Identification of superfluous Energy Demand Model variables

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EXECUTIVE SUMMARY

This analysis intends to facilitate a better understanding of DECC's Energy Demand Model, identifying those variables that are significant to the outputs of the model, those that are of little significance and those that are redundant. Our approach is based on three analytical methodologies: precedence analysis, global sensitivity analysis and tangential space analysis. These have been complemented with an ad hoc analysis aided by logic maps.

The precedence analysis depends on the set of model outputs; we analysed two sets of variables: a radical case looking at the effect on the 3 output variables deemed most important to DECC (electricity demand, total CO2 emissions and total energy consumption) and a conservative case with a list of 934 variables that were initially deemed to be outputs out of the 3923 variables in the model. The radical case variables have a total 1878 precedents, 665 of which are terminal. None of these precedents have zero dependents, i.e. none are model output variables (MOV). Indeed, approximately half of the difference between conservative and main MOV only analyses precedent counts is due to MOVs with no precedents being included in the former analysis. The conservative case showed that EDM has at most 2519 precedents for 970 Model Outputs. 842 of the precedents are terminal, i.e. they have no further precedents. Of the 3923 variables, 1404 of them definitely have no effect on the 970 selected outputs. It will be noted that a significant number of the precedents have zero dependents or have no MOV dependents (because the EDM is passing through data to the summary sheets). Whether they are deemed redundant or not is a matter for DECC to decide, but there is no reason for them to be included in the model, as they are not involved in calculations that affect the other MOV.

The Method of Morris sensitivity analysis focussed on 89 input variables which were found to vary over six scenarios provided by DECC. These 89 input variables are terminal precedents for 1852 variables, of which 455 are in the preferred list provided by DECC. However, these inputs do not explain any changes in 515 of the preferred output variables. These remaining outputs require information for almost 280 inputs, for which we currently do not have data. When lags are included for the 89 inputs (doubling the inputs to 178), almost half of the lagged variables influence the preferred outputs.

The tangential space analysis shows that any binary classification of the non-redundant variables into insignificant and significant is likely to be sensitive itself to the assumptions made. This is because: the range of effects of perturbing a given terminal precedent by even a small amount spans half a dozen orders of magnitude, there are a large number of Model Output variables, the effect of different amounts of variation in the variables, the continuous distribution of effects across these orders of magnitude, and the fact that many precedent variables are connected to tens or even hundreds of Model Output Variables whose relative importance is unknown to us.

We then performed a large sensitivity analysis over the varying proportion of the 665 variables which are terminal precedents for the high level output variables. We removed terminal precedents that are unlikely to vary, as well as other inputs deemed unimportant for the analysis. This reduced the number of input variables to ~200. We obtained an approximate ranking of input variables, predicated on an arbitrary 5% variation around the value in the reference scenario for each of these inputs. A brief analysis of the 89 input variables we did have data for suggested that this is likely to raise both Type I and Type II errors, wrongly identifying non-

influential variables and influential and *vice versa*. Given the lack of data, the results from this analysis should be treated with healthy skepticism.

While our method is shown to be robust in identifying inputs of importance, all sensitivity methods depend upon a reasonable quantification of input ranges to provide results that are reliable. To offer globally robust and general conclusions, it is essential that the input range for each of the inputs is quantified. Nevertheless, we have developed a set of tools and an approach that DECC can apply to further explore the Energy Demand Model, in order to better understand the significance of different input variables under various assumptions and decide on which variables to focus their data gathering efforts.

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1 Introduction

DECC, like many government departments, has to address new complexities and pressures in evaluating their policies. A combination of limited resource capacity and increased accountability make transparency and efficiency an institutional challenge. For DECC and other government departments with strong modelling elements in their activities, the challenge of understanding the effects of key variables in their models are non-trivial.

The Energy Demand Model (EDM) is at the heart of DECC's Energy and Emissions Projection (EEP) model suite. It can be used in isolation, but for key forecasting work it is used in conjunction with other EEP models, particularly the Dynamic Dispatch Model (DDM) which forecasts wholesale electricity prices. The EDM is central to assessments of the UK's likely progress against legally established carbon budgets. DECC are seeking to simplify the EDM as part of an overall redevelopment programme to streamline and better integrate the EEP model suite.

The main objective of this analysis is to measure the sensitivity of the model outputs to the rest of the variables in the EDM, in order to identify those which are significant, insignificant and redundant. This will improve the overall efficiency of the modelling process by removing unnecessary model variables and will also aid understanding of the key variables in the model.

2 Overview of the Energy Demand Model

The Energy Demand Model (EDM) calculates energy demand on a final fuel use basis and calculates all non-electricity generation transformation processes.

There are number of inputs that feed into the EDM that are either statistics or external projections (external to DECC's energy and emissions projections team). These are:

- Fossil fuel price projections
- Socio-economic growth projections
- Historic fuel uses from DUKES
- Historic greenhouse gas emissions
- Past and projected savings in fuel use due to emissions abatement policies
- Past and projected carbon emissions factors

Aside from these, a number of workbooks based on outputs from other models are also imported:

- CHP projections
- Retail price uplifts
- Non-energy non-CO_2 and LULUCF projections

The EDM predicts fuel demand for energy by sector and sub-sector using a series of econometric and behavioural equations. This is done on a final energy demand basis. To estimate the econometric equations, a regression analysis of historic fuel demand against drivers of energy use is undertaken.

The EDM also calculates primary fuel inputs into energy transformation processes using similar equations. This is with the exception of electricity generation and CHP generation. The associated emissions from energy use are also calculated inside the model, using exogenously derived carbon emission factors.

The EDM exports key outputs from itself and the Dynamic Dispatch Model (DDM) into a set of spreadsheet workbooks. Here, another level of transformation and aggregation takes place to provide the most commonly needed outputs for the update on energy and emission projections (UEP). These workbooks are:

- Final energy demand
- Primary energy demand
- Energy balance
- Electricity supply – generation, capacity and build (Major Power Producers and non-MPPs)
- CO2 emissions
- Fossil fuel and electricity prices (wholesale and retail)
- Non-CO_2 greenhouse gas emissions
- Emissions by a traded / non-traded split
- Greenhouse gas inventory tables
- UK net carbon account

“Values” worksheet

This is the main worksheet in EDM since it’s where all the calculations take place. There are 3923 rows representing time series, the values for which correspond to just two types of data:

- a constant value, pasted in from an external workbook or calculation
- an equation, which may contain constants, and which refers to other rows

Each column in the “Values” sheet represents a different year. For some rows, past years contain a constant value, and future years contain equations, which extrapolate from these historical values (sometimes indirectly). For other rows, historical years contain equations, while future years contain constant data, such as projections input from other models. Cells that contain fixed data values are shaded grey, while those that contain equations are un-shaded (white).

With the exception of historical years, each column is identical to the next. This allows some reduction in the number of cells in the spreadsheet that we need to investigate.

In our analyses, we use the year column 2020 as a central year of analysis, thus reducing the size of the analysis, while not sacrificing our ability to extrapolate to the rest of the model (we have checked this assumption by running otherwise identical analyses on other years).

Each row variable is named according to a logical convention, although capitalisation of the variable names could be more strictly enforced. A sector, or variable type (e.g. 'AGRIC', 'CONSTANT') is separated from a descriptive name by a double underscore ('__'). These sectors provide a convenient means to disaggregate the large amount of results in our analysis.

Variable Values

	Sector	Variable	Unique Name	Var Description/Units	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
7	AGRIC	BOIL_CO2	AGRIC__BOIL_CO2	tC	10,308	0	0	0	0	0	0	0	0	0	0	0	0	0
8	AGRIC	BOIL_CO2_COEF	AGRIC__BOIL_CO2_COEF	tC/MTh	1,963	1,963	1,963	1,963	1,963	1,963	1,963	1,963	1,963	1,963	1,963	1,963	1,963	1,963
9	AGRIC	BOIL_FIN_DEM	AGRIC__BOIL_FIN_DEM	MUThs	3	0	0	0	0	0	0	0	0	0	0	0	0	0
10	AGRIC	BOIL_FIN_MTH	AGRIC__BOIL_FIN_MTH	MTh	5	0	0	0	0	0	0	0	0	0	0	0	0	0
11	AGRIC	BOIL_FIN_MTN	AGRIC__BOIL_FIN_MTN	Mtonne	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	AGRIC	BOIL_KTOE	AGRIC__BOIL_KTOE	Ktoe	14	0	0	0	0	0	0	0	0	0	0	0	0	0
13	AGRIC	BOIL_TOT_DEM	AGRIC__BOIL_TOT_DEM	MUThs	3	0	0	0	0	0	0	0	0	0	0	0	0	0
14	AGRIC	CHP_ELECGEN	AGRIC__CHP_ELECGEN	GWh														
15	AGRIC	CHP_EXPORT	AGRIC__CHP_EXPORT	GWh														
16	AGRIC	COAL_CO2	AGRIC__COAL_CO2	tC	2,556	3,195	0	906	964	975	0	0	0	0	0	0	0	0
17	AGRIC	COAL_CO2_COEF	AGRIC__COAL_CO2_COEF	tC/MTh	2,408	2,408	2,408	2,408	2,408	2,408	2,408	2,408	2,408	2,408	2,408	2,408	2,408	2,408
18	AGRIC	COAL_FIN_DEM	AGRIC__COAL_FIN_DEM	MUThs	1	1	0	0	0	0	0	0	0	0	0	0	0	0
19	AGRIC	COAL_FIN_MTH	AGRIC__COAL_FIN_MTH	MTh	1	1	0	0	0	0	0	0	0	0	0	0	0	0
20	AGRIC	COAL_FIN_MTN	AGRIC__COAL_FIN_MTN	Mtonne	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	AGRIC	COAL_KTOE	AGRIC__COAL_KTOE	Ktoe	3	3	0	1	1	1	1	1	1	1	1	1	1	1
22	AGRIC	COAL_TOT_DEM	AGRIC__COAL_TOT_DEM	MUThs	1	1	0	0	0	0	0	0	0	0	0	0	0	0
23	AGRIC	COKE_CO2	AGRIC__COKE_CO2	tC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	AGRIC	COKE_CO2_COEF	AGRIC__COKE_CO2_COEF	tC/MTh	2,911	2,911	2,911	2,911	2,911	2,911	2,911	2,911	2,911	2,911	2,911	2,911	2,911	2,911
25	AGRIC	COKE_FIN_DEM	AGRIC__COKE_FIN_DEM	MUThs	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	AGRIC	COKE_FIN_MTH	AGRIC__COKE_FIN_MTH	MTh	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	AGRIC	COKE_FIN_MTN	AGRIC__COKE_FIN_MTN	Mtonne	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	AGRIC	COKE_KTOE	AGRIC__COKE_KTOE	Ktoe														
29	AGRIC	COKE_TOT_DEM	AGRIC__COKE_TOT_DEM	MUThs	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	AGRIC	ELEC_CCP_TOTAL	AGRIC__ELEC_CCP_TOTAL	MUThs	0	0	0	0	0	0	0	1	1	1	1	1	2	2
31	AGRIC	ELEC_FIN_DEM	AGRIC__ELEC_FIN_DEM	MUThs	125	125	117	124	121	119	119	119	119	119	118	118	118	117
32	AGRIC	ELEC_FIN_MTH	AGRIC__ELEC_FIN_MTH	MTh	138	139	130	137	135	132	132	132	132	132	131	131	131	130
33	AGRIC	ELEC_FIN_TWH	AGRIC__ELEC_FIN_TWH	TWh	4	4	4	4	4	4	4	4	4	4	4	4	4	4
34	AGRIC	ELEC_KTOE	AGRIC__ELEC_KTOE	Ktoe	349	350	327	346	339	333	334	334	334	334	334	334	334	334
35	AGRIC	ELEC_PROD_RP	AGRIC__ELEC_PROD_RP	p/Th (1995px)														
36	AGRIC	ELEC_TOT_DEM	AGRIC__ELEC_TOT_DEM	MUThs	125	125	117	124	121	119	119	119	119	119	119	119	119	119
37	AGRIC	ELEC_TOT_TWH	AGRIC__ELEC_TOT_TWH	TWh	4	4	4	4	4	4	4	4	4	4	4	4	4	4
38	AGRIC	ENG_LUB_CO2	AGRIC__ENG_LUB_CO2	tC	2,747	2,083	2,085	2,371	2,007	1,685	1,685	1,685	1,685	1,685	1,685	1,685	1,685	1,685
39	AGRIC	FIN_SHARE_BOIL	AGRIC__FIN_SHARE_BOIL	%	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	AGRIC	FIN_SHARE_COAL	AGRIC__FIN_SHARE_COAL	%	1	1	0	1	1	1	1	1	1	1	1	1	1	1
41	AGRIC	FIN_SHARE_COKE	AGRIC__FIN_SHARE_COKE	%	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	AGRIC	FIN_SHARE_FOIL	AGRIC__FIN_SHARE_FOIL	%	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	AGRIC	FIN_SHARE_GOIL	AGRIC__FIN_SHARE_GOIL	%	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44	AGRIC	FIN_SHARE_LPG	AGRIC__FIN_SHARE_LPG	%	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	AGRIC	FIN_SHARE_OSF	AGRIC__FIN_SHARE_OSF	%	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	AGRIC	FOIL_CO2	AGRIC__FOIL_CO2	tC	8,484	21,096	7,032	9,669	14,032	12,439	12,748	10,103	9,311	8,486	7,493	6,984	6,984	6,984
47	AGRIC	FOIL_CO2_COEF	AGRIC__FOIL_CO2_COEF	tC/MTh	2,141	2,141	2,141	2,141	2,141	2,141	2,141	2,141	2,141	2,141	2,141	2,141	2,141	2,141
48	AGRIC	FOIL_FIN_DEM	AGRIC__FOIL_FIN_DEM	MUThs	2	5	2	2	4	3	3	3	2	2	2	2	2	2
49	AGRIC	FOIL_FIN_MTH	AGRIC__FOIL_FIN_MTH	MTh	4	10	3	5	7	6	6	5	4	4	3	3	3	3
50	AGRIC	FOIL_FIN_MTN	AGRIC__FOIL_FIN_MTN	Mtonne	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	AGRIC	FOIL_KTOE	AGRIC__FOIL_KTOE	Ktoe	10	25	8	11	17	15	15	12	11	10	9	8	8	8
52	AGRIC	FOIL_TOT_DEM	AGRIC__FOIL_TOT_DEM	MUThs	2	5	2	2	4	3	4	3	3	3	3	3	3	3

Figure 1. Screenshot of the Values worksheet in EDM

1.1 Understanding the relationship between inputs and outputs

We classify model variables as fixed data or equations. Fixed data are almost always classed as inputs, although they may not vary between scenarios (constants and historical data). Equations may be used as model output variables (MOVs) but by definition are never inputs. The majority of equation variables are used as outputs, as shown by the precedence analysis in the next section.

