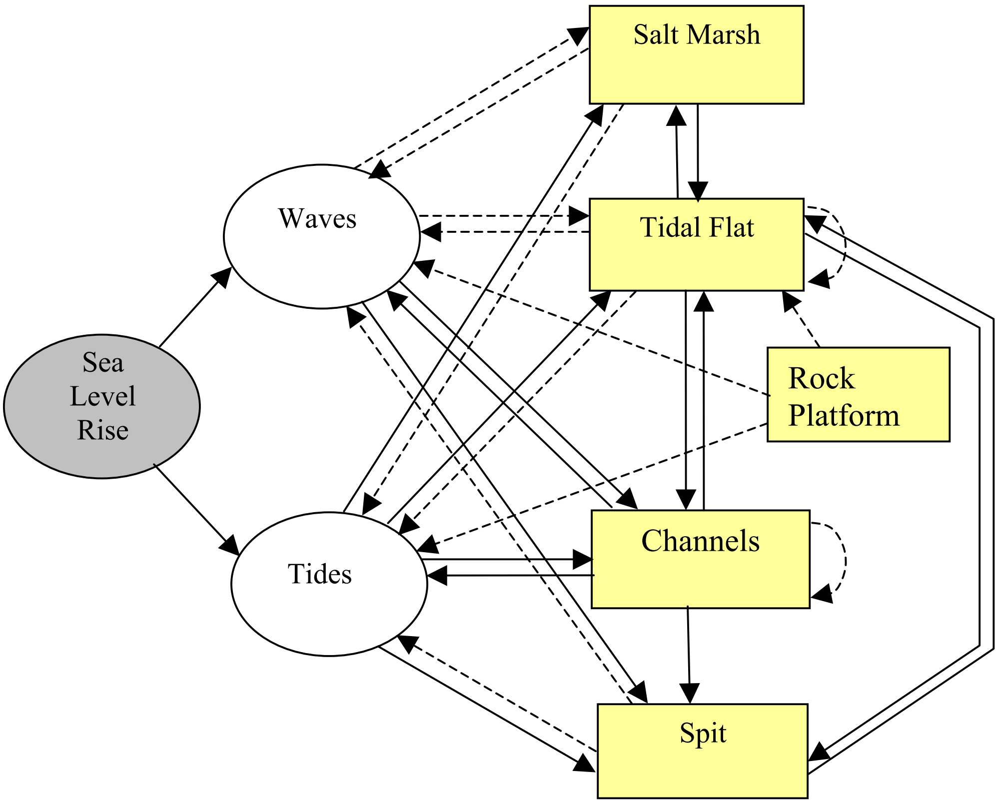
Van Goor, M.A.; Stive, M.J.F.; Wang, Z.B.; and Zitman, T.J., 2001. Influence of Relative Sea Level Rise on Coastal Inlets and Tidal Basins, *Proc. 4th Conference on Coastal Dynamics*, ASCE, Lund, Sweden, 2001, pp. 242-252.

Wang, Z.B., Karssen, B., Fokkink, R.J. and Langerak, A., 1998. A dynamic/empirical model for long-term morphological development of estuaries. In: J. Dronkers and M. Scheffers (eds.), *Physics of estuaries and coastal seas*, Balkema, Rotterdam, pp. 279-286.

Appendix B. Boolean Analysis of Generic Glacial Valley Estuary

Glacial Valley Estuary

- Abundant flow of external sediment
- No control structure present



Logical framework:

W = rp' + sm' + tf' + cc + ss'
T = rp' + sm' + tf' + cc + ss'
SM = w' * t * tf
TF = w' + t * (sm + cc + ss) + (rp' + tf')
CC = (w + t) + (tf * t) + cc'
SS = w + (tf * t)
RP = rp

Network element	Boolean variable	Boolean function
waves	*w*	*W*
tides	*t*	*T*
salt marsh	*sm*	*SM*
tidal flats	*tf*	*TF*
channels	*cc*	*CC*
sand spit	*ss*	*SS*
rock platform	*Rp*	*RP*

1. One stable state is reached

    1101111

    - Estuary reaches equilibrium against SLR without no or little salt marshes

2. Wave dominated, no sand spit

    1011101 → 1100111 → 1101111

    - sand spit develops but salt marshes and tidal flat recede against SLR
    - tidal flats re-develops gradually with sediment inflow from outside.
    - estuary reaches stable state with little or no salt marshes

