Post 16

# Five year review of standards 

GCSE mathematics

## Introduction

Every summer, the publication of GCSE and A level examination results prompts public interest in the standards of those examinations.

In 1996, Lord Dearing in his Review of Qualifications for 16-19 Year Olds made several recommendations to ensure that 'there is a basis and accepted procedure ... for monitoring and safeguarding standards over time'. In the same year, SCAA (one of QCA's predecessors) and the Office for Standards in Education jointly investigated standards in English, mathematics and science (chemistry) in 16+ and $18+$ public examinations over time. ${ }^{1}$

The outcomes of this work were published in Standards in Public Examinations 1975 to 1995. One of the recommendations was that there should be:
'... a rolling programme of reviews on a five-year cycle to ensure examination demands and grade standards are being maintained in all major subjects. Physics, history, French and German should be included in the programme at an early stage.'

The five-yearly review of standards programme is a response to these recommendations. It is run by QCA in collaboration with the regulatory authorities for Wales and Northern Ireland, ACCAC and CCEA, and is designed to investigate the standards in A level and GCSE examinations. It aims to find out if:
the demand of syllabuses and their assessment instruments has changed over the last 20 years (examination demand);
the level of performance required of candidates at grade boundaries has changed over the last 20 years (grade standard).

Organised to run in five-year cycles, the programme was structured to cover every major subject during its first cycle. Each year, up to 100 independent specialists review around 2,000 exam scripts, drawn from all the awarding bodies, together with their associated syllabuses, question papers and mark schemes. ${ }^{2}$

[^0]
## Methodology

Each study was organised in two stages:

- stage one - investigating changes in examination demand;
- stage two - investigating changes in standards of performance.

Each covered two sample years: the year of the study and 1995, the year used for the SCAA/Ofsted study.

## Stage one: examination demand


#### Abstract

Aim The aim of this review was to establish whether the demand of syllabuses and their assessment instruments changed over the period of the review.


## Evidence base

The awarding bodies were asked to supply, for each subject, copies of one major syllabus from the most recent year. They were also asked to provide the related question papers, mark schemes, examiners' reports, and details of the procedures in operation at the time of each examination. The materials used in the SCAA/Ofsted study were available for comparison.

## The process

A coordinator and three reviewers - independent experts from a variety of backgrounds - were appointed for each subject. Each coordinator was given a framework and asked to use it to describe the main differences between the syllabuses from the different years. This description was given to the reviewers, who were asked to study the syllabuses, question papers and mark schemes and independently judge whether the differences between years affected the demand of the examination. After the material had been reviewed, the team for each subject area met and discussed any issues. The coordinator then reported on the findings and identified any conclusions.

## Stage two: standards of performance

## Aim

The aim of the second stage was to find out if the level of performance required of candidates at grade boundaries has changed over the period of the study. The review focused on the performance of candidates at grades A and E at A level, and grades A, C and, sometimes, F for 16+ examinations.

## Evidence base

The awarding bodies were asked to provide 15 examples of candidates' work at the defined boundaries from the most recent year of examination. They were asked to submit the complete examination work of candidates, including all examination papers, coursework and any oral examinations. The materials used in the SCAA/Ofsted study were available for comparison.

## The process

A team of up to 12 reviewers was recruited for each subject. The reviewers came from a variety of backgrounds, including universities, selective and non-selective schools, maintained and independent schools, and further education institutions (including sixth form colleges). Some of them had backgrounds working for the various awarding bodies.

The coordinator from stage one was used again in this stage and the syllabus reviewers normally participated.

The review took place over two days. Before the meeting, each coordinator produced a general description of the standards expected for the grade boundaries in the study. Where these were available, published grade descriptions normally formed the basis of the performance descriptors. The coordinators were asked to take into account the fact that they would be looking at borderline performance rather than that comfortably in grade which is the intention of grade descriptions. The performance descriptors were discussed and agreed by the team at the start of the meeting.

Reviewers were each given a batch of scripts for a particular year, grade and awarding body. Working independently, they were asked to judge if the scripts matched the agreed grade description. They could categorise the work as:

- above the expected standard;
- slightly above the expected standard;
- at the expected standard;
- slightly below the expected standard;
- below the expected standard.

They were then given another batch of scripts of the same grade, either from another awarding body or of a different year from the same awarding body. They categorised these scripts and compared them with the first batch to identify any significant differences between candidates' performance. A sampling framework ensured adequate coverage of the sample. A copy of part of one framework is provided on page 4.

At the end of the two days, a plenary session was held and the reviewers discussed their findings and any significant issues. As with stage one, the coordinator reported on the findings and conclusions.

## Limitations of the study

Comparing examination standards over time is a complex task, heavily dependent on the evidence available and the ability of reviewers to make valid judgements on it. When considering the findings and conclusions, several limitations need to be kept in mind.

## Changes in syllabus and examination content

Syllabuses and examination papers changed significantly over the period of the review. For example, in assessing GCSE science examinations, the three tiers of entry of 1995 had been reduced to two. Fundamental changes make it difficult for reviewers to make valid judgements about relative standards because they are not comparing like with like.

## Individual opinion

Each individual places different values on each part of a subject. Agreed definitions of standards and frameworks show reviewers the standards they should work to, but it is difficult for them to avoid applying their own values. This can lead to differences in opinion about the same syllabus or piece of candidate's work.