Identification of fixed data or equations, and discerning whether the data is a fixed assumption or forecasted data from another model cannot be automatically extracted from variable names. This means that an ad-hoc approach to assessing model inputs is required, and dramatically increases the volume of work required to carry out any meta-analyses, such as the sensitivity methods in this report.

A model can be considered as a mathematical relationship between an arbitrary number of inputs and outputs. In the case of the EDM model, rows of cells consist of a mix of fixed data and equations, the latter referring to other rows of cells. Approximately 970 of the rows are considered as MOVs, and are used in successor spreadsheets for compilation of statistics. The remaining ~3000 rows consist of 850 rows which contain fixed data for future years, while the remainder contain equations.

Categorising the rows (variables) using this logic, gives in the sets and subsets that follow:

All variables

- *All inputs* [850]
    - *varying inputs* [89]

    - *non-varying inputs* (may vary, but not observed in scenarios) [252]
    - *constant inputs* [509]
- *All outputs*/equations [2742]
    - *preferred outputs* [900]
        - Affected by *varying inputs* [400]
        - Not affected by *varying inputs* [500]
            - affected by some or all of *non-varying inputs* [?]
            - redundant [?]
    - *other outputs*/equations [2150]
        - Affected by *varying inputs* [?]
        - Not affected by *varying inputs* [?]
            - affected by some of all of *non-varying inputs* [?]
            - redundant [1404]

We cannot fully populate this figure without actually doing the analysis, so there is some uncertainty as to what are the numbers represented by question marks. However, this outlines our thinking on the problem. The deliverables specify the subset of *all variables* which correspond with *redundant* variables - those that can be safely removed without affecting the *preferred outputs*; *insignificant* variables - those inputs that do not manifestly affect the *preferred outputs* and the dependents thereof; *significant* variables - those inputs which fundamentally drive the variation of the *preferred outputs*.

2 Our approach

In this study we use a three-stage process to classify the significance of internal variable and inputs on model outputs:

1) Identify internal variables and inputs that are not linked by formula chains to the model output. Such cell formula dependency trees can be automatically constructed by recursively applying the Excel VBA Range precedence property.

2) Apply, in parallel, a global sensitivity analysis technique and a deterministic technique to identify variable effects:

a) Elementary effects approach (global sensitivity analysis);

b) Linearized tangent space analysis (local deterministic technique).

3) The results of (2a) and (2b) are cross correlated and where any discrepancies appear are further investigated using ad hoc methods e.g. focussed simulation/Monte Carlo tests or direct analysis of equations by hand and/or using the graphical technique detailed below. Before beginning our sensitivity analysis, we established a regression testing framework to ensure that our adjustments to the model do not materially affect the result outputs. We have amended the model through removal of equations and variables deemed to be uninfluential by our sensitivity analysis, and demonstrated the effect of the change through comparison of models before-and-after our amendments.

2.1 Precedence Analysis

Precedence analysis involves recursively tracing the variables in equations defining the model output variables until they reach variables which are not defined by formulae. We call these latter variables ***terminal precedents*.** The converse of this, dependency analysis, traces variables that depend upon an initial set of variables; in the context of the EDM this is usually a subset of the terminal precedents.

Excel and the VBA API have a number of tools for performing these types of analyses, e.g. the Trace Precedents and Trace Dependents options on the Formulas ribbon in Excel. Unfortunately, these tools are not suitable for analysing nearly 4000 time series variables visually, so we developed a number of tools in VBA to automate these analyses.

All of the analyses we have performed rely upon filtering and manipulating information provided by the (Direct)Precedents and (Direct)Dependents properties of Excel Range objects. The EDM uses named ranges in its formulae, and the named ranges referring to the variables on the “Values” Worksheet, where the model is implemented, reference entire rows, i.e. since Excel 2007 this amounts to 16384 cells per named range. As any tracing of links between cells will be recursive in nature, referencing entire rows leads to very slow running macros and, on occasion, out of memory errors and similar. To circumvent this, we have provided a macro, `RestrictEDMNamedRanges`, which alters an EDM’s variables’ named ranged to the first 52 columns of the “Values” worksheet. This does not alter the output of the model.

The Formula Analysis Tools workbook contains 3 methods for tracing precedents and dependents. The first of these, `IdentifyRelatedCells` contained in the `General_Precedents_Tools` code module, is a generic formula precedent tracing tool whose input is a collection of ranges from a single worksheet. Users are requested to manually select ranges and asked about how they wish to identify precedents or dependents (cell interior colouring, or fill with diagonal lines, or both) and terminal precedent or dependents (cell interior colouring, or fill with crisscross hatching, or both).

`IdentifyRelatedCells` was our first attempt at identifying precedents. Although it is more powerful than the other two methods detailed below, as it can trace precedents across sheets and even across open workbooks and its logic is not specific to the EDM, it is quite resource intensive and we found that, due to the deep and wide linkage of some output variables, it ran into resource allocation problems. Because of this we had to develop other methods which were less general and relied on specific details of the EDM and the precedent analysis results presented were not obtained from this macro. It has, however been retained in the delivered code base as it is useful for tracing precedents and dependents in more restricted circumstances.

The second macro, `IdentifyCellsRelatedToEDMOutputs` contained in the `EDM_Precedents_Tools` code module relies upon the VBA's `Range.Precedents` and `Range.Dependents` properties. These properties fail to follow precedents and dependents beyond the worksheet containing the initial Range. However, this turns out to be satisfactory for the purpose of analysing the EDM, as the model's equations are contained on a single worksheet. Furthermore, the equations for the model are, *mutatis mutandis*, identical for all years post 2012. Because of this observation, the second macro focusses on a single, user selected, year for each variable and traces all precedents/dependents arising from that instance of the variable. It marks the results in a similar fashion to the first macro.

The results of either of these first two macros may be collated into the AllPrecedents and TerminalPrecedents worksheets in the Formula Analysis Tools workbook using the `GatherSelectedVariables` macro. These worksheets, along with the AllVars and ModelOutput worksheets are then used in the performance of further analyses.

The final macro method for tracing precedents and dependents works by recording only the direct precedents/dependents (*sensu* `Range.DirectPrecedents`) of a given variable, i.e. the cells that are either directly referenced in the formula (precedent case) or which directly reference the cell (dependent case). The macro `CreateDirectPrecedentMatrix` (resp. `CreateDirectDependentMatrix`) creates a worksheet DirectPrecedentMatrix which contains a square matrix recording which variables are precedents (resp. dependents) of each variable. The precedent and dependent matrices are transposes of each other. We can construct the directed precedence and dependences graph of variables by recursively following the links in these matrices.

Each of these methods has advantages and disadvantages, and each has been utilised in the preparation of this report.

Finally, the DirectPrecedentMatrix worksheet can also be used to derive the sets of edges for directed graphs illustrating how all the variables/equations in the EDM are linked together. The NodeXL diagrams are generated by the `CalculatePrecedentGraphEdges` macro applied to this matrix.

2.2 Sensitivity Analysis

We use two approaches to attempt to determine a measure of the sensitivity of the model output variables to changes in the model input values. The Method of Morris is a global approach, and gives an estimate of the true influence of each input variable for each output variable. The term 'global' means that interactions between two or more variables are taken into account, thus variables which result in cancellation effects or multiplicative effects are identified. The Tangential Space Analysis is a local approach, which perturbs individual inputs by a small amount one-at-a-time. This gives an exact indication of the importance of an input for all the outputs. Both approaches will provide similar results if i) the model is linear ii) there are no interaction affects and iii) the same range data is used for the inputs.

2.2.1 Method of Morris Analysis

Global sensitivity analysis techniques quantify the degree to which model inputs influence model outputs. The associated metrics are global, in that they are robust over all inputs for all combinations of possible input values. Given the large number of input parameters to the EDM, we have used a screening approach based on the Method of Morris[1] to determine the subset of the parameters that are most influential, and those that are not influential. The main advantage of the global sensitivity analysis technique we have used over alternatives is the allowance for models that are non-linear and formulations that are non-additive. Techniques based upon linear regression or other statistical approaches such as principal components analysis, make unrealistic assumptions regarding the relationships between variables, e.g. that the output values are a linear function of the input values. These assumptions can strongly bias the results of a sensitivity analysis, particularly for models which contain polynomial or quadratic components. The Method of Morris technique is computationally efficient, with the elementary effects method requiring N(k+1) runs of the model, where k is the number of model input parameters. Typically, the screening analysis will use N=10. Thus the typical running times for the analysis have been at most 10 * (200 + 1) * 1 sec ≈ 30 minutes.

This methodology involved the following development tasks:

- Module to generate input samples, based upon the Method of Morris. The Method of Morris requires only lower and upper bounds on input data, rather than full probability distributions.
- VBA module to load input samples into input parameter sheets of the EDM, run the model, and export results for each result metric
- Module to analyse results of EDM sensitivity analysis and produce graphics.

Initial analytical work was performed using the Python library SALib, and the cluster maps in the Appendix were drawn using the Seaborn library for Python. Subsequently, the necessary code has been ported to VBA so that DECC may run their own analyses.

2.2.1.1 Obtaining input samples

Initially, we identified 89 inputs which varied across six scenario files provided to us. An initial sensitivity analysis was run using only this data.

Further investigation raised the issue that several types of variables were unlikely to be of interest. These included:

- Coefficients - 'COEF' in the name

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[1] Morris, M. D. (1991). Factorial Sampling Plans for Preliminary Computational Experiments. *Technometrics*, *33*, 161–174. doi:10.2307/1269043

Campolongo, F., Cariboni, J., & Saltelli, A. (2007). An effective screening design for sensitivity analysis of large models. *Environmental Modelling & Software*, *22*(10), 1509–1518. doi:10.1016/j.envsoft.2006.10.004

- Combined heat and power variables - 'CHP' in the name
- Policy variables – CCP in the name
- Share variables – while it would have been good to include these in the analysis, no easily identifiable logic was found to enable a computational solution to these
- Calorific values – GCV – these should be included under 'constants' or 'COEF names
- Constant variables – a sector of variables in the model names 'CONSTANT_'

There were also a number of variables whose value is zero in 2020. With no range data to draw from, these had to be excluded from the analysis.

The final analysis was run using the inputs for the high-level output variables derived from the precedence analysis described in Section 2.1. The input ranges were generated with an arbitrary 5% variation around the 2020 value from the reference scenario spreadsheet, since we were unable to successfully identify robust input ranges for the model inputs (see Figure 2). Note that the results are entirely a function of this range assumption, and that this is likely to result in both Type I and Type II errors; thus the results should be viewed as speculative.

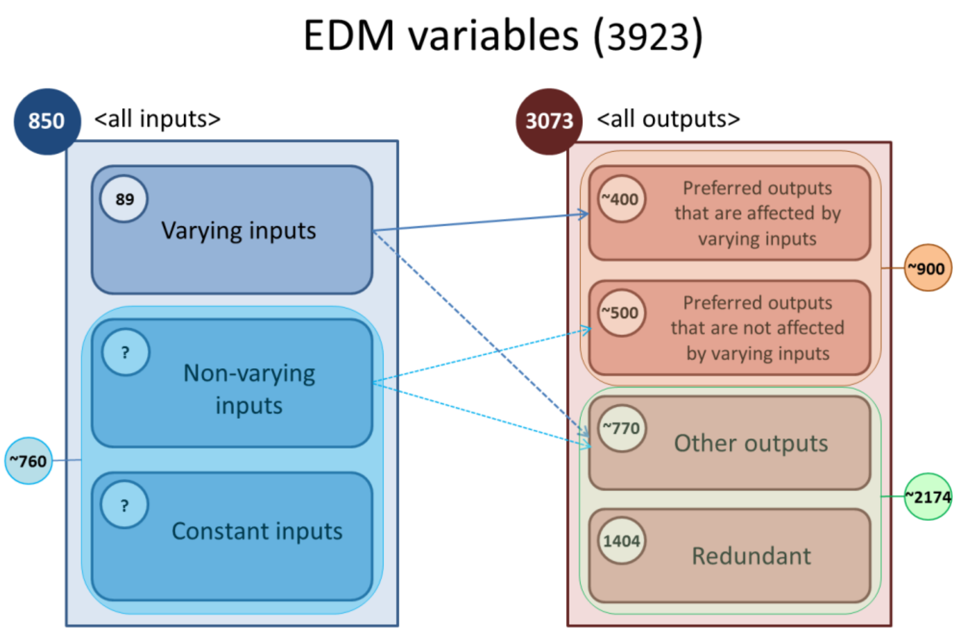


Figure 2. Limited information on input data ranges. This diagram illustrates the relationship between input and output variables in the model, highlighting in particular the importance of gathering range information on varying and non-varying inputs in order to better understand their impact on output variables.

2.2.1.2 Generating samples for the sensitivity analysis

Once the inputs and ranges for those inputs have been identified, the sample is generated using the following procedure.

Given a model with k independent inputs X_i, i = 1,...,k, each of which is divided into p discrete levels to produce a grid Ω, the objective of the sampling method is to generate a panel of N trajectories. N is an exogenously defined sample value and defines the total size of the sampling procedure and interacts with the number of levels p into which the inputs are divided. p defines the 'resolution' of the analysis, with higher values of p (higher resolution) requiring a bigger input sample for the same degree of confidence. A p of 4 requires an N of 10 or more.

Each trajectory is represented here by matrix $B^{\star}$ and has dimensions (k + 1) × k which provides one randomly selected elementary effect per input parameter:

$$B^{\star} = (J_{k+1,1}x^{\star} + (\Delta/2)\,[(2B - J_{k+1,k})\,D^{\star} + J_{k+1,k}])\,P^{\star}$$

where $D^{\star}$ is a k-dimensional diagonal matrix in which each element is either +1 or −1 with equal probability, $P^{\star}$ is a k-by-k random permutation matrix in which each row contains one element equal to one, all others are zero, and no two columns have 1's in the same position. $x^{\star}$ is a randomly chosen base value of X from the p-level grid Ω and is thus a vector of dimensions k. $x^{\star}$ provides an initial starting point for each trajectory, upon which the randomly generated permutation matrix $P^{\star}$ operates. B is a (k + 1) × k matrix with elements that are either 0 or 1 and has the key property that for every column index j,j = 1,...,k there are two rows of B that differ only in the jth entry. This is fulfilled through using a strictly lower triangular matrix of 1s. $J_{k+1,k}$ is a (k + 1) × k matrix of 1s. Finally, Δ is computed as a function of the number of levels p/2 (p − 1) when p is even and denotes the order in which the inputs move in each row of the $B^{\star}$ matrix.

