GCE AS and A Level Subject Criteria for Mathematics

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Contents

The criteria

Introduction

AS and A level subject criteria set out the knowledge, understanding, skills and assessment objectives common to all AS and A level specifications in a given subject.

They provide the framework within which the awarding organisation creates the detail of the specification.

Aims and objectives

1. AS and A level specifications in Mathematics should encourage learners to:

   - develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;
   - develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;
   - extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;
   - develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;
   - recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and standard and other mathematical models and how these can be refined and improved;
   - use mathematics as an effective means of communication;
   - read and comprehend mathematical arguments and articles concerning applications of mathematics;
   - acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;
   - develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;

- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

Specification content

2. Mathematics is, inherently, a sequential subject. There is a progression of material through all levels at which the subject is studied. The criteria therefore build on the knowledge, understanding and skills established in GCSE Mathematics. The core content for AS is a subset of the core content for A level.

3. Progression in the subject will extend in a natural way beyond AS and A level, into Further Mathematics or into related courses in higher education.

Knowledge, understanding and skills

4. *Proof*

4.1 AS and A level specifications in Mathematics should require:

- construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;

- correct understanding and use of mathematical language and grammar in respect of terms such as ‘equals’, ‘identically equals’, ‘therefore’, ‘because’, ‘implies’, ‘is implied by’, ‘necessary’, ‘sufficient’, and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$.

4.2 In addition, A level specifications in Mathematics should require:

- methods of proof, including proof by contradiction and disproof by counter-example.

4.3 These requirements should pervade the core content material set out in Sections 5 and 6

5. Core content material for AS and A level examinations in Mathematics is listed below. AS core content is listed in the second column with A2 core content in the right-hand column.

6. *Algebra and functions*

	AS core content	A2 core content
6.1	Laws of indices for all rational exponents	
6.2	Use and manipulation of surds	
6.3	Quadratic functions and their graphs; the discriminant of a quadratic function; completing the square; solution of quadratic equations	
6.4	Simultaneous equations: analytical solution by substitution, e.g. of one linear and one quadratic equation	
6.5	Solution of linear and quadratic inequalities	
6.6	Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem and the Remainder Theorem	Simplification of rational expressions including factorising and cancelling, and algebraic division
6.7	Graphs of functions; sketching curves defined by simple equations; geometrical interpretation of algebraic solution of equations; use of intersection points of graphs of functions to solve equations	
6.8		Definition of a function; domain and range of functions; composition of functions; inverse functions and their graphs

6.9		The modulus function
6.10	Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$	Combinations of these transformations
6.11		Rational functions; partial fractions (denominators not more complicated than repeated linear terms)

7. *Coordinate geometry in the (x,y) plane*

7.1	Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; conditions for two straight lines to be parallel or perpendicular to each other	
7.2	Coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$, and including use of the following circle properties: a) the angle in a semicircle is a right angle; b) the perpendicular from the centre to a chord bisects the chord; c) the perpendicularity of radius and tangent	
7.3		Parametric equations of curves and conversion between Cartesian and parametric forms

8. *Sequences and series*

8.1	Sequences, including those given by a formula for the *n*th term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$	
8.2	Arithmetic series, including the formula for the sum of the first *n* natural numbers	
8.3	The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$	
8.4	Binomial expansion of $(1+x)^n$ for positive integer *n*; the notations $n!$ and $\binom{n}{r}$	
8.5		Binomial series for any rational *n*

9. *Trigonometry*

9.1	The sine and cosine rules, and the area of a triangle in the form 1/2absinC	
9.2	Radian measure, including use for arc length and area of sector	
9.3	Sine, cosine and tangent functions; their graphs, symmetries and periodicity	

9.4		Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs and appropriate restricted domains
9.5	Knowledge and use of $\tan\theta = \sin\theta/$, $\operatorname{Cos}\theta$ and $\sin^2\theta + \cos^2\theta = 1$	Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$
9.6		Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$
9.7	Solution of simple trigonometric equations in a given interval	

10. *Exponentials and logarithms*

10.1	$y = a^x$ and its graph	The function e_x and its graph
10.2	Laws of logarithms: $\log_a x + \log_a y = \log_a(xy)$ $\log_a x - \log_a y = \log_a(x/y)$ $k \log_a x = \log_a x^k$	The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x
10.3	The solution of equations of the form $a^x = b$	
10.4		Exponential growth and decay

11. *Differentiation*

11.1	The derivative of f(x) as the gradient of the tangent to the graph of y = f (x) at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives	
11.2	Differentiation of x^n, and related sums and differences	Differentiation of e^x, $\ln x$, sin$x$, cos$x$, tan$x$ and their sums and differences
11.3	Applications of differentiation to gradients, tangents and normals, maxima and minima and stationary points, increasing and decreasing functions	
11.4		Differentiation using the product rule, the quotient rule, the chain rule and by the use of $\mathrm{d}y/\mathrm{d}x=1/(\mathrm{d}x/\mathrm{d}y)$
11.5		Differentiation of simple functions defined implicitly or parametrically
11.6		Formation of simple differential equations

