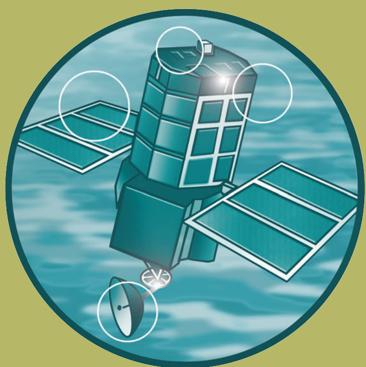
Joint Defra/EA Flood and Coastal Erosion Risk Management R&D Programme

Understanding the lowering of beaches in front of coastal defence structures, Stage 2 Technical Note 7

R&D Project Record FD1927/PR7

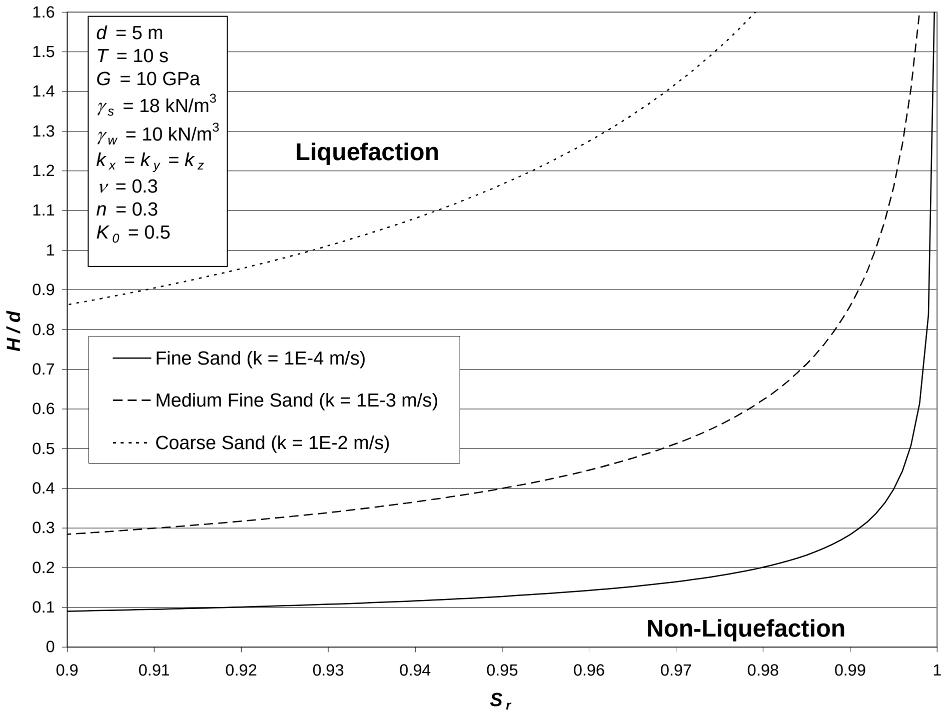


Environment Agency

defra
Department for Environment Food and Rural Affairs

Understanding the lowering of beaches in front of coastal defence structures, Phase 2

Wave-induced Liquefaction of Sediment in front of Coastal Structures



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Document Information

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Document History

Date	Revision	Prepared	Approved	Authorised	Notes
12/09/06	3.0	PL Vun	Sutherland	Whitehouse	Simplified assessment approach added, plus minor edits
05/09/06	2.0	PL Vun	Sutherland	Whitehouse	Extended introduction and conclusions
26/07/06	1.0	PL Vun	Sutherland	Whitehouse	Internal review version

Contents

1. Introduction

The role of scour in removing sediment from the toe of coastal structures has received much attention (Whitehouse, 1998; Hoffmans & Verheij, 1997; Sumer and Fredsøe, 2002) but the potential for the liquefaction of sediment by waves has been studied less. This situation prompted a recently completed research project on seabed liquefaction, which has been reported in ASCE (2006).

In soil mechanics, liquefaction starts to occur when the effective stress of the seabed becomes zero. A useful introduction to liquefaction is given by Sumer and Fredsøe (2002, Chapter 10). Seabed liquefaction may be caused by the passage of waves (Jeng, 1998), earthquakes and other shocks (de Groot et al., 2006a) or the rocking of coastal structures subjected to wave action (de Groot et al., 2006b). Two types of liquefaction have been observed in laboratory test and field trials, namely residual liquefaction and momentary liquefaction. Liquefaction can lead to the reduction in bearing capacity of the soil adjacent to the foundation of a coastal structure. The potential consequences of this include the seabed flowing as a liquid, reduced resistance to the slipping of a coastal structure and settlement of armour stones into the seabed.

Residual liquefaction occurs in loose sand beds due to the progressive increase of residual excess pore pressure. Under a wave crest the pressure at the bed is greater than hydrostatic, so the bed is compressed. Under a wave trough the pressure at the bed is less than hydrostatic, so the bed is dilated. This creates shear stresses in the soil, which will lead to some rearrangement of the grains and a building up of the pore pressure (dissipated by draining). If the pore-water pressure builds up to such an extent that it exceeds the overburden pressure, the soil will liquefy. Residual liquefaction hardly occurs in dense seabeds due to its high shearing resistance, which prevents the excessive build up in pore pressure. Therefore the occurrence of residual liquefaction was not investigated in this study as marine structures are usually founded on a medium dense to dense sand layer.