2.2.1.3 Running the model

The model is run, permuting each of the input variables under analysis, and recording the value of the outputs being measured.

The sample values for each of the input variables was automatically pasted into the Values sheet in the reference scenario. The results were observed and recorded for each of the Model Output Variables (MOVs).

2.2.1.4 Analyzing the results

This analysis process is run for each MOV individually.

A prior step to computing the sensitivity indices is to calculate the elementary effects. The input sample and output results are compared, and the change in each of the output variable compared to the change in each input variable.

The elementary effect of the ith input parameter computed along trajectory j is given by:

$$EE_i^j\left(x^{(l)}\right) = \frac{\left[y\left(x^{(l+1)}\right) - y\left(x^{(l)}\right)\right]}{\Delta}$$

if the ith component of $x^{(l)}$ is increased by Δ, and

$$EE_i^j\left(x^{(l+1)}\right) = \frac{\left[y\left(x^{(l)}\right) - y\left(x^{(l+1)}\right)\right]}{\Delta}$$

, if the ith component of $x^{(l)}$ is decreased by Δ.

Once the elementary effects have been computed, it is a simple matter to compute the desired sensitivity metrics.

Mu (μ) is the mean of the effect of each variable.

$$\mu_i = \frac{1}{r}\sum_{j=1}^{N} EE_i^j$$

Mu_star (μ*) is the mean of the absolute effect of each variable.

$$\mu_i^\star = \frac{1}{r}\sum_{j=1}^{N} \mid EE_i^j \mid$$

Sigma (σ^2) is the standard deviation of the effect of each variable.

$$\sigma_i^2 = \frac{1}{r-1}\sum_{j=1}^{N} \left(EE_i^j - \mu\right)^2$$

We use **μ***to compare results in this analysis. Usefully, the units of **μ*** are the same as that of the output being measured.

Two types of plots are used, clustermaps and scatter diagrams.

Clustermaps are used for comparing the relationship between multiple model inputs and multiple model outputs. The centre of the plot contains values for **μ***, which are subsequently normalised across the inputs. Thus a ranking of input variables for each output is easily identified by the darkness of the colour. So for output x, inputs a, b and c can be ranked. However, comparisons between outputs are not possible, it is only possible to obtain identify that a certain input is relatively important for one or more outputs. In other words, the normalised results are only relevant locally. For the global picture i.e. across the entire model, a small number, and ideally one, high level output is required. On the margins of the clustermap, there are dendrograms for both inputs and outputs. The dendrograms indicate clustering between inputs and outputs , which indicate similar sensitivity. Thus the position of the inputs and outputs relative to one another gives some indication of their relative importance. Finally, the coloured blocks associated with inputs and outputs are assigned according to the sector to which the variable belongs. Thus the relationship across sectors of inputs to outputs can be observed together with the proxy for sensitivity.

The scatter diagrams are only plotted for one output at a time. These have been provided for a small number of headline model output variables only and allow direct comparison between inputs. However, only those inputs (the terminal precedents) that affect the output under consideration can be included in these plots. It is therefore important to select as aggregate an output as possible which covers as many inputs as possible. However, there is another trade-off here, as aggregate variables may miss some cancellation effects. We therefore present results for the highest level aggregate variables, and those variables that directly feed into this output variable.

2.2.2 Tangent Space Analysis

Tangent space analysis attempts to understand the effects of internal variables and inputs by investigating the strength of their individual marginal effects on **model output variables** (**MOV**s) at a given point on the model. In the case of the EDM, for a given model scenario, and a given year in the data time series, we alter the values of each of the input variables (i.e. precedent variables denoted PVs) in turn by a small amount whilst maintaining all other values constant. The model is recalculated and the effect of this alteration on a subset of interest of the output variables is measured. For convenience we will refer to such a subspace of output variables as the **response space**.

Tangent space analysis identifies the effects of internal variables and inputs deterministically by investigating the strength and form of their individual marginal effects on output variables. We have developed in VBA a marginal effect analysis tool (MEAT) to perform the tangent space analysis. MEAT detects the effect of small (in percentage terms) perturbations of inputs and internal variables on the output variables by automatically working through the EDM altering a single variable at a time and letting the model update the relevant output cells, before resetting and moving onto the next input cell. These are identified automatically by detecting cell status in the used ranges if appropriate, or by user selected ranges if not. The results of this analysis are reported in a detailed log workbook together with a snap shot of the EDM workbook with non-redundant input variables' cells colour coded. This information is also graphically recorded (e.g. by fill effects) in the snap shot of the EDM workbook. The sizes of marginal effects, however, are not sufficient to determine which variables and data provide significant effects; the magnitude of the potential effect requires some knowledge of the likely variation in the variable/data. Although the actual variance in the data is unknown, having 'baseline' and 'reference' scenario input data is enough to obtain pseudo relative "coefficients of variation" which in conjunction with the linearized analysis proposed gives a rough cut ranking of the variables/input data's importance.

The MEAT tool is documented to a level where DECC staff can easily repeat analyses performed.

Specifically, in the case of the EDM, the tangent space analysis has been implemented as follows. (1) The modeller selects a subset of the MOVs and a subset of the PVs usually chosen from the terminal precedents as all other precedents ultimately rely upon the terminal precedents. (2) A year range over which the effects of altering precedent values on MOVs are recorded is selected, together with a year range over which the PVs are perturbed by a small, fixed, amount.

Two important issues arise in the case of multidimensional response spaces. First, the issue of commensurability: the importance of effects should not be dependent upon the units of measurement chosen for each of the dimensions. This has been addressed by measuring the induced deviation in terms of absolute proportional effects. This works in most cases, but can be misleading when reference values are "accidentally" close to zero.

Proportional effects cannot be derived for variables with zero value, and we have recorded their deviations separately. Obviously, input variables which are not precedents should not give rise to non-zero absolute deviations. This provides a useful check for the precedent analysis results.

The second issue is that of the intrinsic relative importance of the various dimensions and how to amalgamate them into a one dimensional ranking measure. We have approached this in a number of ways. The issue is partially addressed by focussing on a limited subset of important variables suggested by DECC, but even restricting the output variables to 8 or 9 dimensions leads to an almost intractable ranking problem. We have also used multiple metrics on the response space measurements: sum of absolute deviations (**aggregate deviation**), sum of absolute deviations averaged by the number of dependent variable (**mean deviation**), and the maximum absolute deviation (**maximum deviation**.) The last of these is the most appropriate for identifying variables which have universal negligible effect.

For example, if a pair of MOVs had original values (1, 1000), and the values (0.8, 1010) the aggregate deviation would be:

$$\frac{|0.8-1|}{1}+\frac{|1010-1000|}{1000} = 0.2+0.1=0.3$$

The "mean deviation" is defined as the ratio of the aggregate deviation over the number of MOVs dependent upon the perturbed PV. If the two MOVs mentioned above were the only MOVs dependent upon the perturbed PV, then the mean deviation would be 0.3/2 = 0.15. The maximum deviation for the above MOVs is Max(0.1,0.2)= 0.2.

A third issue arising from the fact that the EDM consists of a number of time series, is that varying an input variable may not affect an output variable in the same year, due to temporal lags in formulae. Because of this, the option to measure induced deviations for years other than the focal year for the analysis is also included. Indeed most of the results that are presented in the results section are based on analyses that calculate effects not only at the focal year in the time series, but on a year either side.

The variability of input variables must also be considered. The tangent space analysis, as outlined above, performs calculations on the assumption that each input variable naturally varies to the same degree as the others. There is no *a priori* reason to suspect that this is the case, and analysis of 6 scenarios provided by DECC suggests that the natural variability as a proportion of the reference value may range over many orders of magnitude, or even be zero in the case of well-known constants. If at all possible a tangent analysis must take the natural variability of the underlying input variables into account. This is done by multiplying the absolute deviation recorded for an input variables percentage variation by a measure of the input variables natural relative variability. We call this a **pseudo coefficient of variation** (abbreviated as **CV**) and have defined it to be the half width of the maximum and minimum range in the variable observed, divided by the average of the maximum and minimum.

In the case of the EDM, this information is not available for most of the terminal precedents. This is a serious problem for ranking the importance of variables in the EDM by both the tangential analysis method and the method of Morris, or any other method for that matter, as it appears that the underlying input variables have a wide range of natural variability. The effect of this is discussed and quantified in detail in the results section. We wish to make it very clear that whilst our methods are excellent at detecting variables with zero effect on the focal output variables, the lack of information about relative variability of the remaining input variables makes our conclusions about their relative importance tenuous at best.

The calculations described above are performed by the parameterized procedure `MarginalEffectAnalysis` in the `EDM_Precedent_Tools` module of the Formula Analysis Tool spreadsheet, which is called by a number of macros that supply appropriate sets of parameters.

2.3 Ad hoc analysis and logic maps

The results of the global and the local sensitivity analyses have been synthesized and where discrepancies were found, they have been investigated further using ad hoc methods. In particular we are using NodeXL, a free, open-source network graphing template for Excel (http://nodexl.codeplex.com/) in conjunction with an extension of the VBA formula precedence software discussed in (3.1) to generate formula network graphs to aid this stage of analysis. An example of these graphs appears in section 3.3.

We have also provided NodeXL workbooks which map the relationships between each variable within each sector, as well as a workbook containing a graph of all the variables linked to the 970 model output variables. These workbooks allow interaction with the graphs, such as zooming, following links, redrawing and other forms of analysis, either provided by NodeXL or bespoke. It should be noted that not all variables appear in this last graph (e.g. redundant variables aren't linked to anything.)

3 Results

As described in section 2, our analysis consists of three methodologies that inform the classification of variables based on their significance to model outputs. Whilst these three processes are independent and provide different insights to the analysis, they have also been useful to corroborate the robustness of our results.

3.1 Precedence Analysis

3.1.1 Precedence Analysis – Conservative results

The results from the precedence analysis depend upon the set of model outputs. DECC initially provided us with an Excel spreadsheet a list of 934 variables that were deemed to be outputs out of the 3923 variables in the model. Further questions led to a number of other variables, all prefixed with the sector identifier Output_Var as being output variables of interest. As a conservative analysis we assumed the original set of 934 variables together with all remaining variables prefixed by Output_Var as our MOV set; this amounted to 970 variables, which are listed in the **ModelOutputs** worksheet in the **EDM Conservative Precedents** workbook.

A precedent analysis centred on the 2020 formulae in the EDM indicates that there are 2519 variables that are either in this conservative MOV set or are precedents of this MOV set. These are listed in the **AllPrecedents** worksheet of the EDM Conservative Precedents workbook. This leaves 1404 redundant variables that are not related to the MOV set, listed in the **RedundantVariables**. An important subset of the precedents consists of those which have no further precedents. There are 842 of these **terminal precedents** and they are listed in the **TerminalPrecedents** worksheet, 744 of which are not MOVs (see below).

The AllPrecedents and TerminalPrecedents worksheets have columns labelled "Dependent Variable Count" and "MOV Dependent Count." These columns record the number of time series variables which are dependent on the given variable (i.e. variables for which the focus variable is a precedent) from the entire set of variables and from the set of model output variables respectively. It will be noted that a significant number of the precedents have zero dependents or have no MOV dependents. This may at first seem like a contradiction; how can a variable be a precedent but not have any dependents, or at least none in the MOV set? This is because all precedents that have a zero MOV dependent count are members of the MOV itself. Whether they are deemed redundant or not is thus a matter for DECC to decide, but there is no reason for them to be included in the model, as they are not involved in calculations that affect the MOV.

1. That the 1404 variables that have been identified as redundant are actually redundant has been confirmed in a number of ways.
2. The macro **CheckRedundancyGlobal** in the EDM_Precedent_Tools module, alters the value of all the variables in the time series variables identified as redundant, lets the model recalculate and then compares the “before” and “after” values of all the non-redundant variables. No difference has been detected in any of the runs of this macro.
3. The tangent space analysis predicts that the absolute deviation in any of the model output variables arising from a change in a redundant variable should be zero, which is always observed.
4. Both the direct precedent matrix and the VBA precedents property identify the same sets of redundant variables.
5. Manual checking of a random selection of the redundant variables confirms that they are redundant.
6. The same set of precedents and redundant variables arise from performing the same analysis on another year (2025).

3.1.2 Precedence Analysis – Main MOVs
DECC has identified 3 output variables that it deems to be particularly important:

- Output_Vars_DDM_ELEC_TWH (Electricity demand)
- Output_Vars_Total_CO2 (Total CO2 emissions)
- Output_Vars_Total_Energy_Final_Cons (Total energy consumption)

These 3 variables have a total 1878 precedents, 665 of which are terminal. None of these precedents have zero dependents, i.e. none are MOVs. Indeed, approximately half of the difference between conservative and main MOV only analyses precedent counts is due to MOVs with no precedents being included in the former analysis.

The outcomes of the main MOVs only analysis are reported in the **EDM Radical Precedents** workbook.

3.2 Sensitivity Analysis
Both of the sensitivity analysis methods were run using ranges extracted from the six scenarios provided to us by DECC. This highlighted 89 variables that vary across these scenarios, and gives an estimate of the range of variation. We also ran tangent space analysis under the assumption of variables having identical levels of variability.

A second sensitivity analysis run was performed using a subset of the ~842 terminal precedents of the main output variables. Coefficient, constant, GCV, share, CCP and CHP variables were removed, leaving 200 variables which were subsequently varied by ±5% around the value in the reference scenario.

3.2.1 Method of Morris Analysis

We developed a Python-based prototype of the sensitivity analysis based on the Method of Morris. As inputs to the analysis, the values from 6 separate scenario sheets provided by DECC have been used. This analysis showed that only eighty-nine of the 851 entries in the 'Equations' sheet (of the 3923 equations in total) actually change between the scenarios. Therefore, we have performed the sensitivity analysis for these eighty-nine inputs for the year 2020, and only over the ranges covered in these five scenarios. This study could be expanded if we receive further ideas on ranges for the remaining 761 variables (although around 500 of these 'inputs' are constant values and are unlikely to vary).

