



Department  
for Education

# **Multiplicative reasoning professional development programme: evaluation**

**Research brief**

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## The Multiplicative Reasoning Project

The Multiplicative Reasoning Project (MRP) delivered by the National Centre for Excellence in the Teaching of Mathematics (NCETM) in 2013/14 focused on developing teachers' understanding and capacity to teach topics that involved multiplicative reasoning to Key Stage 3 (KS3) pupils. Multiplicative reasoning refers to the mathematical understanding and capability to solve problems arising from proportional situations often involving an understanding and application of fractions as well as decimals, percentages, ratios and proportions. The aspects of multiplicative reasoning addressed in the NCETM project were proportional and fractional relationships. The teaching approaches encouraged were the use of mathematical models, visual approaches and problem solving strategies.

Approximately 60 teachers in 30 schools engaged in 3 regional professional development networks, led by professional development leaders, supported by university researchers. Specific project materials were created by a team of curriculum developers.

## The Evaluation

The impact of MRP on pupil outcomes was evaluated using a 3-level Clustered Randomised Control Trial (CRT) research design. The trial ran between October 2013 and June 2014 and involved 8,777 year 7 (Y7), year 8 (Y8) and year 9 (Y9) pupils (level 1) clustered into 418 Y7, Y8 and Y9 mathematics classes (level 2) clustered into 60 secondary schools. Approximately half the schools, teachers and pupils participated in the intervention and half formed a control group. Progress was compared between the two groups of pupils using Key Stage 2 (KS2) data as a base line and GL Assessment Progress in Mathematics Test (PiM) tests as an outcome measure. This is a measure of general mathematical attainment that is correlated with national tests.

Further analysis was completed specifically in relation to multiplicative reasoning. The GLA PiM tests were analysed and items related to the project materials were identified. Outcomes on these items in the intervention and control samples were analysed. In addition a fidelity analysis was carried out. Outcomes were considered for the sub-sample of pupils who had been taught by teachers who had fully participated in the project, for example, by excluding pupils who had moved classes.

In addition, a process evaluation was conducted using a mixed methods approach of surveys, observations and interviews.

Please see the full report and the accompanying technical report for further details.

## Key Findings

### Research Question 1

*What is the impact of the programme on pupil outcomes on both general mathematical attainment as measured by GL assessment Progress in Mathematics (PiM) tests and on those items in the GL PiM associated specifically with multiplicative reasoning?*

During the timescale of the trial (13 October 2014 to May 2015) the programme did not have any statistically significant impacts on general mathematical attainment as measured by PiM tests or on items on the tests specifically associated with multiplicative reasoning.

### Research Question 2

*What are the impacts (if any) on: pupils' relationships to mathematics; teacher beliefs and practice including on lesson planning; teacher knowledge of multiplicative reasoning pedagogy; capacity of core teachers' to lead professional development?*

The project had a positive impact on pupils' relationship with mathematics as reported by teachers in surveys and interviews and pupils in focus group interviews.

A range of changes in teacher beliefs and practices were identified:

- thinking more deeply about mathematics prior to teaching
- using models when teaching MR and extending models to other areas
- developing questioning
- promoting independence
- focussing on student learning
- questioning assumptions

Many participants, who responded to surveys or participated in case study visits, reported an increased awareness of the importance of multiplicative reasoning and its relationship to other areas of mathematics. Some discussed specific issues about the relationship between additive and multiplicative understanding.

With regard to core teachers leading professional development, many participants shared materials with other members of their department. However, opportunities to lead more extended professional development activities were limited. Nevertheless approximately a quarter of participants involved another member of the department in the first Lesson Study activity. There was some evidence of development of leadership capacity and it appeared the project developed a desire to lead professional development in others.