## Lack of evidence

While reviewers had syllabuses and examination papers (although not always mark schemes) for all the years in the study, they did not have all the evidence they needed to analyse standards of performance. This applies particularly to examination scripts. What was used in the SCAA/Ofsted study was work for separate components of the examination rather than the whole work of candidates. Coursework and any oral examinations were usually missing.

Table 1: Sampling framework for part of a typical A level study

DAY 1

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \(8: 30\)
10:00 \& \[
\begin{gathered}
\text { BOARD A, GRADE } \\
\text { A } \\
1996 \\
1-7
\end{gathered}
\] \& \[
\begin{gathered}
\text { BOARD A, GRADE } \\
\text { E } \\
1996 \\
1-7
\end{gathered}
\] \& \begin{tabular}{l}
BOARD F, GRADE \\
A \\
1996 \\
1-7
\end{tabular} \& \[
\begin{gathered}
\text { BOARD F, GRADE } \\
\text { E } \\
1996 \\
7-1
\end{gathered}
\] \& \[
\begin{gathered}
\text { BOARD C, GRADE } \\
\text { A } \\
1996 \\
1-7
\end{gathered}
\] \& \begin{tabular}{l}
BOARD C, GRADE E \\
1996 \\
15-8
\end{tabular} \\
\hline \(10: 10\)
11:30 \& \[
\begin{gathered}
\text { BOARD A, GRADE } \\
\text { A } \\
1991 \\
1-3
\end{gathered}
\] \& \[
\begin{gathered}
\text { BOARD A, GRADE } \\
\text { E } \\
1991 \\
1-3
\end{gathered}
\] \& \[
\begin{gathered}
\text { BOARD F, GRADE } \\
\text { E } \\
1996 \\
8-15
\end{gathered}
\] \& \[
\begin{gathered}
\text { BOARD F, GRADE } \\
\text { A } \\
1996 \\
7-1
\end{gathered}
\] \& \[
\begin{gathered}
\text { BOARD C, GRADE } \\
\text { A } \\
1991 \\
1-7
\end{gathered}
\] \& \begin{tabular}{l}
BOARD C, GRADE E \\
1991 \\
15-8
\end{tabular} \\
\hline \(11: 50\)
1:05 \& \[
\begin{gathered}
\text { BOARD A, GRADE } \\
\text { E } \\
1996 \\
1-7
\end{gathered}
\] \& \[
\begin{gathered}
\text { BOARD A, GRADE } \\
\text { A } \\
1996 \\
15-8
\end{gathered}
\] \& \begin{tabular}{l}
BOARD C, GRADE \\
E \\
1996 \\
1-7
\end{tabular} \& \[
\begin{gathered}
\text { BOARD C, GRADE } \\
\text { A } \\
1996 \\
8-15
\end{gathered}
\] \& \[
\begin{array}{|c}
\hline \text { BOARD E, GRADE } \\
\text { A } \\
1996 \\
1-7
\end{array}
\] \& \begin{tabular}{l}
BOARD D, GRADE A \\
1996 \\
15-8
\end{tabular} \\
\hline \(2: 15\)

3.30 \& $$
\begin{gathered}
\text { BOARD A, GRADE } \\
\mathrm{E} \\
1991 \\
1-3
\end{gathered}
$$ \& BOARD A, GRADE

A
1991

$3-1$ \& $$
\begin{gathered}
\text { BOARD A, GRADE } \\
\text { E } \\
1996 \\
15-8
\end{gathered}
$$ \& BOARD B, GRADE

E
1996

$15-8$ \& \[
$$
\begin{gathered}
\text { BOARD E, GRADE } \\
\text { E } \\
1996 \\
1-7
\end{gathered}
$$

\] \& | BOARD D, GRADE E |
| :--- |
| 1996 |
| 15-8 | <br>

\hline 3:30

4:45 \& \begin{tabular}{l}
BOARD B, GRADE <br>
A <br>
1996 <br>
1-7

 \& 

BOARD D, GRADE <br>
E <br>
1996 <br>
1-7

 \& 

BOARD B, GRADE <br>
A <br>
1996 <br>
15-8

 \& 

BOARD D, GRADE <br>
E <br>
1991 <br>
4-1

 \& 

BOARD D, GRADE <br>
A <br>
1996 <br>
7-1

 \& 

BOARD E, GRADE <br>
A <br>
1996 <br>
8-15
\end{tabular} <br>

\hline 5:05

6:20 \& \[
$$
\begin{gathered}
\text { BOARD B, GRADE } \\
\text { E } \\
1996 \\
1-7
\end{gathered}
$$

\] \& | BOARD D, GRADE |
| :--- |
| E |
| 1991 |
| 1-4 | \& | BOARD B, GRADE |
| :--- |
| E |
| 1996 |
| 8-15 | \& \[

$$
\begin{gathered}
\text { BOARD D, GRADE } \\
\text { E } \\
1986 \\
4-1
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\hline \text { BOARD D, GRADE } \\
\text { E } \\
1996 \\
8-15
\end{gathered}
$$

\] \& \[

$$
\begin{gathered}
\hline \text { BOARD E, GRADE } \\
\text { A } \\
1991 \\
1-3
\end{gathered}
$$
\] <br>