To illustrate this analysis, we show the sensitivity of gas consumption in the Chemical sector to various inputs. As shown in Figure 2, plotting **μ*** (mean variation) against σ^2 (standard deviation) for each of the output variables gives a quick overview of the results obtained using this method. If you split the plot into quadrants, top right are inputs that are highly influential (**μ*** on the x-axis) and also interact with other variables (*σ* on the y-axis); bottom-left, unimportant variables; top-left, interactions, but small effect and bottom-right, large effect, small interactions.

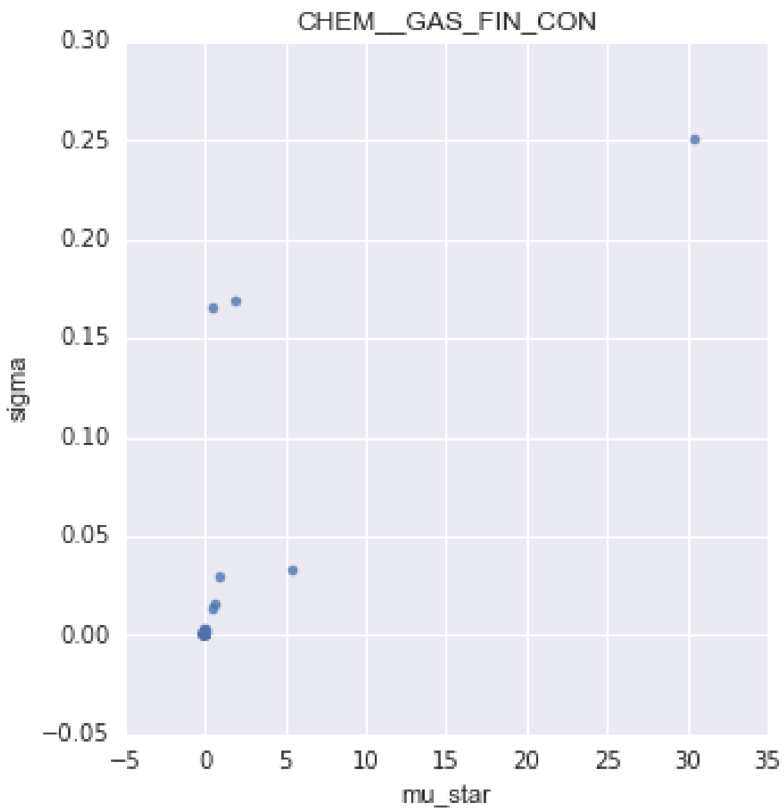


Figure 3. Sensitivity of Gas Consumption in the Chemical Sector to various inputs

Final consumption of gas in the chemicals sector in 2020 is 393 MuThs. The **μ*** metric expresses the amount variance in gas consumption could be reduced by fixing the different inputs to a value within its range. The input with the highest input is CON_OMAN__INDUS_GAS_CCP with a value of 30 MuThs.

The equation for CHEM__GAS_FIN_CON is as follows:

```
CHEM__GAS_FIN_CON = CHEM__Gas_Tot_Dem - CHEM__GAS_CCP_TOTAL
```

So this approach shows the effect of inputs which may not be immediate precedents of the output in question. In this example, CON_OMAN__INDUS_GAS_CCP is a component of CHEM__GAS_CCP_TOTAL.

As part of this analysis, we have investigated the ranking of each input variable for each output on a sectoral basis. For this purpose we have produced clusterplots (sector-by-sector) of our preliminary results excluding CCP variables and a high-level version with just the aggregate output variables provided by DECC.

See Figure 3 as an example for the Chemical sector. **μ*** is the mean variation in the output, that would be reduced through fixing the input. However, in these plots, **μ*** has been normalised over the inputs, so that the chart should be read as follows:

"For output *a* (read from the right-hand column), inputs *x, y* and *z* are important and ranked in a particular order, and the remainder are unimportant."

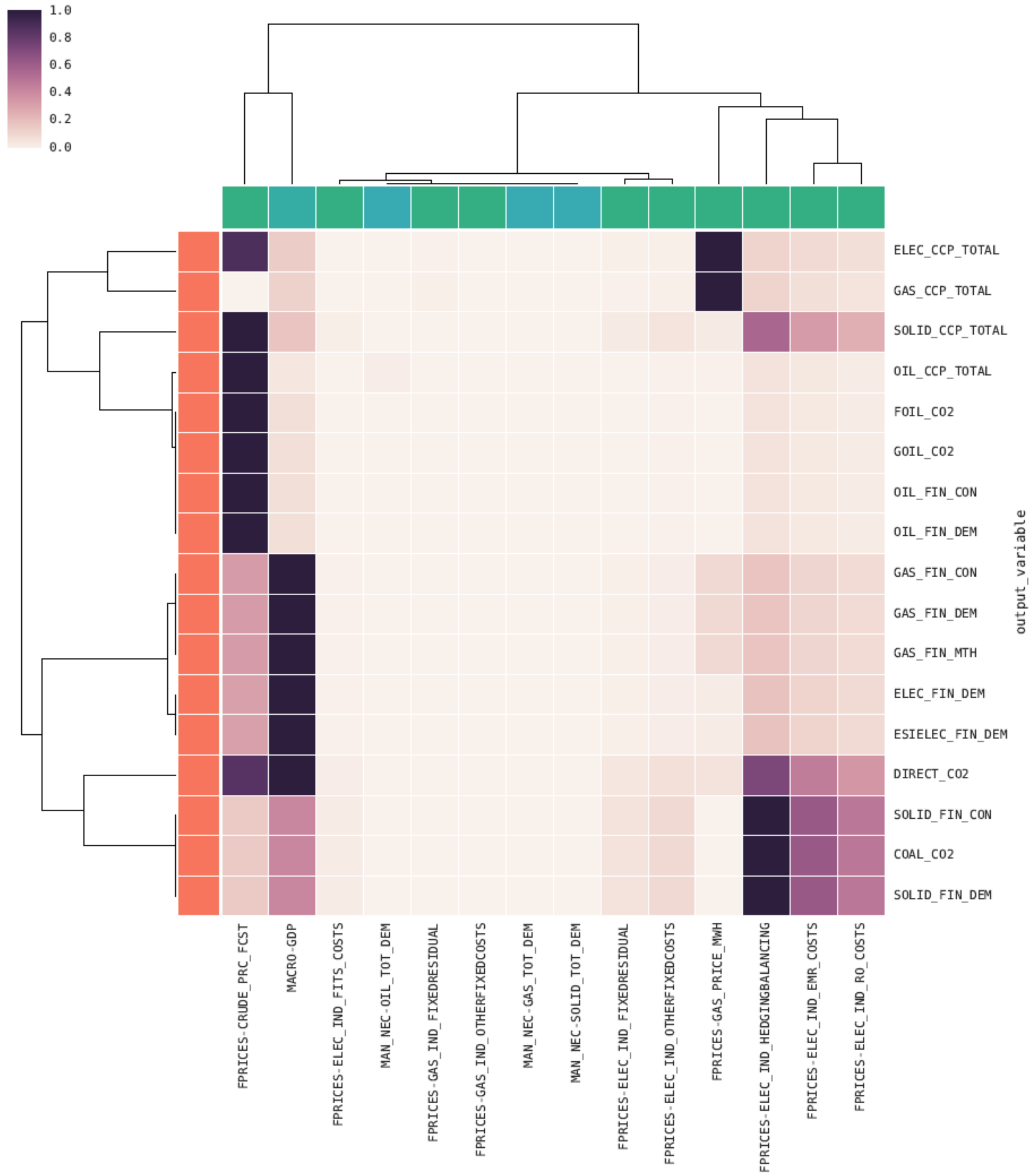


Figure 4. Chemical sector sensitivity analysis – measure of the variation in outputs for select inputs

However, one can only say that a given input is important for a certain number of outputs, but no ranking is implied (due to the difficult in comparing across different units - say CO_2 emissions versus TWh of electricity). This 'importance' is specific to the each output only, and the scales across these outputs may significantly vary. In short, unlike the tangential space analysis, no dimensionless proxies for sensitivity are offered by the Method of Morris, and sensitivity is measured directly in the units and scale of each output.

We attempted to give a more useful ranking of the inputs by investigating only some high-level output variables. As an example, Figure 4 shows the relationship between **μ*** and **σ** for TOTAL_ENERGY_FINAL_CONS. **σ** shows interaction, so an input which appears in the top left or right of the plot will interact with other variables to cause an effect. These types of plots give a quick indication, through glancing in the top right-hand corner, for the most important variables. We've facetted these plots across the CCP dimension - so that CCP variables are excluded from the left-hand plots, and included in the right-hand plots.

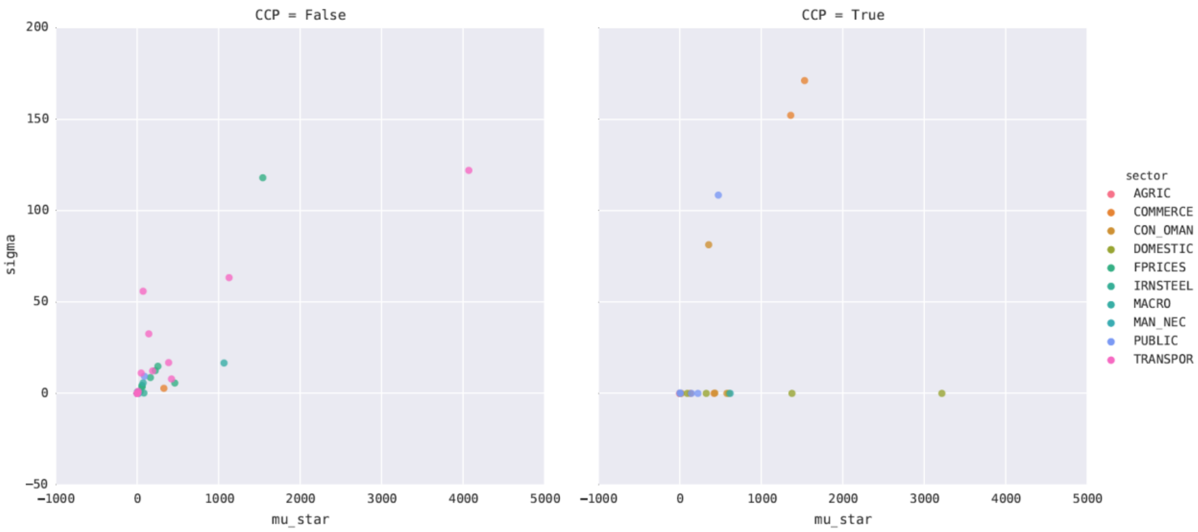


Figure 5. Sensitivity of Total Final Energy Consumption to different input variables

These CCP variables are often very influential in determining the model results - and they often vary across the six scenarios given to us. It is important to understand that often, these variables cause more variation than any other variables in the model. In short, the graphs which exclude the CCP variables may give a false impression of importance attributable to some input, when in fact it is dominated by a CCP variable. This effect is particularly noticeable in the FC_ELECTRICITY plot.

Out of the 89 inputs investigated so far, some have zero effect upon TOTAL_ENERGY_FINAL_CONS:

Variables
COMMERCE__EMPLOYMENT
CON_OMAN__FIN_SHARE_FOIL
CON_OMAN__OUT_INDEX
FPRICES__CL_PRC_TON_FCST
FPRICES__ELEC_DOM_FIXEDRESIDUAL
FPRICES__ELEC_DOM_HEDGINGBALANCING
FPRICES__ELEC_DOM_OTHERFIXEDCOSTS
MACRO__HH_INCOME_GDP_RATIO
MAN_NEC__FIN_SHARE_PETCOKE

PUBLIC__GVA_CONTRIB
TRANSPOR__BIODERV_VOLSH
TRANSPOR__BIOETH_VOLSH
TRANSPOR__CAR_MSPERC_SHAR
TRANSPOR__COM_MSPERC_SHAR

The top 10 non-CCP variables are:

Variables	μ*
TRANSPOR__CAR_EFF	4,045
FPRICES__GAS_PRICE_MWH (lagged)	2,749
FPRICES__CRUDE_PRC_FCST	1,507
TRANSPOR__CAR_FF_EFF	1,080
MACRO__GDP	1,059
FPRICES__GAS_DOM_OTHERFIXEDCOSTS	457
MACRO__GDP (lagged)	433
TRANSPOR__HGV_EFF	424
TRANSPOR__LGV_EFF	396
COMMERCE__GVA_CONTRIB	320

We have noticed that the range of values for Car Efficiency over the 6 scenarios shared with us, explain the vast majority of the variation in total energy consumption, more than twice times the variation induced by the range of oil prices explored.

In summary, these are the results of the global sensitivity analysis performed for the 89 inputs:

- We found 89 input variables whose range changes over six scenarios provided by DECC
- These 89 input variables are terminal precedents for 400 output variables. However, these inputs do not explain the change in the remaining 570 of the preferred output variables.
- These remaining outputs require information for 200 inputs, for which we currently do not have data.
- When lags are included for the 89 inputs (doubling the inputs to 178), almost half of the lagged variables do influence the preferred outputs.

We then performed a larger sensitivity analysis over the 140 variables which are terminal precedents for the high level output variables (corresponding to the radical precedents). We obtain an approximate ranking of input variables, predicated on an arbitrary 5% variation around the value in the reference scenario for each of these inputs. A brief analysis of the 89 input variables we did have data for suggested that this is likely to raise both Type I and Type II errors, wrongly identifying non-influential variables and influential and *vice versa*. Given the lack of data, this seemed to be the only course of action.

Table 1 shows the top 14 input variables that influence the total final consumption of energy in the EDM model, given the assumption that all 140 inputs that influence this output variable only vary over a 5% symmetric range around the reference scenario value. Note that the year suffix on each variable name allows lag variables to be identified. Thus the value of GDP in 2019 explains half the variation in the output variable than GDP in 2020 does. The final column gives the sensitivity metric as a normalized percentage of the central reference scenario output. Thus the 5% range of HOUSEHOLDS_2020 explains 3% of the total 23% variation caused by the inputs.