12. *Integration*

12.1	Indefinite integration as the reverse of differentiation	
12.2	Integration of x^n	Integration of e^x, 1/x, sin$x$, cos$x$

12.3	Approximation of area under a curve using the trapezium rule; interpretation of the definite integral as the area under a curve; evaluation of definite integrals	
12.4		Evaluation of volume of revolution
12.5		Simple cases of integration by substitution and integration by parts; these methods as the reverse processes of the chain and product rules respectively
12.6		Simple cases of integration using partial fractions
12.7		Analytical solution of simple first order differential equations with separable variables

13. *Numerical methods*

13.1		Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous
13.2		Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$
13.3		Numerical integration of functions

14. *Vectors*

14.1		Vectors in two and three dimensions
14.2		Magnitude of a vector
14.3		Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations
14.4		Position vectors; the distance between two points; vector equations of lines;
14.5		The scalar product; its use for calculating（the angle between two lines.

Assessment objectives

15. The assessment objectives and the associated weightings for AS and A level are the same.

16. All learners must be required to meet the following assessment objectives. The assessment objectives are to be weighted in all specifications as indicated. The maximum weighting for any assessment objective should not normally be more than ten per cent greater than the minimum weighting. The assessment objectives apply to the whole specification.

Assessment objectives		**Minimum weighting**
AO1	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%
AO2	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%

AO3	Recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; and present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models.	10%
AO4	Comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications	5%
AO5	Use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; and understand when not to use such technology, and its limitations. Give answers to appropriate accuracy.	5%

Scheme of assessment

Internal assessment

17. The maximum weighting for internal assessment in AS and A level specifications in Mathematics is 20 per cent.

Synoptic assessment

18. All specifications must include a minimum of 20 per cent synoptic assessment.

19. All synoptic assessment units must be externally assessed. Synoptic assessment is addressed in the assessment objectives as parts of assessment objectives 1, 2, 3 and 4. The synoptic requirements must be met in full for the basic six-unit qualification.

20. The definition of synoptic assessment in the context of mathematics is as follows:

    - Synoptic assessment in Mathematics addresses learners' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the A level course through using and applying methods developed at earlier stages of study in solving problems. Making and understanding connections in this way is intrinsic to learning mathematics.

21. In papers which address the A2 core content, synoptic assessment requires the use of methods from the AS core content. In papers which address mathematical content outside the core content, synoptic assessment requires the use of methods from the core content and/or methods from earlier stages of the same aspect of mathematics (pure mathematics, mechanics, statistics or discrete mathematics). In determining what content may appropriately be required, the rules of dependency for modules as set out in the specification (paragraph 23) should be observed.

22. All AS and A level specifications in Mathematics must explicitly refer to the importance of learners using clear, precise and appropriate mathematical language. These references must draw attention to the relevant demands of assessment objective 2.

23. AS and A level specifications in Mathematics must:

- explicitly include all the material in the relevant knowledge, understanding and skills section of the criteria. Specifications are permitted to elaborate on details in different ways but all specifications must show a very high degree of consistency and comparability in addressing the material in sections 4 - 14. For both AS and A level, the knowledge, understanding and skills must attract two-thirds of the total credit for the qualification;

- designate units assessing the core content as C1-C4; further units assessing pure mathematics should be designated FP1, FP2 etc.;

- explicitly state that at least one area of the application of mathematics must be addressed. This is not specified in terms of content in section 4 - 14 but is required under assessment objective 3 (see section 16). The application of mathematics must count for at least 30 per cent of the total credit for the qualification. Specifications must provide full details of the application area(s) to be addressed;

- provide full details of any extension material in pure mathematics;

- set out clear rules of dependency for the available modules which indicate appropriate pathways and prohibit incoherent combinations of modules;

- include an element of the assessment, addressing assessment objectives 1 and 2, in which learners are not permitted to use any calculating aid in a paper addressing core content. This element must comprise one AS unit; all other units must permit the use of graphic calculators;

- encourage the appropriate use of graphic calculators and computers as tools by which the teaching and learning of mathematics may be enhanced;

- identify formulae which learners are expected to know. The list of formulae relating to core material which learners will be expected to know is attached to this document at Appendix 1. Specifications must identify any additional formulae of comparable significance which relate to other parts of the specification and which learners will be expected to know. Other formulae will be available on a formulae sheet, to be drawn up by the regulators in collaboration with the awarding organisations;

- indicate the mathematical notation that will be used. Normally this should be the agreed list of notation that was included with the original 1983 core and which was reaffirmed with the 1993 core. The list is attached to this document at Appendix 2.