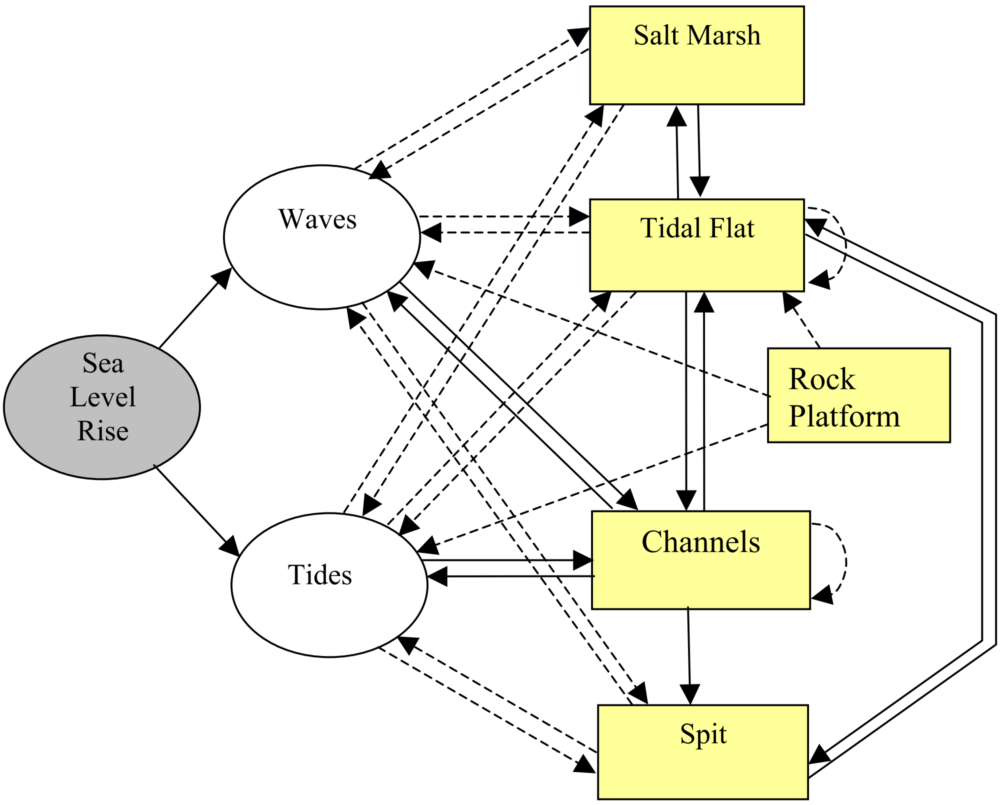
3. Tide dominated, no sand spit

    0111101 → 1111111 → 1101111

    - wave energy increases and a sand spit develops when sea level rises
    - estuary reaches stable state with little or no salt marshes

Glacial Valley Estuary

- Constrained flow of external sediment
- No control structure present



Logical framework:

W = rp' + sm' + tf' + cc + ss'
T = rp' + sm' + tf' + cc + ss'
SM = w' * t * tf
TF = (w' + t') + (sm + cc + ss) * t + (rp' + tf')
CC = (w + t) + (tf + cc')
SS = w' * tf * t
RP = rp

Network element	**Boolean variable**	**Boolean function**
Waves	*w*	*W*
Tides	*t*	*T*
salt marsh	*sm*	*SM*
tidal flats	*tf*	*TF*
Channels	*cc*	*CC*
sand spit	*ss*	*SS*
rock platform	*Rp*	*RP*

1. One stable state is reached

   1101101

   - Estuary reaches equilibrium against SLR without no or little salt marshes

2. Wave dominated, no sand spit

   1011101 → 1101101

   - sand spit develops but salt marshes and tidal flat recede against SLR
   - tidal flats re-develops gradually due to sediment exchange between elements.
   - estuary reaches stable state with little or no salt marshes