### Research Question 3:

*How was the programme conducted and how did this differ from the planned programme in what way and why?*

The programme was conducted as planned at national and regional levels involving collaboration between NCETM leads, curriculum developers, university researchers and

professional development (PD) leads. At school level the programme was implemented by two core teachers in each school. Originally it was planned to have a stronger focus on core teachers leading PD in their own departments. During the recruitment phase this was judged by the NCETM and Department for Education (DfE) to be more appropriately something to be encouraged rather than required. There was a degree of loss of fidelity at regional level in relation to the extent to which materials were a resource to be drawn on or a recommended coherent curriculum. The extent to which teachers were able to fully participate in the programme depended on support in schools. In some schools the project was actively supported by senior and departmental leadership and there was a good fit with departmental approaches to mathematics teachers and/or strategic developments in the subject. In other schools there were various issues that meant alignment was lower and this created constraints on implementation.

#### Research Question 4

*What are the views of teachers/development teams on the programme including its effectiveness?*

Participants responding to surveys or participating in interviews judged both the curriculum materials and professional development activities to be, on the whole, effective. Aspects of the professional development activities that were viewed positively were:

- discussion of lessons
- time to plan with the other core teacher
- input on the pedagogical approach
- engaging in mathematics by trialling lesson activities

Aspects of the materials and resources that were produced that were viewed positively were the variety of materials; the use of realistic contexts, and visual models.

Issues were raised for improvement such as having the materials in advance of the training events and having the events nearer to the participants' schools. However, these may be related to the pilot nature of the project.

#### Research Question 5:

*If/how was the programme effective and what lessons can be learnt for scalability?*

The programme was effective as a curriculum design project and pilot project for teacher professional development. It did not meet the intention to impact on pupil attainment (which was measured by the GL PiM tests within the timespan of the project).

Given this, there is no rationale, in terms of an intention to impact on pupil attainment in the short term, for repeating the project in the same form and for the same duration with a larger number of schools. With further development or refinement a similar approach might lead to impact on those aspects of pupil learning measured by PiM tests. One possibility would be to increase the project length to allow time for professional learning to impact on pupil learning and then to re-evaluate.

A project with all the features of the MRP may not be scalable. For example, a positive feature was the opportunity for face to face dialogue between teachers and curriculum developers. This may difficult to repeat at scale. However, support could be given to collaborative professional development communities based on the TIME model or different forms of teacher networks using specific curriculum materials in relation to MR or other areas of the curriculum. There are examples in both recent and medium term past of this being done successfully.

Features of the project that were found to be effective and are reproducible at scale in future projects are:

- teacher professional development focused on research-informed curriculum materials
- teacher collaborative learning communities that draw on the expertise of teacher leaders, curriculum developer and researchers

The MRP brought together a team of curriculum developers whose designs were informed by different but complementary pedagogical principles. Developments were supported by other university researchers and teacher professional development leaders. This was recognised by the national development team members, including teacher leaders, as a powerful and productive approach. Teachers interviewed also valued the different contributions.

The project aimed to address concerns about pupil understanding of multiplicative reasoning in KS3. Teachers found the models used in the projects as useful approaches to teaching multiplicative reasoning. Given this, there is the potential to evaluate specific materials in a more focused way - for example using the bar model and ratio table. This would then allow for an assessment of whether these models should be adopted more widely.

The project materials were found to be a useful resource for KS3 and potentially KS2 curriculum. They were developed to be used as part of a professional development project, however, they have the potential to be adapted as a stand-alone resource or potentially with on-line or similar PD materials. This might follow the Bowland approach<sup>1</sup> or develop alternatives such as video presentation and discussion of materials by developers or through webinars.

#### Research Question 6:

*Are there any patterns of differences in effectiveness for particular groups of pupils, teachers, schools, or across the three TIME teams?*

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<sup>1</sup> [Bowland Maths website](#)

In relation to the impact analysis, there are no significant patterns of differences between particular groups of pupils, teachers, and schools or across the three TIME teams.

Analysis of qualitative data indicates that some of the materials were perceived to be particularly useful for younger or relatively low attaining pupils, with the bar model and ratio tables being accessible ways to develop multiplicative reasoning. Other materials were used with high attaining pupils or adapted for KS4. Teachers who worked in schools in which there was strong senior leadership or departmental support for the programme were able to engage more fully. The project appeared more successful where school cultures supported innovation and problem solving pedagogies.