Momentary liquefaction usually occurs in dense seabeds due to the damping of amplitude and the development of phase lag between the pressures at the seabed surface and lower in the bed. Under the wave trough the pressure at the bed is less than hydrostatic. This pressure decays with depth through the seabed, creating a pressure gradient. If the pressure gradient is sufficiently large it can generate more lift than the submerged weight of the soil above, resulting in momentary liquefaction, which will occur for a fraction of a wave period only (see, for example, Sumer and Fredsøe, 2002, §10.1.2). The rate at which the pressure decays with depth depends on the degree of saturation of the seabed: the greater the degree of saturation, the lower the rate. The pressure gradient decays most slowly with depth in a fully saturated seabed.

Here, an analytical solution for the wave-induced pore pressure response in an isotropic infinite thickness seabed in front of a breakwater, proposed by Jeng (1998) was used to study the liquefaction potential of the seabed in front of coastal defence structures subjected to various wave loadings. The liquefaction potential was determined by calculating the minimum total wave height to depth ratio that will cause the momentary liquefaction of the top 0.05m of a sandy seabed in front of a vertical seawall. Calculations were made for fine, medium fine and coarse sand with the degree of saturation between 0.90 and 1.0, for a range of water depths and a typical storm wave period of 8s. The results can be used to indicate whether liquefaction of the seabed in front of a coastal structure is likely to occur. If so, a more detailed study should be carried out.

2. Methodology

The analytical solution proposed by Jeng (1998) was based on Biot's poro-elastic theory (Biot, 1941) for an infinite and homogenous seabed with assumptions that the soil follows elastic stress-strain law and the fluid is compressible and its motion follow Darcy's law. The degree of saturation in the poro-elastic sediment was governed by the compressibilities of water and gas, which was proposed by Fredlund (1976). Esrig and Kirby (1977) reported that the in-situ values of the degree of saturation S_r for marine sediment normally lie in the range of 90%-100%. Intertidal sediments are more likely to have trapped air (Mory et al, 2004, 2006). Mory *et al* (2006) observed from their field data that the S_r-value of sand bed on the Atlantic coast of France ranged from 94% to 100% for the top 0.5m of the sand bed. Sandven et al. (2006) describe how the gas content can be measured in soils.

The basic assumptions used in the development of the analytical solution for the problem described in this study are:

- Linear wave theory is used to calculate the wave-induced pressure on the surface of the seabed (referred to as the mudline in the geotechnical literature, even for a sandy seabed).
- The wave pressure acting on the seabed is calculated based on the combined wave height, H, from an incident wave (H_i) and reflected wave (H_r) assuming a reflection coefficient of 1.0 (so $H = H_r + H_i = 2H_i$). The maximum standing wave height (H_{max}) is assumed to be 1.6 times the water depth (d).
- The porous seabed is homogeneously unsaturated, hydraulically isotropic and has an infinite thickness.
- The compressibilities of water and gas in the poro-elastic sediment are governed by the degree of saturation.
- The soil skeleton and the pore fluid are uniformly compressible.
- Despite phase lag of the pore pressure in very fine sediments, the soil skeleton generally obeys Hooke's law, implying linear, reversible and non-retarded mechanical properties.
- The flow in the porous bed is assumed to be governed by Darcy's law.
- The porous bed is sandy and the pore pressure in the soil is a result of elastic interaction between soil and water and, thus, neglects dilation effects.
- The structure is assumed to be deeply embedded into the soil matrix.
- Liquefaction is defined as occurring when the mean effective stress is equal to zero at an elevation of 0.05m below the seabed level. The depth of 0.05m was chosen as the smallest depth of liquefaction worth considering.

3. Theoretical solution

The analytical solution was implemented into a MathCAD calculation sheet to determine the wave heights required to cause liquefaction to the soil. The developed equations and coefficients for the analytical solution are given in Appendix A.

The effects of wave height (H) and degree of saturation of the seabed (S_r) on the occurrence of liquefaction to fine sand, medium fine sand and coarse sand beds for four different water depths (2m, 5m, 10m and 15m) were investigated in this study. The degree of saturation of the seabed ranges between 0.9 and 1.0. The adopted permeabilities of fine sand, medium fine sand and coarse sand seabeds are 10^{-4} m/s, 10^{-3} m/s and 10^{-2} m/s respectively (Jeng, 1998). As the model is applicable to homogenous soil, constant permeability and soil stiffness are adopted in the analysis although it is expected that the permeability and the stiffness of seabed vary with depth (permeability decreases and soil stiffness increases with depth). Other typical parameters of the medium dense sand seabed adopted in this study can be found in Table 1.

Table 1 Typical material properties of sand bed

Description	Symbol	Unit	Value
Shear modulus	G	GPa	10
Poisson's ratio	ν		0.3
Porosity	n'		0.3
Coefficient of earth pressure at rest	K_0		0.5
Unit weight of sand	γ_s	kN/m^3	18
Unit weight of water	γ_w	kN/m^3	10

For the wave condition, the wave period is fixed to be 8s and the wave lengths for four different adopted water depths were calculated using linear wave theory (see Appendix A). The wave conditions adopted in the study are shown in Table 2.

Table 2 Wave conditions

Wave Period, T (s)	Water Depth, d (m)	Wave Length, L (m)
8	2	35
8	5	53
8	10	71
8	15	82

A series of parametric studies was carried out in this study and the cases are listed in Table 3. A total of 12 analyses were carried out to examine the minimum wave heights required to liquefy three different types of seabed under the wave condition generated in four different water depths ($H \leq 1.6d$). The degree of saturation of the seabed, S_r, ranged from 0.9 to 1.0. Other input parameters for this study are given in Tables 1 and 2.