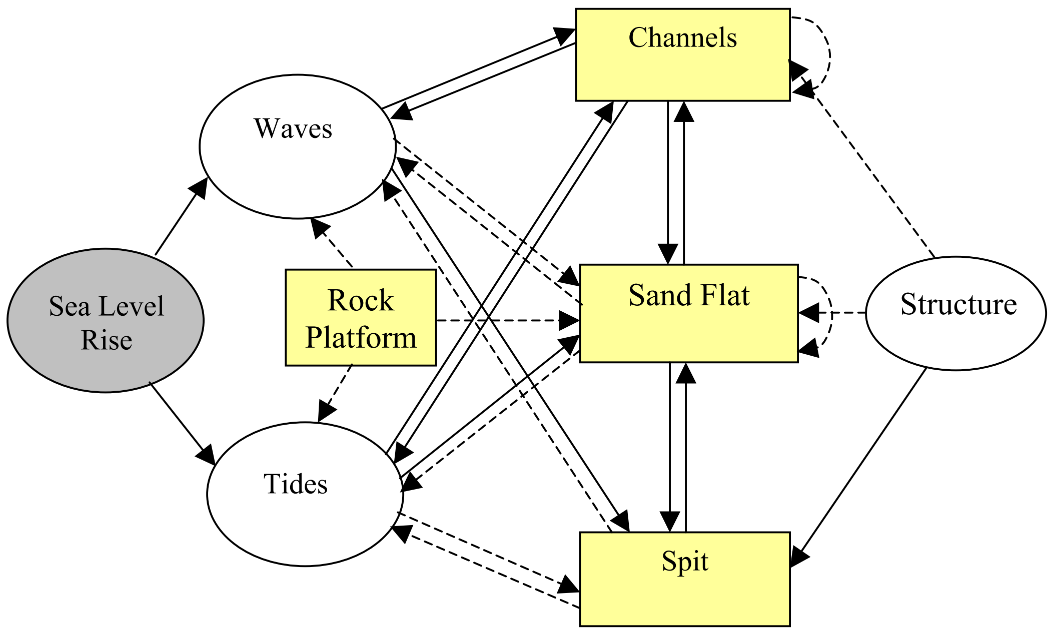
3. Tide dominated, no sand spit

   0111101 → 1111111 → 1101101

   - sand spit develops
   - sand spit recedes again when sea level rises further due to lack of sediment inflow. Salt marshes recede too and the estuary reaches stable state with little or no salt marshes

Glacial Valley Estuary

- Constrained flow of external sediment
- Control structure present



Logical framework:

W = rp' + st' + sf' + cc + ss'
T = rp' + st' + sf' + cc + ss'
CC = (w + t) + (sf * t) + cc' + st'
SF = (w' + t *ss) + (st' + sf' + rp')
SS = w + (sf * t) + st'
RP = rp
ST = st

Network element	Boolean variable	Boolean function
waves	*w*	*W*
tides	*t*	*T*
channels	*cc*	*CC*
sand flat	*sf*	*SF*
sand spit	*ss*	*SS*
rock platform	*Rp*	*RP*
structure	*St*	*ST*

1. The estuary evolves in a cyclic pattern by accretion and depletion of sand flats if it stars as a high energy system with SLR

$$1110111 \rightleftarrows 1111111$$

- At the initial stage of the estuary, there is little or no sand flats
- With SLR the sand flats accrete
- But, estuary evolves back into its original position and continue to evolve in a cyclic pattern with sediment exchange between estuary elements

2. Wave dominated estuary with fully developed sand flats

$$1011111 \rightarrow 1110111 \rightleftarrows 1111111$$

- same evolutionary process as in (1).

3. Tide dominated estuary with fully developed sand flat

$$0111111 \rightarrow 1110111 \rightleftarrows 1111111$$

- same evolutionary process as in (2).

4. Wave dominated, no sand flat

$$1010111 \rightarrow 1111111 \rightleftarrows 1110111$$

- same evolutionary process as in (2).