#### **Research Question 7:**

*What was delivered through the programme including: activities; quality and quantity of professional development; the reach (teachers and pupils) including beyond those directly involved and impacts on organisational capacity?*

The programme delivered a total of 1,250 professional development (PD) days that were potentially accessible by participants. The quality of PD was judged to be high by participants. In addition, a set of curriculum resources and supporting documentation were produced. Materials were organised into 'lessons' and 'units'. Each lesson could take 2-3 hours of teaching time. Thus approximately 36-54 hours' worth of high quality research-informed curriculum materials were produced. These could potentially form the basis of a coherent KS3 curriculum strand in relation to multiplicative reasoning. Approximately 60 teachers attended PD events, with a further group of 15 teachers in schools participating in Lesson Study activity. In the majority of schools materials were shared with other teachers.

Approximately 3,400 KS3 pupils were taught in classes identified with nominated core teachers in intervention schools. Approximately 2,450 of these pupils were identified through an 'on-treatment' analysis. These 2,450 pupils had experienced a minimum level of MRP curriculum materials and were taught by teachers who had engaged in PD activity. The other 950 pupils had only experienced one or the other. Additional pupils in KS2 and KS4 experienced some of the materials.

One aspect of the project was the hosting of TIME events by Maths Hubs and the contribution of these schools in providing professional development leaders. The project provided a focus for the Maths Hubs programme and provided lessons that have informed the Maths Hub programme. Teaching schools involved in the programme have indicated they intended to provide further professional development on multiplicative reasoning.

The project also brought together curriculum developers working from different approaches to curriculum design.

#### **Research Question 8: *Was the programme cost effective?***

Given the lack of evidence with regard to impact on pupil attainment as measured by PiM tests, a cost benefit analysis is not appropriate. As an alternative, a cost effectiveness

analysis based on a comparison of the MRP with other forms of professional development was undertaken.

It found that the project appeared to be delivered in a cost effective manner. Considering the direct PD costs rather than developmental costs, the programme was cost effective in comparison with alternative PD priced as one day courses. The MRP has offered a range of added value with PD focused on a specific recognised need - multiplicative reasoning and developing research based mathematics materials.

## Discussion of results

It is important to recognise the limitations of the results of both the RCT and process evaluation. Schools involved in the project were not representative of the national population of schools and in the case of the process evaluation there is likely to be some sampling bias with those schools more favourable to the project being over represented.

A measured impact on pupil attainment using the PiM tests was not found, this may be due to a combination of:

- unobserved significant variables
- attrition
- sensitivity/appropriateness of the test measures
- issues related to the theory of change and/or the intervention

Most significantly, in relation to the latter, were the complexity of the project and the short time scale in which it took place.

Despite the lack of impact on pupil attainment on the PiM tests, the MRP embodied an approach that drew on the evidence base of effective professional development as well as research on multiplicative reasoning.

Characteristics of the MRP that were important were:

- a focus on subject matter
- professional learning focused on student learning
- sustained duration
- collective participation of teachers from the same school

One additional area that could have been included was to directly address teachers' beliefs about mathematics and the teaching and learning of mathematics. Definitions or frameworks could have been introduced at the start of the project and, in particular, the definition of multiplicative reasoning upon which the project was based could have been made more explicit.

Important to the theory of change that underlay the project was the need to address teachers' understanding of multiplicative structures and MR pedagogy. There is the potential to develop further targeted interventions and professional development materials



related to this. One approach to doing this would be to draw on international developments in relation to understanding and assessing teachers' multiplicative reasoning knowledge<sup>2</sup>. This could lead to interventions that were assessed by impact on teachers in the first instance as a pre-requisite for evaluating impact on pupils.

The regional collaborative communities were led by schools and hosted by Maths Hubs. The Maths Hubs pilot (initially known as Mathematics Education Strategic Hubs) has evolved into the Maths Hub programme and this has already been funded by the DfE. The MRP evaluation provides evidence that supports this decision. Maths Hubs involved in the project, as well as other teaching schools, recognised the potential of MRP model and materials as an approach to professional development in their hubs.

A design approach to curriculum development has potential to address other areas of concern in mathematics. In addition to the curriculum developers involved in MRP there are other recent and significant examples of curriculum design in mathematics in England<sup>3</sup>. The NCETM has a particular role as broker in relation to different contributors to mathematics professional development, as evidenced by its approach in the MRP.