\hline
\end{tabular}

DAY 2

| $8: 30$ 9:45 | BOARD C, GRADE <br> E <br> 1996 <br> 7-1 | $\begin{gathered} \text { BOARD E, GRADE } \\ \text { E } \\ 1996 \\ 15-8 \end{gathered}$ | BOARD E, GRADE <br> A <br> 1996 <br> 1-7 | EDEC , GRADE A <br> 1996 <br> 7-1 | BOARD F, GRADE A 1996 $8-15$ | BOARD A, GRADE E 1996 $15-8$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9:45 11:00 | $\begin{gathered} \text { BOARD C, GRADE } \\ \text { E } \\ 1991 \\ 1-7 \end{gathered}$ | BOARD E, GRADE <br> E <br> 1991 <br> 3-1 | $\begin{gathered} \text { BOARD E, GRADE } \\ \text { A } \\ 1991 \\ 3-1 \end{gathered}$ | $\begin{gathered} \text { BOARD B, GRADE } \\ \text { E } \\ 1996 \\ 8-15 \end{gathered}$ | $\begin{gathered} \text { BOARD F, GRADE } \\ \text { E } \\ 1996 \\ 8-15 \end{gathered}$ | $\begin{gathered} \text { BOARD A, GRADE } \\ \text { E } \\ 1986 \\ 7-1 \end{gathered}$ |
| 11:20 12:35 | BOARD C, GRADE <br> A <br> 1996 <br> 7-1 | BOARD E, GRADE <br> A <br> 1996 <br> 7-1 | BOARD E, GRADE E 1996 $8-15$ | BOARD E, GRADE <br> A <br> 1996 <br> 8-15 | BOARD C, GRADE A 1996 $15-8$ | BOARD A, GRADE <br> A <br> 1996 <br> 1-7 |
| $1: 45$ $3: 00$ | BOARD C, GRADE A 1991 $7-1$ | $\begin{gathered} \text { BOARD E, GRADE } \\ \text { A } \\ 1991 \\ 1-3 \end{gathered}$ | $\begin{gathered} \text { BOARD E, GRADE } \\ \text { E } \\ 1991 \\ 1-3 \end{gathered}$ | BOARD E, GRADE A 1991 $3-1$ | BOARD C, GRADE A 1991 $15-8$ | $\begin{gathered} \text { BOARD A, GRADE } \\ \text { A } \\ 1991 \\ 3-1 \end{gathered}$ |

# GCSE mathematics: review of standards 1995-99 

## Introduction

Two major factors led to changes in GCSE mathematics between 1995 and 1999:

- the national curriculum was revised in 1995, leading to revised GCSE subject criteria and syllabuses, first examined in 1998;
- the SCAA/Ofsted report, Standards in Public Examinations, 1975-1995 (SCAA, 1996), contained several recommendations which influenced the composition of GCSE examination papers from 1998.

The 1995 GCSE examinations in mathematics were the second series of examinations assessing performance against the 1991 version of the national curriculum. Awarding bodies offered centres a range of syllabuses which varied in the nature and composition of their examination components, but which shared the same mathematical content and included a 20 per cent assessment of Using and Applying Mathematics (Ma1), all defined by the national curriculum.

The 1999 GCSE examinations were the second series based on the 1995 version of the national curriculum. One key difference between the two versions of the national curriculum was the merger of Number and Algebra into one attainment target (double weighted). This allowed Number and Algebra to carry different weightings at each tier. There was also a small reduction in content at the higher grades. Awarding bodies were restricted (in general) to two syllabuses, with the main difference being the assessment of Ma 1 , which nevertheless retained a weighting of 20 per cent.

The CCEA syllabus differed from those offered from awarding bodies in England and Wales in the way it assigned its national curriculum topics to GCSE tiers. This was particularly noticeable at the Intermediate and Higher Tiers, which had less material overall at grades $B, A$ and $A^{*}$.

At the time of this study, there were known changes affecting the summer 2000 GCSE mathematics examinations and further developments for examinations in summer 2003. All aspects of these changes informed this report.

## Syllabus and examination demand

## Materials available

For both years being considered in this review, the syllabus used for each awarding body was its largest in terms of the number of candidates. In most cases the 1999 syllabus was the direct descendant of the 1995 syllabus; in the others, there was sufficient correspondence between the components to allow comparisons over time to be made with some confidence. Evidence was collected from the syllabus
documents, its assessment components, mark schemes and examiners' reports. Full details of the syllabuses used are given in Annex A.

About 692,000 candidates took GCSE mathematics in 1999. About 61 per cent of those entered for the syllabuses used for that year in this study.

## Assessment pattern

In 1999, the assessment pattern for GCSE mathematics across awarding bodies was approaching uniformity. The most common model was to assess Number, Algebra, Shape, Space and Measure, and Data Handling through two parallel papers of one and a half hours at the Foundation Tier and two hours at both the Intermediate and Higher Tiers, each paper carrying 100 marks. There was also wide use of questions common to adjacent tiers, for example grade C and B questions appearing on both Intermediate and Higher Tier papers. Ma1 was assessed through coursework or terminal tasks. Details of significant changes in assessment pattern and any differences specific to individual awarding bodies are summarised in the table below.

| Awarding body | Significant features of assessment in 1995 <br> which had changed in 1999 | Atypical elements in 1999 |
| :--- | :--- | :--- |
| AQA/N | Double weighting of level 4 at Foundation Tier |  |
| AQA/A | Double weighting of level 4 at Foundation Tier | Aural assessment worth 5\% |
| CCEA | Four papers of increasing difficulty offered in <br> pairs leading to three tiers of entry (plus an <br> extension paper) | 140 marks per paper at Intermediate <br> Tier and 200 at Higher Tier. <br> Aural assessment worth $10 \%$ |
| Edexcel | Papers included material below the target levels <br> for each tier | Different coursework scaling |
| OCR | Six papers of increasing difficulty offered in <br> pairs, leading to five tiers of entry |  |
| WJEC | Papers divided by content: Number and Algebra <br> on Paper 1; Shape and Space and Data <br> Handling on Paper 2. Double weighting of level <br> 4 at Foundation Tier | $1 / 4$ hours at Higher Tier |

