Department
for Environment
Food & Rural Affairs

Modelling agriculture in the UK

Documentation for the UK Agricultural Market Model (UKAMM)

February 2021

Modelling agriculture in the UK

Defra, like all government departments, uses economic analysis to better understand the sectors we work with and regulate. Modelling is one key part of this. We created, maintain and operate the UK Agricultural Market Model (UKAMM) to inform our understanding of UK agriculture.

We treat its results as 'expert in the room' alongside qualitative analysis, social research, stakeholder insights and more to supply evidence to policymakers.

This document explains the key economic concepts underpinning the way UKAMM approaches the UK's agricultural economy.

It then gives an in-depth breakdown of how it models each sector and commodity. By lifting the lid on one of our key agricultural economic tools, we hope to promote the kind of dialogue and scrutiny that challenges our analysis to be as current and robust as possible.

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Modelling agriculture in the UK

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How does the size of the UK's dairy herd change over time? How does a change in GDP affect how much food we buy? How likely are arable farmers to change what they sow when crop prices change? Each of these questions could be an analysts' lifetime's work. Each question hides untold complexity. The dairy herd varies by region, has a variety of breeds and faces a range of health challenges.

Changes in GDP arising from construction, finance or healthcare will filter down to people's food consumption patterns in different ways, or not at all. Arable farmers change what they sow and the way they make decisions evolves from one year to the next.

Understanding agriculture is critical to Defra's role with the sector. Defra economists work closely with other government departments, academics and industry to answer these questions and hundreds more.

The UK Agricultural Market Model (UKAMM) lets Defra economists explore interconnections in a measured and systematic way. It creates simplified best estimates to these hundreds of questions into projections for the future of UK agriculture. Mapping out the five critical parts of the UK agricultural system: meat, dairy, arable, oilseeds, sugar including how their production and consumption in the UK relates to their international trade.

UKAMM allows Defra economists to imagine changes in one corner of the system and examine how ripples might spread.

There is no exact forecasting of the future as there no telling what the future holds. But this model is used as expert in the room with a variety of other insights including government analytical professions, stakeholder intelligence, academic research. This model is a simplified overview of the sector is an excellent tool for exploring system-wide impacts and avoiding unintended consequences.

UKAMM is part of a wider modelling ecosystem, compared and contrasted with other models across Whitehall and feeds into the Aglink-Cosimo model developed the Organisation for Economic Co-operation and Development (OECD) and the Food and Agriculture Organization (FAO).
This documentation is part of that global dialogue and we look forward to further collaboration with the OECD and our counterparts around the world in years to come.

Foreword – a global perspective

Marion Jansen

Director, OECD Trade and Agriculture Directorate

The COVID-19 crisis and significant recent trade policy disruptions have highlighted the need to anticipate future developments in policies and markets and to prepare for potential shocks.

The OECD addresses this demand by collecting data, analysing risks and policies, and exploring trends using a range of models like Aglink-Cosimo in the case of agricultural markets. Developing and improving these tools is a never-ending and complex undertaking relying on valued partnerships across organisations to bring together the necessary expertise and experience.

The long-standing collaboration with Defra is one such partnership, which has resulted in the creation of the UK Agricultural Market Model. It will allow Defra to assess the potential market impacts of policy proposals before they are implemented.

This is especially important as the United Kingdom is redesigning its agricultural policy. The OECD stands ready to support this undertaking with its broad range of evidence and analysis.

1 Introducing UKAMM

The UK Agricultural Market Model – UKAMM for short – is a set of mathematical equations that describe economic relationships in the arable crops, livestock, dairy, oilseed processing and sugar sectors.

These economic relationships include production processes, consumption patterns and international trade flows. We use specialist software to solve these equations simultaneously using data about UK agriculture today and in the past.

The result is a projection of the state of UK agriculture for the next ten years. This projection provides annual estimates of prices, quantities produced, and quantities demanded of the agricultural commodities in the model.

This is a projection, not a forecast. We cannot foresee the events that will shape the future of UK agriculture over the next ten years. Nor can we perfectly predict how policies currently in formulation will end up shaping Defra sectors.

What we can provide are benchmark orders of magnitude for key variables. We use this baseline for comparison when analysing the impacts of policy proposals or hypothetical scenarios.

This is a living model. The agricultural economy and the agricultural policy affecting it change and our model changes with it, not least when new data comes out each year.

Defra economists constantly update and review UKAMM. That is particularly true when we are using the model for policy analysis, which usually involves making ad hoc changes to the model.

It may also mean that this documentation does not reflect the exact modified version of the model used when analysing policies and scenarios.

UKAMM is a dynamic partial equilibrium model. As a **partial** equilibrium model, it examines the way markets work in one sector of the economy without looking more widely at the relationship between agriculture and other sectors.

It estimates **equilibrium** prices: these are the prices at which the amount of each commodity supplied by producers would equal the amount demanded by consumers.

As a **dynamic** model, UKAMM projects how UK agriculture might change over time. UKAMM deals in years nots weeks or months. It accounts for ways in which economic conditions one year might affect conditions the next without covering short term horizons in which prices may fluctuate around equilibrium.

UKAMM is closely related to the OECD and FAO’s Aglink-Cosimo model of global agriculture.

Many of UKAMM’s sectors draw on Aglink-Cosimo’s structure and naming system and Defra’s UKAMM team play an active part in the associated global conversations on agricultural modelling.

One of UKAMM’s key uses is providing projections which are then used as UK variables in Aglink-Cosimo.

Another model related to UKAMM is Food and Agricultural Policy Research Institute (FAPRI), a dynamic partial equilibrium model of UK agriculture operated by Agri-Food and Biosciences Institute of the Northern Ireland Executive. The two models cover similar terrain in different ways.

UKAMM has more of a trade focus, for example, and FAPRI separates results between the four UK nations unlike UKAMM.

1.1 Assumptions

Like every model, UKAMM makes simplifying assumptions. We need to understand them to know when UKAMM is a suitable tool and when it is not.

Specific assumptions about commodities and sectors are explained in the sector-specific sections of this document.

Table 1: General economic assumptions underpinning UKAMM's illustration of the agricultural economy.

Assumption	Implications
The UK is a 'small' agricultural economy and does not affect other countries prices.	UKAMM treats overseas prices as fixed. In markets where the UK is a significant player, some effects might be missing. For example, an increase in UK demand for wheat imports would appear to have no impact on EU or world wheat prices.
Agriculture does not have an impact on the rest of the economy.	Macro-economic variables like GDP are exogenous, meaning that even if UKAMM projects the size of the UK agricultural economy to increase, GDP will not change. GDP is a factor in UKAMM's demand equations so the equilibrium points at which markets clear would not reflect macroeconomic effects of sector growth.
In the end, demand always equals supply.	There are many reasons markets might not clear to equilibrium: incomplete information, barriers to firm entry or exit and supply chain disruption to name a few. For some commodities UKAMM does model the way stocks can be used to balance supply and demand in any given year.
Past behaviour and relationships are a good guide to the future.	UKAMM's parameters are generated using analysis of historic data. The scenarios it generates assume actors respond to change in the future in similar ways to how they have responded to change in the past.
Each unit of a given commodity is identical in quality and consumer appeal.	By assuming goods are homogenous, UKAMM may overstate how likely imports are to replace UK produce and how likely UK produce is to replace imports. The model assumes traders only differentiate broadly between EU and non-EU goods. They are assumed to have no specific preferences for goods from one country or another.

Production is based on current and past economic conditions, not foresight of future conditions.	UKAMM does not simulate economic 'agents' with forward-looking, rational expectations. It assumes production depends on economic conditions in current or previous years, akin to behaviour based on adaptive expectations. This means future policy changes cannot be expected in the model and therefore does not affect production and consumption prior to their actual implementation in the model's time horizon.

1.2 The model piece by piece

UKAMM models agricultural sectors separately while accounting for ways in which they interact. The model covers:

- cereals
- livestock
- dairy
- oilseeds
- sugar

Each commodity has its own set of equations covering prices, production, consumption and trade.

1.2.1 Commodities

UKAMM covers agricultural commodities in the arable, oilseeds, sugar, livestock and dairy sectors. The model covers around 87% of the value of UK agricultural production. There is a list of all commodities modelled in UKAMM in the 'abbreviations' section. There are several important agricultural sectors not covered by UKAMM such as horticulture, potatoes and non-animal proteins like nuts and pulses.

1.2.2 Prices

In UKAMM, the price paid to producers for their goods is what 'clears' the market. Let us unpack what that means. When we solve the model, we find the 'equilibrium' point at which the amount of each commodity produced or imported supplied equals the amount demanded or exported.

Every equation describing how much of a given commodity is produced, imported, demanded or exported depends on price. When we solve the model, our specialist software adjusts the price level for each commodity until it finds the point at which each sector is in equilibrium.

We will explain this again using UKAMM's equations in the sections for each sector.

UKAMM also calculates an estimate of consumer prices.

It does this differently for each commodity, but it often does this by multiplying producer prices by an externally sourced estimate for how much more consumers pay for a product than producers get paid for it, for example a mark-up.

1.2.3 Production

Each agricultural sector is completely different. As such UKAMM uses hugely different equations to describe these hugely different economic and material relationships. What these equations have in common is that they will incorporate inputs used in production like energy, animal feed, fertilisers, seeds and more.

Exactly what these inputs are is specific to each sector. Some sectors need to account for hard limits, such as the amount of land area available.

UKAMM's production equations also account for how producers make decisions and how long they take to implement.

Often, UKAMM assumes producers make adaptive decisions, meaning they decide how much to produce each year based on outcomes in previous years.

Where poultry farmers can adjust how many birds they keep within a matter of months, it takes years for cattle farmers to adjust the size of their herds. So, each sector takes different amounts of time to adjust to changes in the market.

Subsidies are another factor in many production equations. Chapter 3 offers a dedicated explanation of how UKAMM accounts for market support.

1.2.4 Consumption

Each commodity in UKAMM is used for different purposes. Each of these purposes is reflected in the equations describing how much of each commodity is demanded each year.

Demand for meat or sugar, for example, depends on food demand alone. UKAMM models food demand as dependent on population, economic growth and price elasticities of demand.

Price elasticities are a measure of how responsive a given economic variable is to price. Price elasticities of demand are measures of how likely consumers are to increase or reduce our consumption in response to changes in price.

Products have low elasticities if consumers are reluctant to change how much they buy as prices change. If a product has a high elasticity, consumers are more likely to change their habits, reducing their consumption when the price rises and increasing it when the price falls.

1.2.5 Interlinkages

There are interlinkages between different agricultural sectors and many of these are captured in UKAMM's equations. One example is the use of arable crops as animal feed. The cost of arable crops is a factor in the production equations for livestock. The demand for animal feed is a factor in the demand equation for each arable crop.

Sectors are also linked by 'cross-price elasticities' between commodities. These describe how readily buyers will switch between products when one becomes cheaper than the other.

So, if pigmeat gets cheaper than other types of meat, these elasticities give a measure for how likely consumers are to switch meats. UKAMM also contemplates this sort of substitution by producers, with relative prices being a factor in livestock farmers' decisions about what to feed their animals and in arable farmers' decisions about how much of their land to dedicate to each crop, to give two examples.

1.2. 6 International trade

UKAMM's equations describe flows of imports and exports to and from the UK and how these flows affect domestic production and consumption. The model accounts for tariffs, quotas and other trade policies.

The structure of the trade equations is broadly the same for each commodity, so we summarise them in a resolute 'trade' section rather than alongside each sector's price, production and consumption equations.

UKAMM is a 'three-country' model. Rather than modelling each of the UK's trading partners separately, UKAMM aggregates trade with the EU into one bloc and trade with the rest of the world into a third bloc.

UKAMM only models the UK agricultural economy. EU and world prices are drawn from external data sources.

1.2.7 Data

UKAMM uses annual data to solve its equations and make projections. Defra's annual Agriculture in the UK compendium of statistics is the main data source, compiling statistics from Defra, the Welsh Government, Scottish Government and Northern Ireland Executive.

Other data comes from the Agriculture and Horticulture Development Board (AHDB), the OECD's Aglink model and other government departments. UKAMM also draws on outside sources for parameters in equations.

Some of these come from academic literature, some come from the OECD and some come from Defra's own analysis.

1.3 What a UKAMM equation looks like

UKAMM has equations of two types: linear and logarithmic. The linear equations all have the same structure.

The logarithmic equations vary. Many of the equations that describe firm or consumer behaviour are 'log-log,' meaning that both the dependent and independent variables are expressed as logarithms like this:

$$\mathbf{log}[\boldsymbol{Y}] = \alpha + \beta * \log[X] + \log[R]$$

Where Y is the dependent **variable**, α is the **constant**, β is the **parameter**, X is the independent **variable**, and R is the **residual**.

You will see lots of equations of this form in the documentation, such as the following equation for butter production. In this equation logarithms are taken of both the dependent variable, $QP_{BT,t}$, the quantity of butter produced, and the independent variables, $PP_{BT,t}$ and $PP_{FAT,t}$, the producer prices of butter and the fat component of milk respectively:

$$Log(QP_{BT,t}) = \boldsymbol{\alpha} + \boldsymbol{\beta_{QP_{BT}PP_{BT}}} \cdot \boldsymbol{Log}\left(\frac{\boldsymbol{PP_{BT,t}}}{\boldsymbol{PP_{FAT,t}}}\right) + \boldsymbol{R_{QP,BT}}$$

Variables

There are two types of variable in UKAMM. The value of *endogenous* variables is calculated by equations within the model.

The value of **exogenous** variables come from outside the model, sometimes directly from data sources like Agriculture in the UK and sometimes calculated by Defra analysts.

Each variable has specific values for different regions, commodities and years. The variable's label refers to the economic concept it is seeking to describe, such as crop yield.

The region can be any of the three blocks in UKAMM: UK, EU or the rest of the world.

Not all variables exist for every commodity: crop yield is available for wheat and oats, for example, but clearly not for sheepmeat or butter.