Table 3 Parametric study

Case	Seabed Type	Permeability (m/s)	Water depth, d (m)	Degree of Saturation, S_r
1a	Coarse sand	10^{-2}	2	0.9-1.0
1b	Medium fine sand	10^{-3}	2	0.9-1.0
1c	Fine sand	10^{-4}	2	0.9-1.0
2a	Coarse sand	10^{-2}	5	0.9-1.0
2b	Medium fine sand	10^{-3}	5	0.9-1.0
2c	Fine sand	10^{-4}	5	0.9-1.0
3a	Coarse sand	10^{-2}	10	0.9-1.0
3b	Medium fine sand	10^{-3}	10	0.9-1.0
3c	Fine sand	10^{-4}	10	0.9-1.0
4a	Coarse sand	10^{-2}	15	0.9-1.0
4b	Medium fine sand	10^{-3}	15	0.9-1.0
4c	Fine sand	10^{-4}	15	0.9-1.0

4. Results and discussion

The parametric study results for cases 1a-1c, 2a-2c, 3a-3c and 4a-4c are presented in Figures 1, 2, 3 and 4 respectively. The wave height, H, presented in the figures is the wave height of the combined incident and reflected waves. Liquefaction occurs to the seabed when both wave condition and seabed condition fall into the area above the line. For instance, the medium fine sand bed, with S_r-value of 0.98, starts to liquefy under the wave condition with wave height greater than 1.3*d* (*d* = 2m, see Figure 1). In Figure 1, it shows that no liquefaction will occur to coarse sand seabed under the extreme wave condition of $H_{max} = 1.6d$.

Figures 1 to 4 show that the liquefaction potential increases with a decrease in permeability. The results show that the liquefaction hardly occurs to the coarse sand seabed with a water depth less than 5m (shallow water). Due to low permeability of fine sand, the fine sand seabed tends to liquefy more easily than the coarser sand bed with a higher permeability. Moreover, the wave height leading to liquefaction increases with an increase in degree of saturation. This is mainly due to the damping of amplitude of pore pressure as the amplitude of the pore pressure gradient in the seabed decreases with an increasing degree of saturation.

Generally, the wave height for the occurrence of liquefaction in fine sand seabed increases sharply when the degree of saturation is greater than 99%. No momentary liquefaction can possibly occur to fully saturated coarse sand seabed under the most severe defined wave condition ($H = 1.6d$).

Table 4 shows the minimum fully reflected wave height required to liquefy the seabed to a depth of 0.05m, for different water depths and with degrees of saturation of 90%, 95% and 100%. For the unsaturated fine sand seabed with a sea depth of 2m, the seabed could liquefy with a wave height as small as 0.4m. For the deep water case (d = 15m), the unsaturated fine sand seabed could liquefy under the wave condition with a wave height of 0.9m. No liquefaction can possibly occur to fully saturated seabed in shallow water ($d \leq 5$m).

Table 4 Minimum wave height required to cause the occurrence of liquefaction to seabed

Wave Height (m) / Water depth (m) / Sand Type	**2**	**5**	**10**	**15**
Coarse (S_r = 90%)	3.4	4.3	6.1	8.3
Medium fine (S_r = 90%)	1.1	1.4	2.0	2.7
Fine (S_r = 90%)	0.4	0.5	0.6	0.9
Coarse (S_r = 95%)	4.4	5.8	8.3	11.4
Medium fine (S_r = 95%)	1.6	2.0	2.8	3.8
Fine (S_r = 95%)	0.5	0.6	0.7	1.2
Coarse (S_r = 100%)	-	-	-	-
Medium fine (S_r = 100%)	-	-	-	24.0
Fine (S_r = 100%)	-	-	14.2	18.7

5. Simplified assessment approach

Figure 5 presents the minimum height of standing wave required to induce momentary liquefaction to a depth of 0.05m in a seabed with Sr-value of 0.95 at various water depths. This figure can be used to estimate the minimum wave height required to induce liquefaction to seabed. An S_r-value of 0.95 was selected in the plot because the typical air content of an inter-tidal sand bed is approximately 5% (Mory *et al*, 2006).

To assess liquefaction potential with Figure 5 first select the water depth, d, and then determine the combined wave height, H from the incident wave height, Hi, i.e. H = 2Hi. If the value of H is greater than 1.6d then H is limited to H = 1.6d. Select the most representative bed sediment grading and if the value of H is equal to or greater than the value of H on the y-axis then momentary liquefaction can occur.