## Tiering structure

A further difference between the GCSE examination in 1995 and 1999 was in the range of grades awarded at each tier. This may have changed entry patterns. The table summarises the grades awarded.

| 1995 |  |  |  |  |  |  |  | Tier <br> Higher | 1999 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A* | A | B | C | D |  |  |  |  | A* | A | B | C |  |  |  |  |
|  |  | B | C | D | E | F |  | Intermediat e |  |  | B | C | D | E |  |  |
|  |  |  |  | D | E | F | G | Foundation |  |  |  |  | D | E | F | G |

There was also provision to award some exceptional grades above the grades listed for 1995, but it was general practice in mathematics to award only those grades shown. There was no such provision in 1999, but CCEA continued to award grade D at the Higher Tier.

## Assessment of Ma1

In 1999, assessment of Ma1 was the area in which schemes of assessment varied most across awarding bodies with alternative routes even within awarding bodies. Options included:

- awarding body set and/or centre set coursework tasks, marked by the centre;
- awarding body set and marked coursework tasks;
- awarding body set terminal tasks - restricted time or exam conditions.

Clearly candidates had differing degrees of support due to the different conditions of assessment which meant that there was varying demand on candidates depending on the route for which their centre opted.

There is also an issue over the allowed tolerance for centre-marked coursework. This is sufficiently wide to allow a grade D piece of coursework to be assessed anywhere between grade E to grade C . The situation was exacerbated by the scaling of coursework marks, which, in the worst case (at Higher Tier), could affect a candidate's performance by five per cent of the overall mark. More typically, the variation was of the order of one or two per cent. The criteria for assessing Ma1 were similar in 1995 but were measured in national curriculum levels. The tolerance was smaller and the marks were scaled differently, generally leading to less variation.

There is now a document agreed by the awarding bodies, Exemplification of Ma1 criteria, in which the demands of the Ma1 criteria have been made clearer. For some criteria, evidence requirements are more demanding than in 1999. The document was in use within awarding bodies in the 1999 examination but had greatest impact on awarding body-marked coursework and terminal tasks. It was being made available to centres for use from 2000 onwards. ${ }^{2}$

## Questions set in context

A substantial proportion (between 40 per cent and 80 per cent) of the questions in all the examination papers in 1995 and 1999 were set in context. These fell into three main groups:

- contexts which were meaningful to the candidates, to help give them purchase on the mathematics involved in solving a particular problem;
- more unusual or unfamiliar contexts, intended to test whether candidates can apply their mathematical knowledge appropriately and interpret their results in the light of the context (this second group is clearly the most demanding);
- questions in which the context is trivial or even completely irrelevant.

[^1]The last type simply increases the reading demand of the paper with no added value to the mathematical assessment. There were too many questions of this type.

## Mark schemes

By 1999 mark schemes were very similar with close agreement on the mark tariff for particular questions and the distribution of marks within the solutions.

A very small (and decreasing) proportion of marks is used to assess candidates' ability to explain methods or comment on findings. Such expressions of understanding are difficult for candidates to write (and for examiners to mark) but searching questions which probe understanding need to be asked. ${ }^{3}$

## The examination papers at Higher Tier

The relative weighting for Number and Algebra changed from 1:1 in 1995 to $1: 2$ in 1999. This, coupled with strong emphasis on manipulative algebra, had increased demand significantly at this tier. The Algebra questions also replaced questions on a very limited selection of Number topics at grades A and A*. The predominance of multi-step questions within Shape and Space at this tier had also increased demand in this section of the curriculum. Several topics were removed from this tier: linear programming and critical path analysis, matrix transformations, and solving quadratic equations by iteration and by completing the square. This reduction in content partly offset the increased demand identified above. It also led to a limited number of Data Handling topics at the highest grades: this did not affect demand due to the difficulty of some of the questions posed, but increased predictability. Overall, demand at this tier was judged to have increased.

## The examination papers at Intermediate Tier

There was no change in weighting between Number and Algebra at this tier but there was an increased amount of manipulative algebra. This was particularly noticeable at grades $C$ and $B$ and most marked in AQA/N and Edexcel which used most overlapping questions between adjacent tiers. Increased focus on manipulative algebra skills more than compensated for the removal of flow diagrams, networks and 3D coordinates from the national curriculum. The movement of trigonometry from level 8 to grade C and the introduction of the factorisation and solution of quadratic equations of the form $x^{2}+b x+c=0$ had increased demand. Overall, demand at this tier was also considered to have increased.

## The examination papers at Foundation Tier

The change in weighting for Number to Algebra from 1:1 to 2:1 may appear at first glance to have decreased demand at this tier. However, the algebra topics where the emphasis on assessment had been reduced tended to be pre-algebra (for example, function machines) and non-manipulative algebra (for example, pattern spotting). The proportion of manipulative algebra was maintained or slightly increased in all cases. At this tier, the multi-step questions were generally in Number and Shape and Space and there had been an increase in sophistication in the style of some questions.