In this documentation we use subscript after the variable label to denote the region, commodity and time period referred to. By default, we refer to the UK, so if no region is specified, we are referring to the UK version of the variable. To give an example:

$$YLD_{UKM,WTS,2019}$$

The label is crop yield, denoted YLD. The region is United Kingdom, denoted UKM. The commodity is wheat, denoted WTS. The year in question is 2019. A full list of UKAMM abbreviations is in section 1.4.

Constants

Constants shift an equation's values without altering what effect a change in the independent variable's value has on the value of the dependent variable. The constants in UKAMM calibrate the model to historical data by shifting the trend line for each variable up or down to ensure it starts from a position in line with recent historic levels.

We use specialist software to calculate these calibrating constants as part of the process of using UKAMM to generate projections. In this documentation we label this constant α in all equations. The model calculates a unique constant for each equation in the model.

Elasticities

An elasticity shows the size by which the dependent variable would change after a given change in the independent variable, assuming other variables are held constant. The value of UKAMM's elasticities stays constant across the whole simulation period.

For example, the elasticity between wheat yield ($YLD_{WTS,t}$) and wheat price ($PP_{WTS,t}$) tells us by what degree wheat yield would change were the wheat price to rise or fall. In the documentation we format this parameter as $\beta_{YLD_{WTS}PP_{WTS}}$.

Residuals

Residuals are also referred to as error terms. Usually the residual terms in each equation will be zero, so have no impact on modelling results.

Residuals come in handy when we want to make manual adjustments to the model. This might be because we know a new change in a sector is create a long-lasting trend that would not be predicted from the historic data.

Residuals are particularly useful when we want to 'shock' the model, generating projections for hypothetical scenarios. We can alter the residual in each logarithmic equation to quickly change its behaviour.

Were we to increase the value of a residual from 1 to 1.2, for example, this would increase the equation's output by 20% because of the logarithmic form of the equation.

In this documentation we format residuals in a comparable way to other equation terms. So, the residual for wheat yield is abbreviated to $R_{WTS,YLD}$.

1.4 Commodities and their abbreviations

Table 1: commodities in UKAMM and their abbreviations

Commodity Name	Commodity Code	Commodity Name	Commodity Code
Barley	BA	**Rapeseed Meal**	RM
Butter	BT	**Rapeseed**	RP
Beef and Veal	BV	**Soybean**	SB
Coarse Grains	CG	**Sugar Beet**	SBE
Cheese	CH	**Raw Sugar (Cane)**	SUR
Dried Distiller's Grains	DDG	**Sheepmeat**	SH
Ethanol	ET	**Soybean Oil**	SL
Fresh Dairy Products	FDP	**Soybean Meal**	SM
Maize	MA	**Skimmed Milk Powder**	SMP
Milk	MK	**White Sugar**	SUW
Oats	OT	**Sweetener**	SW
Pork	PK	**Whole Milk Powder**	WMP
Protein Meal	PM	**Wheat**	WTS
Poultrymeat	PT	**Whey Powder**	WYP
Rapeseed Oil	RL		

1.5 Variables and their labels

Table 2: variables in UKAMM and their labels

Label Name	Label Code	Label Name	Label Code
Quantity Produced	QP	**Returns from Production**	RET
Quantity Consumed	QC	**Returns per Hectare**	RH
Producer Price	PP	**Consumer Price**	CP
Cow Inventory	CI	**Livestock Inventory**	LI
Feed Conversion Ratio	FCR	**Cost of Feed Index**	FECI
Food Use	FO	**Feed Use**	FE
Other Use	OU	**Use for Crushing**	CR
Crush Margin	CRMAR	**Single Farm Payment**	SFP
Cost of Production Index	CPCI	**Stocks**	ST
Intervention Stocks	IST	**Private Stocks**	PRST
Value of Production	VP	**Imports**	IM
Imports from the EU	IM, EU	**Import Price from the EU**	IMP, EU
Imports from the Rest of the World	IM, WLD	**Import Price from the Rest of the World**	IMP, WLD
Exports	EX	**Net Trade**	NT
Exports to the EU	EX..EU	**Export Price to the EU**	EXP..EU

Label Name	Label Code	Label Name	Label Code
Exports to the Rest of the World	EX..WLD	**Export Price to the Rest of the World**	EXP..WLD
Import Tariff on the EU	TAVI..EU	**Export Tariff from the EU**	TAVE..EU
Import Tariff on the Rest of the World	TAVI..WLD	**Export Tariff from the Rest of the World**	TAVE..WLD
In-quota Import Tariff on the Rest of the World	TAVI..IQ..WLD	**In-quota Specific Import Tariff on the Rest of the World**	TSP..IQ..WLD
Tariff Revenue from the EU	TREV..EU	**Tariff Revenue from the Rest of the World**	TREV..WLD
Pound to Euro Exchange Rate	XR	**Pound to Dollar Exchange Rate**	XR..USA
Population	POP	**Gross Domestic Product Index**	GDPI
Gross Domestic Product Deflator	GDPD	**Payment equivalent relevant to production**	EPQ
Payment Equivalent Relevant to Area/Herd Size	EPA	**Payment Equivalent of all Payments Affecting Inventory**	EPI

2 Sector by sector

2.1 Arable

UKAMM's arable sector includes wheat, barley, oats, rapeseed and sugar beet. UKAMM does not model fruit, vegetable, potato or ornamental growing. By land area this represents around 80% of commercial growing in the UK. By value of production it represents about half.

2.1.1 Producer prices

UKAMM uses prices to clear the UK arable market. When simulating, UKAMM adjusts prices until the amount of each arable crop produced, imported and stockpiled from the previous year equals the amount of each arable crop consumed, exported and stockpiled for the following year. This works as all of these variables are dependent on price. In the model, this is expressed algebraically:

$$\boldsymbol{PP_{WTS,t}} : 0 = QP_{WTS,t} + IM_{WTS,t} + ST_{WTS,t-1} - QC_{WTS,t} - EX_{WTS,t} - ST_{WTS,t}$$
$$\boldsymbol{PP_{BA,t}} : 0 = QP_{BA,t} + IM_{BA,t} + ST_{BA,t-1} - QC_{BA,t} - EX_{BA,t} - ST_{BA,t}$$
$$\boldsymbol{PP_{OT,t}} : 0 = QP_{OT,t} + IM_{OT,t} + ST_{OT,t-1} - QC_{OT,t} - EX_{OT,t} - ST_{OT,t}$$
$$\boldsymbol{PP_{RP,t}} : 0 = QP_{RP,t} + IM_{RP,t} - QC_{RP,t} - EX_{RP,t}$$

These market balancing equations include production ($QP_{c,t}$), consumption ($QC_{c,t}$), imports ($IM_{c,t}$), exports ($EX_{c,t}$) and stocks from the current ($ST_{c,t}$) and previous period ($ST_{c,t-1}$). The elasticities in the trade equations keep UK domestic prices tied to import and export prices depending on UK's net-trade position.

Coarse grains are used primarily for animal feed and brewing. They include barley, maize and oats but exclude wheat and rice. UKAMM calculates the aggregate price of coarse grains as a weighted average of barley, oat and maize prices multiplied by their share of coarse grain feed demand:

$$\boldsymbol{PP_{CG,t}} = \left(PP_{BA,t} \cdot FE..SHR_{BA,t}\right) + \left(PP_{MA,t} \cdot FE..SHR_{MA,t}\right) + \left(PP_{OT,t} \cdot FE..SHR_{OT,t}\right)$$

Dried distillers' grains are a by-product of brewing used as animal feed. Soybean meal is also used as animal feed. Their prices are taken as equal to the European producer price multiplied by the pound-euro exchange rate:

$$\boldsymbol{PP_{DDG,t}} = PP_{DDG,EUN,t} \cdot XR_{EUN,t}$$
$$\boldsymbol{PP_{SM,t}} = PP_{SM,EUN,t} \cdot XR_{EUN,t}$$

Protein meal can be made from soybeans, rapeseed or sunflower seeds. UKAMM assumes the UK price of protein meal to be equal to the soybean protein meal price:

$$\boldsymbol{PP_{PM,t}} = PP_{SM,t}$$

The sugar beet price equation can be found in the sugar section of this document.

The average protein feed producer price is a weighted average of the prices of low-protein feed, medium-protein feed, and high-protein feed. UKAMM assumes the price of these three types of feed to be the price of their principle components – respectively wheat, dried distiller's grains and soybean meal:

$$\boldsymbol{PP_{APF,t}} = \frac{\left(FE_{LPF,t} \cdot PP_{WTS,t}\right) + \left(FE_{MPF,t} \cdot PP_{DDG,t}\right) + \left(FE_{HPF,t} \cdot PP_{PM,t}\right)}{FE_{APF,t}}$$

2.1.2 Consumer prices

UKAMM calculates an estimate for wheat consumer prices using standard ratios for the mark-up between consumer and producer prices in European countries. This ratio is multiplied by UK producer prices to estimate consumer prices.

$$\boldsymbol{CP_{WTS,t}} = PP_{WTS,t} \cdot CP..E15_{WTS,t}$$

Here $CP_{WTS,t}$ is the wheat consumer price, $PP_{WTS,t}$ is the wheat producer price and $CP..E15_{WTS,t}$ is the ratio between the European consumer and producer price.

For coarse grains, the consumer price is assumed equal to the producer price:

$$\boldsymbol{CP_{CG,t}} = PP_{CG,t}$$

2.1.3 Production

To calculate the amount of each arable crop produced, UKAMM first creates a measure of the cost's farmers incur to produce each crop.

UKAMM has indices of various standard types of costs incurred by farmers. It draws on these to calculate each commodity's measure of production costs. The overall production cost index is different for each crop, but all are constructed from a combination of five input sub-indices: seed, fertiliser, energy, other tradable inputs and non-tradable inputs.

$$\boldsymbol{CPCI_{c,t}} = CPCI..SHSD_{c,t} \cdot \left(\frac{PP_{c,t-1}}{PP_{c,2008}}\right) + CPCI..SHEN_{c,t} \cdot \left(\frac{XP_{oil,wld,t}}{XP_{oil,wld,2008} \cdot XR_{USA,2008}}\right) + CPCI..SHFT_{c,t} \cdot \left(\frac{XP_{ft,wld,t} \cdot XR_{USA,t}}{XP_{ft,wld,2008} \cdot XR_{USA,2008}}\right) + CPCI..SHTR_{c,t} \cdot \left(\frac{GDP_{EUN,t} \cdot XR_{EUN,t}}{GDPD_{EUN,2008} \cdot XR_{EUN,2008}}\right) + CPCI..SHNT_{c,t} \cdot \left(\frac{GDPD_t}{GDPD_{2008}}\right)$$

In this equation:

- $\boldsymbol{CPCI_{c,t}}$ is the commodity production cost index for commodity ***c*** in year *t*
- $\boldsymbol{CPCI..SHNT_{c,t}}$ is the share of non-tradable inputs in total base commodity production costs
- $\boldsymbol{CPCI..SHEN_{c,t}}$ is the share of energy in total base commodity production costs
- $\boldsymbol{CPCI..SHFT_{c,t}}$ is the share of fertiliser in total base commodity production costs
- $\boldsymbol{CPCI..SHTR_{c,t}}$ is the share of other tradable inputs in total base commodity production costs
- $\boldsymbol{CPCI..SHSD_{c,t}}$ is the share of seed input in total base commodity production costs

Table 4: Costs of arable production covered in UKAMM

UKAMM cost index	Costs covered
Energy	Electricity and fuels
Fertilisers	Fertilisers and soil improvers
Seeds	Seeds and seedlings bought as well as those produced on the farm
Other tradeables	Crop protection products, other specific crop costs, machinery and buildings
Non-tradeables	Contract work, other farming overheads, depreciation, wages and own work

Production ($QP_{c,t}$) of grains (wheat, barley, and oats), oilseeds (rapeseed) and sugar beet is calculated by multiplying the yield ($YLD_{c,t}$) by the area harvested ($AH_{c,t}$).

$$\boldsymbol{QP_{WTS,t}} = AH_{WTS,t} \cdot YLD_{WTS,t}$$
$$\boldsymbol{QP_{BA,t}} = AH_{BA,t} \cdot YLD_{BA,t}$$
$$\boldsymbol{QP_{OT,t}} = AH_{OT,t} \cdot YLD_{OT,t}$$
$$\boldsymbol{QP_{RP,t}} = AH_{RP,t} \cdot YLD_{RP,t}$$
$$\boldsymbol{QP_{SBE,t}} = AH_{SBE,t} \cdot YLD_{SBE,t}$$

Farmers are assumed to decide on their inputs based on their margin expectations. This is reflected in the yield of the crop, which is based on the ratio of the price to the input costs from the previous year. This equation has the same form for wheat, barley, oats and rapeseed – here is the wheat yield equation:

$$\boldsymbol{Log(YLD_{WTS,t})} = \alpha + \beta_{YLD_{WTS}PP_{WTS}} \cdot Log\left(\frac{PP_{WTS,t-1}}{\beta_{CPCI_{WTS,t*(t-1)}} \cdot CPCI_{WTS,t-1} + \left(1 - \beta_{CPCI_{WTS,t*(t-1)}}\right) \cdot CPCI_{WTS,t}}\right) + \beta_{YLD_{WTS}TRND} * trend + R_{YLD,WTS}$$

In this equation:

- $YLD_{c,t}$ denotes the yield of commodity c at time t
- α is a constant
- $\beta_{YLD,PP}$ is the price elasticity of the yield to the price. This has small effect.
- $PP_{c,t-1}$ is the price of the commodity c in time $t-1$
- $\beta_{CPCI_{c,t*(t-1)}}$ is the weighting of the production cost index in time t relative to the lagged production cost index in time $t-1$
- $CPCI_{c,t}$ and $CPCI_{c,t-1}$ are the cost of production index for commodity c in year t and $t-1$ respectively
- $Trend$ is a time trend, scaled for respective yields by $\beta_{YLD_{c}TRND}$. Technological change is assumed to have a positive effect on yield and is captured by the time trend. This trend variable has a bigger effect on yield than it does on most other variables.
- $R_{Y,e}$ is the residual for the yield of commodity c

Area is allocated to a given crop based on its relative competiveness with wheat, the major UK crop.