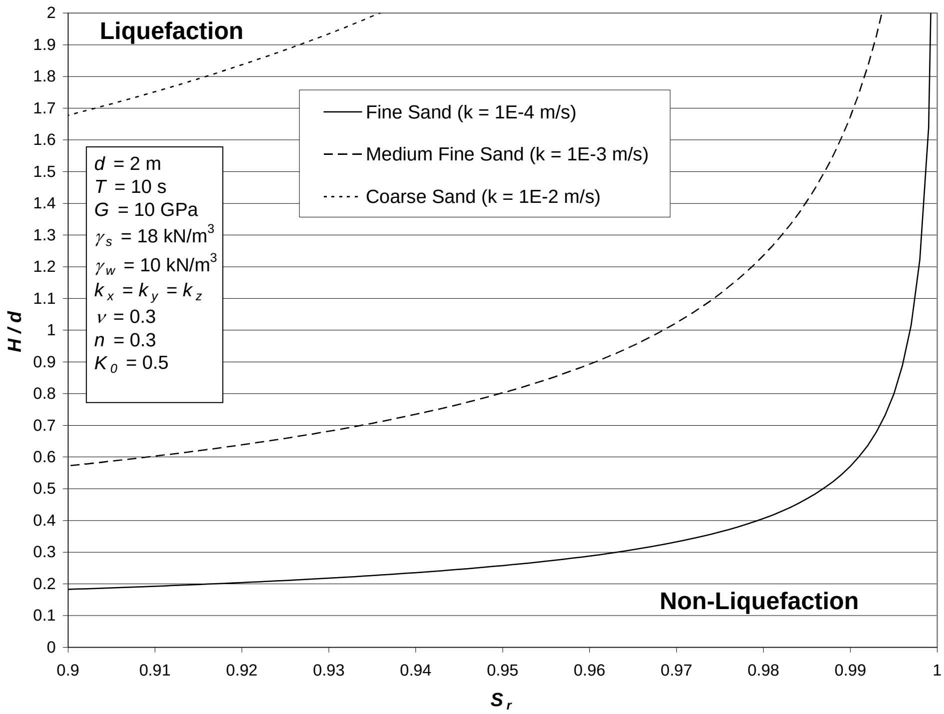

Figure 1 Wave-induced liquefaction potential around a marine structure founded on a saturated/unsaturated seabed subjected to various standing wave loadings with a water depth of 2m

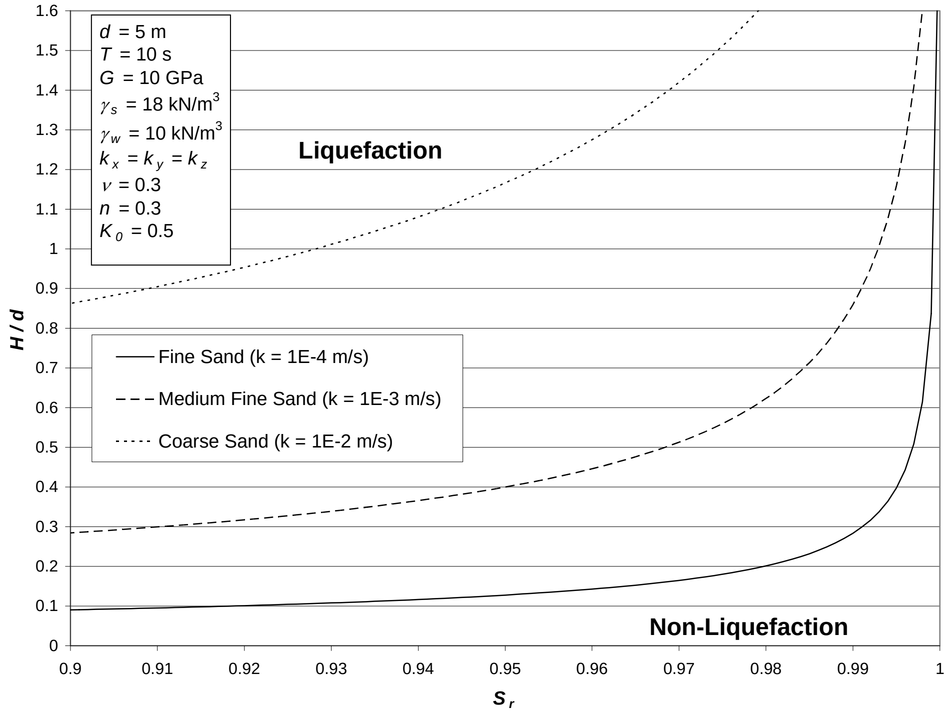

Figure 2 Wave-induced liquefaction potential around a marine structure founded on a saturated/unsaturated seabed subjected to various standing wave loadings with a water depth of 5m

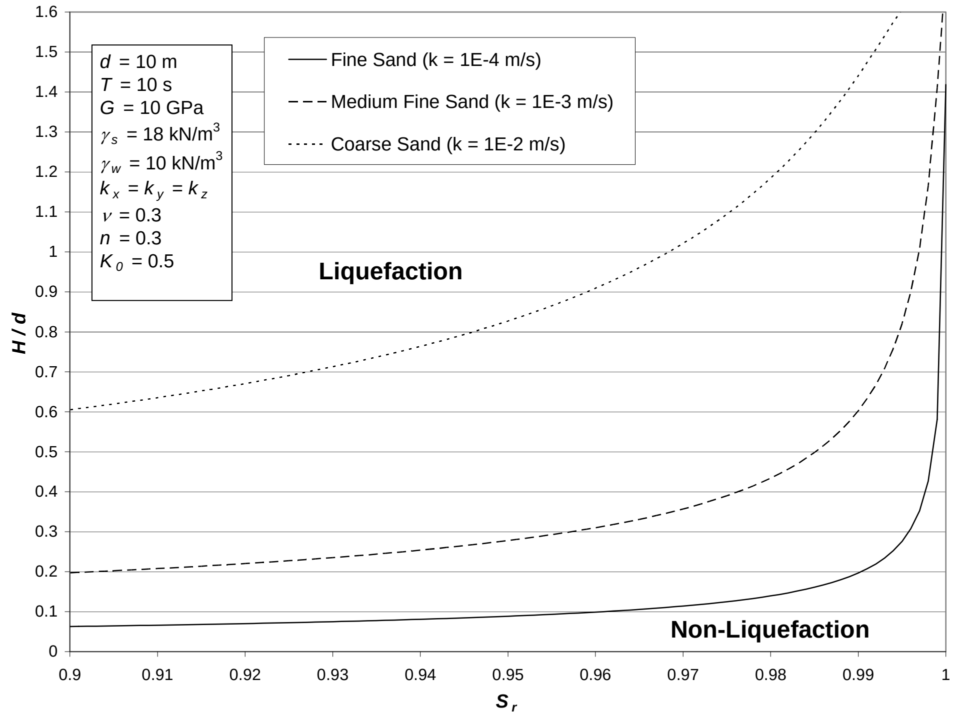

Figure 3 Wave-induced liquefaction potential around a marine structure founded on a saturated/unsaturated seabed subjected to various standing wave loadings with a water depth of 10m