[^2]In some syllabuses, there had been a significant reduction in the proportion of questions targeted at the lower grades (caused by the change in the way the syllabuses related to the national curriculum). Overall, demand at this tier was also considered to have increased.

## Impact of Standards in Public Examinations, 1975-1995

The 1996 report, Standards in Public Examinations, 1975-1995, contained a number of specific recommendations for GCSE mathematics. The current review provided an opportunity to consider how far these recommendations had been taken into account in the latest examinations. However, it is important to note that the subject criteria and syllabus structures governing the 1999 syllabuses had already been agreed at the time of the publication of the report. This made it impossible for some points to have been addressed.

## Manipulative algebra

There should be 'increased emphasis given to the skills of algebraic manipulation'. The 1999 examination papers address this requirement at all tiers but, appropriately, most heavily in the Higher Tier. The need for greater levels of algebraic fluency had an immediate impact on demand.

Most of the manipulative algebra questions were structured, rather than requiring candidates to develop independently the two or three steps necessary to solve the problem. There are some valid examining reasons for this, principally centring on the effect that early mistakes have on the subsequent algebra, and the difficulty in following through and rewarding the candidates' work.

There were similar difficulties with setting algebra questions in context. Whilst many of the array of algebraic conventions and techniques which candidates will have experienced can be assessed appropriately in context-free examination questions, there is also a need for candidates to be required to model situations algebraically. Such models do not always have to be developed into full solutions in an examination. The ability to model a mathematical situation algebraically was not assessed effectively in 1999.

## Multi-step problem solving

There should an 'increase in the proportion of questions demanding the unprompted solution of multi-step problems'. GCSE questions in 1995 were almost all carefully structured, leading candidates through the various, and often linked, stages of a problem. In the 1999 examination papers, there were some unstructured problems at all tiers. At the higher grades these were almost always Shape and Space problems, often set in context. The inclusion of multi-step problems, which are more difficult than the equivalent structured question where part solutions (and hence part marks) may often be obtained, raised the demand of the examination papers. There is a second aspect which was not fully recognised in the 1999 examinations. Where the individual parts of the question might be at one grade, C say, then developing the whole solution independently would be of a higher order, possibly grade $B$.

## Use of a calculator

There should be 'some demanding mathematical tasks which require [candidates] to demonstrate effective use of the capabilities of calculators'. A few questions in both

1995 and 1999 were designed to test the ability to use a calculator correctly to evaluate an expression, for example knowing when and how to use brackets, powers and roots. At other times, final answers to a problem required the correct use of a calculator, for example in trigonometry, in finding areas and volumes, or in calculations with numbers in standard form. Using statistical functions on a calculator is a desirable skill and demonstrates efficient use but does not lend itself to effective testing in an examination where a minor error, for example, in keying in data can have major consequences since there is no evidence of correct method. ${ }^{4}$ Questions on graph sketching and transformations tend to be set using general functions so that candidates with graphical calculators are not advantaged. Overall, this recommendation was not realised in GCSE examinations in 1999. ${ }^{5}$

## Formulae Sheets

There should be agreement on 'the wider range of formulae that candidates for different tiers should be able to recall'. There were some changes in moving to a common formulae sheet in 1999 which affected some awarding bodies more than others. CCEA provided very few formulae. Some of the formulae sheets were loose leaf, which may be more helpful to candidates. ${ }^{6}$

## Summary

There was strong evidence of converging practice between awarding bodies in terms of their examination papers and associated mark schemes. There was some variation between and within awarding bodies in the optional routes for assessing Ma1.

At Higher Tier, demand increased due to the emphasis on manipulative algebra and the proportion of multi-step questions, particularly within Shape and Space. This was only partly offset by a reduction in syllabus content. At Intermediate Tier, demand increased slightly at the higher grades due to the increase in manipulative algebra at those grades. At Foundation Tier, demand increased due to the increased weighting of Number at the expense of some of the accessible pre-algebra topics and, in some awarding bodies, to a reduced proportion of questions on straightforward topics.

Thought needs to be given to the use of context within examination papers. It should be purposeful, giving purchase on the mathematics at one end of the spectrum and increasing demand at the other. Context also gives the necessary link between mathematics and its wider applications. It is also appropriate that a significant proportion of questions should be context-free.

[^3]
## Standards of performance at grades A, C and F

## Materials available

In this part of the review, candidates' examination performance at each of the key boundaries $A / B, C / D, F / G$ was analysed for both years. Full details of the materials used are given in Annex $B$.

In 1995 each script was a boundary script for that particular examination component, whereas in 1999 all the work for boundary candidates (examination papers and coursework, where available) was considered. This difference favoured the 1995 scripts at grade C (Intermediate Tier) and grade A when compared with 1999 scripts, but it was decided that the reviewers would consider all the scripts at face value.

The following comparisons were made at each key boundary:

- different awarding bodies in 1999;
- same awarding body between 1995 and 1999;
- grade $C$ at the Intermediate Tier with grade $C$ at Higher Tier within the same awarding body in 1999.


## Standards of performance expected

Descriptors were developed, using an agreed document setting out the likely difficulty of a question on a given topic. This helped to identify the mathematical content which is typical of material assessed at the two grades in each borderline. True borderline work was expected to demonstrate quite securely aspects of the lower grade and begin to exhibit some features of the higher grade. In any GCSE examination, mathematical topics are sampled so not all the content listed in the performance descriptor would be assessed. The detailed descriptors are provided at Appendix A.