This reduces the number of elasticities needed in the area share equations but means that competition with anything other than wheat for land (from grazing livestock to solar farms) is not considered.

The following form of equation is repeated for barley, oats, rapeseed and sugar beet – here the barley area share equation:

$$\boldsymbol{Log\left(AH..SHR_{BA,t}\right)} = \alpha + \beta_{AH..SHR_{BA}RH_{WTS}} \cdot Log\left(\frac{\frac{RH_{BA,t-1} + EPA_{BA,t-1}}{0.5 \cdot CPCI_{BA,t-1} + 0.5 \cdot CPCI_{BA,t}}}{\frac{RH_{WTS,t-1} + EPA_{WTS,t-1}}{0.5 \cdot CPCI_{WTS,t-1} + 0.5 \cdot CPCI_{WTS,t}}}\right) + \beta_{AH..SHR_{BA}TRND} * trend + R_{AH..SHR,BA}$$

In this equation:

- $\beta_{AH..SHR_{c}RH_{WTS}}$ denotes the elasticity of the area share of crop c to the returns per hectare of wheat
- $RH_{c,t-1}$ is the return per hectare in the previous year
- $EPA_{c,t-1}$ is the area payment equivalent in the previous year. See chapter 4 on market support for the derivation of the subsidy payments.
- $CPCI_{c,t}$ represents the cost of production index
- $\beta_{AH..SHR_{c}TRND}$ denotes the weighting attached to a trend for the share of arable area allocated to crop c

The area share for wheat is calculated by subtracting the sum of all the other area shares from 1:

$$\boldsymbol{AH..SHR_{WTS,t}} = 1 - \sum_{c \neq WTS} \left(AH..SHR_{c,t}\right)$$

To find the total harvested area, the area share is multiplied by the total cropped land. This equation has the same form for wheat, barley, oats, rapeseed and sugar beet – for brevity we include the wheat harvested area equation only:

$$\boldsymbol{AH_{WTS,t}} = AH..SHR_{WTS,t} \cdot AH_{CR,t}$$

The revenue per crop is composed of two elements: the returns per hectare and the area payment equivalent. Crop returns are calculated as a three year weighted average to negate volatility in both yield and prices. This equation has the same form for wheat, barley, oats, rapeseed and sugar beet – here is the wheat harvested area equation:

$$\boldsymbol{RH_{WTS,t}} = \alpha_t \cdot PP_{WTS,t} \cdot YLD_{WTS,t} + \alpha_{t-1} \cdot PP_{WTS,t-1} \cdot YLD_{WTS,t-1} + \alpha_{t-2} \cdot PP_{WTS,t-2} \cdot YLD_{WTS,t-2}$$

Here $PP_{WTS,t}$ is the price of wheat in year t and $YLD_{WTS,t}$ is the wheat yield in year *t*.

2.1.5 Consumption

Each crop in UKAMM has different uses, summarised in the table.

Table 5: Uses of Arable Crops

Commodity	Code	Biofuel	Crushing	Feed	Food	Other
Barley	BA			X	X	X
Maize	MA	X		X	X	X
Oats	OT			X	X	X
Rapeseed	RP		X		X	X
Sugar beet	SBE	X			X	
Wheat	WTS	X		X	X	X

UKAMM models domestic demand for each crop as the sum of its uses: biofuel use ($BF_{c,t}$), crush use (CR_{ct}), feed use ($FE_{c,t}$), food use ($FO_{c,t}$), and other uses ($OU_{c,t}$):

$$QC_{WTS,t} = BF_{WTS,t} + FE_{WTS,t} + FO_{WTS,t} + OU_{WTS,t}$$

$$QC_{BA,t} = FE_{BA,t} + FO_{BA,t} + OU_{BA,t}$$

$$QC_{OT,t} = FE_{OT,t} + FO_{OT,t} + OU_{OT,t}$$

$$QC_{RP,t} = CR_{RP,t} + OU_{RP,t}$$

$$QC_{CG,t} = FE_{CG,t} + FO_{CG,t} + OU_{CG,t}$$

Biofuel use is capped at an estimate of the UK's limited manufacturing capacity. Below this level UKAMM determines biofuel use using a function of the sterling value of the world fossil oil reference price ($XP_{OIL,t} * XR..USA_t$) relative to the UK wheat producer price ($PP_{WTS,t}$). This is because biofuel is a substitute for fossil oil derived fuels.

$$\boldsymbol{BF_{WTS,t}} = \min\left[BF..CPC_{WTS,t}\,, BF..SIM_{WTS,t}\right]$$

$$\log\left[\boldsymbol{BF..SIM_{WTS,t}}\right] = \alpha + \beta_{BF_{WTS}PP_{WTS}} * \log\left[\frac{\alpha_{BF..SIM_{WTS}} * XP_{OIL,t} * \ XR..USA_t}{PP_{WTS,t}}\right] + R_{BF..SIM_{WTS}}$$

Crushing is the processing of oilseeds like rapeseed and soybean into oil and protein meal. The **crush margin** is the difference between the value of the raw crop and its processed products. UKAMM determines the amount of rapeseed crushed through the margins from crushing rapeseed $CRMAR_{RP,t}$ the elasticity of rapeseed and soybean margins $\beta_{CR_{RP,t},CRMAR_{SB,t}}$, and soybean crush margins $CRMAR_{SB,t}$.

$$\boldsymbol{Log(CR_{RP,t})} = \alpha + \beta_{CR_{RP},CRMAR_{SB}} * Log(CRMAR_{SB,t}) + \beta_{CR_{RP},CRMAR_{RP}} * Log(CRMAR_{RP,t}) + \beta_{CR_{RP}TRND} * \beta_{CR_{RP}TRND} * trend + R_{CR,RP}$$

Food use for wheat and the coarse grains are functions of the consumer prices of wheat, coarse grains and rice; the consumer price index; the GDP index; and population:

$$\boldsymbol{Log(FO_{WTS,t})} = \alpha + \beta_{FO_{WTS},CP_{WTS}} * Log\left(\frac{CP_{WTS,t}}{GDPD_t}\right) + \beta_{FO_{WTS},CP_{CG}} * Log\left(\frac{CP_{CG,t}}{GDPD_t}\right) + \beta_{FO_{WTS},CP_{RI}} * Log\left(\frac{CP_{RI,t}}{GDPD_t}\right) + \beta_{FO_{WTS},GPDI} * Log\left(\frac{GDPI_t}{POP_t}\right) + POP_t + \beta_{FO_{WTS}TRND}trend + R_{FO,WTS}$$

$$\boldsymbol{Log(FO_{CG,t})} = \alpha + \beta_{FO_{CG},CP_{WTS}} * Log\left(\frac{CP_{WTS,t}}{GDPD_t}\right) + \beta_{FO_{CG},CP_{CG}} * Log\left(\frac{CP_{CG,t}}{GDPD_t}\right) + \beta_{FO_{CG},CP_{RI}} * Log\left(\frac{CP_{RI,t}}{GDPD_t}\right) + \beta_{FO_{CG},GPDI} * Log\left(\frac{GDPI_t}{POP_t}\right) + POP_t + \beta_{FO_{CG}TRND}trend + R_{FO,CG}$$

UKAMM breaks down this overall demand for the food use of coarse grains into food use for barley, maize and oats. It does this by dividing it by each crop's share ($FO..SHR_{c,t}$) of overall coarse grain food usage ($FO_{CG,t}$):

$$\boldsymbol{FO_{BA,t}} = FO_{CG,t} * FO..SHR_{BA,t}$$

$$\boldsymbol{FO_{OT,t}} = FO_{CG,t} * FO..SHR_{OT,t}$$

$$\boldsymbol{FO_{MA,t}} = FO_{CG,t} * FO..SHR_{MA,t}$$

UKAMM has specific functions to calculate each crop's share of coarse grain food use. The maize food use share is exogenous. Oats' food use share is treated as a function of the relative price of oats and barley:

$$\boldsymbol{Log(FO..SHR_{OT,t})} = \alpha + \beta_{FO..SHR_{OT},PP_{CG}} * Log\left(\frac{PP_{OT,t}}{PP_{BA,t}}\right) + R_{FO..SHR,OT}$$

The barley food use share is 1 minus the oat share and the maize share:

$$\boldsymbol{FO..SHR_{BA,t}} = 1 - FO..SHR_{OT,t} - FO..SHR_{MA,t}$$

Other usage captures all of the miscellaneous uses for arable crops. The other usage is exogenous for maize, rapeseed, barley and oats. Other use for wheat is a function of the wheat price and a trend variable:

$$\boldsymbol{Log(OU_{WTS,t})} = \alpha + \beta_{OU_{WTS,t}PP_{WTS,t}} \cdot Log\left(\frac{PP_{WTS,t}}{GDPD_t}\right) + \beta_{OU_{WTS}TRND} * trend + R_{OU,WTS}$$

Coarse grain other usage is the sum of the other use of barley, maize and oats:

$$\boldsymbol{OU_{CG,t}} = OU_{BA,t} + OU_{OT,t} + OU_{MA_t}$$

2.1.6 Animal feed demand

UKAMM models demand for animal feed ($FE_{c,t}$) as a function of beef, sheepmeat and milk production ($QP_{c,t}$); non-ruminant feed use ($FE_{NR,t}$); aquaculture feed use ($FE_{FHA,t}$); and the prices of wheat, coarse grains, protein meal and dried distiller's grains. For wheat:

$$\begin{aligned}
\boldsymbol{Log}(\boldsymbol{FE_{WTS,t}}) = {} & \alpha + \beta_{APF_{FE},QP_{SH}} * Log(QP_{SH,t}) + \beta_{APF_{FE},QP_{BV}} \cdot Log(QP_{BV,t}) \\
& + \beta_{APF_{FE},QP_{MK}} \cdot Log(QP_{MK,t}) + \beta_{APF_{FE},FHA_{FE}} \cdot FE_{FHA,t} \\
+ (1 - \beta_{APF_{FE},QP_{SH}} - \beta_{APF_{FE},QP_{BV}} & - \beta_{APF_{FE},QP_{MK}} - \beta_{APF_{FE},FHA_{FE}}) \cdot Log(FE_{NR,t}) \\
& + \beta_{FE_{WTS}PP_{CG}} \cdot Log\left(\frac{PP_{CG,t}}{GDPD_t}\right) + \beta_{FE_{WTS}PP_{WTS}} \cdot Log\left(\frac{PP_{WTS,t}}{GDPD_t}\right) \\
& + \beta_{FE_{WTS}PP_{PM}} \cdot Log\left(\frac{PP_{PM,t}}{GDPD_t}\right) + \beta_{FE_{WTS}PP_{DDG}} \cdot Log\left(\frac{PP_{DDG,t}}{GDPD_t}\right) \\
& + \beta_{FE_{WTS}TRND} trend + R_{FE,WTS}
\end{aligned}$$

For coarse grains:

$$\begin{aligned}
\boldsymbol{Log}(\boldsymbol{FE_{CG,t}}) = {} & \alpha + \beta_{APF_{FE},QP_{SH}} * Log(QP_{SH,t}) + \beta_{APF_{FE},QP_{BV}} \cdot Log(QP_{BV,t}) \\
& + \beta_{APF_{FE},QP_{MK}} \cdot Log(QP_{MK,t}) + \beta_{APF_{FE},FHA_{FE}} \cdot FE_{FHA,t} \\
+ (1 - \beta_{APF_{FE},QP_{SH}} - \beta_{APF_{FE},QP_{BV}} & - \beta_{APF_{FE},QP_{MK}} - \beta_{APF_{FE},FHA_{FE}}) \cdot Log(FE_{NR,t}) \\
& + \beta_{FE_{CG}PP_{CG}} \cdot Log\left(\frac{PP_{CG,t}}{GDPD_t}\right) + \beta_{FE_{CG}PP_{WTS}} \cdot Log\left(\frac{PP_{WTS,t}}{GDPD_t}\right) \\
& + \beta_{FE_{CG}PP_{PM}} \cdot Log\left(\frac{PP_{PM,t}}{GDPD_t}\right) + \beta_{FE_{CG}PP_{DDG}} \cdot Log\left(\frac{PP_{DDG,t}}{GDPD_t}\right) \\
& + R_{FE,CG}
\end{aligned}$$

For dried distiller's grains:

$$\begin{aligned}
\boldsymbol{Log}(\boldsymbol{FE_{DDG,t}}) = {} & \alpha + \beta_{APF_{FE},QP_{SH}} * Log(QP_{SH,t}) + \beta_{APF_{FE},QP_{BV}} \cdot Log(QP_{BV,t}) \\
& + \beta_{APF_{FE},QP_{MK}} \cdot Log(QP_{MK,t}) + \beta_{APF_{FE},FHA_{FE}} \cdot FE_{FHA,t} \\
+ (1 - \beta_{APF_{FE},QP_{SH}} - \beta_{APF_{FE},QP_{BV}} & - \beta_{APF_{FE},QP_{MK}} - \beta_{APF_{FE},FHA_{FE}}) \cdot Log(FE_{NR,t}) \\
& + \beta_{FE_{DDG}PP_{CG}} \cdot Log\left(\frac{PP_{CG,t}}{GDPD_t}\right) + \beta_{FE_{DDG}PP_{WTS}} \cdot Log\left(\frac{PP_{WTS,t}}{GDPD_t}\right) \\
& + \beta_{FE_{DDG}PP_{PM}} \cdot Log\left(\frac{PP_{PM,t}}{GDPD_t}\right) + \beta_{FE_{DDG}PP_{DDG}} \cdot Log\left(\frac{PP_{DDG,t}}{GDPD_t}\right) \\
& + R_{FE,DDG}
\end{aligned}$$

For protein meal:

$$\boldsymbol{Log}(\boldsymbol{FE_{PM,t}}) = \alpha + \beta_{APF_{FE},QP_{SH}} * Log(QP_{SH,t}) + \beta_{APF_{FE},QP_{BV}} \cdot Log(QP_{BV,t}) + \beta_{APF_{FE},QP_{MK}} \cdot Log(QP_{MK,t}) + \beta_{APF_{FE},FHA_{FE}} \cdot FE_{FHA,t} + (1 - \beta_{APF_{FE},QP_{SH}} - \beta_{APF_{FE},QP_{BV}} - \beta_{APF_{FE},QP_{MK}} - \beta_{APF_{FE},FHA_{FE}}) \cdot Log(FE_{NR,t}) + \beta_{FE_{PM}PP_{CG}} \cdot Log\left(\frac{PP_{CG,t}}{GDPD_t}\right) + \beta_{FE_{PM}PP_{WTS}} \cdot Log\left(\frac{PP_{WTS,t}}{GDPD_t}\right) + \beta_{FE_{PM}PP_{PM}} \cdot Log\left(\frac{PP_{PM,t}}{GDPD_t}\right) + \beta_{FE_{PM}PP_{DDG}} \cdot Log\left(\frac{PP_{DDG,t}}{GDPD_t}\right) + R_{FE,PM}$$

Similarly, to food use, the feed use of barley, maize and oats are calculated using a share of aggregated coarse grain feed use:

$$\boldsymbol{FE_{BA,t}} = FE_{CG,t} * FE..SHR_{BA,t}$$

$$\boldsymbol{FE_{OT,t}} = FE_{CG,t} * FE..SHR_{OT,t}$$

$$\boldsymbol{FE_{MA,t}} = FE_{CG,t} * FE..SHR_{MA,t}$$

Here $FE_{c,t}$ is the feed use, $FE_{CG,t}$ is the feed use of coarse grains and $FE..SHR_{c,t}$ is the share of total coarse grain feed use.