One of the effective features of the project, as identified by participants, was utilising the expertise of university-based developers and researchers. The Maths Hub programme aligns with this, with higher education institutions (HEIs) involved in strategic bodies within Hubs. The NCETM has also previously encouraged teacher innovators to draw on HEI expertise when awarding direct grants.

## Lessons for evaluation design

There are lessons to be learnt for the approach to design evaluations of this type in current educational contexts, as well as in relation to practical matters. A key lesson is the importance of ensuring that any specific intervention has gone through a design and pilot stage, preferably with evidence of positive effect. Longer lead in times would help to ensure greater balance at the point of randomisation.

Longitudinal studies would allow for use of national test data rather than additional testing of pupils, so reducing the burden on schools. Trials lasting longer would also allow for impact over a longer period than nine months to be assessed.

More generally, the appropriateness of randomised controlled trials as an evaluation approach should be considered for complex PD interventions.

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<sup>2</sup> For example the [Diagnosing Teachers' Multiplicative Reasoning Project](#)

<sup>3</sup> For example, [Bowland Mathematics](#), the [Effecting Principled Improvement in Science and Mathematics project](#), the [Cre8ate maths project](#), the [NRich project](#); and the Nuffield applying mathematical processes.

## Recommendations

### Improving pupils' multiplicative reasoning skills

The programme did not have any statistically significant impacts on general mathematical attainment as measured by PiM tests, within the timescales of the project. Given the evidence of previous studies that multiplicative reasoning can be improved through targeted intervention<sup>4</sup>, and the possibility that the lack of impact was due to the timescale of the project, further development in this area is justified:

Recommendation 1: For the DfE to consider commissioning smaller scale quasi-experimental trials of specific MR materials or models, aiming for a more consistent approach to material use, with outcome measures designed specifically for the trial to measure MR knowledge, and to allow more time to detect impact.

Recommendation 2: For the DfE to consider developing and evaluating interventions that focus specifically on teachers' multiplicative reasoning knowledge and pedagogy using specific measures of impact that are suitable for this purpose.

Recommendation 3: For the DfE to consider lessons learnt for the use of RCT and similar methodologies in relation to: pilot and development prior to testing; early appointment of external evaluators to advise on intervention design; and longer lead in times to allow for greater balance at randomisation and a longer trial period.

### Impact on teacher professional development

Teachers judged both the curriculum materials and activities to be, on the whole, effective in terms of their professional development, and useful in the classroom.

Recommendation 4: That the NCETM is encouraged to refine the MRP materials for use in KS3 and potentially KS2, providing guidance on how they could be used effectively, potentially with on-line PD activities.

Recommendation 5: For the NCETM to make available MRP materials and approaches to teaching MR through the Maths Hub network.

### Impact on new approaches to PD in the school-led system

Participants were generally positive about the form of collaborative professional development with teacher leaders being key to delivery and focused on issues of concern to them.

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<sup>4</sup> Hodgen, J. et al. (2014) Improving students' understanding of algebra and multiplicative reasoning: Did the ICCAMS intervention work? Pope, S. (Ed.) Proceedings of the 8th British Congress of Mathematics Education 2014, 167-174

Recommendation 6: For the DfE to support collaborative professional development with features of the MRP model and similar teacher-led professional development networks through the Maths Hub programme.

Recommendation 7: That the NCETM seeks further opportunities to engage curriculum developers with Maths Hubs and other NCETM activities and potentially to develop future curriculum design projects that address the needs of teachers, schools and pupils.

Recommendation 8: For the NCETM to encourage HEI involvement in specific projects developed by Maths Hubs.

## Conclusion

The Multiplicative Reasoning Project was a pilot professional development project. Given that it was a pilot and it was conducted over a short time scale it is not surprising that it did not lead to pupil impact as evidenced by external tests. The project was effective in relation to teacher professional development and supported the development of the Maths Hub programme. Those effective features of the project in relation to professional development should be developed and replicated. However, further evaluation of the specific curriculum models is needed to evaluate if they are useful more widely.



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