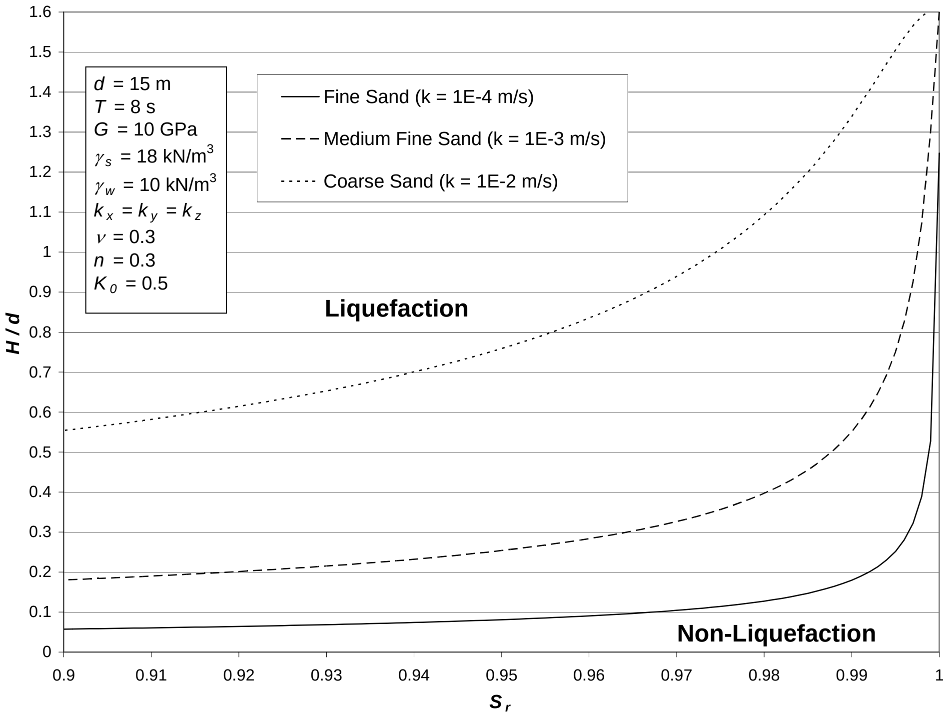

Figure 4 Wave-induced liquefaction potential around a marine structure founded on a saturated/unsaturated seabed subjected to various standing wave loadings with a water depth of 15m

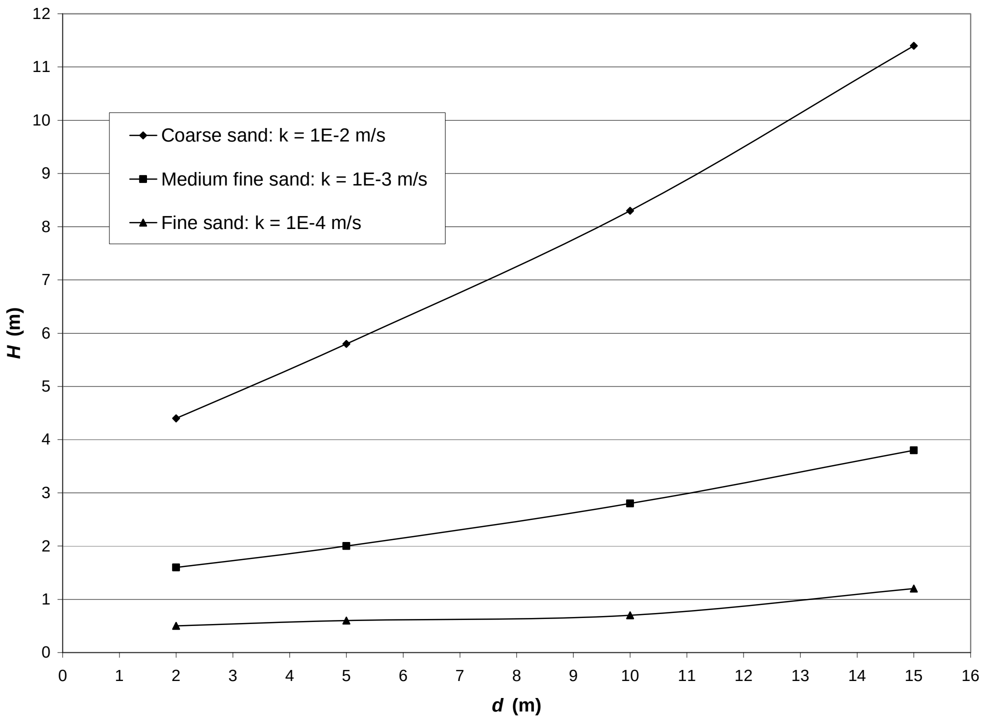

Figure 5 Wave heights required to liquefy 3 different types of sand bed (with degree of saturation of 0.95) at various water depths

6. Conclusion

The following conclusion about wave induced momentary liquefaction can be drawn from this study:

- The likelihood of the occurrence of momentary liquefaction increases with a decrease in seabed permeability, which is associated with a decrease in grain size. A seabed of fine sand is therefore more likely to experience momentary liquefaction than a seabed of coarse sand;
- The likelihood of the occurrence of momentary liquefaction increases with a decrease in the degree of saturation of the seabed;
- The wave height required to liquefy a fine sand seabed increases significantly when the degree of saturation of the seabed increases higher than 0.995;
- An S_r-value of 0.95 is recommended for the estimation of the minimum wave height required to liquefy the seabed, in the absence of a site-specific study;
- Figures 1 to 5 can be used to provide a quick check on the potential for momentary liquefaction of the top 0.05m of the seabed. If the potential for momentary liquefaction exists, a more detailed, site-specific study can be carried out by adapting the Mathcad code provided in Appendix A or using another liquefaction model.