The oat feed share is inputted exogenously. The barley feed share is again calculated by subtracting the oat and maize share from 1:

$$\boldsymbol{FE..SHR_{BA,t}} = 1 - FE..SHR_{OT,t} - FE..SHR_{MA,t}$$

An average protein feed demand estimate ($FE_{APF,t}$) is calculated as the sum of low-protein feed ($FE_{LPF,t}$), medium-protein feed ($FE_{MPF,t}$) and high-protein feed ($FE_{HPF,t}$):

$$\boldsymbol{FE_{APF,t}} = FE_{LPF,t} + FE_{MPF,t} + FE_{HPF,t}$$

UKAMM totals the commodities belonging to each category (as shown in Table 4) to give the feed use for each level of protein content.

Table 6: Arable Feed types

Low-Protein Feeds	Medium-Protein Feeds	High-Protein Feeds
Coarse Grains, Wheat	**Dried Distillers Grains**	**Protein Meal**
$\boldsymbol{FE_{LPF,t}} = FE_{CG,t} + FE_{WTS,t}$	$\boldsymbol{FE_{MPF,t}} = FE_{DDG,t}$	$\boldsymbol{FE_{HPF,t}} = FE_{PM,t}$

Ruminant feed ($FE_{RU,t}$) is calculated by the total feed use ($FE_{APF,t}$) minus non-ruminant feed demand ($FE_{NR,t}$) :

$$\boldsymbol{FE_{RU,t}} = FE_{APF,t} - FE_{NR,t}$$

Non-ruminant feed use ($FE_{NR,t}$) is calculated as:

$$\boldsymbol{FE_{NR,t}} = \frac{QP_{PK,t}}{PK..LWT_{UK,t}} \cdot FCR_{PK,t} + \frac{QP_{PT,t}}{1} \cdot FCR..DW_{PT,t} + QP_{EG,t} \cdot FCR_{EG,t}$$

Here $QP_{PK,t}$ and $QP_{PT,t}$ are pork and poultry production. $FCR_{PK,t}$ and $FCR_{PT,t}$ are their respective feed conversion ratios.

Feed conversion ratios say how much weight farmed animals gain, on average, for each unit of feed they consume.

These ratios are historical averages exogenous to UKAMM. Animal weights can be ‘liveweight’, referring to the weight just before slaughter, or ‘deadweight’, referring to the weight of saleable meat from each carcass.

For poultrymeat the feed conversation ratio is in deadweight equivalent, but for pigmeat the feed ratio is given in liveweight. This means an additional exogenous scaling factor, $LWT_{UK,t}$, is needed to convert the feed conversion ratio to a deadweight basis.

Beef feed conversion ratios are assumed to be equal to the general ruminant feed ratio and sheep feed conversion ratios are assumed to be a fixed proportion of the general ruminant ratio:

$$\boldsymbol{FCR_{BV,t}} = FCR_{RU,t}$$

$$\boldsymbol{FCR_{SH,t}} = \frac{FCR_{RU,t}}{\alpha}$$

In UKAMM the feed cost index is split into indexes for ruminant feed and non-ruminant feed. The non-ruminant feed cost index is modelled exogenously, and the ruminant feed cost index ($FECI_{RU,t}$) is equal to the average price of feed: $\boldsymbol{FECI_{RU,t}} = PP_{APF,t}$.

2.1.7 Stocks

Many arable crops can be stored from year to year. This is one way producers adjust to periods of excess supply or price volatility. In UKAMM's arable component, only wheat and barley stocks are modelled endogenously. Oat stocks are exogenous for simplicity, given the lower amount produced.

Aggregate stock levels are the sum of private stocks ($PRST_{c,t}$) and intervention stocks ($IST_{c,t}$):

$$\boldsymbol{ST_{c,t}} = PRST_{c,t} + IST_{c,t} \ .$$

Private stocks are modelled as a function of private stocks from the previous period ($PRST_{c,t-1}$) and the price in the current period compared to the previous three periods:

$$\begin{aligned}\boldsymbol{Log(PRST_{WTS,t})} = \alpha + \beta_{PRST_{WTS}QP_{WTS}} \cdot Log\left(QP_{WTS,t} + PRST_{WTS,t-1}\right) \\ + \beta_{PRST_{WTS}PP_{WTS}} \cdot Log\left(\frac{3 \cdot PP_{WTS,t}}{PP_{WTS,t-1} + PP_{WTS,t-2} + PP_{WTS,t-3}}\right) \\ + \beta_{PRST_{WTS}TRND} trend + R_{PRST,WTS}\end{aligned}$$

$$\begin{aligned}\boldsymbol{Log(PRST_{BA,t})} = \alpha + \beta_{PRST_{BA}QP_{BA}} \cdot Log\left(QP_{BA,t} + PRST_{BA,t-1}\right) \\ + \beta_{PRST_{BA}PP_{BA}} \cdot Log\left(\frac{3 \cdot PP_{BA,t}}{PP_{BA,t-1} + PP_{BA,t-2} + PP_{BA,t-3}}\right) \\ + \beta_{PRST_{WTS}TRND} trend + R_{PRST,BA}\end{aligned}$$

The modelling of intervention stocks is discussed in chapter 3.2.

2.2 Oilseeds

2.2.1 Producer prices

UKAMM uses prices to clear the UK oilseeds market. When simulating, UKAMM adjusts prices until the amount produced ($QP_{c,t}$) and imported ($IM_{c,t}$) equals the amount consumed ($QC_{c,t}$) and exported ($EX_{c,t}$). This works as all of these variables are dependent on price. In the model, this is expressed algebraically:

$$\boldsymbol{PP_{RM,t}}:\ 0 = QP_{RM,t} + IM_{RM,t} - QC_{RM,t} - EX_{RM,t}$$

$$\boldsymbol{PP_{RL,t}}:\ 0 = QP_{RL,t} + IM_{RL,t} - QC_{RL,t} - EX_{RL,t}$$

2.2.2 Production

Oilseeds like rapeseed and soybean are crushed to produce oil and seed 'meal'. The quantity of vegetable oil produced ($QP_{c,t}$) is equal to the amount of seed crushed ($CR_{c,t}$), multiplied by the oil yield ($YLD_{c,t}$):

$$\boldsymbol{QP_{RL,t}} = CR_{RP,t} \cdot YLD_{RL,t}$$

$$\boldsymbol{QP_{SL,t}} = CR_{SB,t} \cdot YLD_{SL,t}$$

The amount of seed meal produced ($QP_{c,t}$) is equal to the amount of seed crushed ($CR_{c,t}$), multiplied by the meal yield ($YLD_{c,t}$):

$$\boldsymbol{QP_{RM,t}} = CR_{RP,t} \cdot YLD_{RM,t}$$

$$\boldsymbol{QP_{SM,t}} = CR_{SB,t} \cdot YLD_{SM,t}$$

The crush margin is the difference between the value of raw oilseed and the value of the oil and meal produced. The amount of rapeseed and soybean crushed is a function of the crush margins ($CRMAR_{c,t}$) and of elasticities measuring the extent to which they effect the amount crushed ($\beta_{CR_c,CRMAR_c}$). There is also a trend variable picking up technological innovation.

$$\begin{aligned}\boldsymbol{Log(CR_{RP,t})} = \ & \alpha + \beta_{CR_{RP},CRMAR_{SB}} * Log(CRMAR_{SB,t}) \\ & + \beta_{CR_{RP},CRMAR_{RP}} * Log(CRMAR_{RP,t}) \\ & + \beta_{CR_{RP}TRND} * trend + R_{CR,RP}\end{aligned}$$

$$\begin{aligned}\boldsymbol{Log(CR_{SB,t})} = \ & \alpha + \beta_{CR_{SB},CRMAR_{SB}} * Log(CRMAR_{SB,t}) \\ & + \beta_{CR_{SB},CRMAR_{RP}} * Log(CRMAR_{RP,t}) \\ & + \beta_{CR_{SB}TRND} * trend + R_{CR,SB}\end{aligned}$$

These crush margins indicate how profitable it is to process the seeds into oils and meals rather than just selling the seeds. For this reason they are modelled as a function of the meal price ($PP_{c,t}$) and yield ($YLD_{c,t}$); the oil price ($PP_{c,t}$) and yield ($YLD_{c,t}$); and the seed price ($PP_{c,t}$). As soybeans are not widely grown in the UK, their crush margin depends on the import prices ($IMP_{c,t}$) for meal, oil and beans.

$$CRMAR_{RP,t} = \left(\frac{PP_{RM,t} \cdot YLD_{RM,t} + PP_{RL,t} \cdot YLD_{RL,t}}{PP_{RP,t}}\right)$$

$$CRMAR_{SB,t} = \left(\frac{IMP_{SM,t} \cdot YLD_{SM,t} + IMP_{SL,t} \cdot YLD_{SL,t}}{IMP_{SB,t}}\right)$$

2.2.3 Consumption

The total quantity of rapeseed oil consumed is the sum of the biofuel use (modelled exogenously) and the food use. The food use is modelled as a function of the consumer price of butter ($CP_{BT,t}$), the consumer price of rapeseed oil ($CP_{RL,t}$), economic growth ($GDPI_T$) and the population (POP_t):

$$\boldsymbol{Log}(\boldsymbol{RL_{FO}}) = \alpha + \beta_{FO_{RL}CP_{BT}} \cdot Log\left(\frac{CP_{BT,t}}{GDPD_t}\right) + \beta_{FO_{RL}CP_{RL}} \cdot Log\left(\frac{CP_{RL,t}}{GDPD_t}\right) + \beta_{FO_{RL}GDPI} \cdot Log\left(\frac{GDPI_T}{POP_T}\right) + Log(POP_t) + R_{FO,RL}$$

The consumer price of rapeseed oil is a behavioural equation that is a function of the GDP deflator and the producer price:

$$\boldsymbol{Log}(\boldsymbol{CP_{RL,t}}) = \alpha + \beta_{CP_{RL}GDPD} \cdot Log(GDPD_t) + \left(1 - \beta_{CP_{RL}GDPD}\right) \cdot Log\left(PP_{RL,t}\right) + R_{CP,RL}$$

Protein meal feed use is a function of the production of sheepmeat, beef and milk, as well as non-ruminant feed use. It is also a function of the prices of coarse grain, wheat, protein meal and dried distillers' grains:

$$\boldsymbol{Log}\left(\boldsymbol{FE_{PM,t}}\right) = \alpha + \beta_{FE_{APF}QP_{SH}} \cdot Log\left(QP_{SH,t}\right) + \beta_{FE_{APF}QP_{BV}} \cdot Log\left(QP_{BV,t}\right) + \beta_{FE_{APF}QP_{MK}} \cdot Log\left(QP_{MK,t}\right) + \beta_{FE_{APF}FE_{FHA}} \cdot Log\left(FE_{FHA,t}\right) + \left(1 - \beta_{FE_{APF}QP_{SH}} - \beta_{FE_{APF}QP_{BV}} - \beta_{FE_{APF}QP_{MK}} - \beta_{FE_{APF}FE_{FHA}}\right) \cdot Log\left(FE_{NR,t}\right) + \beta_{FE_{PM}PP_{PM}} \cdot \left(\frac{PP_{CG,t}}{GDPD_t}\right) + \beta_{FE_{PM}PP_{WTS}} \cdot \left(\frac{PP_{WTS,t}}{GDPD_t}\right) + \beta_{FE_{PM}PP_{PM}} \cdot \left(\frac{PP_{PM,t}}{GDPD_t}\right) + \beta_{FE_{PM}PP_{DDG}} \cdot \left(\frac{PP_{DDG,t}}{GDPD_t}\right) + R_{FE,PM}$$

Rapeseed meal is solely demanded for feed use. Rapeseed meal feed use ($FE_{RM,t}$) is calculated by multiplying the share of rapeseed meal ($FE..SHR_{RM,t}$) by total protein meal feed use ($FE_{PM,t}$):

$$\boldsymbol{FE_{RM,t}} = FE..SHR_{RM,t} \cdot FE_{PM,t}$$

This share variable is calculated as a function of the relative prices of rapeseed meal and soybean meal ($\frac{PP_{RM,t}}{IMP_{SM,t}}$):

$$\boldsymbol{Log}(\boldsymbol{FE..SHR_{RM,t}}) = \alpha + \beta_{FE..SHR_{RM}IMP_{SM}} \cdot Log\left(\frac{PP_{RM,t}}{IMP_{SM,t}}\right) + R_{FE..SHR,RM}$$