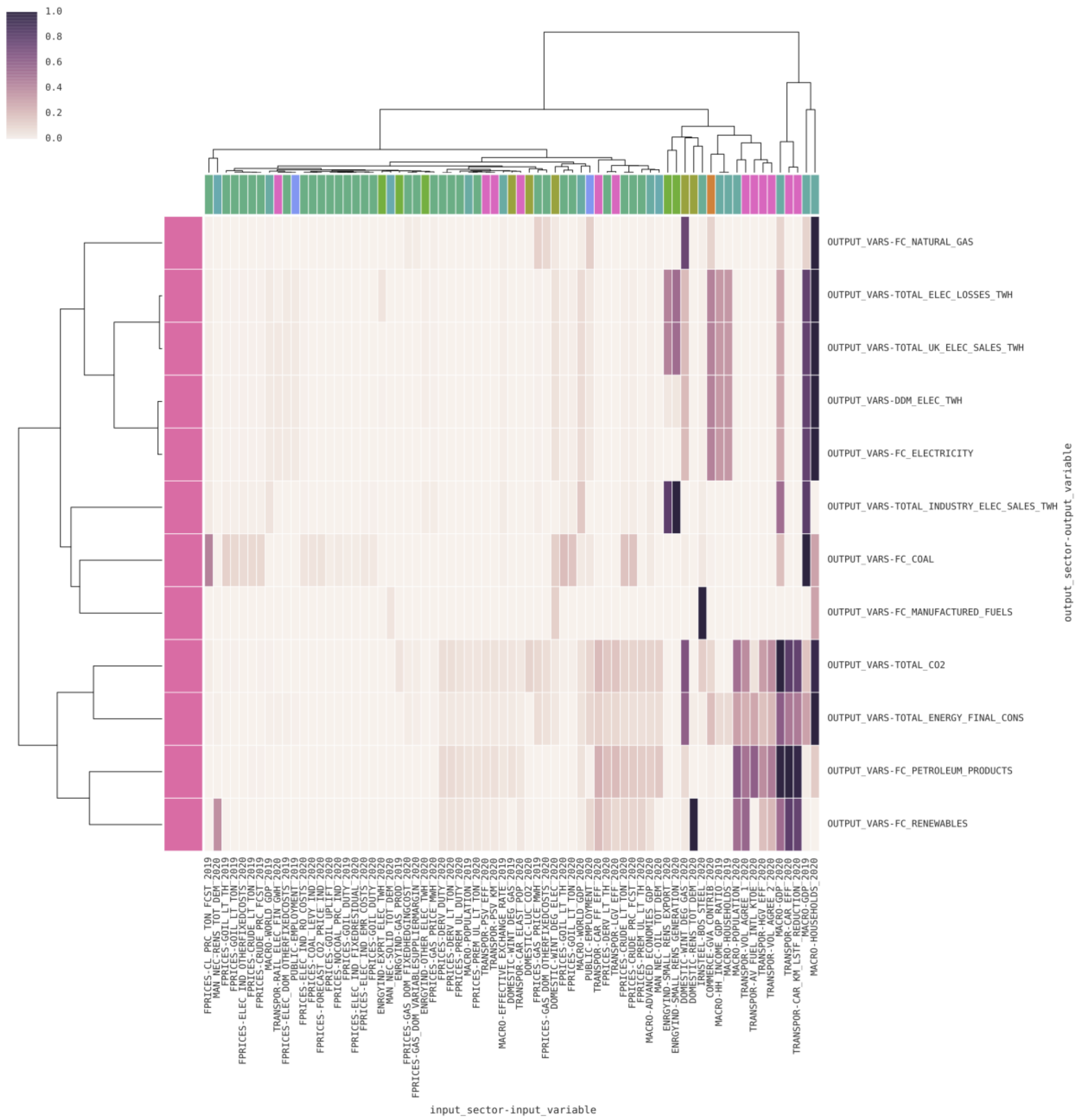
Table 1 Results for TOTAL_ENERGY_FINAL_CONSUM for year 2020

Sector	Variable	Mu_star	Sigma	
MACRO	HOUSEHOLDS_2020	4,654.81	87.97	3%
MACRO	GDP_2020	2,893.45	704.48	2%
DOMESTIC	WINT_DEG_GAS_2020	2,791.56	54.05	2%
TRANSPOR	CAR_EFF_2020	2,251.40	141.22	2%
TRANSPOR	CAR_KM_LSTF_REDUCTION_2020	2,166.83	134.36	2%
MACRO	POPULATION_2020	1,453.81	171.72	1%
MACRO	GDP_2019	1,422.14	52.41	1%
TRANSPOR	AV_FUEL_INTL_KTOE_2020	1,309.65	0.00	1%
TRANSPOR	VOL_AGREE_1_2020	1,149.45	86.97	1%
TRANSPOR	VOL_AGREE_2_2020	1,030.47	264.82	1%
COMMERCE	GVA_CONTRIB_2020	906.07	74.31	1%
TRANSPOR	HGV_EFF_2020	866.45	43.23	1%
TRANSPOR	CAR_FF_EFF_2020	570.36	59.64	0%
PUBLIC	EMPLOYMENT_2020	473.90	115.62	0%

Taking just those inputs that are more sensitive than average, we plotted a cluster map of the output variables. This clearly shows that the ranking for TOTAL_ENERGY_FINAL_CONSUM is consistent across most of the other output variables, with some exceptions.

This plot also illustrates how some inputs are extremely important in determining the value of certain outputs e.g. `FPRICES__CL_PRC_TON_FCST_2019` is important for `FC_COAL` but within the higher level aggregation of `TOTAL_ENERGY_FINAL_CONSUM`, this input is proportionally less important, because `FC_COAL` makes up just a fraction (1.3%) of the higher level output variable. Thus issue is missed when normalizing the results and comparing outputs of different scales.

Table 2 Inputs of above average influence for output variables



The full ranking of the 140 variables is given for each of the output variables in the supplementary spreadsheet (see Appendix F).

3.2.2 Tangent Space Analysis

Tangent space analyses have been performed on a number of sets of model outputs and inputs, ranging from all terminal precedents and model outputs, to single model outputs and the restricted set of terminal precedents for which some estimate of variation was obtainable from provided scenarios.

As an illustrative example, consider the following model:

- MOV: *Output_Vars__Total_Energy_Final_Cons*
- PVs: *89 terminal precedents for which we have range data, as described in 4.2.1*
- Perturbations: *we consider perturbations on the 2020 value for each of the PVs by increasing the original by 10%. (There were no original values equal zero.)*
- Aggregate deviation: *We record the deviations in the MOV for the years 2019, 2020, and 2021 to catch lagging effects.*
- CV: *We estimate a pseudo coefficient of variation for each PV by taking the absolute value of the difference between the maximum and minimum values recorded for 2020 across scenarios divided by the absolute value of the average of the maximum and minimum values. (Note that the definition of the CV in this analysis differs by a factor of 2 from the one defined in the methods section, as these calculations were performed earlier in project.)*

The results of this analysis indicated that, of the 89 PVs, 77 of them affected the MOV. The magnitudes of these effects on aggregate deviations ranged over 5 orders of magnitude and nearly 7 orders of magnitude once scaled by the CVs. With the exception of Output_Vars_Total_CO2, we found a very close relationship between the results of the two approaches to measuring variable effect.

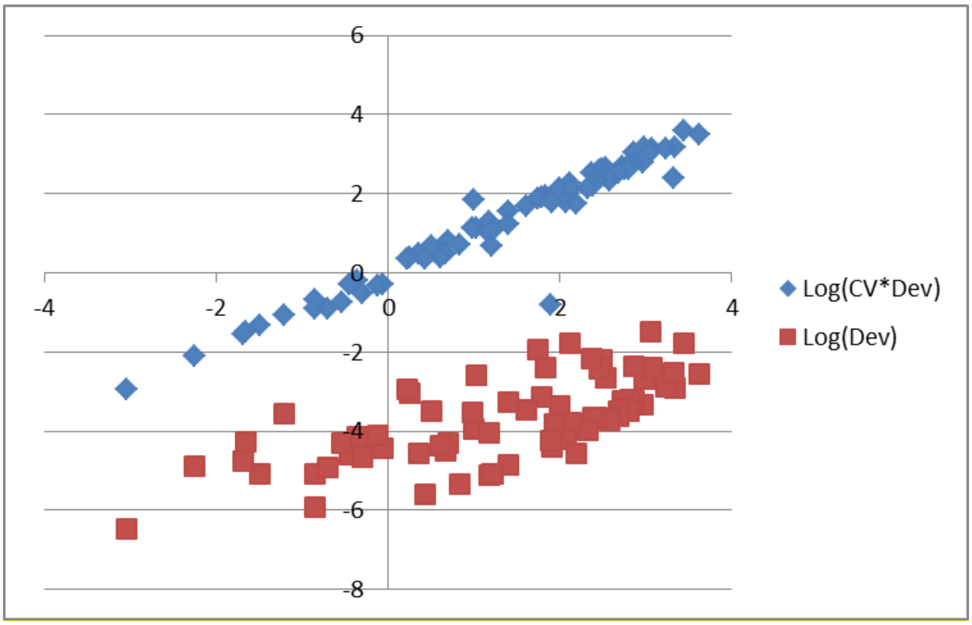


Figure 6. Graph of Log10(CV*Dev) and Log10(Dev) versus the results of the Method of Morris Analysis after scaling by a constant for absolute deviation of Total_Energy_Final_Cons for 89 terminal precedents. Note the strong correlation between the CV*Dev results and those obtained by the method of Morris. As the method of Morris is a global approach and the tangent analysis is a local approach, this strong correlation cross validates the two methods and indicates that their results are robust. Note also the continuous nature of the aggregate deviation across the set of PVs making a demarcation between significant and insignificant PVs somewhat arbitrary. The looser correlation between Log10(Dev) and the method of Morris arise from different assumptions about the relative variability of the PVs in the two models. This highlights the importance of understanding the likely range of variation in the data before concluding a variable is significant or insignificant. Moreover, a variable may be significant for one MOV and insignificant for another.

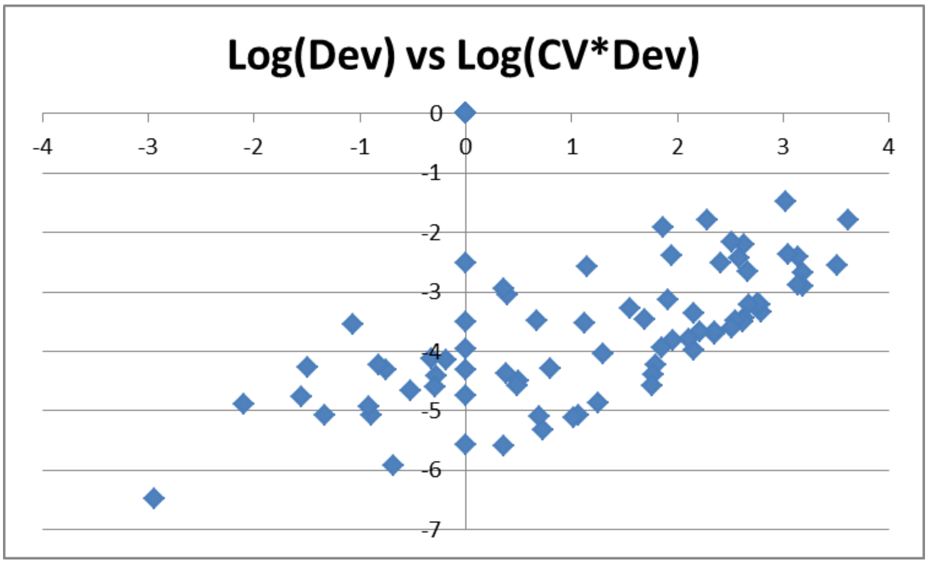


Figure 7. Plot of Log(Dev) versus Log(CV*Dev) for the same set of data as the previous figure. Correlation coefficient for the data is approximately 0.73. The inclusion of CV information has in many cases changed the relative magnitude of the deviations by as much as 5 orders of magnitude. Ranking without a knowledge of data variability can lead to serious mistakes regarding relative importance of variables.

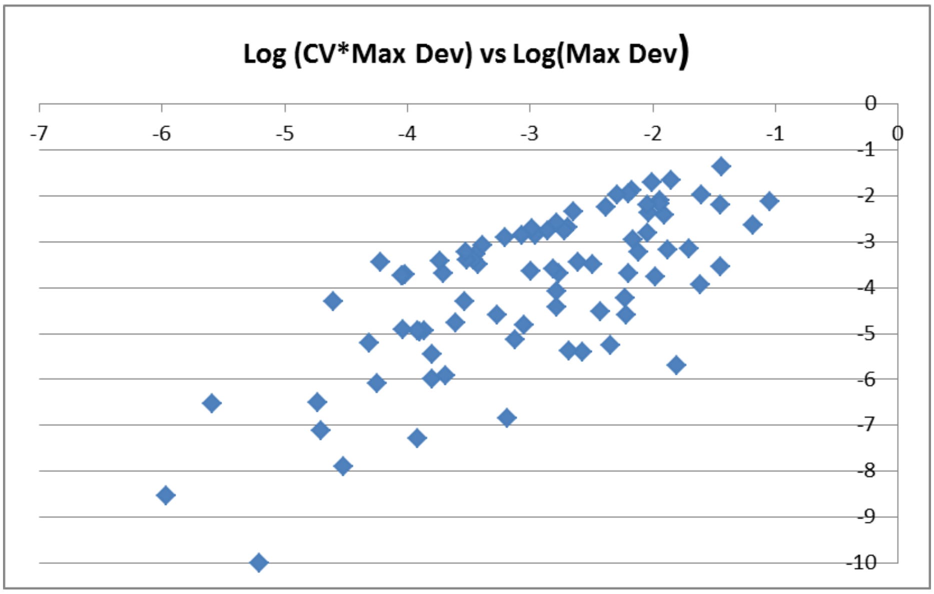


Figure 8. Plot of Log(CV* Max Dev) versus Log(Max Dev) for the same set of data as the previous figure. Similar observations are to be made about the importance of understanding the differences in expected amounts of variation between terminal precedents.

To see whether mean deviation and aggregate deviation are closely related, i.e. a variable which has a strong effect on one of its dependent variables has a similarly strong effect on all of its dependent variables, we graphed the ranks (1 = smallest effect, 89 = largest effect) of the PVs under these two measures against each other on the for the set of MOVs consisting of all model outputs. It does seem that although here is some consistency (high total deviation is correlated with high mean deviation) the scatter is very broad, again supporting the idea that without a good grip of variable variation and which outputs are the most important, it is difficult to classify variables as universally unimportant or important, and that a given PV will affect some variables strongly and others weakly.

Similarly, we found only moderate correlation between the number of dependents a variable had and its mean and aggregate deviations.