7. References

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Appendix A MathCAD Calculation Sheet

Project	LOWER BEACHES	Calc no.		Project Engineer	JAS
Contract		Filename ref		Designer	PLV
Section	Review	Job No.	CBS0726	Department	Engineering
Subject	Wave-induced liquefaction of sediment in the vicinity of coastal structures				

	Total Sheets	Mathcad Made by	Date	Cal Made by	Date	Checked by	Date		
ORIGINAL	8	PLV	17/07/06	PLV	17/07/06	SDU	18/07/06		
REV									
REV									
REV									
REV									
REV									
Superseded by Calculation no.				Date					

Objective of Calculation:

Liquefied depth in the vicinity of costal structures found in a seabed with degree of saturation ranges from 0.9 to 1.0

Description of Calculation:

Linear wave theory to calculate the wave pressure acting on the seabed

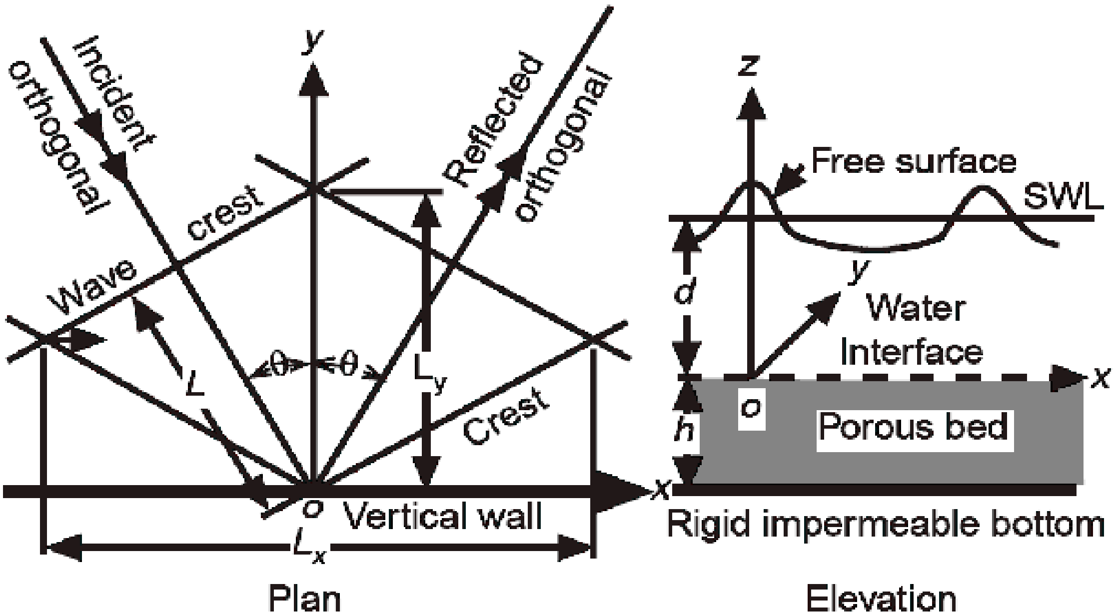
Calculating the pore pressure in seabed at different depths

Calculating the maximum liquefied depth

Calculating the minimum wave heights required to liquefy seabed at various degrees of saturation of seabed

METHOD

Analytical solutions for wave-induced soil response in an either saturated or unsaturated seabed of infinite thickness seabed developed by Jeng (1998). This solution is only application for isotropic soil condition.



INPUT DATA

Sea and wave conditions

$d := 5m$	Water depth
$H_s := 2m$	Wave height
$T_s := 8s$	Wave period
$\gamma_w := 10\frac{kN}{m^3}$	Unit weight of water
$\theta_s := 0deg$	Wave obliquity

Seabed condition

$\gamma_s := 18 \frac{kN}{m^3}$	Unit weight of seabed
$K_w := 2GPa$	Bulk modulus of elasticity
$p_{atm} := 1atm$	Atmospheric pressure
$S_r := 0.95$	Degree of saturation of seabed
$\begin{pmatrix} k_{x1} \\ k_{y1} \\ k_{z1} \end{pmatrix} := \begin{pmatrix} 10^{-4} \\ 10^{-4} \\ 10^{-4} \end{pmatrix} \frac{m}{s}$	Permeabilities of fine sand in x, y and z directions
$\begin{pmatrix} k_{x2} \\ k_{y2} \\ k_{z2} \end{pmatrix} := \begin{pmatrix} 10^{-3} \\ 10^{-3} \\ 10^{-3} \end{pmatrix} \frac{m}{s}$	Permeabilities of medium fine sand in x, y and z directions
$\begin{pmatrix} k_{x3} \\ k_{y3} \\ k_{z3} \end{pmatrix} := \begin{pmatrix} 10^{-2} \\ 10^{-2} \\ 10^{-2} \end{pmatrix} \frac{m}{s}$	Permeabilities of coarse sand in x, y and z directions
$\nu := 0.3$	Poisson's ratio of seabed
$G_s := 10GPa$	Shear modulus of seabed
$n' := 0.3$	Seabed porosity
$K_0 := 0.5$	Earth pressure coefficient at rest

Output control

$\Delta t := \frac{T_s}{4}$	Time interval
$x := 0m$	Distance in x direction
$y := 0m$	Distance in y direction
$ST := 1.0$	Maximum degree of saturation
$END := 0.9$	Minimum degree of saturation
$INT := -0.001$	Interval of defined degree of saturation

ASSUMPTIONS

Linear wave theory is used to calculate the wave-induced pressure on mudline.