Soybean feed use ($FE_{SM,t}$) makes up the other part of protein meal feed use, and as such is calculated by subtracting the rapeseed meal share ($FE..SHR_{RM,t}$) from one and multiplying it by the protein meal feed use ($FE_{PM,t}$):

$$\boldsymbol{FE_{SM,t}} = \left(1 - FE..SHR_{RM,t}\right) \cdot FE_{PM,t}$$

2.3 Sugar

2.3.1 Producer prices

UKAMM uses prices to clear the UK white sugar market. When simulating, UKAMM adjusts prices until the amount produced ($QP_{c,t}$) and imported ($IM_{c,t}$) equals the amount consumed ($QC_{c,t}$) and exported ($EX_{c,t}$). This works as all these variables are dependent on price. In the model, this is expressed algebraically:

$$\boldsymbol{PP_{SUW,t}}\text{: } 0 = QP_{SUW,t} + IM_{SUW,t} - QC_{SUW,t} - EX_{SUW,t}$$

White sugar can be made from sugar cane, which cannot be grown in the UK, and sugar beet, which can be grown in the UK. UKAMM assumes the producer price of sugar beet to be driven by the producer price for white sugar (from both cane and beet). To calculate the producer price of sugar beet its multiples the white sugar producer price by the sugar yield per unit of sugar beet and a historical average profit margin.

$$\boldsymbol{PP_{SBE,t}} = YLD..SBE_{SUW,t} \cdot PP_{SUW,t} \cdot MAR_{SBE,t}$$

2.3.2 Consumer prices

The consumer price for sweeteners like sugar is modelled as a function of the GDP deflator ($GDPD_t$) and the producer price of white sugar ($PP_{SW,t}$):

$$\begin{aligned}\boldsymbol{Log(CP_{SW,t})} = {} & \alpha + \beta_{CP_{SW}GDPD} \cdot Log(GDPD_t) \\ & + \left(1 - \beta_{CP_{SW}GDPD}\right) \cdot Log\left(PP_{SW,t}\right) + R_{CP,SW}\end{aligned}$$

2.3.3 Production

White sugar can be made from sugar cane, which cannot be grown in the UK, and sugar beet, which can be grown in the UK.

Sugar beet production is modelled in the same way as other arable crops. We explain how UKAMM models arable crops, including sugar beet, in section 2.1.4.

The amount of white sugar produced from sugar beet ($QP..SBE_{SUW,t}$) is modelled as a function of sugar beet production ($QP_{SBE,t}$), biofuel use of sugar beet ($BF_{SBE,t}$), and the sugar yield per unit of beet ($YLD..SBE_{SUW,t}$):

$$\boldsymbol{QP..SBE_{SUW,t}} = (QP_{SBE,t} - BF_{SBE,t}) \cdot YLD..SBE_{SUW,t}$$

White sugar production from raw sugar cane ($QP..SUR_{SUW,t}$) is a function of raw cane imports ($IM_{SUR,t}$) and the conversion factor from cane to sugar ($SUR..W_{SUW,t}$), with a constraint imposed to reflect the refining industry's maximum capacity:

$$\boldsymbol{QP..SUR_{SUW,t}} = \min(CPC_{SUR}, IM_{SUR,t} \cdot SUR..W_{SUW,t})$$

The overall amount of white sugar produced ($QP_{SUW,t}$) is the sum of the white sugar produced from sugar beet ($QP..SBE_{SUW,t}$) and from raw sugar cane ($QP..SUR_{SUW,t}$):

$$\boldsymbol{QP_{SUW,t}} = QP..SBE_{SUW,t} + QP..SUR_{SUW,t}$$

2.3.4 Consumption

The consumption of white sugar is modelled as a function of the food use of sweeteners ($SW_{FO,t}$) and a constant (α) to account for non-food uses of sugar:

$$\boldsymbol{QC_{SUW,t}} = \frac{SW_{FO,t}}{\alpha}$$

The food use of sweeteners is modelled a function of their consumer price ($CP_{SW,t}$), economic growth ($GDPI_t$), and the population (POP_t):

$$\begin{aligned}\boldsymbol{Log(FO_{SW,t})} = \alpha + \beta_{FO_{SW}CP_{SW}} \cdot Log\left(\frac{CP_{SW,t}}{GDPD_t}\right) \\ + \beta_{FO_{SW}GDPI} \cdot Log\left(\frac{GDPI_t}{POP_t}\right) + Log(POP_t) \\ + \beta_{FO_{SW}TRND} \cdot trend + R_{FO,SW}\end{aligned}$$

2.4 Livestock

2.4.1 Producer prices

UKAMM uses prices to clear the UK beef, sheepmeat, pigmeat and poultry markets. When simulating, UKAMM adjusts prices until the amount produced ($QP_{c,t}$) and imported ($IM_{c,t}$) equals the amount consumed ($QC_{c,t}$) and exported ($EX_{c,t}$).

This works as all these variables are dependent on price. In the model, this is expressed algebraically in the same way for beef, sheepmeat, pigmeat and poultry:

$$\boldsymbol{PP_{c,t}}:\ 0 = QP_{c,t} + IM_{c,t} - QC_{c,t} - EX_{c,t}$$

2.4.2 Consumer prices

UKAMM models livestock consumer prices separately to producer prices to differentiate between price changes for producers and consumers. UKAMM models consumer prices ($CP_{c,t}$) as a function of the GDP deflator ($GDPD_t$) and the producer price ($PP_{c,t}$). This equation has the same form for beef, sheepmeat, pigmeat and poultry:

$$\boldsymbol{Log}(\boldsymbol{CP_{c,t}}) = \alpha + \beta_{CP_c GDPD} \cdot Log(GDPD_t) + (1 - \beta_{CP_c GDPD}) \cdot Log(PP_{c,t}) + R_{CP,c}$$

2.4.3 Production

To calculate the amount of each meat produced, UKAMM first creates a measure of the cost's farmers incur in meat production.

Table 2: Costs of livestock production covered in UKAMM

UKAMM cost index	Costs covered
Energy	Electricity and fuels
Other tradables	Veterinary costs and other specific livestock costs, machinery and buildings
Non-tradables	Contract work, other farming overheads, depreciation, wages and own work

UKAMM has indices of various standard types of costs incurred by farmers. It draws on these to calculate each commodity's measure of production costs. To calculate costs of livestock production, UKAMM draws on the indices for energy, non-tradable inputs and other tradable inputs.

The energy sub-index is affected by world oil prices. The non-tradable input costs are affected by the domestic GDP deflator. Inflation affects the index for tradable inputs. These give the following equation for the costs of livestock production:

$$\boldsymbol{CPCI_{c,t}} = \frac{CPCS..SHEN_{c,t} \cdot \left(\mathrm{XP_{oil,wld,t}} \cdot XR_{USA,t}\right)}{\left(XP_{oil,wld,2008} \cdot XR_{USA,2008}\right)} + \frac{CPCS..SHNT_{c,t} \cdot GDPD_t}{GDPD_{2008}} + \frac{\left(CPCS..SHTR_{c,t}\right) \cdot \left(GDPD_{EUN,t} \cdot XR_{EUN,t}\right)}{\left(GDPD_{EUN,2008} \cdot XR_{EUN,2008}\right)}$$

In this equation:

- $CPCI_{c,t}$ is the commodity production cost index for commodity c in year t.
- $CPCI..SHNT_{c,t}$ is the share of non-tradable inputs in total production costs.
- $CPCI..SHEN_{c,t}$ is the share of energy in total production costs.
- $CPCI..SHTR_{c,t}$ is the share of other tradable inputs in total production costs.

UKAMM models the amount of meat produced as a function of the contemporary producer price, a series of past producer prices ($PP_{c,t-i}$), subsidies ($EPQ_{c,t-i}$), feed costs ($FECI_{c,t-i}$), the commodity's production cost index ($CPCI_{c,t}$) and an exogenous trend.

For beef production, UKAMM also accounts for the size of the UK beef herd from the previous two years and the size of the UK dairy herd from the previous year.

$$\boldsymbol{Log(QP_{BV,t})} = \alpha + \beta_{QP_{BV},PP_{BV}} \cdot Log\left(\frac{PP_{BV,t} + EPQ_{BV,t}}{CPCI_{BV,t}}\right) + \beta_{QP_{BV},PP1_{BV}} \cdot Log\left(\frac{PP_{BV,t-1} + EPQ_{BV,t-1}}{CPCI_{BV,t-1}}\right) + \beta_{QP_{BV},PP2_{BV}} \cdot Log\left(\frac{PP_{BV,t-2} + EPQ_{BV,t-2}}{CPCI_{BV,t-2}}\right) + \beta_{QP_{BV},FECI1_{RU}} \cdot Log\left(\frac{FECI_{RU,t-1}}{CPCI_{BV,t-1}}\right) + \beta_{QP_{BV},FECI2_{RU}} \cdot Log\left(\frac{FECI_{RU,t-2}}{CPCI_{BV,t-2}}\right) + \beta_{QP_{BV},FECI3_{RU}} \cdot Log\left(\frac{FECI_{RU,t-3}}{CPCI_{BV,t-3}}\right) + \beta_{QP_{BV},CI1_{BV}} \cdot Log(CI_{BV,t-1}) + \beta_{QP_{BV},CI2_{BV}} \cdot Log(CI_{BV,t-2}) + \beta_{QP_{BV},CI1_{MK}} \cdot Log(CI_{MK,t-1}) + \beta_{QP_{BV}TRND} \cdot trend + R_{QP,BV}$$

For **sheepmeat**, UKAMM calculates the quantity produced by multiplying the number of sheep slaughtered by the average carcass weight:

$$\boldsymbol{QP_{PT,t}} = CW_{SH,t} * MKT_{SH,t}$$

UKAMM's more detailed sheepmeat production function models the number of sheep slaughtered. The production function includes the standard factors in other livestock sectors' production functions as well as the previous year's production ($QP_{c,t-i}$).

$$\boldsymbol{Log(MKT_{SH,t})} = \alpha + \beta_{QP_{SH},PP1_{SH}} \cdot Log\left(\frac{PP_{SH,t-1} + EPQ_{SH,t-1}}{CPCI_{SH,t-1}}\right) + \beta_{QP_{SH},FECI1_{RU}} \cdot Log\left(\frac{FECI_{RU,t-1}}{CPCI_{SH,t-1}}\right) + \beta_{QP_{SH},FECI2_{RU}} \cdot Log\left(\frac{FECI_{RU,t-2}}{CPCI_{SH,t-2}}\right) + \beta_{QP_{SH}QP_{SH1}} \cdot Log(QP_{SH,t-1}) + \beta_{QP_{SH}TRND} \cdot trend + R_{QP,SH}$$

Chicken production ($QP_{CK,t}$) and **other poultry** production ($QP_{OP,t}$) are modelled by UKAMM in a similar way to beef and sheepmeat. Production is modelled as a function of the price, feed costs and production from the previous period:

$$\boldsymbol{Log(QP_{CK,t})} = \beta_{QP_{CK},PP_{CK}} \cdot Log\left(\frac{PP_{CK,t}}{CPCI_{PT,t}}\right) + \beta_{QP_{CK},FECI_{NR}} \cdot Log\left(\frac{FECI_{NR,t}}{CPCI_{PT,t}}\right) + \beta_{QP_{CK},LAG} \cdot Log\left(QP_{CK,t-1}\right) + R_{QP,CK}$$

$$\boldsymbol{Log(QP_{OP,t})} = \beta_{QP_{OP},PP_{OP}} \cdot Log\left(\frac{PP_{OP,t}}{CPCI_{PT,t}}\right) + \beta_{QP_{OP},FECI_{NR}} \cdot Log\left(\frac{FECI_{NR,t}}{CPCI_{PT,t}}\right) + \beta_{QP_{OP},LAG} \cdot Log\left(QP_{OP,t-1}\right) + R_{QP,OP}$$

Overall poultry production ($QP_{PT,t}$) is the sum of chicken and other poultry production:

$$\boldsymbol{QP_{PT,t}} = QP_{CK,t} + QP_{OP,t}$$

For pigmeat, UKAMM calculates the quantity produced by multiplying the number of animals slaughtered ($SLH_{PK,t}$) by the average carcass weight ($CW_{PK,t}$):

$$QPS_{PK,t} = SLH_{PK,t} \cdot \frac{CW_{PK,t}}{1000}$$

UKAMM has a more detailed production function to model the number of pigs slaughtered:

$$Log(SLH_{PK,t}) = \alpha + \beta_{SLH_{PK}PP_{PP}} \cdot Log\left(\frac{PP_{PK,t-1}}{CPCI_{PK,t-1}}\right) + \beta_{SLH_{PK}FECI_{NR}} \cdot Log\left(\frac{FECI_{PK,t-1}}{GDPD_{t-1}}\right) + \beta_{SLH_{t}LAG_{t-1}} \cdot Log\left(SLH_{PK,t-1}\right) + R_{SLH,PK}$$

In this equation:

- $\boldsymbol{\frac{PP_{PK,t-1}}{CPCI_{PK,t-1}}}$ is the ratio of price to cost of production
- $\boldsymbol{\frac{FECI_{PK,t-1}}{GDPD_{t-1}}}$ is the cost of feed accounting for inflation
- $\boldsymbol{SLH_{PK,t-1}}$ is the number of pigs slaughtered in the previous year.