In addition to the evidence found within examination scripts, the reviewers considered coursework, but this was available for 1999 only. Coursework sometimes provided additional evidence supporting performance at the borderline. The coursework scores awarded for the mathematical processes involved in developing task solutions were also recorded.

## Performance at grade $\boldsymbol{A}$

In 1999, the performance of candidates at this grade boundary met the expected standard well, showing secure performance in almost all aspects expected for grade B. Performance in grade A material was, as expected, variable with strongest performances in Shape and Space and Data Handling, in spite of the number of multi-step questions set on Shape and Space topics. There were many questions assessing skills in manipulative algebra at grade A . The borderline candidates experienced some difficulties, but there was evidence of partial success within these questions. This suggested that the emphasis on these topics is reflected in the candidates' preparation for GCSE.

The principal difference between the work of candidates in 1995 and 1999 related to syllabus coverage. Very little manipulative algebra was assessed in 1995 (except by CCEA) making direct comparison inappropriate. Performance in Shape and Space was broadly comparable, being slightly stronger in 1999, with better performances in the sine and cosine rules, areas and volumes, and the circle theorems. Congruent triangles and similar figures were weak topics in both years. The reduction in Data Handling topics at grades A/A* between 1995 and 1999 resulted in complete coverage in 1999 in most awarding bodies. Performance in these topics was comparable.

## Performance at grade C

## Intermediate Tier

In 1999, the performance of Intermediate Tier candidates at the C/D grade boundary matched the expected standard. Performance in Number, Shape and Space, and Data Handling was particularly secure. In Algebra, candidates could form simple algebraic expressions and solve simple equations but had more difficulty when brackets were involved or solutions were not positive integers. They could simplify linear expressions but not use the laws of indices. Most were able to draw straight line graphs and simple curves when given algebraically. However, they were not able to generalise linear or quadratic sequences under examination conditions, despite their ability to do so within coursework tasks.

The performance at this tier and grade boundary was closely comparable with that in 1995, although there were some differences in syllabus coverage. More algebra topics were assessed in 1999, but where similar questions occurred in 1995, performance was similar. There was improvement in the calculation of lengths, areas and volumes, and in bearings, but use of trigonometry was slightly weaker. However, as with the Higher Tier, most of the unstructured questions were set in Shape and Space contexts: this appeared to deny access to candidates of this ability. In Data Handling, the calculation of a mean from a frequency distribution appeared to remain beyond these candidates. Other aspects of data handling were comparable, although giving written interpretations of graphs or results continued to cause difficulty.

## Higher Tier

Applying the borderline grade C descriptor to the Higher Tier scripts was more difficult as grade D mathematical content could not be the focus of any examination question at this tier (except for CCEA). However, some grade D evidence was found within the solutions to problems involving higher level mathematics. It was also clear that candidates had been inappropriately entered at this tier.

In 1999, the performance of Higher Tier candidates at the C/D grade boundary matched expectation. Allowing for the more limited evidence, reviewers found that performance in Number, Shape and Space, and Data Handling was generally secure. However, candidates' algebra was weak. This was not expected as these candidates would have been exposed to a wide array of algebraic techniques, suggesting that wider experience on its own does not lead to proficiency. There was some evidence that they could form simple algebraic expressions but they were not able to solve linear equations. Nor could they use the laws of indices. They were able to draw curves when given algebraically and could use trial and improvement
methods to solve cubic equations. However, they were not able to generalise quadratic sequences except within coursework tasks.

Most the candidates' scripts also contained part solutions to higher grade questions which help to secure the grade C performance, but rarely would such part responses contain significant mathematical understanding.

The performance at this tier and grade boundary was almost identical to that in 1995.

## Performance at grade $F$

Performance at this borderline was well-matched to expectation, with Data Handling being the strongest area. There was secure evidence in almost all aspects of the grade G part of the descriptor. In Number, candidates showed understanding of multiples, factors and squares, and some evidence of directed numbers in context and of fractional and percentage parts. Non-calculator multiplication and division was a significant weakness. Describing and using patterns was a strength at this level within both examination and coursework. Candidates had difficulty using specialised mathematical language, imperial units and their approximate metric equivalents and averages; and in writing about angles or shapes and their properties. Interpreting statistical diagrams and work on symmetry and basic area, perimeter and volume was secure.

The only scripts available from 1995 at this borderline (CCEA) showed a similar overall level of achievement, Number being slightly stronger in 1995 and Data Handling stronger in 1999.

## Summary

There was no evidence of a decrease in performance at any tier over time. Where the content and style of questions were similar, performance was comparable or slightly stronger in 1999. At the Higher Tier, performance at the A/B boundary demonstrated candidates' increased exposure to manipulative algebra, and the pattern of success was similar to other areas of the curriculum. There was no evidence of inequality of performance at grade C from the two alternative routes (Higher and Intermediate Tiers) although the mathematical experiences of candidates were markedly different.

The introduction of multi-step questions and the increased emphasis on manipulative algebra should establish a stronger mathematical background for further study of mathematics. However, the evidence from the scripts suggested that candidates at grade C from either tier displayed none of the characteristics desirable for such further study. This perhaps suggests that caution should be exercised over increasing breadth of content at the expense of depth.