CALCULATIONS

Design parameters

$$f(k_t) := g \cdot k_t \cdot \tanh(k_t \cdot d) - \frac{4 \cdot \pi^2}{T_s^2}$$

$k_t := 0.1m^{-1}$

$r0 := \mathrm{root}(f(k_t), k_t)$

$k := r0$	Wave number	$k = 0.1184 \frac{1}{m}$
$L_s := \frac{2 \cdot \pi}{k}$	Wave length	$L_s = 53.0714m$
$p_0 := \frac{\gamma_w \cdot H_s}{2 \cdot \cosh(k \cdot d)}$	Wave pressure amplitude	$p_0 = 8.4718 \times 10^3$ Pa
$m_s := \sin(\theta_s)$		$m_s = 0$
$n_s := \cos(\theta_s)$		$n_s = 1$
$\beta := \frac{1}{K_w} + \frac{1 - S_r}{p_{atm} + d \cdot \gamma_w}$	Compressibility of pore fluid	$\beta = 330.9147 \frac{1}{GPa}$
$\omega := \frac{2 \cdot \pi}{T_s}$	Angular frequency of wave	$\omega = 0.7854 \frac{1}{s}$
$k1 := \begin{pmatrix} k_{x1} \\ k_{y1} \\ k_{z1} \end{pmatrix}$	Permeabilities of fine sand in x, y and z directions	$k1 = \begin{pmatrix} 1 \times 10^{-4} \\ 1 \times 10^{-4} \\ 1 \times 10^{-4} \end{pmatrix} \frac{m}{s}$
$k2 := \begin{pmatrix} k_{x2} \\ k_{y2} \\ k_{z2} \end{pmatrix}$	Permeabilities of medium fine sand in x, y and z directions	$k2 = \begin{pmatrix} 1 \times 10^{-3} \\ 1 \times 10^{-3} \\ 1 \times 10^{-3} \end{pmatrix} \frac{m}{s}$
$k3 := \begin{pmatrix} k_{x3} \\ k_{y3} \\ k_{z3} \end{pmatrix}$	Permeabilities of coarse sand in x, y and z directions	$k3 = \begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix} \frac{m}{s}$

Output control

Type :=
Fine Sand
Medium Fine Sand
Coarse Sand

$$k_x := \begin{vmatrix} k1_0 & \text{if Type} = 1 \\ k2_0 & \text{if Type} = 2 \\ k3_0 & \text{otherwise} \end{vmatrix} \qquad k_y := \begin{vmatrix} k1_1 & \text{if Type} = 1 \\ k2_1 & \text{if Type} = 2 \\ k3_1 & \text{otherwise} \end{vmatrix} \qquad k_z := \begin{vmatrix} k1_2 & \text{if Type} = 1 \\ k2_2 & \text{if Type} = 2 \\ k3_2 & \text{otherwise} \end{vmatrix}$$

$$\text{Range2Vec}(st, end, int) := \begin{vmatrix} N \leftarrow \text{ceil}\left(\frac{end - st}{int}\right) \\ v_0 \leftarrow st \\ \text{for } j \in 1..N \\ \quad v_j \leftarrow j \cdot int + st \\ \text{return } v \end{vmatrix}$$

$$t1 := \text{Range2Vec}(0, T_s, \Delta t)$$

$$t1^T = (0\ \ 2\ \ 4\ \ 6\ \ 8)\ s$$

$$\Delta z := \frac{-L_s}{20}$$

$$z := \text{Range2Vec}\left(0, \frac{-L_s}{2}, \Delta z\right)$$

$z^T =$

0	-2.65	-5.31	-7.96	-10.61	-13.27	-15.92	-18.58	-21.23	-23.88	-26.54

m

$$t := t1_0 \rightarrow 0$$

Coefficients

$$\delta_1 := \sqrt{k^2\cdot\left(\frac{k_x}{k_z}\cdot m_s{}^2 + \frac{k_y}{k_z}\cdot n_s{}^2\right) - \frac{i\gamma_w\cdot\omega}{k_z}\left[n'\cdot\beta + \frac{1-2\cdot\nu}{2\cdot G_s\cdot(1-\nu)}\right]}$$

$$\lambda_1 := \frac{(1-2\cdot\nu)\cdot\left[k^2\cdot\left(1-\frac{k_x}{k_z}\cdot m_s{}^2-\frac{k_y}{k_z}\cdot n_s{}^2\right)+\frac{\gamma_w\cdot\omega}{k_z}\cdot n'\cdot\beta\cdot i\right]}{k^2\cdot\left(1-\frac{k_x}{k_z}\cdot m_s{}^2-\frac{k_y}{k_z}\cdot n_s{}^2\right)+\frac{\gamma_w\cdot\omega}{k_z}\left(n'\cdot\beta+\frac{1-2\cdot\nu}{G_s}\right)\cdot i}$$

$$C_1 := \frac{-\lambda_1\cdot\left[\nu\cdot(\delta_1-k)^2-\delta_1\cdot(\delta_1-2\cdot k)\right]}{k\cdot(\delta_1-k)\cdot(\delta_1-\delta_1\cdot\nu+k\cdot\nu+k\cdot\lambda_1)}$$

$$C_2 := \frac{\delta_1-\delta_1\cdot\nu+k\cdot\nu}{\delta_1-\delta_1\cdot\nu+k\cdot\nu+k\cdot\lambda_1}$$

$$C_3 := \frac{k\cdot\lambda_1}{(\delta_1-k)(\delta_1-\delta_1\cdot\nu+k\cdot\nu+k\cdot\lambda_1)}$$