UKAMM models the carcass weight of pigs ($CW_{PK,t}$) in a similar way to other meat production as a function of price ($PP_{PK,t}$), subsidy ($EPQ_{PK,t}$) and feed costs ($FECI_{PK,t}$):

$$Log(CW_{PK,t}) = \alpha + \beta_{CW_{PK},PP_{PK}} \cdot Log\left(\frac{PP_{PK,t} + EPQ_{PK,t}}{CPCI_{PK,t}}\right) + \beta_{CW_{PK}FECI_{NR}} \cdot Log\left(\frac{FECI_{PK,t}}{GDPD_t}\right) + \beta_{CW_{PK}TRND} \cdot trend + R_{CW,PK}$$

UKAMM also models livestock inventories, the total number of animals of each species currently living on UK farms. For pigs and poultry, livestock inventories ($LI_{c,t}$) are functions of a constant (α), production ($QP_{c,t}$), a trend variable ($trend$) and a residual (R_{LI}) such that:

$$\boldsymbol{Log}(\boldsymbol{LI_{PK,t}}) = \alpha + Log(QP_{PK,t}) + \beta_{LI_{PK}TRND} \cdot trend + R_{LI,PK}$$

$$\boldsymbol{Log}(\boldsymbol{LI_{PT,t}}) = \alpha + Log(QP_{PT,t}) + \beta_{LI_{PT}TRND} \cdot trend + R_{LI,PT}$$

The total sheep population is the sum of the numbers of ewes and lambs, each of which is driven by the number of sheep slaughtered that year.

$$\boldsymbol{POP_{SH,t}} = LI_{SH,t} + LMB_{SH,t}$$

$$\boldsymbol{Log}(\boldsymbol{LI_{SH,t}}) = \alpha + \beta_{LI_{SH}MKT} Log(MKT_{SH,t}) + R_{LI,SH}$$

$$\boldsymbol{Log}(\boldsymbol{LMB_{SH,t}}) = \alpha + \beta_{LMB_{SH}MKT} Log(MKT_{SH,t}) + R_{LMB,SH}$$

The beef cow inventory is a measure of the size of the breeding herd of calving animals for slaughter. It is calculated using the returns from beef production for the period and the two preceding periods, the feed costs for the preceding three periods and a lagged variable.

$$\boldsymbol{Log}(\boldsymbol{CI}_{\boldsymbol{BV,t}}) = \alpha + \beta_{CI_{BV},RET_{BV}} \cdot Log\left(\frac{RET_{BV,t}}{CPCI_{BV,t}}\right) + \beta_{CI_{BV},RET1_{BV}} \cdot Log\left(\frac{RET_{BV,t-1}}{CPCI_{BV,t-1}}\right) + \beta_{CI_{BV},RET2_{BV}} \cdot Log\left(\frac{RET_{BV,t-2}}{CPCI_{BV,t-2}}\right) + \beta_{CI_{BV},FECI1_{BV}} \cdot Log\left(\frac{FECI_{BV,t-1}}{CPCI_{BV,t-1}}\right) + \beta_{CI_{BV},FECI2_{BV}} \cdot Log\left(\frac{FECI_{BV,t-2}}{CPCI_{BV,t-2}}\right) + \beta_{CI_{BV},FECI3_{BV}} \cdot Log\left(\frac{FECI_{BV,t-3}}{CPCI_{BV,t-3}}\right) + \beta_{CI_{BV},LAG1} \cdot Log(CI_{BV,t-1}) + \beta_{CI_{BV,t}CI1_{MK}} \cdot Log(CI_{MK,t-1}) + \beta_{CI_{BV}TRND} \cdot trend + R_{CI,BV}$$

Farmers' returns from beef production are a function of the producer price ($PP_{BV,t}$), subsidy ($EPQ_{BV,t}$) and the carcass weights ($CW_{BV,t}$).

$$\boldsymbol{RET}_{\boldsymbol{BV,t}} = (PP_{BV,t} + EPQ_{BV,t}) \cdot CW_{BV,t}$$

See section 3.1 for the discussion on subsidies.

2.2.4 Consumption

Beef, sheepmeat, pigmeat and poultrymeat are used exclusively as food. In UKAMM, their total consumption is therefore equal to the quantity demanded from the food market.

UKAMM models the food demand for these four types of meat as a function of economic growth $\left(\frac{GDPI_t}{POP_t}\right)$, population growth (POP_t) and two types of price elasticity.

The price elasticity of demand is a measure of how much consumer demand changes in response to a given change in price. Cross-price elasticities in this part of UKAMM give a measure of the extent to which consumer demand for one type of meat changes in response to changes in the price of other types of meat $\left(\frac{CP_{c,t}}{GDPD_t}\right)$.

Beef, sheepmeat, poultry and pigmeat consumption all have the following form – for brevity we have only included the beef consumption equation:

$$\begin{aligned}\boldsymbol{Log(FO_{BV,t})} = {} & \alpha + \beta_{FO_{BV}CP_{BV}} \cdot Log\left(\frac{CP_{BV,t}}{GDPD_t}\right) \\ & + \beta_{FO_{BV}CP_{PK}} \cdot Log\left(\frac{CP_{PK,t}}{GDPD_t}\right) + \beta_{FO_{BV}CP_{SH}} \cdot Log\left(\frac{CP_{SH,t}}{GDPD_t}\right) \\ & + \beta_{FO_{BV}CP_{PT}} \cdot Log\left(\frac{CP_{PT,t}}{GDPD_t}\right) + \beta_{FO_{BV}GDPI} \cdot Log\left(\frac{GDPI_t}{POP_t}\right) \\ & + Log(POP_t) + \beta_{FO_{BV}TRND} \cdot trend + R_{FO,BV}\end{aligned}$$

2.5 Dairy

Milk can be used to produce a variety of various products. UKAMM's modelling of the dairy sector considers these various interlocking markets. In UKAMM's model of the dairy sector, prices adjust until production equals demand for four processed products:

- butter
- cheese
- skimmed milk powder
- whole milk powder.

Two components of milk are used in dairy processing to make these products: fats and non-fat solids. Fats are used to make butter and cheese.

Non-fat solids are more prevalent in skimmed and whole milk powder. After calculating the demand for each dairy product at market equilibrium, UKAMM calculates what this implies for the demand for raw milk for processing.

It does this by calculating the amount of fats and non-fat solids required to satisfy the demand for each product.

2.5.1 Producer prices

UKAMM uses prices to clear the UK markets for butter, cheese, skimmed milk powder and whole milk powder. When simulating, UKAMM adjusts prices until the amount of each of these products produced $(QP_{c,t})$, imported $(IM_{c,t})$ and stockpiled from the previous year $(ST_{c,t-1})$ equals the amount consumed $(QC_{c,t})$, exported $(EX_{c,t})$ and stockpiled for the following year $(ST_{c,t})$.

This works as all these variables are dependent on price. In the model, this is expressed algebraically:

$$\boldsymbol{PP_{BT,t}}\text{: } 0 = QP_{BT,t} + IM_{BT,t} + ST_{BT,t-1} - QC_{BT,t} - EX_{BT,t} - ST_{BT,t}$$

$$\boldsymbol{PP_{CH,t}}\text{: } 0 = QP_{CH,t} + IM_{CH,t} + ST_{CH,t-1} - QC_{CH,t} - EX_{CH,t} - ST_{CH,t}$$

$$\boldsymbol{PP_{SMP,t}}\text{: } 0 = QP_{SMP,t} + IM_{SMP,t} + ST_{SMP,t-1} - QC_{SMP,t} - EX_{SMP,t} - ST_{SMP,t}$$

$$\boldsymbol{PP_{WMP,t}}\text{: } 0 = QP_{WMP,t} + IM_{WMP,t} + ST_{WMP,t-1} - QC_{WMP,t} - EX_{WMP,t} - ST_{WMP,t}$$

Having simulated the price and quantity at which these markets are in equilibrium, UKAMM calculates the implications for the two components of milk used in these different dairy manufacturing processes:

- fat
- non-fat solids

UKAMM models price of the fat component as a function of the fat content of milk $(FAT_{MK,t})$, the quantity of milk used in manufacturing $(MAN_{MK,t})$, and the amount of fat used in the production of dairy products $(QP_{c,t} \cdot FAT_{c,t})$:

$$\begin{aligned}\boldsymbol{PP..FAT_t}: 0 = & \left(FAT_{MK,t} \cdot \left(\frac{MAN_{MK,t}}{\alpha}\right)\right) \\ & - \left(QP_{BT,t} \cdot FAT_{BT,t}\right) - \left(QP_{CH,t} \cdot FAT_{CH,t}\right) \\ & - \left(QP_{SMP,t} \cdot FAT_{SMP,t}\right) - \left(QP_{WMP,t} \cdot FAT_{WMP,t}\right) \\ & - \left(QP_{FDP,t} \cdot FAT_{FDP,t}\right) - \left(QP_{WYP,t} \cdot FAT_{WYP,t}\right)\end{aligned}$$

UKAMM models the price of non-fat solids as a function of the non-fat solids content of milk $(NFS_{MK,t})$, the milk demand from manufacturing $(MAN_{MK,t})$, and the amount of non-fat solids used in the production of dairy products $(QP_{c,t} \cdot NFS)$:

$$\begin{aligned}\boldsymbol{PP..NFS_t}: 0 = & \left(NFS_{MK,t} \cdot \left(\frac{MAN_{MK,t}}{\alpha}\right)\right) \\ & - \left(QP_{BT,t} \cdot NFS_{BT,t}\right) - \left(QP_{CH,t} \cdot NFS_{CH,t}\right) \\ & - \left(QP_{SMP,t} \cdot NFS_{SMP,t}\right) - \left(QP_{WMP,t} \cdot NFS_{WMP,t}\right) \\ & - \left(QP_{FDP,t} \cdot NFS_{FDP,t}\right) - \left(QP_{WYP,t} \cdot NFS_{WYP,t}\right)\end{aligned}$$

To calculate the price of milk $(PP_{MK,t})$, UKAMM takes an average of the price of the fat $(PP..FAT_{MK,t})$ and non-fat $(PP..NFS_{MK,t})$ components. This average is then divided by the milk margin $(MAR_{MK,t})$, which is based on historical pricing of raw milk compared to the fat and non-fat components:

$$\boldsymbol{PP_{MK,t}} = \left(\frac{\left(PP..FAT_{MK,t} \cdot FAT_{MK,t} + PP..NFS_{MK,t} \cdot NFS_{MK,t}\right)}{MAR_{MK,t}}\right)$$

2.5.2 Consumer prices

Butter and cheese consumer prices are modelled separately to producer prices in order to be able to differentiate between price changes for producers and consumers. Consumer price ($CP_{c,t}$) is modelled as a function of the GDP deflator ($GDPD_t$) and the producer price ($PP_{c,t}$):

$$\boldsymbol{Log}(\boldsymbol{CP}_{\boldsymbol{BT},\boldsymbol{t}}) = \alpha + \beta_{CP_{BT}GDPD} \cdot Log(GDPD_t) + \left(1 - \beta_{CP_{BT}GDPD}\right) \cdot Log\left(PP_{BT,t}\right) + R_{CP,BT}$$

$$\boldsymbol{Log}(\boldsymbol{CP}_{\boldsymbol{CH},\boldsymbol{t}}) = \alpha + \beta_{CP_{CH}GDPD} \cdot Log(GDPD_t) + \left(1 - \beta_{CP_{CH}GDPD}\right) \cdot Log\left(PP_{CH,t}\right) + R_{CP,CH}$$

Skimmed and whole milk powder consumer prices are assumed to be equal to the producer prices:

$$\boldsymbol{CP}_{\boldsymbol{SMP},\boldsymbol{t}} = PP_{SMP,t}$$

$$\boldsymbol{CP}_{\boldsymbol{WMP},\boldsymbol{t}} = PP_{WMP,t}$$

2.5.3 Production

Milk production ($QP_{MK,t}$) is calculated as the product of the cow inventory ($CI_{MK,t}$) and the milk yield ($YLD_{MK,t}$), adjusted for on-farm use of milk:

$$\boldsymbol{QP}_{\boldsymbol{MK},\boldsymbol{t}} = [CI_{MK,t} \cdot YLD_{MK,t}] - FU_{mk,t}$$

The **cow inventory** for dairy cows is a function of the price and subsidy in the dairy market over the current and previous period $\left(PP_{MK,t} + EPI_{MK,t}\right)$, the price and subsidy from beef production for the current period $\left(PP_{BV,t} + EPI_{BV,t}\right)$, the feed cost for ruminants over the current and previous periods $\left(FECI_{RU,t}\right)$, milk costs of production $\left(CPCI_{MK,t}\right)$and a lagged variable for the cow inventory in the previous year:

$$CI_{MK,t} = \alpha + \beta_{CI_{MK}PP_{MK}} \cdot Log\left(\frac{PP_{MK,t} + EPI_{MK,t}}{CPCI_{MK,t}}\right)$$
$$+ \beta_{CI_{MK}PP1_{MK}} \cdot Log\left(\frac{PP_{MK,t-1} + EPI_{MK,t-1}}{CPCI_{MK,t-1}}\right)$$
$$+ \beta_{CI_{MK}PP_{BV}} \cdot Log\left(\frac{PP_{BV,t} + EPQ_{BV,t}}{CPCI_{MK,t}}\right)$$
$$+ \beta_{CI_{MK}FECI_{RU}} \cdot Log\left(\frac{FECI_{RU,t}}{CPCI_{MK,t}}\right) + \beta_{CI_{MK}FECI1_{RU}} \cdot Log\left(\frac{FECI_{RU,t-1}}{CPCI_{MK,t-1}}\right)$$
$$+ \beta_{CI_{MK}LAG1} \cdot Log(CI_{MK,t-1}) + R_{CI,MK}$$

Subsidy discussion and derivation can be found in section 3.1.