5. Tide dominated, no sand flat

$$0110111 \rightarrow 1111011 \rightarrow 1110111 \rightleftarrows 1111111$$

- at the initial stage, little or no sand flats exist
- with SLR, sand flat accrete and sand spit depletes due to sediment exchange between them
- with further SLR, sand flats deplete again but spit accretes
- if enough sediment is brought to the system from out side, both sand spit and sand flats develop
- estuary continues to evolve between the last two states

6. Wave dominated, no sand spit

$$1011011 \rightarrow 1110111 \rightleftarrows 1111111$$

- If depletion rate of sand flat is faster than the accretion of sand spit, estuary will follow the following path:

$$1011011 \rightarrow 1110011 \rightarrow 1111111 \rightleftarrows 1110111$$

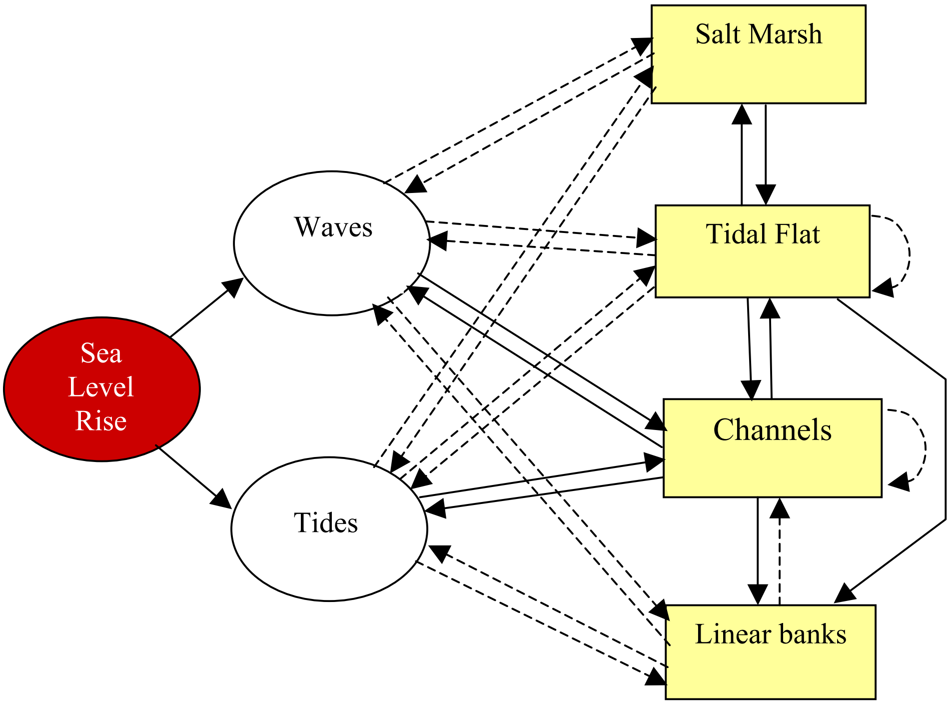
- If sand spit accretes faster than the depletion of sand flat, the estuary will follow the following path:

$$1011011 \rightarrow 1111111 \rightleftarrows 1110111$$

Appendix C. Boolean Analysis of Drowned River Valley Estuary

Drowned River Valley

- Constrained flow of external sediment
- No control structure present



Logical framework:

W	= sm' + tf' + cc + lb'
T	= sm' + tf '+ cc + lb'
SM	= (w' * t * tf) + (tf'* t') + (t' *w')
TF	= (w' + t') + (sm + cc) * t + (tf' * w')
CC	= (w + t) * (tf + lb') + cc'
LB	= w' * t * tf

Network element	Boolean variable	Boolean function
waves	*w*	*W*
tides	*t*	*T*
salt marsh	*Sm*	*SM*
tidal flat	*tf*	*TF*
channels	*cc*	*CC*
linear banks	*lb*	*LB*

1. One stable state is observed:

   110110

   - estuary remains stable with little or no salt marshes and sand banks
   - sand spit depletes against sea level rise
   - sediment exchange between tidal flats and channels maintain the stability of the system
   - development of salt marshes take place due to sediment inflow from outside

2. Tide dominated estuary with fully developed estuary elements

   011111 ⟶ 111111 ⟶ 110110

   - wave energy increases as the initial response
   - salt marshes and sand banks erode when sea level rises and the estuary reaches a stable state with little or no salt marshes and sand banks due to lack of sediment supply.