Liquefaction Depth

$$f(z_L) := -\frac{1+2\cdot K_0}{3}\cdot(\gamma_s-\gamma_w)\cdot z_L - p_0 \cdots$$
$$+\left|\frac{p_0}{1-2\cdot\nu}\cdot\left[(1-2\cdot\nu-\lambda_1)\cdot C_2\cdot e^{k\cdot z_L}+\frac{\delta_1{}^2-k^2}{k}\cdot(1-\nu)\cdot C_3\cdot e^{\delta_1\cdot z_L}\right]\cdot\cos\left[n_s\cdot k\cdot y\cdot e^{(m_s\cdot k\cdot x-\omega\cdot t)\cdot i}\right]\right|$$

$$z_L := -10\,\mathrm{m}$$

$$z_L := \mathrm{root}(f(z_L), z_L)$$

$$z_L = -1.5085\,\mathrm{m}$$

Minimum Wave Height for the Occurrence of Liquefaction

$z_L := -0.001\ \text{m}$

$$
\begin{aligned}
\text{Answer} :=\ & N \leftarrow \frac{END - ST}{INT} \\
& \text{for } j \in 0..N \\
& \quad S_{r_j} \leftarrow ST + INT \cdot j \\
& \quad \beta_j \leftarrow \frac{1}{K_w} + \frac{1 - S_{r_j}}{p_{atm} + d \cdot \gamma_w} \\
& \quad \delta_j \leftarrow \sqrt{k^2 \cdot \left(\frac{k_x}{k_z} \cdot m_s^{\ 2} + \frac{k_y}{k_z} \cdot n_s^{\ 2}\right) - \frac{i \cdot \gamma_w \cdot \omega}{k_z} \cdot \left[n' \cdot \beta_j + \frac{1 - 2 \cdot \nu}{2 \cdot G_s \cdot (1 - \nu)}\right]} \\
& \quad \lambda_j \leftarrow \frac{(1 - 2\cdot\nu)\cdot\left[k^2 \cdot \left(1 - \frac{k_x}{k_z}\cdot m_s^{\ 2} - \frac{k_y}{k_z}\cdot n_s^{\ 2}\right) + \frac{\gamma_w \cdot \omega}{k_z}\cdot n' \cdot \beta_j \cdot i\right]}{k^2 \cdot \left(1 - \frac{k_x}{k_z}\cdot m_s^{\ 2} - \frac{k_y}{k_z}\cdot n_s^{\ 2}\right) + \frac{\gamma_w \cdot \omega}{k_z}\cdot\left(n' \cdot \beta_j + \frac{1 - 2\cdot\nu}{G_s}\right)\cdot i} \\
& \quad C_{2_j} \leftarrow \frac{\delta_j - \delta_j \cdot \nu + k \cdot \nu}{\delta_j - \delta_j \cdot \nu + k \cdot \nu + k \cdot \lambda_j} \\
& \quad C_{3_j} \leftarrow \frac{k \cdot \lambda_j}{(\delta_j - k)\cdot(\delta_j - \delta_j \cdot \nu + k \cdot \nu + k \cdot \lambda_j)} \\
& \quad H_{L_j} \leftarrow \frac{\frac{1 + 2 \cdot K_0}{3} \cdot (\gamma_s - \gamma_w) \cdot z_L \cdot 2 \cdot \cosh(k \cdot d)}{\gamma_w \cdot \left[-1 + \left|\frac{1}{1 - 2\cdot\nu}\cdot\left[(1 - 2\cdot\nu - \lambda_j)\cdot C_{2_j} \cdot e^{k \cdot z_L} + \frac{(\delta_j)^2 - k^2}{k}\cdot(1 - \nu)\cdot C_{3_j}\cdot e^{\delta_j \cdot z_L}\right]\cdot \cos\left[n_s \cdot k \cdot y \cdot e^{(m_s \cdot k \cdot x - \omega \cdot t)\cdot i}\right]\right|\right]} \\
& \text{return } H_L
\end{aligned}
$$

Answer = (m)

	0
0	9.7819
1	4.1847
2	3.0731
3	2.5415
4	2.2151
5	1.9889
6	1.8202
7	1.6883
8	1.5814
9	1.4925
10	1.4172
11	1.3522
12	1.2954
13	1.2452
14	1.2004

$$\frac{\gamma_w\cdot H}{2\cdot\cosh(k\cdot d)}\cdot\left[1-\left|\frac{1}{1-2\cdot\nu}\cdot\left[(1-2\cdot\nu-\lambda_j)\cdot C_{2_j}\cdot e^{k\cdot z_L}+\frac{(\delta_j)^2-k^2}{k}\cdot(1-\nu)\cdot C_{3_j}\cdot e^{\delta_j\cdot z_L}\right]\cdot\cos\left[n_s\cdot k\cdot y\cdot e^{(m_s\cdot k\cdot x-\omega\cdot t)\cdot i}\right]\right|\right]$$

Plotting

$$xS_r := \left| \begin{array}{l} N \leftarrow \dfrac{END - ST}{INT} \\ \text{for } j \in 0..\,N \\ \quad S_{r_j} \leftarrow ST + INT\cdot j \\ \text{return } S_r \end{array} \right.$$

x-axis: Degree of saturation

$H_{wave} :=$ Answer

y-axis: Wave height