Milk's costs of production are calculated in much the same way as livestock costs, as an index based on energy costs ($CPCI..SHEN_{c,t}$), the cost of non-tradables ($CPCI..SHNT_{c,t}$), and other tradable inputs ($CPCI..SHTR_{c,t}$). See the livestock production section 2.4.3 for more details on how these indexes are calculated.

$$CPCI_{c,t} = \frac{CPCI..SHEN_{c,t} \cdot (\mathrm{XP}_{\mathrm{oil,wld,t}} \cdot XR_{USA,t})}{(XP_{oil,wld,2008} \cdot XR_{USA,2008})}$$
$$+ \frac{CPCI..SHNT_{c,t} \cdot GDPD_t}{GDPD..2008_{2008}}$$
$$+ \frac{(CPCI..SHTR_{c,t}) \cdot (GDPD_{EUN,t} \cdot XR_{r,t})}{(GDPD_{EUN,2008} \cdot XR_{r,2008})}$$

Milk yield ($YLD_{MK,t}$) is calculated using the milk producer price for the current period ($PP_{MK,t}$), the feed costs index ($FECI_{RU,t}$), and a trend variable ($trend$):

$$Log(YLD_{MK,t}) = \alpha + \beta_{YLD_{MK}PP_{MK}} \cdot Log(PP_{MK,t})$$
$$+ \beta_{YLD_{MK}FECI_{RU}} \cdot Log(FECI_{RU,t})$$
$$+ \beta_{YLD_{MK}TRND} \cdot trend + R_{YLD,MK}$$

Butter production is calculated as a function of the relative price between butter and the fat component of milk ($\frac{PP_{BT,t}}{PP_{FAT,t}}$):

$$\boldsymbol{Log(QP_{BT,t})} = \alpha + \beta_{QP_{BT}PP_{BT}} \cdot Log\left(\frac{PP_{BT,t}}{PP_{FAT,t}}\right) + R_{QP,BT}$$

Cheese production is calculated as a function of the relative price between cheese and the fat ($\frac{PP_{CH,t}}{PP_{FAT,t}}$) and non-fat solid ($\frac{PP_{CH,t}}{PP_{NFS,t}}$) components of milk:

$$\boldsymbol{Log(QP_{CH,t})} = \alpha + \beta_{QP_{CH}PP_{CH}} \cdot Log\left(\frac{PP_{CH,t}}{PP_{FAT,t}}\right) + \beta_{QP_{CH}PP2_{CH}} \cdot Log\left(\frac{PP_{CH,t}}{PP_{NFS,t}}\right) + R_{QP,CH}$$

Whey powder is a by-product of the cheese manufacturing process, so is therefore a function of cheese production:

$$\boldsymbol{Log(QP_{WYP,t})} = \alpha + \beta_{QP_{WYP}QP_{CH}} \cdot Log(QP_{CH,t}) + R_{QP,WYP}$$

Skimmed and whole milk powder production is a function of the relative price between the producer price of the powder and the price of the non-fat solid component of milk:

$$\boldsymbol{Log(QP_{SMP,t})} = \alpha + \beta_{QP_{SMP}PP_{SMP}} \cdot Log\left(\frac{PP_{SMP,t}}{PP_{NFS,t}}\right)$$

$$\boldsymbol{Log(QP_{WMP,t})} = \alpha + \beta_{QP_{WMP}PP_{WMP}} \cdot Log\left(\frac{PP_{WMP,t}}{PP_{NFS,t}}\right)$$

2.5.4 Consumption

Total milk demand ($QC_{MK,t}$) is taken as the sum of the demand for liquid milk ($LU_{MK,t}$) and the demand from manufacturing ($MAN_{MK,t}$).

$$\boldsymbol{QC_{MK,t}} = LU_{MK,t} + MAN_{MK,t}$$

Liquid use is modelled as depending on population and price:

$$\boldsymbol{LOG(\ LU_{MK,t})} = \alpha + \beta_{LU_{MK}PP_{MK}} \cdot Log(PP_{MK,t}) + Log(POP_{ME,t}) + R_{LU,MK}$$

UKAMM calculates the milk used in manufacturing as the liquid milk demand subtracted from the total milk production, multiplied by a constant, α. This constant capture milk lost through on-farm usage and wastage.

$$\boldsymbol{MAN_{MK,t}} = (QP_{MK,t} - LU_{MK,t}) \cdot \alpha$$

The quantity of butter consumed ($QC_{BT,t}$) is a function of its own consumer price ($CP_{BT,t}$) and how that compares to the consumer price of vegetable oil ($CP_{VL,t}$) and cheese ($CP_{CH,t}$). It is also dependent on economic growth and population.

$$\boldsymbol{Log\left(QC_{BT,t}\right)} = \alpha + \beta_{QC_{BT}CP_{VL}} \cdot Log\left(\frac{CP_{VL,t}}{GDPD_t}\right) + \beta_{QC_{BT}CP_{CH}} \cdot Log\left(\frac{CP_{CH,t}}{GDPD_t}\right) + \beta_{QC_{BT}CP_{BT}} \cdot Log\left(\frac{CP_{BT,t}}{GDPD_t}\right) + \beta_{QC_{BT}GDPI} \cdot Log\left(\frac{GDPI_t}{POP_t}\right) + Log(POP_t) + R_{QC,BT}$$

The quantity of cheese consumed is a function of its own consumer price and how that compares to the consumer price of vegetable oil and butter. It is also dependent on economic growth.

$$\boldsymbol{Log(QC_{CH,t})} = \alpha + \beta_{QC_{CH}CP_{CH}} \cdot Log\left(\frac{CP_{CH,t}}{GDPD_t}\right) + \beta_{QC_{CH}CP_{BT}} \cdot Log\left(\frac{CP_{BT,t}}{GDPD_t}\right) + \beta_{QC_{CH}CP_{FDP}} \cdot Log\left(\frac{CP_{FDP,t}}{GDPD_t}\right) + \beta_{QC_{CH}GDPI} \cdot Log\left(\frac{GDPI_t}{POP_t}\right) + Log(POP_t) + R_{QC,CH}$$

The quantity of skimmed milk powder consumed ($QC_{SMP,t}$) is a function of its own consumer price ($CP_{SMP,t}$), the relative consumer price of whole milk powder ($CP_{WMP,t}$), economic growth, population and an exogenous trend.

$$\boldsymbol{Log(QC_{SMP,t})} = \alpha + \beta_{QC_{SMP}CP_{SMP}} \cdot Log\left(\frac{CP_{SMP,t}}{GDPD_t}\right) + \beta_{QC_{SMP}CP_{WMP}} \cdot Log\left(\frac{CP_{WMP,t}}{GDPD_t}\right) + \beta_{QC_{SMP}GDPI} \cdot Log\left(\frac{GDPI_t}{POP_t}\right) + Log(POP_t) + \beta_{QC_{SMP}TRND} \cdot trend + R_{QC,SMP}$$

The quantity of whole milk powder consumed ($QC_{WMP,t}$) is a function of its own price, its relative price with skimmed milk powder, economic growth, population and an exogenous trend:

$$\boldsymbol{Log(QC_{WMP,t})} = \alpha + \beta_{QC_{WMP}CP_{WMP}} \cdot Log\left(\frac{CP_{WMP,t}}{GDPD_t}\right) + \beta_{QC_{WMP}CP_{SMP}} \cdot Log\left(\frac{CP_{SMP,t}}{GDPD_t}\right) + \beta_{QC_{WMP}GDPI} \cdot Log\left(\frac{GDPI_t}{POP_t}\right) + Log(POP_t) + \beta_{QC_{WMP}TRND} + R_{QC,WMP}$$

3 Market support

Agricultural production, consumption and prices are affected by a wide variety of policy measures from health standards to landscape plans.

These policies often intervene to correct for market failures where, left to its own devices, the market would create an outcome with negative environmental or societal consequences. This could be due to an ‘externality’, such as river pollution or carbon emissions, where the long term costs are not factored into market prices and so intervention is required to improve sub-optimal outcomes.

The impacts of many of these policies will be implicit in market prices or have affects on the agricultural economy not captured by a model like UKAMM.

Subsidies and intervention stocks are two major policies with an impact on UK agricultural markets that UKAMM does model explicitly. Both are examples of market support policies.

3.1 Subsidies

Farming subsidies are changing

UKAMM’s subsidy modelling is a simplified approximation of UK farming support as it stands in February 2021. Over the next few years the government plans to transform the market support farmers receive, laid out in the ‘Path to Sustainable Farming’ roadmap in November 2020. UKAMM is a living model. So as the exact shape of the new system is defined, we will update the model to account of its effects on UK agriculture.

In UKAMM we model subsidies for cereals, beef, sheepmeat and milk endogenously. We treat subsidies for pork and poultry exogenously and assume they are equal to zero.

UKAMM models government subsidies to agriculture in terms of two different types of subsidies: coupled and decoupled payments. Coupled payments are subsidies conditional on how much farmers produce.

Decoupled payments are independent of how much farmers produce. The total subsidy paid to farmers ($SUB_{AG,t}$) is the sum of coupled payments ($SUBC_{AG,t}$) and decoupled payments ($SUBD_{AG,t}$) together.

$$\boldsymbol{SUB_{AG,t}} = SUBC_{AG,t} + SUBD_{AG,t}$$

We capture the effects that decoupled payments have on production by using coupling factors, which is the standard approach in partial equilibrium models like UKAMM. The coupling factor measures the degree to which the single payment can be treated as a coupled payment.

The single farm payment is the means by which the government pays out agricultural subsidies. The decoupled payment received in each sector ($EPA_{c,t}$) is equal to the single farm payment ($SFP_{c,t}$) multiplied by the coupling factor ($SFP..CF_{c,t}$). This is described in following equation for cereals, which is similar to equations for beef, sheepmeat and milk:

$$\boldsymbol{EPA_{AG,t}} = SFP_{AG,t} \cdot SFP..CF_{AG,t}$$

For cereal crops, the decoupled payments are calculated on a sector basis while different coupled payment amounts are calculated for each crop.

The decoupled payments for individual crops ($EPA_{c,t}$) are the sum of the coupled payments ($EPA..DP_{c,t}$) and decoupled payments ($EPA_{AG,t}$). Coupled payments in the arable sector are assumed to be zero. This gives the following equation for wheat, barley, oats, rapeseed and sugar beet:

$$\boldsymbol{EPA_{c,t}} = EPA_{AG,t} + EPA..DP_{c,t}$$

These decoupled payments in the different sectors feed into production functions and livestock inventories – this can be seen in the different commodity sections earlier in this documentation.

UKAMM's subsidy model at present follows the structure the UK inherited from the European Union's Common Agricultural Policy (CAP). Under this policy there are two 'pillars' of payments. Pillar 1 payments provide direct income support to farmers. Pillar 2 payments support a broader range of rural development goals including sustainability, competitiveness and regional employment balance.

Pillar 1 payments ($P1..FI..GBP_{AG,t}$) are equal to the Pillar 1 ceiling set by the EU in euros ($P1..FI_{AG,t}$) multiplied by the pound-euro exchange rate ($XR_{ME,t}$):

$$\boldsymbol{P1..FI..GBP_{AG,t}} = P1..FI_{AG,t} \cdot XR_{ME,t}$$

Pillar 2 payments ($P2..FI..GBP_{AG,t}$) are equal to the Pillar 2 ceiling set by the EU in euros ($P2..FI_{AG,t}$) multiplied by the pound-euro exchange rate ($XR_{ME,t}$):

$$P2..FI..GBP_{AG,t} = P2..FI_{AG,t} \cdot XR_{ME,t}$$

The Pillar 1 and 2 payments feed into the agricultural single farm payment. The latter is a function of the decoupled section of Pillar 1 payments ($SUBD_{AG,t} - P2..FI_{AG,t}$), and the amount of land eligible to receive single farm payments ($AH_{EA,t}$):

$$\boldsymbol{SFP_{AG,t}} = \frac{\left(SUBD_{AG,t} - P2..FI_{AG,t}\right)}{AH_{EA,t} \cdot 1000}$$

The decoupled payments ($SUBD_{AG,t}$) are equal to the decoupled part of Pillar 1 payments ($P1..FI..GBP_{AG,t} - SUBC_{AG,t}$) plus Pillar 2 payments ($P2..FI..GBP_{AG,t}$):

$$\boldsymbol{SUBD_{AG,t}} = \left(P1..FI..GBP_{AG,t} - SUBC_{AG,t}\right) + P2..FI..GBP_{AG,t}$$

3.2 Intervention stocks

UKAMM includes provision for intervention stocks. The logic of this policy is that if the price of a commodity gets too low, the government would purchase stocks of the commodity in order to increase demand and stabilise the price. The intervention price acts as a floor price in the model.

Intervention stock equations are included in UKAMM for wheat, skimmed milk powder and butter. Intervention stocks ($IST_{c,t}$) are a function of the amount that producer prices have fallen below the intervention price ($SP..GBP_{c,t} - PP_{c,t}$), and a constant ($ISTC_{c,t}$). If producer prices are not below the intervention price, intervention stocks are not triggered.

$$\begin{aligned}\boldsymbol{IST_{WTS,t}} &= \text{if } PP_{WTS,t} > SP..GBP_{WTS,t} \\ &\Rightarrow \text{then } 0 \\ &\Rightarrow \text{else } \left(\left(SP..GBP_{WTS,t} - PP_{WTS,t}\right) \cdot ISTC_{WTS,t}\right)\end{aligned}$$

$$\begin{aligned}\boldsymbol{IST_{SMP,t}} &= \text{if } PP_{SMP,t} > SP..GBP_{SMP,t} \\ &\Rightarrow \text{then } 0 \\ &\Rightarrow \text{else } \left(\left(SP..GBP_{SMP,t} - PP_{SMP,t}\right) \cdot ISTC_{SMP,t}\right)\end{aligned}$$

$$\begin{aligned}\boldsymbol{IST_{BT,t}} &= IF\ PP_{BT,t} > SP..GBP_{BT,t} \\ &\Rightarrow \text{then } 0 \\ &\Rightarrow \text{else } \left(\left(SP..GBP_{BT,t} - PP_{BT,t}\right) \cdot ISTC_{BT,t}\right)\end{aligned}$$

In the model, the intervention price is currently determined at the EU level, so the UK intervention price ($SP..GBP_{c,t}$) is simply the EU intervention price ($SP_{EUN,WTS,t}$) multiplied by the exchange rate ($XR_{EUN,t}$). This may change now the UK has left the EU as the choice of intervention price and the wider policy is a UK policy decision.

$$\boldsymbol{SP..GBP_{WTS,t}} = SP_{EUN,WTS,t} \cdot XR_{EUN,t}$$

$$\boldsymbol{SP..GBP_{SMP,t}} = SP_{EUN,SMP,t} \cdot XR_{EUN,t}$$

$$\boldsymbol{SP..GBP_{BT,t}} = SP_{EUN,BT,t} \cdot XR_{EUN,t}$$

4 Trade

UKAMM is a three-country model, aggregating the UK's trading partners into two blocs: the European Union (EU) and the rest of the world (RoW). Each sector includes a standard set of equations describing imports and exports between the UK and these two blocs.