## Appendix A1: standards expected at grade C

## Number

$$
\begin{array}{ll}
\text { Generally secure in: } \quad \begin{array}{l}
\text { Use of whole numbers to make estimates; } \\
\text { Understanding of simple fraction, percentage, decimal equivalence; } \\
\text { Ability to calculate result of percentage or fractional change; }
\end{array} \\
& \begin{array}{l}
\text { Calculations with simple ratios in context; } \\
\text { Solution of problems with metric or imperial units; } \\
\text { Ability to carry out simple speed calculations. }
\end{array} \\
\text { Some evidence of: } \quad \begin{array}{l}
\text { Use of } 1 \text { s.f. in making estimates for } x \text { and } \div \\
\text { Correct use of calculator for numerical problems; } \\
\text { Calculations with directed numbers; } \\
\text { Expression of one number as a fraction or percentage of another; } \\
\text { Understanding of equivalent fractions, decimals and percentages; } \\
\text { Calculations using ratios; } \\
\text { Use of repeated proportional change. }
\end{array}
\end{array}
$$

## Algebra

Generally secure in: ability to describe rule for $n$th term of a linear sequence;
Solution of simple equations (eg $x^{2}=20$ ) using trial and improvement;
Formation of simple algebraic expressions and equations;
Solution of simple linear equations;
Ability to draw straight line graphs given algebraically.

Some evidence of: Ability to describe rule for $n$th term of a quadratic sequence;
Solution of simple cubic equations using trial and improvement;
Formation and solution of linear equations;
Ability to represent simple inequalities on a number line;
Simple use of laws of indices (eg $a^{2} \times a^{3}=a^{5}$ );
Ability to draw and use simple curves.

## Shape and Space

Generally secure in: Angles in parallel lines and in quadrilaterals;
Bearings;
Area or perimeter of compound rectilinear shape;
Straightforward circumference and area of circle;
2D representation of 3D;
Enlargement by whole number S.F. from centre ( 0,0 );
Rotation about origin or reflection in $y$ or $x= \pm c$

| Generally secure in: $\quad$Angles in parallel lines and in quadrilaterals; <br>  <br> Bearings; |  |
| :--- | :--- |
|  | Area or perimeter of compound rectilinear shape; |
|  | Straightforward circumference and area of circle; |
|  | 2D representation of 3D; |
|  | Enlargement by whole number S.F. from centre $(0,0) ;$ |
|  | Rotation about origin or reflection in $y$ or $x= \pm c$ |

Some evidence of: Pythagoras' Theorem;
Trigonometry;
Angles in polygons;
Length and area calculations, including circle and trapezium;
Volume of prism;
Enlargement from non-origin centre or by fractional S.F.;
Rotation about non-origin or reflection in $y= \pm x$
Upper and lower bounds of measurements;
Compound measures (speed and density);
Simple loci.
Data Handling
Generally secure in:
Plotting of scatter diagram;
Ability to comment on correlation;
Understanding of mean of discrete frequency distribution;
Ability to draw grouped frequency diagram;
Construction of pie charts;
Surveys, observation sheets, questionnaires;
Probability based on equally likely events;
$P\left(A^{\prime}\right)=1-P(A)$

Some evidence of: Line of best fit;
Calculation and use of relative frequency;
Understanding of mean of grouped frequency distribution;
Ability to compare distributions using frequency diagrams, averages and spread;
Identification or avoidance of bias in surveys, questionnaires, hypotheses.

## Appendix A2: standards expected at grade A

## Number

Generally secure in: Standard form;
Calculations involving powers and roots (numerical);
Reverse percentage problems;
State upper and lower bounds of numbers.

Some evidence of: Rational and irrational numbers;
Effects of rounding on calculations.

## Algebra

Generally secure in: Multiplication of two linear expressions;
Ability to find common factors and factorise $x^{2}+b x+c$
Simultaneous equations;
Rearrangement of formulae (subject appearing once);
Manipulation of algebraic expressions, eg $\left(3 x^{4} y^{2}\right) \times\left(x^{2} y^{3}\right)$;
Solution of linear and simple quadratic inequalities;
Representation of inequalities graphically;
Ability to sketch linear, quadratic, cubic and reciprocal functions;
Understanding of gradient and intercept of straight line.

Some evidence of: $\quad$ Factorising $a x^{2}+b x+c$
Solution of $a x^{2}+b x+c=0$ by factorising or by the formula;
Formation and manipulation of algebraic expressions and equations;
Rules of indices for negative and fractional powers (algebraic);
Proportionality;
Graphical solutions of equations.

## Shape and Space

Generally secure in: Similar triangles (LSF);
Volume of compound prisms;
Multiple application of 2D Trig or Pythagoras (unstructured);
Dimensions to distinguish formulae.

Some evidence of: $\quad$ Sin, cos and tan graphs;
3D Trig and Pythagoras;
Congruent triangles;
Sine and cosine rules;
Lengths of arcs and areas of sectors;
Volumes of cones, spheres and S.A. of cylinders;
Similar shapes (areas and volumes);
Angles in circles.

## Data Handling

Generally secure in: Ability to draw and interpret C.F. graphs;
Tree diagrams for compound events or AND and OR rules.

Some evidence of: Ability to draw and interpret histograms;
Sampling;
Conditional probability.

## Appendix A3: standards expected at grade $F$

## Number

Generally secure in: Simple fractions and percentages of a whole;
Addition and subtraction of decimals to 2 d.p.;
Simple estimation.

Some evidence of: Use of place value to $x$ and $\div$ by 10, 100, 1000;
Ability to order, add and subtract directed numbers in context;
Calculation of fraction or percentage parts;
Non-calculator x and $\div$ of 3 digit by 2 digit;
Use of estimation or inverse operations;
Multiples and factors, squares.