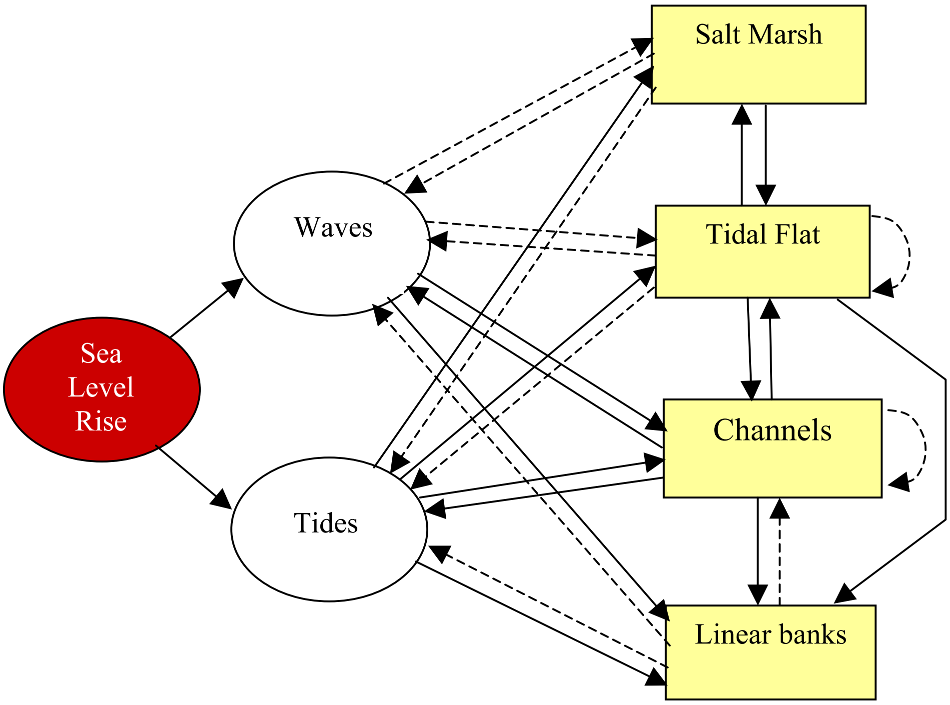
3. Wave dominated, fully developed estuary

   101111 ⟶ 110010 ⟶ 110110

   - salt marshes, tidal flats and sand banks erode when sea level goes up
   - tidal flats regain due to sediment exchange with salt marshes and the estuary reaches a stable state.
   - erosion of salt marshes will take place after depletion of tidal flats. Also, erosion rate of tidal flats would be different from the erosion rate of sand banks. Therefore, following options are available.

Drowned River Valley

- Abundant flow of external sediment
- No control structure present



Logical framework:

W	= sm' + tf' + cc + lb'
T	= sm' + tf '+ cc + lb'
SM	= (w' + t) * tf
TF	= (w' * tf') + (sm + cc) * t
CC	= (w + t) * (tf + lb') + cc'
LB	= w + (t * tf)

Network element	**Boolean variable**	**Boolean function**
waves	*w*	*W*
tides	*t*	*T*
salt marsh	*Sm*	*SM*
tidal flat	*tf*	*TF*
channels	*cc*	*CC*
linear banks	*lb*	*LB*

Structural control is applied on the estuary where sediment distribution in channels, tidal flats and salt marshes is affected by the structure.