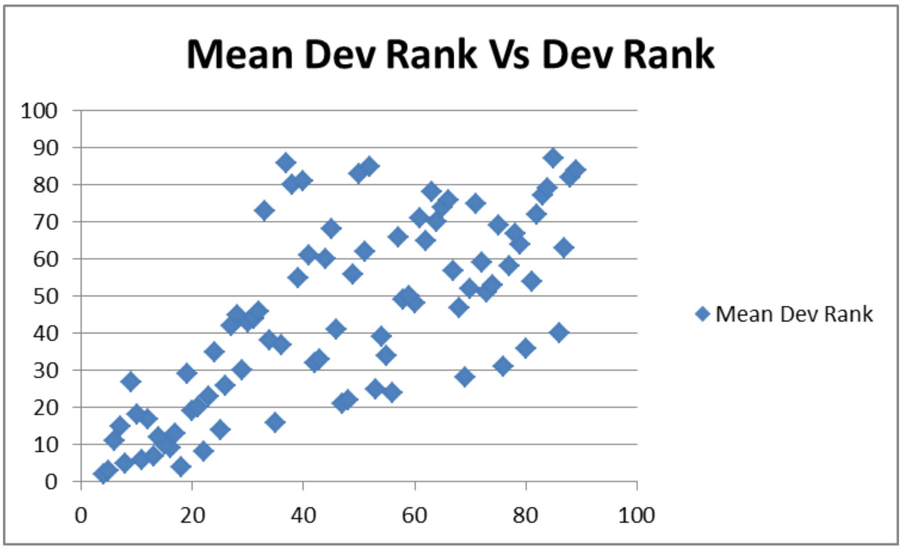


Figure 9. Graph of Aggregate Deviation rankings of the 89 PVs with non-trivial CV versus Mean Deviation rankings for the same variables. Note that, whilst there is a positive correlation between these variables, the correlation is not particularly strong.

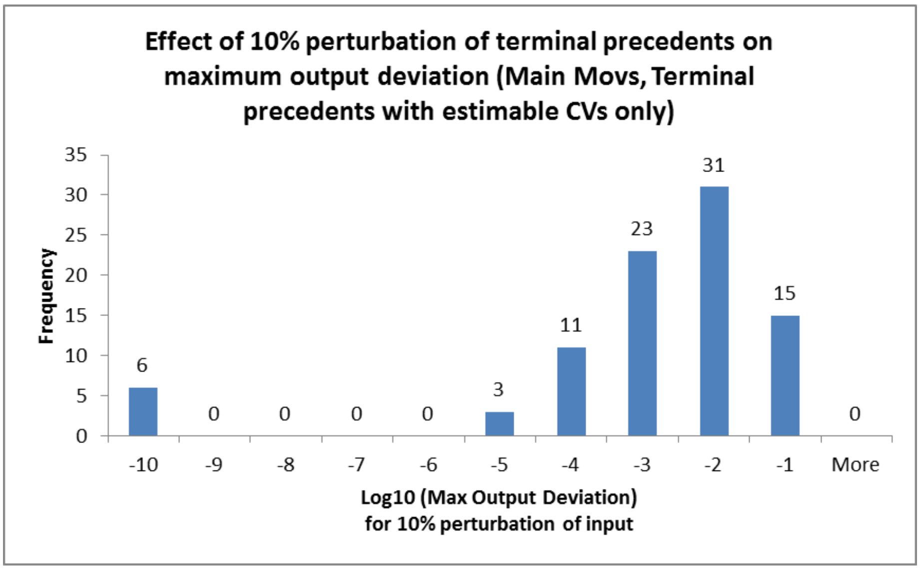


Figure 10. The distribution of the base 10 logarithm of the size of the largest absolute deviation in the 9 main MOVs induced by a 10% increase in the value of the MOVs' 89 terminal precedents for which CVs were estimable from DECC scenarios. The -10 bin includes 6 variables which had no effect. The bin labelled -1 includes those variables that had an effect of between 1% and 10%. The bin labelled -2, those that had an effect between 1% and 0.1% etc.

Table 1. Comparison of the top 15 variables when ranked by maximum deviation and when ranked by maximum deviation multiplied by the pseudo coefficient of variation (CV). Only 7 out of 15 terminal precedents are common between the two lists.

Top 15 MD	MD	CV*MD	Top 15 CV*MD	MD	CV*MD
IRNSTEEL__BOS_STEEL	0.0900	0.0076	TRANSPOR__BIODERV_VOLSH	0.0365	0.0419
MACRO__GDP	0.0661	0.0023	DOMESTIC__ELEC_CCP_1	0.0142	0.0218
TRANSPOR__BIODERV_VOLSH	0.0365	0.0419	MAN_NEC__RENS_CCP	0.0098	0.0197
TRANSPOR__CAR_KM_LSTF_REDUCTION	0.0353	0.0003	COMMERCE__RENS_CCP	0.0067	0.0134
TRANSPOR__CAR_EFF	0.0353	0.0062	COMMERCE__ELEC_CCP_1	0.0064	0.0115
FPRICES__CL_PRC_TON_FCST	0.0251	0.0105	FPRICES__CL_PRC_TON_FCST	0.0251	0.0105
MACRO__POPULATION	0.0244	0.0001	DOMESTIC__RENS_CCP_1	0.0052	0.0103
COMMERCE__GVA_CONTRIB	0.0198	0.0007	TRANSPOR__BIOETH_VOLSH	0.0114	0.0079
MACRO__HH_INCOME_GDP_RATIO	0.0156	0.0000	IRNSTEEL__BOS_STEEL	0.0900	0.0076
DOMESTIC__ELEC_CCP_1	0.0142	0.0218	DOMESTIC__SOLID_CCP_1	0.0114	0.0065
TRANSPOR__HGV_EFF	0.0132	0.0006	FPRICES__Gas_Price_MWh	0.0091	0.0064
DOMESTIC__GAS_CCP_1	0.0125	0.0038	TRANSPOR__CAR_EFF	0.0353	0.0062
TRANSPOR__BIOETH_VOLSH	0.0114	0.0079	COMMERCE__GAS_CCP_1	0.0042	0.0056
DOMESTIC__SOLID_CCP_1	0.0114	0.0065	PUBLIC__RENS_CCP	0.0023	0.0045
PUBLIC__EMPLOYMENT	0.0105	0.0002	FPRICES__CRUDE_PRC_FCST	0.0092	0.0043

As in the Method of Morris analysis, we also found that a few of the variables that had been varied in DECC scenarios did not affect any of the MOVs, at least for the years 2019-2021. Although suggested by the form of the model, the assumption that the magnitude of effect of perturbations was more or less independent of year after 2014, we tested this by running perturbations centred on different years. A typical result of the comparison of the effect of PVs on 2020 and 2030 data and their associated outputs is given in the graph below:

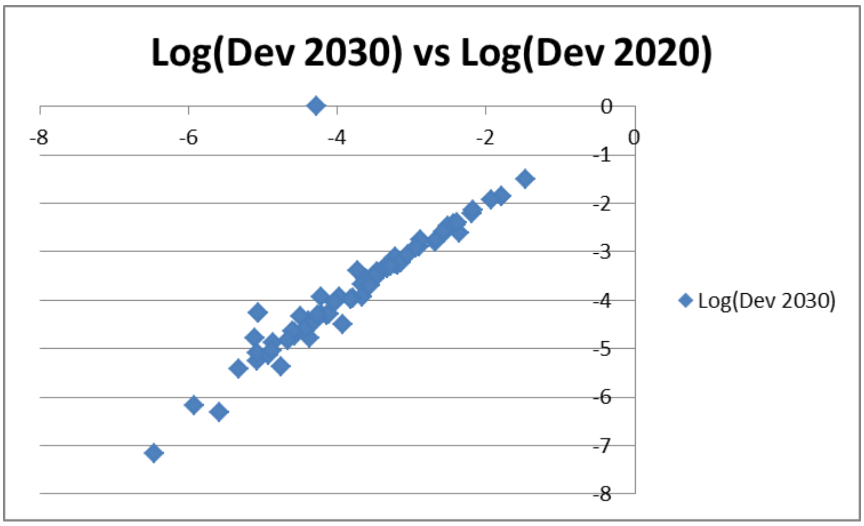


Figure 11. A typical result of the comparison between the effect of the perturbation of PVs on 2020 and 2030 data. The tight correlation suggests that performing a tangential analysis by perturbing any one year post 2014 is as good as perturbing any other.

In conclusion, although the range of effects of perturbing a given terminal precedent by even a small amount spans half a dozen orders of magnitude, the large number of Model Output variables, the effect of different amounts of variation in the variables as evidenced by the effect of including CV on the magnitude of effects, the continuous distribution of effects across these orders of magnitude, and the fact that many PVs are connected to tens or even hundreds of Model Output Variables whose relative importance is unknown to us, means that any binary classification of the non-redundant variables into insignificant and significant is likely to be sensitive itself to the assumptions made.

3.2.3 Tangent Space Analysis – Comparison of Radical and Conservative results

In the radical case analysis we set our output variables to the highly restricted set:

- Output_Vars_DDM_ELEC_TWH (Electricity demand)
- Output_Vars_Total_CO2 (Total CO2 emissions)
- Output_Vars_Total_Energy_Final_Cons (Total energy consumption)

and the variables which partition total energy consumption:

- Output_Vars__FC_Coal
- Output_Vars__FC_Electricity
- Output_Vars__FC_Manufactured_Fuels
- Output_Vars__FC_Natural_Gas
- Output_Vars__FC_Petroleum_Products
- Output_Vars__FC_Renewables

As detailed in the radical precedent analysis (i.e. Main MOVs only), this set of variables has 665 terminal precedents. Perturbing the 2020 values of each of these terminal variables and noting the maximum absolute deviation of the 9 MOV variables for the years 2019, 2020 and 2021 we found the following distribution of the magnitude of effects:

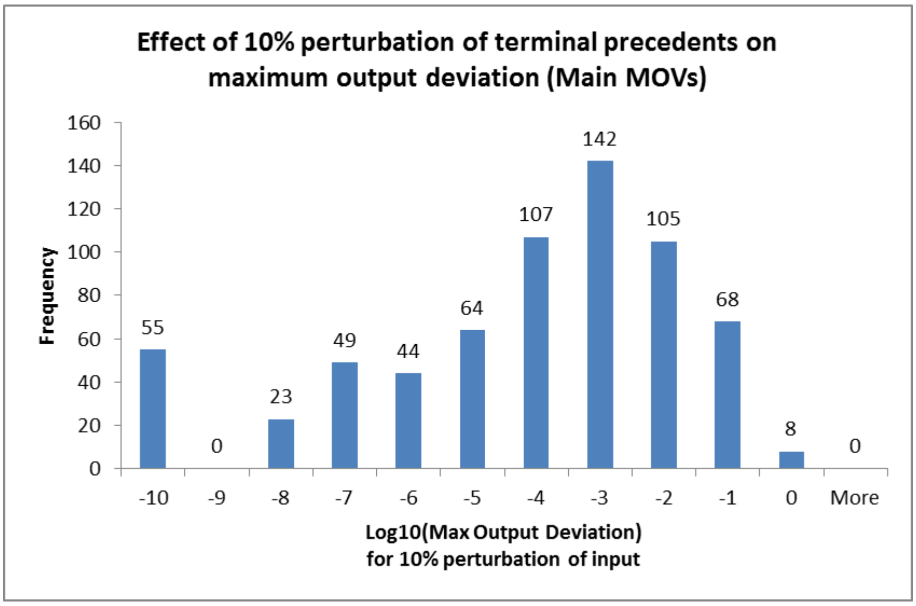


Figure 12. The distribution of the base 10 logarithm of the size of the largest absolute deviation in MOVs induced by a 10% increase in the value of the MOVs' 665 terminal precedents for the 9 main MOVs. The -10 bin includes 54 variables which had no effect. The 0 bin includes all the variable that had a maximum deviation greater than 10% (maximum induced deviation = 31.5%). The bin labelled -1 includes those variables that had an effect of between 1% and 10%. The bin labelled -2, those that had an effect between 1% and 0.1% etc.

Similarly, perturbing the 2020 values of each of the terminal precedent variables for the conservative case we found the distribution of the magnitude of effects shown in Figure 13.

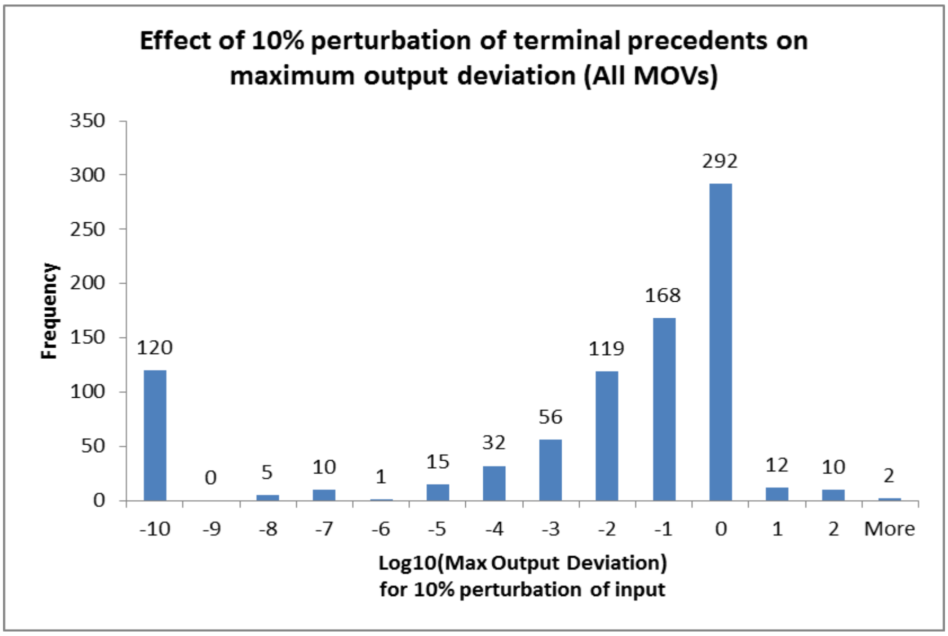


Figure 13. The distribution of the base 10 logarithm of the size of the largest absolute deviation in MOVs induced by a 10% increase in the value of the MOVs' 842 terminal precedents for all 970 MOVs. Note that in this analysis many variables have an effect of the same order of magnitude or greater than the initial perturbation.

Table 2 includes the 30 most influential variables in the precedent analysis for the 9 most important MOVs, while Table 3 shows the 75 most influential variables in the precedent analysis for all 970 MOVs.