#### Abstract

Algebra Generally secure in: Use of simple formulae expressed in words; Coordinates in first quadrant; Ability to predict next term or diagram in a pattern.

Some evidence of: Description and use of number patterns to find later terms; Formation and use of simple formulae; Conversion graphs; Coordinates in four quadrants.


## Shape and Space:

| Generally secure in: $\quad$Ability to draw 2D shapes in different orientation and identify congruent <br> shapes; |  |
| :--- | :--- |
|  | Ability to reflect shapes in a mirror line; <br> Ability to find order of rotational symmetry; |
|  | Ability to read scales on measuring instruments; <br> Ability to find perimeters, areas and volumes by counting; <br> Knowledge of meaning of 'parallel' and 'perpendicular'; |
| Some evidence of: $\quad$Naming of shapes. |  |
| Construction of angles and use of language; <br> Identification of all symmetries of a shape; <br> Conversion of metric units; <br> Knowledge of rough metric/lmperial equivalents and ability to estimate <br> everyday measures. |  |

## Data Handling

Generally secure in: Collation of discrete data in a frequency table, grouping where appropriate;
Interpretation of frequency diagrams (various) and line graphs;
Ability to draw frequency diagrams (various);
Mode and median;
Use of simple vocabulary of probability, eg 'fair', 'certain' and 'likely'.

Some evidence of: Understanding of mean of discrete data;
Comparison of two distributions;
Interpretation of pie charts and other diagrams;
Use of probability scale 0 to 1 ;
Ability to make subjective estimates of probability;
Probability based on equally likely outcomes and experiments.

## Annex B

Details of syllabuses used in the syllabus review.

| Year | Awarding Body and Syllabus |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1995 | London <br> 1384 A | NEAB <br> Syllabus A | SEG <br> $2410 T$ | MEG <br> SMP 11-16 | WJEC <br> Syllabus A | NISEAC <br> Syllabus A |
| 1999 | Edexcel | AQA/N | AQA/A | OCR | WJEC | CCEA |
| 1385 A | Syllabus A | 2500T | Syllabus A | Syllabus A | Syllabus A |  |

## Annex C

Details of scripts used for the script review

| Edexcel |  | AQA/N |  | AQA/A |  | OCR |  | WJEC |  | CCEA |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1995 | 1999 | 1995 | 1999 | 1995 | 1999 | 1995 | 1999 | 1995 | 1999 | 1995 | 1999 |
| A | A |  | A |  | A | A | A |  | A | A | A |
| C(H) | C(H) |  | C(I) |  | C(H) | C(I) | C(I) |  | C(H) | C(H) | C(H) |
| C(I) | C(I) |  | F |  | C(I) |  | F |  | C(I) | C(I) | C(I) |
|  | F |  |  |  | F |  |  |  | F | F | F |

## Key to the awarding bodies

During the period of the reviews, the number of awarding bodies operating fell. There are currently five: AQA, CCEA, Edexcel, OCR and WJEC. However, the three English awarding bodies came together through a number of mergers and a government requirement for unitary awarding bodies which could offer the range of GCSE, A level and GNVQ/VCE qualifications. This means that the qualifications used in the reviews came from a number of earlier examination boards and examining groups.

For the purposes of the reports the following abbreviations will be used:
AQA/A, AQA/N, CCEA, Edexcel, OCR and WJEC.
AQA/A covers AQA legacy A level syllabuses offered by AEB; legacy GCSE syllabuses offered by SEG; and O level syllabuses offered by AEB.

AQA/N covers AQA legacy A level syllabuses offered by NEAB, NEA and JMB; legacy GCSE syllabuses offered by NEAB and NEA; and O level syllabuses offered by JMB.

CCEA covers A level and GCSE syllabuses offered by CCEA, NISEAC and NISEC; and O level syllabuses offered by NISEC and NIGCEEB.

Edexcel covers A level and GCSE syllabuses offered by Edexcel, ULEAC and ULSEB; GCSE syllabuses offered by Edexcel, ULEAC and LEAG; and O level syllabuses offered by ULSEB.

OCR covers A level syllabuses offered by OCEAC, OCSEB, UCLES and UODLE; GCSE syllabuses offered by MEG; and O level syllabuses offered by OCSEB, UCLES and UODLE.

WJEC has retained the same name throughout the period.


[^0]:    ${ }^{1}$ 16+ examinations cover GCE O level and Certificate of Secondary Education (up to 1987), and GCSE (from 1988).
    ${ }^{2}$ For the purposes of this report, the general term awarding bodies is used to cover both the A level examination boards and the GCSE examining groups.

[^1]:    ${ }^{2}$ For GCSE from 2003, all specifications will follow a common route to the assessment of Ma1, half through coursework and half through examination questions.

[^2]:    ${ }^{3}$ For GCSE from 2003, strand 3 of Ma1 concerns reasoning. This should allow some more searching assessment of understanding.

[^3]:    ${ }^{4}$ The topic 'iterative solutions to quadratic equations' has been removed from the Higher Tier Syllabus (except AQA/A) and standard deviation has been removed from the national curriculum for GCSE 2003. Both these are suitable for assessing effective use of a calculator.
    ${ }^{5}$ The parallel recommendation that there should be 'papers which must be completed without the use of a calculator' is in place for GCSE 2000.
    ${ }^{6}$ The formulae sheets for 2000 focus on Shape and Space (with no formulae provided for Foundation Tier) and the number of formulae proposed for 2003 is reduced further still.