24. Specifications with the title Further Mathematics may also be developed. Further Mathematics specifications must meet the criteria for Mathematics specifications, except for the following. They must not contain the A level Mathematics core content material as content which is to be directly assessed, because all Further Mathematics learners are expected to have already obtained (or to be obtaining concurrently) an A level award in Mathematics. Instead, AS Further Mathematics specifications must include at least one unit of pure mathematics, and A level Further Mathematics specifications must include at least two units of pure mathematics.

25. Also, Section 23 (bullet 1) and Section 23 (bullet 5) do not apply to Further Mathematics specifications, and Section 23 (bullet 3) is not a requirement.

26. Specifications with the title Pure Mathematics may also be developed. Pure Mathematics specifications must meet the criteria for Mathematics specifications, except for Section 23 (bullet 3) which does not apply to Pure Mathematic specifications.

27. A level Mathematics specifications must include two or three A2 units, and A level Pure Mathematics and A level Further Mathematics specifications must include at least three A2 units.

Grade descriptions

28. The following grade descriptions indicate the level of attainment characteristic of the given grade at A level. They give a general indication of the required learning outcomes at each specified grade. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the learner has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Grade A	Learners recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts. Learners manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected. Learners recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model. Learners comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended

	pieces of prose having mathematical content. They can comment meaningfully on the mathematical information. Learners make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.
Grade C	Learners recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts. Learners manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations. Learners recall or recognise most of the standard models that are needed and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation. They sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model. Learners comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

	Learners usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.
Grade E	Learners recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts. Learners manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs. Learners recall or recognise some of the standard models that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation. Learners sometimes comprehend or understand the meaning of translations in mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information. Learners often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

Appendix 1: Formulae for AS and A level Mathematics specifications

This appendix lists formulae that learners are expected to remember and that may not be included in formulae booklets.

Quadratic equations

$Ax^2+bx+c=0$ has roots

$$\frac{-b.(+or-)\sqrt{b2-4ac}}{2a}$$

Laws of logarithms

$\log_a x+\log_a y \equiv \log_a(xy)$

$\log_a x-\log_a y \equiv \log_a(x/y)$

$k \log_a x \equiv \log_a (x^k)$

Trigonometry

In the triangle ABC

a/sin A=b/sin B=c/sinC

area = ½ absinC

$\cos^2 + \sin^2 = 1$

$\sec^2 \equiv \tan^2 + 1$
$\mathrm{cosec}^2 \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$

$\tan 2A \equiv 2 \tan A / (1-\tan^2 A)$

Differentiation

Function	Derivative
x^n	nx^{n-1}
sin kx	k cos kx
cos kx	$-k$ sin kx
e^{kx}	$k e^{kx}$
ln x	1/x
f (x) + g (x)	f ‘(x) + g’(x)
f (x) g (x)	f ‘(x) g(x) + f (x) g’(x)
f (g(x))	f ‘(g(x)) g’(x)

Integration

Function	Integral
x^n	$1/(n+1)\ x^{n+1} + c,\ n \neq -1$
cos kx	1/k sin kx + c
sin kx	$-$ 1/k cos kx + c
e^{kx}	1/k e^{kx} + c
1/x	ln x + c, $x \neq 0$
f ‘(x) + g’(x)	f (x) + g (x) + c
f ‘(g (x)) g’(x)	f (g(x)) + c

Area

area under a curve

area = $\int_a^b y\,dx \quad (y \geq 0)$

Vectors

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = ax + by + cz$$

Appendix 2: Mathematical notation

Set notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x : \ldots\}$	the set of all x such that ...
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
ε	the universal set
A'	the complement of the set A
N	the set of natural numbers, $\{1, 2, 3, \ldots\}$
Z	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
Z^+	the set of positive integers, $\{1, 2, 3, \ldots\}$
Z_n	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
Q	the set of rational numbers, $\{p/q : p \in \mathrm{Z}\ q \in \mathrm{Z}^+\}$
Q^+	the set of positive rational numbers, $\{x \in \mathrm{Q} : x > 0\}$
Q^+_0	the set of positive rational numbers and zero, $\{x \in \mathrm{Q} : x \geq 0\}$
R	the set of real numbers
R_+	the set of positive real numbers, $\{x \in \mathrm{R} : x > 0\}$
R^+_0	the set of positive real numbers and zero, $\{x \in \mathrm{R} : x \geq 0\}$
C	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the Cartesian product of sets A and B i.e. $A \times B = \{(a, b) : a \in A, b \in \mathrm{B}\}$