Terminal Variable	Maximum Deviation
FOODDRTO__FIN_SHARE_COG	0.315165904
CHEM__FIN_SHARE_COG	0.305813856
NMETMINP__FIN_SHARE_COG	0.283474113
ENG_VEH__FIN_SHARE_COG	0.180950075
CON_OMAN__FIN_SHARE_COG	0.162442418
PUPAPRPU__FIN_SHARE_COG	0.153781538
NMETMINP__FIN_SHARE_OSF	0.145470266
NMETMINP__FIN_SHARE_COKE	0.145470266
IRNSTEEL__BOS_STEEL	0.089983866
Constant__TWH_MTH	0.087593913
Constant__ELEC_ALL_UEF	0.083232349
Constant__MTH_KTOE	0.082000345
TEXTPROD__FIN_SHARE_COG	0.078985651
DOMESTIC__GAS_HH_Share	0.074121622
MACRO__HOUSEHOLDS	0.074121622
CHEM__FIN_SHARE_COKE	0.072917117
CHEM__FIN_SHARE_OSF	0.072917117
MACRO__GDP	0.066086659
DOMESTIC__WINT_DEG_GAS	0.056132597
Constant__KTOE_MTH	0.055996256
Constant__COKE_IS_TH_TON	0.048578516
NMETMINP__FIN_SHARE_COAL	0.041801661
Constant__COAL_MAN_UEF	0.041077332
Constant__GAS_DOM_UEF	0.040540805
TRANSPOR__BIODERV_VOLSH	0.036492157
TRANSPOR__CAR_EFF	0.03534096
TRANSPOR__CAR_KM_LSTF_REDUCTION	0.03534096
Constant__GWH_KTOE	0.034223679
FPRICES__COAL_VAT_IND	0.029889589
CON_OMAN__FIN_SHARE_COKE	0.027966298

Table 2. The 30 most influential variables in the precedent analysis for the 9 most important MOVs.

Terminal Variable	Maximum Deviation
Constant__GAS_AG_UEF	3.00347E+13
Constant__KTOE_MTH	2.73043E+13
FOODDRTO__FIN_SHARE_COG	93.14314707
CHEM__FIN_SHARE_COG	90.37927184
NMETMINP__FIN_SHARE_COG	83.77705413
Services__DEMSHARE_FOIL	55.90603972
ENG_VEH__FIN_SHARE_COG	53.47742016
CON_OMAN__FIN_SHARE_COG	48.00772474
PUPAPRPU__FIN_SHARE_COG	45.44811542
TEXTPROD__FIN_SHARE_COG	23.34317268
NMETMINP__FIN_SHARE_COKE	23.32156377
CHEM__FIN_SHARE_COKE	11.68995718
NFERRMET__FIN_SHARE_COG	7.809791904
Services__CHPSHARE_COAL	4.929244499
Services__DEMSHARE_GOIL	4.494890909
CON_OMAN__FIN_SHARE_COKE	4.483512734
NMETMINP__FIN_SHARE_OSF	4.065992132
PUPAPRPU__FIN_SHARE_COKE	2.112600054
CHEM__FIN_SHARE_OSF	2.038082626
Constant__GOIL_TH_TON	1.883136331
Constant__MTH_KTOE	1.688440351
TEXTPROD__FIN_SHARE_COKE	1.634314974
ENG_VEH__FIN_SHARE_COKE	1.212299926
FOODDRTO__FIN_SHARE_COKE	1.184008668
AGRIC__GAS_CCP_TOTAL	1
ENG_VEH__CHPSHARE_GOIL	0.931276896
CON_OMAN__FIN_SHARE_OSF	0.781676893
TOTMANUF__GOIL_CO2_COEF	0.656747697
Constant__COKE_IS_TH_TON	0.537981537
IRNSTEEL__BOS_STEEL	0.515538545
PUPAPRPU__FIN_SHARE_OSF	0.368320722
CON_OMAN__CHPSHARE_GOIL	0.360982746
MACRO__GDP	0.3414252
Constant__Trend_1984	0.340952146
TRANSPOR__COM_ELPERC_SHAR	0.336319938
AGRIC__GOIL_CO2_COEF	0.328110459
Constant__OIL_MAN_UEF	0.286484875
TEXTPROD__FIN_SHARE_OSF	0.28493423
Constant__DUM_DUKES13_2008	0.273561348
MAN_NEC__OIL_TOT_DEM	0.264133364
MAN_NEC__FIN_SHARE_OPG	0.258833175
DOMESTIC__WINT_DEG_GAS	0.246923946
Constant__TREND_1970	0.233432407

MAN_NEC__FIN_SHARE_GOIL	0.229206879
PUPAPRPU__CHPSHARE_GOIL	0.220206718
ENG_VEH__FIN_SHARE_OSF	0.211358124
PUBLIC__EMPLOYMENT	0.211322264
TRANSPOR__AV_FUEL_INTL_KTOE	0.208503153
NMETMINP__CHPSHARE_GOIL	0.207326759
FOODDRTO__FIN_SHARE_OSF	0.206425692
DOMESTIC__SOLID_HH_Share	0.173065427
MACRO__HOUSEHOLDS	0.173065427
Constant__TRENDpre04	0.172562891
Constant__COAL_MAN_UEF	0.145484719
AGRIC__FIN_SHARE_BOIL	0.145041428
FPRICES__GOIL_LT_TH	0.136229614
FPRICES__GOIL_VAT	0.136229614
DOMESTIC__OIL_HH_Share	0.133446773
IRNSTEEL__COAL_CO2_COEF	0.129634103
FPRICES__COAL_VAT_IND	0.128024189
TOTMANUF__COAL_CO2_COEF	0.12544897
COMMERCE__GVA_CONTRIB	0.122791183
Constant__TREND_1969	0.122451412
Constant__GWH_MTH	0.122260064
Constant__TJ_MTH	0.121526863
DOMESTIC__GAS_HH_Share	0.120335354
FPRICES__Elec_Ind_VAT	0.120286567
Constant__DUMpre96	0.117799206
Constant__DUMpre93	0.116057062
FPRICES__CRUDE_LT_TON	0.112065959
FPRICES__CRUDE_PRC_FCST	0.112065959
FPRICES__CL_PRC_TON_FCST	0.111950976
PUBLIC__OIL_CCP_1	0.111322264
Constant__COAL_UPLIFT_IND	0.110584226
Constant__TWH_MTH	0.10920165

Table 3. The 75 most influential variables in the precedent analysis for all 970 MOVs.

3.3 Ad hoc analysis and logic maps

In order to further investigate any discrepancies between our methodologies and to confirm our results – in particular the precedence analysis-, we have analysed a selection of equations manually. Additionally, and to better understand the level of interconnection between variables, we have generated formula network graphs to aid this stage of the analysis, using NodeXL in conjunction with an extension of the VBA formula precedence software discussed in section 3.1.

Figure 14 provides a global picture of the relationship between all sectorial variables linked to the 970 MOVs, illustrating how the structure of the model is such that some variables are strongly interconnected.

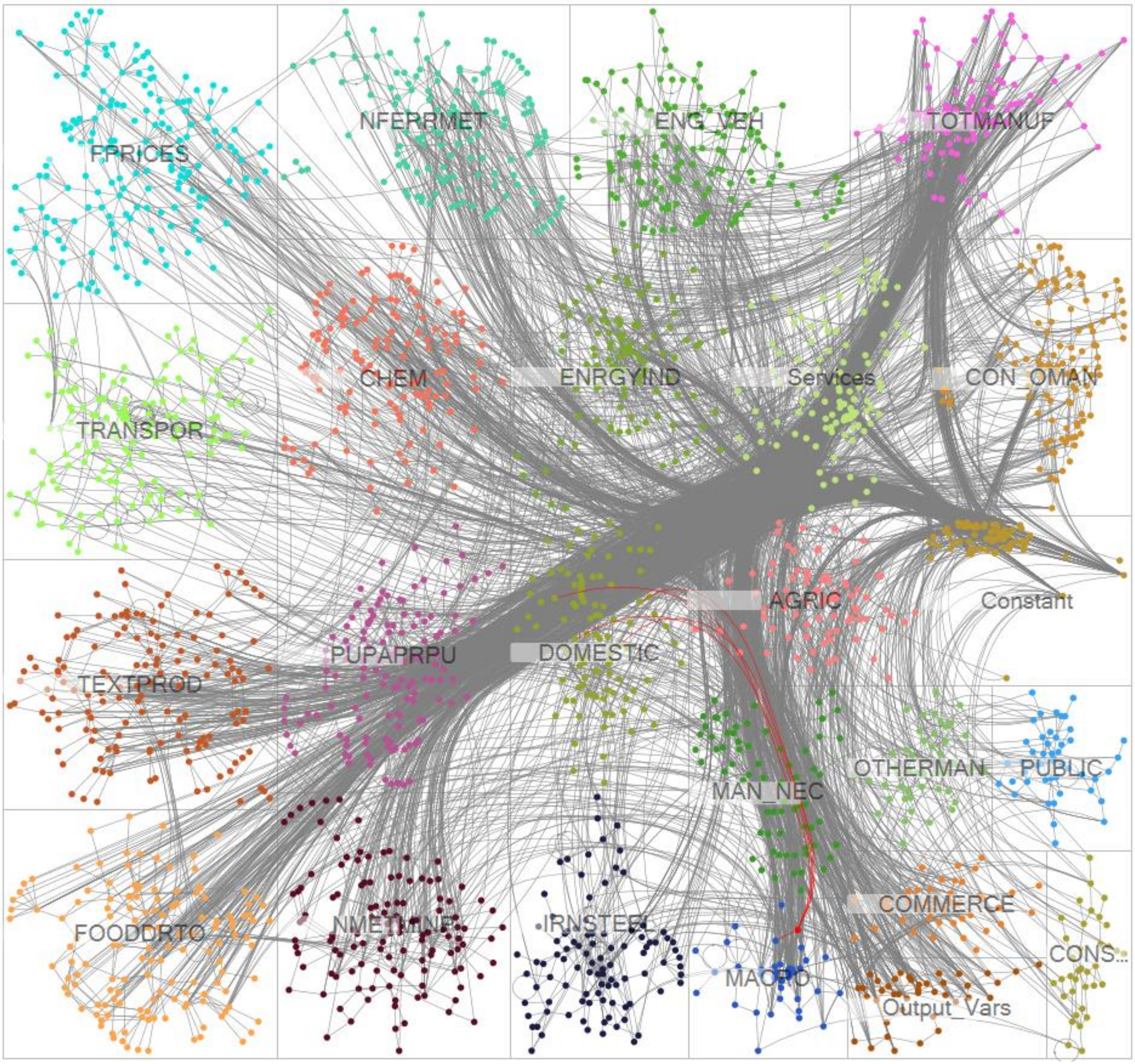


Figure 14. Directed graph of the relationship between all variables linked to the 970 MOVs. The NodeXL tool facilitates investigation of this equation spaghetti by, *inter alia*, allowing a user to zoom in and select an individual variable and trace its links. In the above graphic the variable MACRO_HOUSEHOLDS is selected and one can see the red highlighted edges reassuringly joining to DOMESTIC variables.

To further illustrate the interconnectivity of the model, Figure 15 focuses on one sector (Commerce) and shows how variables within the sector are linked.

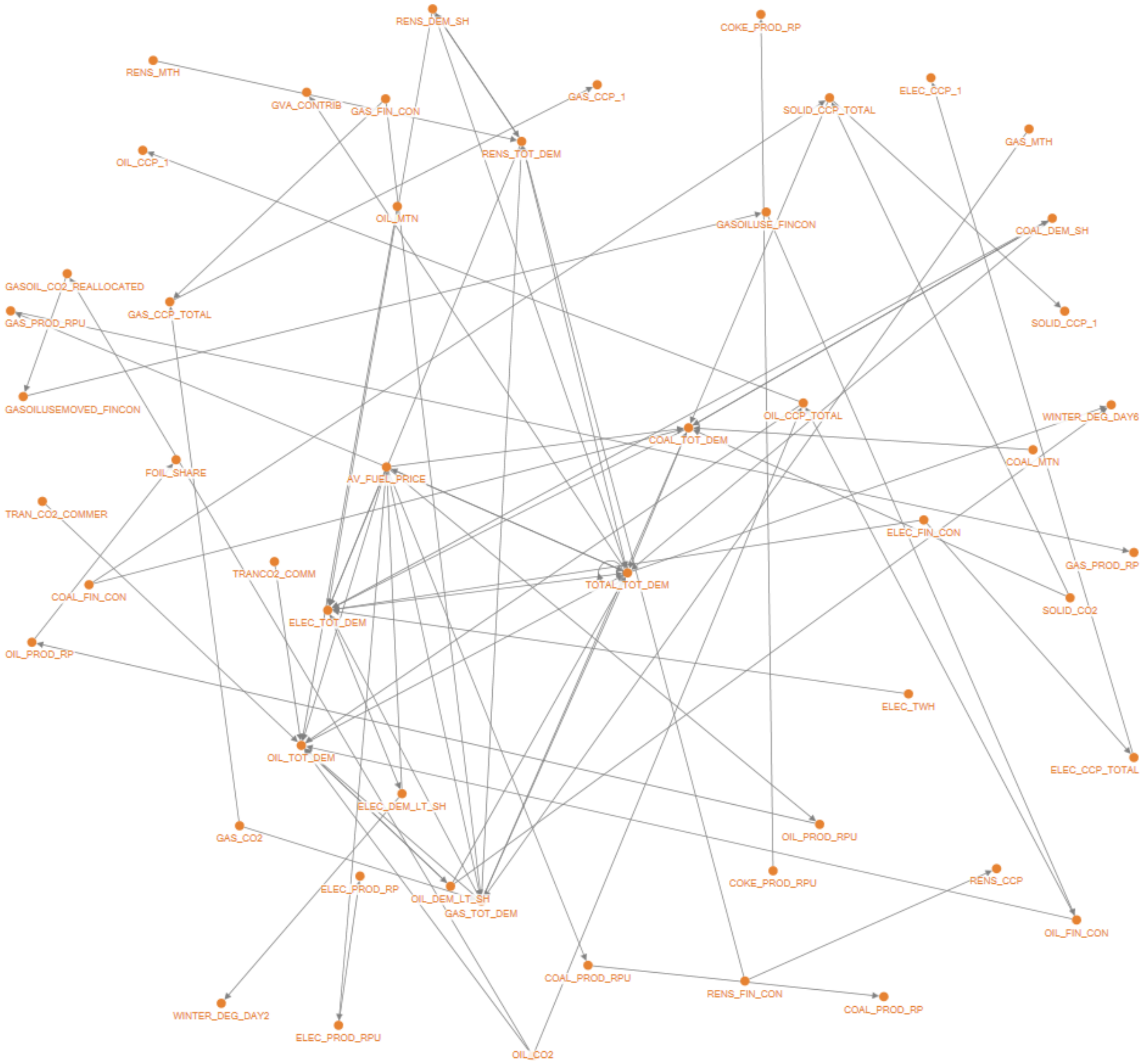


Figure 15. Intra-sector variable linkage graph for COMMERCE. For clarity, links to other sectors have not been included.