1. One stable state is observed:

    1111111

    - estuary remains stable with all generic elements

2. Tide dominated estuary with fully developed estuary elements

    0111111 → 1111111

    - stable state is reached and the estuary stays stable with SLR

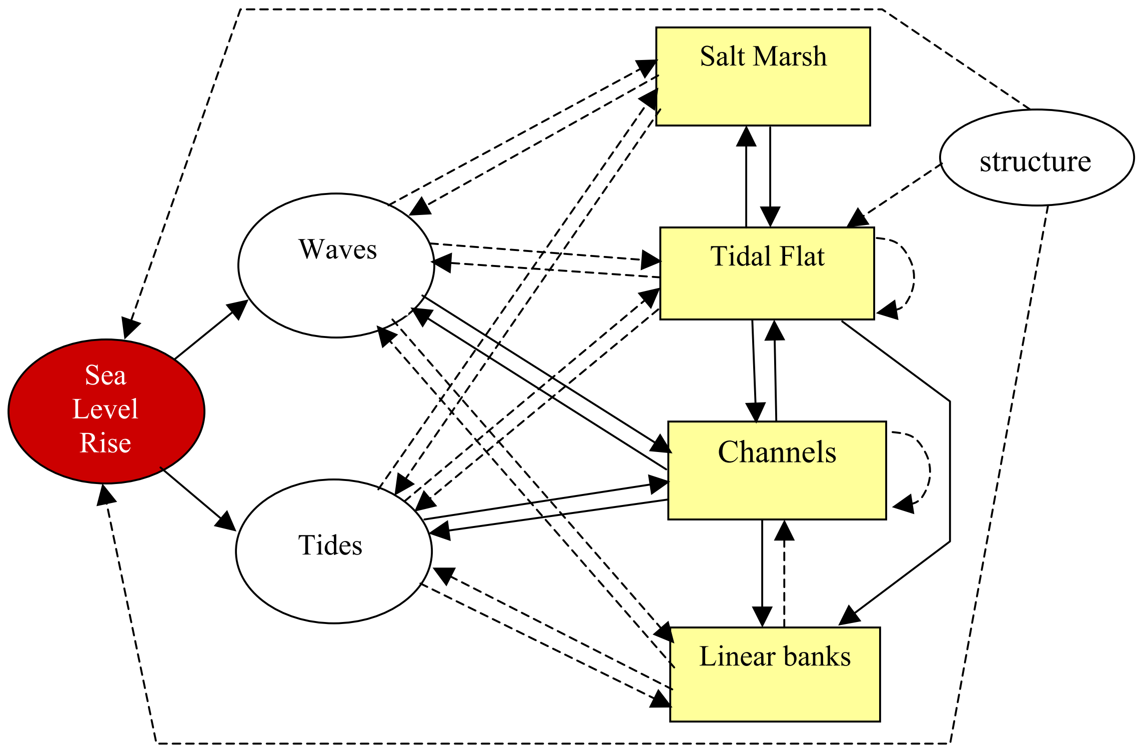
3. Wave dominated, fully developed estuary

    101111 → 110011 → 110101 ⇄ 111011

    - tidal flats and salt marshes recede
    - tidal flat accrete and channels become shallow due to sediment supply from outside
    - tidal flat recede by supplying sediment to salt marshes, channels become deeper again

Drowned River Valley

- Constrained flow of external sediment
- Control structure present



Logical framework:

W = sm' + tf' + cc + lb' + st'
T = sm' + tf '+ cc + lb' + st'
SM = (w' * t * tf) + (tf'* t') + (t' *w') + st'
TF = (w' + t') + (sm + cc) * t + (tf' * w') + st'
CC = (w + t) * (tf + lb') + cc'
LB = w' * t * tf
ST = st

Network element	**Boolean variable**	**Boolean function**
waves	*w*	*W*
tides	*t*	*T*
salt marsh	*Sm*	*SM*
tidal flat	*tf*	*TF*
channels	*cc*	*CC*
linear banks	*lb*	*LB*
structure	*st*	*ST*

1. One stable state is observed:

   1101101

   - estuary remains stable with little or no salt marshes and sand banks
   - sediment exchange between tidal flats and channels maintain the stability of the system
   - development of salt marshes take place due to sediment inflow from outside

2. Wave dominated estuary with fully developed estuary elements

   1011111 → 1100101 → 1101101

   - wave energy increases and salt marsh, tidal flats and sand banks begin to erode when sea level rises
   - tidal flats accrete again due to sediment exchange between channels, flats and salt marshes and the estuary reaches a stable state with little or no salt marshes.
   - salt marsh erosion usually takes place after the estuary loosing tidal flats. Also, erosion rates of tidal flats and sand banks would be different. Therefore, the estuary will have two options to follow:

   1011111 → 1110111 → 1101001 → 1100101 → 1101101

   1011111 → 1111101 → 1101101

3. Tide dominated, fully developed estuary

   0111111 → 1111111 → 1101101

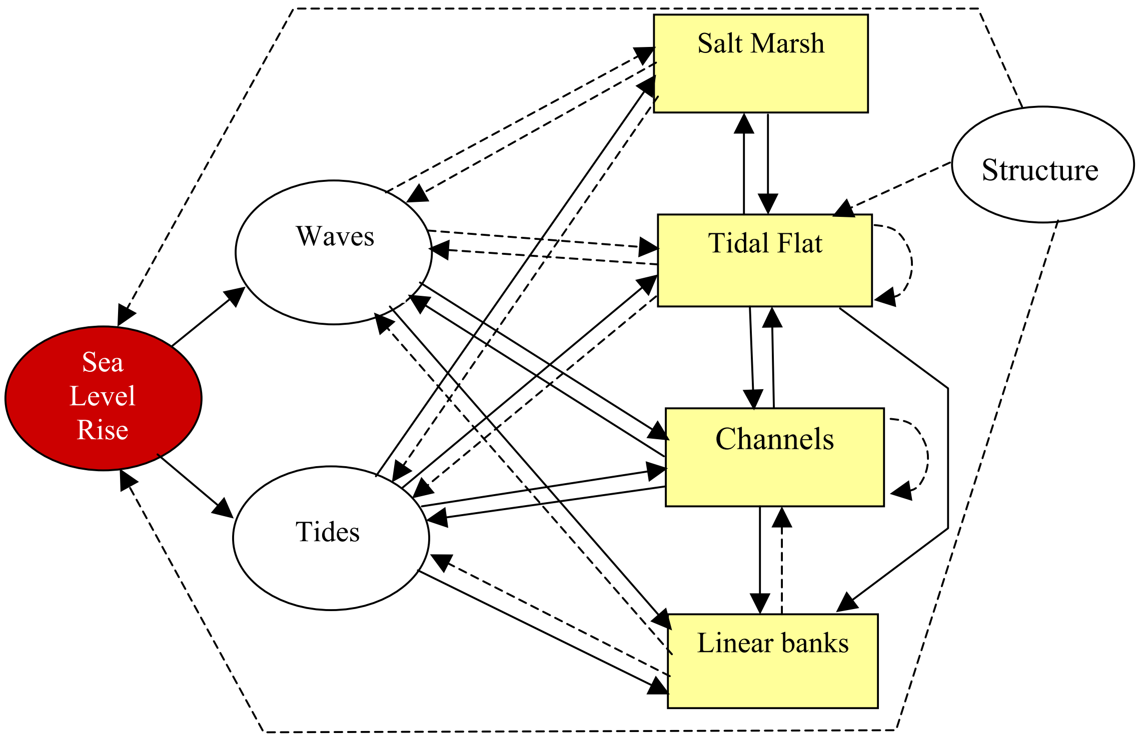
   - salt marshes and sand banks erode when sea level goes up and estuary reaches stable state
   - erosion rate of tidal flats would be different from the erosion rate of sand banks. Therefore, following options are available:

   0111111 → 1111111 → 1101111 → 1101101

   0111111 → 1111111 → 1111101 → 1101101

Drowned River Valley

- Abundant flow of external sediment
- Control structure present



Logical framework:

W	= sm' + tf' + cc + lb' + st'
T	= sm' + tf '+ cc + lb' + st'
SM	= (w' * t * tf) + st'
TF	= (w' * tf') + (sm + cc) * t + st'
CC	= (w + t) * (tf + lb') + cc'
LB	= w + (t * tf)
ST	= st

Network element	**Boolean variable**	**Boolean function**
waves	*w*	*W*
tides	*t*	*T*
salt marsh	*Sm*	*SM*
tidal flat	*tf*	*TF*
channels	*cc*	*CC*
linear banks	*lb*	*LB*
structure	*st*	*ST*

1. One stable state is observed:

   1101111

   - estuary remains stable with little or no salt marshes
   - sediment exchange between tidal flats and channels maintain the stability of the system

2. Wave dominated estuary with fully developed estuary elements

   1011111 $\longrightarrow$ 1100111 $\rightleftarrows$ 1101011

   - wave energy increases and salt marsh and tidal flats begin to erode
   - tidal flats regain but channels become shallower
   - Shallow channels and high tidal flats do not co-exist. Therefore, the estuary reverts back to the previous state and evolve in a cyclic pattern

3. Tide dominated, fully developed estuary

   0111111 $\longrightarrow$ 1111111 $\longrightarrow$ 1101111

   - salt marshes erode and estuary reaches a stable state without salt marshes

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