$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval, $\{x \in \mathrm{R}: a \le x \le b\}$
$[a, b)$, $[a, b[$	the interval $\{x \in \mathrm{R} : a \le x < b\}$
$(a, b]$, $]a, b]$	the interval $\{x \in \mathrm{R}: a < x \le b\}$
(a, b), $]a, b[$	the open interval $\{x \in \mathrm{R} : a < x < b\}$
$y R x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\le$, $\ngtr$	is less than or equal to, is not greater than
$>$	is greater than
$\ge$, $\nless$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q

$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$,	a multiplied by b
$a \div b$, a/b	a divided by b
$\lvert a \rvert$	the modulus of a
$n\,!$	n factorial
	the binomial coefficient
$\prod_{i}^{n} a_i$	$a_1 \times a_2 \times a_3 \ldots \times a_n$
$\sqrt{a}$	the positive square root of a
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + a_3 \ldots + a_n$
$\begin{pmatrix} n \\ r \end{pmatrix}$	the binomial coefficient n!/ r! (n-r)! for n $\in Z^+$ ((n-1_ ... (n-r+1)) /r! for n $\in$ Q

Functions

$f(x)$	the value of the function f at *x*
$f : A \to B$	f is a function under which each element of set *A* has an image in set *B*
$f : x \to y$	the function f maps the element *x* to the element *y*
f^{-1}	the inverse function of the function f
$g \circ f$, gf	the composite function of f and g which is defined by $(g \circ f)(x)$ or $g f(x) = g(f(x))$
$\lim f(x)_{x \to a}$	the limit of $f(x)$ as *x* tends to *a*
Δx, δx	an increment of *x*
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the *n*th derivative of *y* with respect to *x*
$f'(x)$, $f''(x)$, ….$f^n(x)$	the first, second, ... , *n*th derivatives of f(*x*) with respect to *x*
$\int y\,dx$	the indefinite integral of *y* with respect to *x*
$\int_a^b y\,dx$	the definite integral of *y* with respect to *x* between the limits
$\frac{\partial V}{\partial x}$	the partial derivative of *V* with respect to *x*
$\dot{x}, \ddot{x}, \dddot{x}, \ldots$	the first, second, third etc ... derivatives of *x* with respect to *t*

Exponential and logarithmic functions

e	base of natural logarithms
e^x, exp x	exponential function of x
$\log_a x$	logarithm to the base a of x
ln x, $\log_e x$,	natural logarithm of x
lg x, $\log_{10} x$	logarithm of x to base 10

Circular and hyperbolic functions

the circular functions

sin, cos, tan

cosec, sec, cot

the inverse circular functions

$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

cosec^{-1} , $\sec^{-1}$, $\cot^{-1}$

OR

arcsin, arccos, arctan

arccosec, arcsec, arccot

the hyperbolic functions

sinh, cosh, tanh

cosech, sech, coth

the inverse hyperbolic functions

$\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$

cosech^{-1} , sech^{-1} , $\coth^{-1}$

OR

ar(c)sinh,ar(c)cosh,ar(c)tanh

ar(c)cosech,ar(c)sech,ar(c)coth

Complex numbers

i, j	square root of -1
z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
Re z	the real part of z, Re $z = x$
Im z	the imaginary part of z, Im $z = y$
z	the modulus of z, $\lvert z\rvert = \sqrt{x^2 + y^2}$
arg z	the argument of z, arg $z = \theta$, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z, $x - \mathrm{i}y$

Matrices

M	a matrix **M**
$\mathbf{M}^{-1}$	the inverse of the matrix **M**
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix **M**
det **M** or $\lvert\mathrm{M}\rvert$	the determinant of the square matrix **M**

Vectors

a	the vector a
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment *AB*
$\hat{a}$	a unit vector in the direction of a
i, j, k	unit vectors in the directions of the Cartesian coordinate axes
$\lvert \mathrm{a} \rvert$,a	the magnitude of a
$\lvert \overrightarrow{AB} \rvert$,AB	the magnitude of *AB*
a . b	the scalar product of a and b
a $\times$ b	the vector product of a and b

Probability and statistics

A, B, C, etc.	events
$A \cup B$	union of the events *A* and *B*
$A \cap B$	intersection of the events *A* and *B*
$\mathrm{P}(A)$	probability of the event *A*
A'	complement of the event *A*
$\mathrm{P}(A \cdot B)$	probability of the event *A* conditional on the event *B*
X, Y, R, etc.	random variables
x, y, r, etc.	values of the random variables *X, Y, R,* etc.
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable *X*
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable *X*

$\mathrm{f}(x)$, $\mathrm{g}(x)$, ...	the value of the probability density function of a continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)$, ...	the value of the (cumulative) distribution function $\mathrm{P}(X \le x)$ of a continuous random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{G}(t)$	probability generating function for a random variable which takes the values 0, 1, 2, ...
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$,m	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0,1)$
Φ	corresponding cumulative distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
$\mathrm{Cov}(X, Y)$	covariance of X and Y

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