It should be noted that the model makes the simplifying assumption that the UK has no market power in affecting world prices. Whilst a restrictive assumption, this allows for foreign prices (producer prices in the EU and the rest of the world) to be inputted exogenously into the model.

4.1 Imports

Imports ($IM_{c,t}$) are the sum of imports from the EU ($IM..EU_{c,t}$) and imports from the rest of the world ($IM..WLD_{c,t}$). The example equations are the beef (BV) trade equations:

$$\boldsymbol{IM_{BV,t}} = IM..EU_{BV,t} + IM..WLD_{BV,t}$$

Imports from the EU ($IM..EU_{c,t}$) are functions of the relative domestic producer price ($PP_{c,t}$) and the import price from the EU ($IMP..EU_{c,t}$):

$$\boldsymbol{Log(IM..EU_{BV,t})} = \alpha + \beta_{IM..EU_{BV}PP_{BV}} \cdot Log\left(\frac{PP_{BV,t}}{IMP..EU_{BV,t}}\right) + R_{IM..EU,BV}$$

Similarly, imports from the rest of the world are functions of the relative domestic producer price ($PP_{c,t}$) and the import price form the international market ($IMP..WLD_{c,t}$):

$$\boldsymbol{Log(IM..WLD_{BV,t})} = \alpha + \beta_{IM..WLD_{BV}PP_{BV}} \cdot Log\left(\frac{PP_{BV,t}}{IMP..WLD_{BV,t}}\right) + R_{IM..WLD,BV}$$

The import price from the EU is a function of the EU internal market price ($PP_{c,E15,t}$) and any import tariff applied ($TAVI..EU_{c,t}$):

$$\boldsymbol{IMP..EU_{BV,t}} = PP_{BV,E15,t} \cdot XR_{EUN,t} \cdot \left(1 + \frac{TAVI..EU_{BV,t}}{100}\right)$$

The import price from the international market is similarly a function of the world reference price, converted to sterling, and any import tariffs that may have been applied.

$$\boldsymbol{IMP..WLD_{BV,t}} = PP_{BV,WLD,t} \cdot XR_{USA,t} \cdot \left(1 + \frac{TAVI..WLD_{BV,t}}{100}\right)$$

4.1.1 Tariffs

There are three types of tariff. Ad valorem tariffs charge a percentage on the value of imports. Specific tariffs charge a fixed fee per unit imported – a number of pounds per tonne, for example. Compound tariffs combine both, charging both a percentage of the value and additionally a fixed fee per unit.

In many of UKAMM's equations, specific and compound tariffs get recalculated as ad valorem equivalent tariff rates. If prices vary year to year, the ad valorem equivalent of a specific tariff will vary too: if the fixed fee stays the same but price falls, it will be a bigger percentage of the overall post-tariff import cost.

UKAMM converts specific tariffs into ad valorem equivalents by converting their import price into sterling and dividing the specific tariff by this per unit price. This calculates how big the specific import tariff is as a proportion of the current overseas producer price (multiplying by 100 to represent this as a percentage). So, for butter, the in-quota tariff ad valorem equivalent is given by:

$$\boldsymbol{TAVI..IQ..WLD_{BT,t}} = \left(\frac{TSP..IQ..WLD_{BT,t}}{XR_{USA,t} \cdot PP_{BT,WLD,t}}\right) \cdot 100$$

Table 6: Livestock Import Tariff Lines

Commodity	Combined Nomenclature Tariff Code	Label
Beef and Veal	0201 30 00	Meat of bovine animals, fresh or chilled, boneless
Sheep	0204 42 90	Meat of sheep and goats, frozen, other cuts with bone in
Pork	0203 29 55	Meat of swine, frozen, boneless
Poultry	16 02 3219	Prepared or preserved chicken meat
Butter	0405 10 19	Butter
Cheese	0406 90 21	Cheddar
Skimmed Milk Powder	0402 21 9100	-
Whole Milk Powder	0402 21 9100	-
Wheat	10 01 99 00	Common wheat, spelt and muslin grains
Barley	10 03 90 00	Barley grains
Maize	10 05 90 00	Maize grains
Other Cereals	10 07 90 00	Grain sorghum
Raw Cane Sugar	17 01 13 10 17 01 14 10	Cane sugar, for refining Other cane sugar, for refining
White Sugar	17 01 99 10	White Sugar

4.1.2 Tariff rate quotas

For some products, the UK allows a certain volume of imports at one tariff then charges a higher tariff on any imports outside of this quota. This type of policy is called a tariff rate quota.

Cereals and dried milk powders, for example, have tariff rate quotas. If imports from world markets are below their respective quotas ($IM..WLD..TRQ_{c,t}$), then only the in-quota tariff is levied ($TAVI..IQ..WLD_{c,t}$). If imports exceed the respective TRQs, then the higher, out-of-quota tariff is levied ($TAVI..WLD_{c,t}$).

For cereals and milk powders, these tariffs are ad valorem tariffs and are inputted exogenously into the model. UKAMM's equation for the import price for cereals or milk powders from outside the EU is therefore:

$$\boldsymbol{IMP..WLD_{c,t}} = \text{if}\,(IM..WLD_{c,t} < IM..WLD..TRQ_{c,t})$$
$$\Rightarrow \text{then}\left(PP_{c,WLD,t} \cdot XR_{USA,t} \cdot \left(1 + \frac{TAVI..IQ..WLD_{c,t}}{100}\right)\right)$$
$$\Rightarrow \text{else}\left(PP_{c,WLD,t} \cdot XR_{USA,t} \cdot \left(1 + \frac{TAVI..WLD_{c,t}}{100}\right)\right)$$

This type of equation, with discrete limits at which the results drastically change, are known as discontinuous functions. Discontinuous functions in mathematical models can cause problems as they often allow for multiple solutions. So, while the more realistic equation above represents the tariff rate quotas for cereals and milk powders, a smoother function is used in UKAMM's meat, cheese and dairy sectors.

For meat, cheese and dairy tariff rates quotas, UKAMM treats the effective tariff rate as increasing exponentially as imports increase towards the quota before stabilising close to the out-of-quota tariff rate.

Import tariffs for meat, cheese and butter are therefore dependent on the in-quota tariff ($TAVI..IQ..WLD_{c,t}$), the amount of imports from the rest of the world ($IM..WLD_{c,t}$), the quota size ($IM..WLD..TRQ_{c,t}$), the ad valorem component of the out-of-quota tariff ($\alpha_{TAVI..WLD_{c,t}}$), the specific component of the out-of-quota tariff ($TSP..WLD_{c,t}$), and the world reference price in sterling ($PP_{WLD,c,t} \cdot XR_{USA,t}$).

The following beef smoothed TRQ equation is replicated for sheepmeat, poultry, pigmeat, cheese and butter:

$$\boldsymbol{TAVI..WLD_{BV,t}} = TAVI..IQ..WLD_{BV,t} + \gamma_{BV,t} \cdot \left(\alpha_{TAVI..WLD_{BV,t}} + \left(\frac{TSP..WLD_{BV,t}}{PP_{WLD,BV,t} \cdot XR_{USA,t}}\right) \cdot 100\right)$$

Whereby $\boldsymbol{\gamma_{BV,t}} \equiv \frac{e^{\min\left\{0,\frac{IM..WLD_{BV,t}-IM..WLD..TRQ_{BV,t}}{IM..WLD..TRQ_{BV,t}}\right\}\cdot 10}}{1+e^{-1\cdot\frac{\left|IM..WLD_{BV,t}-IM..WLD..TRQ_{BV,t}\right|}{IM..WLD..TRQ_{BV,t}}\cdot 10}}$

For clarity, the scaling factor that smooths the transition between in-quota and out-of-quota tariffs is denoted $\gamma_{c,t}$ and then detailed separately.

4.2 Exports

UKAMM models UK exports to the EU and UK exports to the rest of the world separately. Total UK exports ($EX_{c,t}$) are the sum of these separate simulations of exports to the EU ($EX..EU_{c,t}$), and exports to the rest of the world ($EX..WLD_{c,t}$).

$$\boldsymbol{EX_{BV,t}} = EX..EU_{BV,t} + EX..WLD_{BV,t}$$

UKAMM assumes exports to the EU ($EX..EU_{c,t}$) to be functions of the relative domestic producer ($PP_{c,t}$) and the export price to the EU ($EXP..EU_{c,t}$):

$$\boldsymbol{Log(EX..EU_{c,t})} = \alpha + \beta_{EX..EU_c PP_c} \cdot Log\left(\frac{PP_{c,t}}{EXP..EU_{c,t}}\right) + R_{EX..EU,c}$$

UKAMM models exports to the rest of the world in a similar way, assuming them to be functions of the relative domestic producer ($PP_{c,t}$) and the export price to the international market ($EXP..WLD_{c,t}$).

$$\boldsymbol{Log(EX..WLD_{c,t})} = \alpha + \beta_{EX..WLD_c PP_c} \cdot Log\left(\frac{PP_{c,t}}{EXP..WLD_{c,t}}\right) + R_{EX..WLD,c}$$

UKAMM calculates the export price to the EU market as a function of the European producer price ($PP_{c,E15,t}$) and any export tariff imposed on the UK's exports (inputted as negative ad valorem equivalents, $TAVE..EU_{c,t}$):

$$\boldsymbol{EXP..EU_{c,t}} = PP_{c,E15,t} \cdot XR_{EUN,t} \cdot \left(1 + \frac{TAVE..EU_{c,t}}{100}\right)$$

UKAMM calculates the world export price as a function of the producer price and any export tariffs imposed on UK exports ($TAVE..WLD_{c,t}$):

$$\boldsymbol{EXP..WLD_{c,t}} = PP_{c,WLD,t} \cdot XR_{USA,t} \cdot \left(1 + \frac{TAVE..WLD_{c,t}}{100}\right)$$

4.3 Sugar Trade

UKAMM models trade in sugar differently to other sectors. Sugar imports and exports are split into raw and white sugar, rather than into EU and non-EU categories. Imports are the only source of raw sugar cane into the UK's domestic market as there is no domestic production. These imports are a function of the margin of refining cane into white sugar:

$$\boldsymbol{Log(IM_{SUR,t})} = \alpha + \beta_{IM_{SUR}PP_{SUW}} \cdot Log\left(\frac{PP_{SUW,t}}{(0.5 \cdot IMP_{SUR,t}) + (0.5 \cdot IMP_{SUR,t-1})}\right) + R_{IM,SUR}$$

The import price is equal to the world reference price for raw sugar cane ($XP_{SUR,WLD,t}$), plus any additional tariff ($TAVI_{SUR,t}$):

$$\boldsymbol{IMP_{SUR,t}} = XP_{SUR,WLD,t} \cdot XR_{USA,t} \cdot \left(1 + \frac{TAVI_{SUR,t}}{100}\right)$$

Historically the UK and the EU have accepted sugar imports at a lower tariff rate from a group of 19 African, Caribbean and Pacific ('ACP') countries. For this reason, UKAMM models the 'tariff' on raw sugar imports as a function of the price gap between the world reference price of raw sugar cane and the ACP producer price, to reflect the additional cost of sourcing from these preferential suppliers:

$$\boldsymbol{TAVI_{SUR,t}} = \left(\frac{\frac{ACP..PG_{SUR,t}}{XR_{EUN,t}}}{XP_{SUR,t}}\right) \cdot 100$$

UKAMM models white sugar exports as a function of the relative producer (PP_{SUW}) and export prices(EXP_{SUW}):

$$\boldsymbol{Log(EX_{SUW,t})} = \alpha + \beta_{EX_{SUW}PP_{SUW}} \cdot Log\left(\frac{PP_{SUW,t}}{EXP_{SUW,t}}\right) + R_{EX,SUW}$$

Similarly, UKAMM models white sugar imports as a function of the relative producer and import prices(IMP_{SUW}):

$$\boldsymbol{Log(IM_{SUW,t})} = \alpha + \beta_{IM_{SUW}PP_{SUW}} \cdot Log\left(\frac{PP_{SUW,t}}{IMP_{SUW,t}}\right) + R_{IM,SUW}$$

UKAMM assumes export and import prices to be equal to the European producer prices, plus respective export and import tariffs:

$$\boldsymbol{EXP_{SUW,t}} = PP_{SUW,E15,t} \cdot XR_{EUN,t} \cdot \left(1 + \frac{TAVE_{SUW,t}}{100}\right)$$

$$\boldsymbol{IMP_{SUW,t}} = PP_{SUW,E15,t} \cdot XR_{EUN,t} \cdot \left(1 + \frac{TAVI_{SUW,t}}{100}\right)$$

A note of thanks

There is a very long list of Defra colleagues we would like to thank for their support building this model, but this concluding section is not about us. This is a section to thank those of you beyond Defra's metaphorical four walls who have read this far, engaging in depth with the way we conduct our analysis.

Independent scrutiny is vital to robust analysis in government. The public discussion and debate surrounding every report, statistic and assessment we publish aids our analysis, improving how we do things next time around.

Past public conversations will find their reflection in UKAMM and we hope this documentation sparks more conversation still.

Your views, comments and suggestions on our approach are exactly what we have put out this document to seek. We would also value your views on ways we could improve this document to aid comprehension and transparency.

You can email us at: **trademodels@defra.gov.uk**

About the team

Defra's Trade Modelling team works with a diverse range of macroeconomic and microeconomic modelling approaches including general equilibrium models, gravity models and partial equilibrium models like UKAMM and Aglink-Cosimo.

We contribute these models' insights to the holistic analysis of the diverse group of economists, social researchers, statisticians and operational researchers that make up Defra's wider International Trade Analysis division.

This rounded analytical approach is a critical part of Defra's policy cycle in the trade sphere and beyond.