4 Discussion

The precedence analysis identifies the subset of *all variables* which are *redundant* for the set of *preferred outputs*. Redundant variables are those that are not connected, directly or indirectly in the precedent chain of any of the *preferred outputs*.

The measure of significance is a more difficult step, requiring a means to compute for each of the *preferred outputs* a ranking of the inputs. Given that DECC requires metrics for over 900 *preferred outputs* some means of comparing the influence of inputs upon outputs that cross radically different scales and units is required. A further complication is that the model has a large number of inputs, or terminal precedents, and that the plausible ranges of variance are unknown.

4.1 The pitfalls of blanket percentage changes across model inputs

An approach which ignores the proper range within which an input variable is likely to vary is at risk from both Type I and Type II errors. Type I errors involve identifying an input as influential, when it is in fact non-influential. Type II errors miss identifying an input as influential.

While some input variables in the EDM model could safely be varied by 10% around the value from the reference scenario workbook, there are several examples of inputs which change by orders of magnitude from the reference scenario. In particular, some variables are involved in equations of the form Max(a,b), and thus may appear to have no effect until a threshold is reached. Other variables have a zero value in the reference scenario, but could be non-zero in other scenarios. Also, ranges are not necessarily symmetric around the reference scenario.

4.2 Global sensitivity measures

A safer method to determine sensitivity of the model outputs to inputs, is to first assess the plausible range over which the inputs are likely to be varied. For example, we took the minimum and maximum values for each of the input rows that vary across the six scenarios given to us by DECC. These correspond to scenarios for:

- increased and decreased fossil fuel price projections
- increased and decreased socio-economic growth projections
- a reference scenario
- a baseline scenario (which excludes the effects of savings due to policies)

The resulting sensitivity measures were directly related to the unit of the output variable for which the measure is computed.

One can transform the sensitivity measure across the inputs. The normalising approach does not facilitate comparison across output variables of different units, but does allow inputs to be ranked for each output.

To compare inputs with one another, a high level aggregate variable is required for which all the inputs are more or less significant.

4.3 Example of variables at different scales

If μ^* is normalised by dividing by the mean value of the output variable over all the model results, a measure is obtained which shows variation over the input range as a percentage of the mean output value. However, this still makes it difficult to compare between inputs, as the approach is tripped up by cases such as that of `TRANSPOR__HGV_GASVEH_SHAR` (for 2020) where the input is a very small range of between 0 and 0.03. One of the preferred outputs is `TRANSPOR__GAS_CO2`. The variation is huge (over 200%) compared to the mean value, but in the context of the whole model, the 36 kTCO2 represented by this variation is relatively small.

4.4 Further analysis

The results have shown that the methods are very successful in ranking inputs for individual outputs, and for identifying large sections of the model that are redundant. The agreement between the two independent sensitivity analysis techniques demonstrates that our results are extremely robust. And our results seem both intuitive and sensible when investigated in detail. These results are further reinforced by the combination of precedence analysis and sensitivity analyses. The precedence analysis gives reassurance of the model coverage provided by the techniques we use, together with additional insights that allow us to target specific input and output parameters for further analysis. For example, we can identify the smallest number of inputs we need to target to extend the sensitivity analysis to all 950 output variables.

A remaining challenge is to identify a satisfactory method to allow comparison between all the model input variables. We suggest several options, each of which has advantages and disadvantages.

1. Assume all model outputs are equally important. In this, we treat all non-input variables as outputs and compute sensitivity indices. We then normalise the sensitivity analysis results across both inputs and outputs to provide a matrix of % variation caused by each input variable for each other non-input variable. This gives a rough indication of the influence any particular input has across all the outputs, but it remains very difficult to provide an instant metric that works across all the outputs. The magnitude of % variation is still highly sensitive to the scale of the output variable in question, even though comparison between variables of different units is possible.
2. Use fewer outputs. Perform a sensitivity analysis for the smallest group of outputs that cover the inputs which influence the 950 required outputs. It may be easier to compare a smaller number of outputs in the model (ideally of similar units), to perform a robust ranking of inputs which can then be extrapolated to all the outputs.
3. Try to collate the preferred outputs according to their units, so that the variation caused by the change in an input is understood in relation to the national total. For example, a value for sub-sectoral CO2 emissions would be scaled by the total CO2 emissions. This could be combined with (1) to solve the scaling issue. However, this is a time consuming process and would require manual, rather than automated interventions.

5 Conclusions

Our approach is based on three analytical methodologies: precedence analysis, global sensitivity analysis using the method of Morris and tangential space analysis. These have been complemented with an ad hoc analysis aided by logic maps.

The precedence analysis depends on the set of model outputs; we analysed two sets of variables: a radical case looking at the effect on the 3 output variables deemed most important to DECC (electricity demand, total CO2 emissions and total energy consumption) and a conservative case with a list of 934 variables that were deemed to be outputs out of the 3923 variables in the model. The radical case variables have a total 1878 precedents, 665 of which are terminal. None of these precedents have zero dependents, i.e. none are model output variables (MOV). Indeed, approximately half of the difference between conservative and main MOV only analyses precedent counts is due to MOVs with no precedents being included in the former analysis. The conservative case showed that EDM has at most 2519 precedents for 970 Model Outputs. 842 of the precedents are terminal, i.e. they have no further precedents. Of the 3923 variables, 1404 of them definitely have no effect on the 970 selected outputs.

Preliminary analysis showed that only eighty-nine of the 851 entries in the 'Equations' sheet (of the 3923 equations in total) actually change between the scenarios. Therefore, we have initially performed the sensitivity analysis for these eighty-nine inputs for the year 2020, and only over the ranges covered in these five scenarios. We then performed a large sensitivity analysis over the 140 variables which are terminal precedents for the 9 high level output variables identified by DECC to be the most relevant to them. We obtained an approximate ranking of input variables, predicated on an arbitrary 5% variation around the value in the reference scenario for each of these inputs. A brief analysis of the 89 input variables we did have data for suggested that this is likely to raise both Type I and Type II errors, wrongly identifying non-influential variables and influential and *vice versa*. Given the lack of data, this seemed to be the only course of action.

The tangential space analysis shows that, although the range of effects of perturbing a given terminal precedent by even a small amount spans half a dozen orders of magnitude, the large number of Model Output variables, the effect of different amounts of variation in the variables as evidenced by the effect of including SE on the magnitude of effects, the continuous distribution of effects across these orders of magnitude, and the fact that many PVs are connected to tens or even hundreds of Model Output Variables whose relative importance is unknown to us, means that any binary classification of the non-redundant variables into insignificant and significant is likely to be sensitive itself to the assumptions made.

In particular, while our method is shown to be robust in identifying inputs of importance, no sensitivity method is immune to inconclusive results when information on input data is unavailable. To offer robust and general conclusions, it is essential that the input range for each of the inputs is quantified. Nevertheless, we have developed a set of analytical tools and an approach that DECC can apply to further explore the Energy Demand Model, in order to better understand the significance of different input variables under various assumptions and decide on which variables to focus their data gathering efforts.

ANNEX A. List of significant and insignificant variables

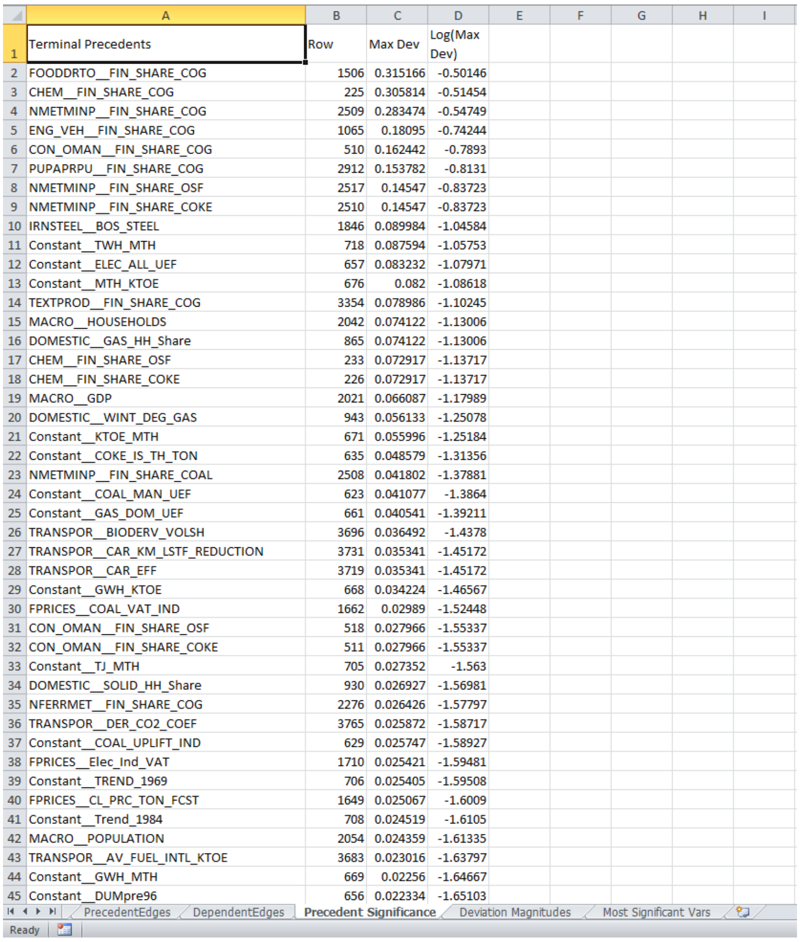
The ranking of variables based on the Morris Method sensitivity analysis can be found in the Excel file “headline_results_5perc_lag” within the folder “Results – 5% - 140 inputs”. Each one of the MOVs has a worksheet (and a ranking) based on the significance of each input variable.

	input_sector	input_variable	mu_star	sigma
2	MACRO	HOUSEHOLDS_2020	4,665.05	77.92
3	MACRO	GDP_2020	3,141.13	78.86
4	DOMESTIC	WINT_DEG_GAS_2020	2,853.46	58.56
5	TRANSPOR	CAR_EFF_2020	2,230.27	129.56
6	TRANSPOR	CAR_KM_LSTF_REDUCTION_2020	2,038.06	482.56
7	MACRO	GDP_2019	1,434.37	55.22
8	MACRO	POPULATION_2020	1,430.22	130.59
9	TRANSPOR	AV_FUEL_INTL_KTOE_2020	1,309.65	0.00
10	TRANSPOR	VOL_AGREE_1_2020	1,171.91	67.76
11	TRANSPOR	VOL_AGREE_2_2020	1,144.39	75.67
12	COMMERCE	GVA_CONTRIB_2020	932.24	9.69
13	TRANSPOR	HGV_EFF_2020	801.77	204.74
14	TRANSPOR	CAR_FF_EFF_2020	593.96	81.00
15	PUBLIC	EMPLOYMENT_2020	514.08	3.36
16	TRANSPOR	LGV_EFF_2020	469.45	56.02
17	FPRICES	DERV_LT_TH_2020	446.24	15.59
18	MACRO	HH_INCOME_GDP_RATIO_2019	426.67	11.98
19	MACRO	HOUSEHOLDS_2019	423.25	17.24
20	FPRICES	GAS_PRICE_MWH_2019	364.46	32.99
21	MAN_NEC	OIL_TOT_DEM_2020	348.03	0.02
22	FPRICES	CRUDE_LT_TON_2020	329.60	18.61
23	FPRICES	PREM_UL_LT_TH_2020	324.53	34.40
24	FPRICES	CRUDE_PRC_FCST_2020	323.77	18.22
25	MACRO	ADVANCED_ECONOMIES_GDP_2020	322.02	32.12
26	MACRO	WORLD_GDP_2020	310.37	13.74
27	FPRICES	GAS_DOM_OTHERFIXEDCOSTS_2020	292.01	30.08
28	FPRICES	DERV_DUTY_2020	211.86	13.18
29	DOMESTIC	WINT_DEG_ELEC_2020	201.39	8.34
30	FPRICES	DERV_LT_TON_2020	197.06	11.91
31	FPRICES	PREM_UL_DUTY_2020	169.41	16.48
32	TRANSPOR	PSV_KM_2020	153.67	4.77
33	DOMESTIC	WINT_DEG_GAS_2019	147.01	5.16
34	MACRO	EFFECTIVE_EXCHANGE_RATE_2019	145.93	5.06
35	TRANSPOR	PSV_EFF_2020	140.97	34.86
36	FPRICES	PREM_UL_LT_TON_2020	130.35	18.34
37	MACRO	POPULATION_2019	123.67	161.34
38	DOMESTIC	RENS_TOT_DEM_2020	107.18	0.00
39	IRNSTEEL	BOS_STEEL_2020	96.20	7.30
40	FPRICES	GAS_DOM_FIXEDHEDGINGCOST_2020	74.77	3.77
41	TRANSPOR	CAR_ELAST_POP_2020	71.44	99.93
42	TRANSPOR	RAIL_GOIL_FIN_GWH_2020	62.59	0.00
43	FPRICES	GAS_DOM_VARIABLESUPPLIERMARGIN_2020	62.28	2.70
44	TRANSPOR	AV_FUEL_COM_DOM_KTOE_2020	57.92	0.00
45	TRANSPOR	CAR_ELAST_POP_2019	53.71	65.01
46	FPRICES	PREM_UL_VAT_2020	52.70	4.03

TOTAL_ENERGY_FINAL_CONS / TOTAL_CO2 / DDM_ELEC_TWH / FC_COAL / FC_ELECTRICITY / FC_MANUFACTURED_FUELS / FC_NATURAL_GAS / FC_PETROLEUM_PRODUCTS / FC_RENEWABLES

Screenshot of the Morris method ranking for the Total Final Energy Consumption output variable

The ranking of variables arising from the conservative tangential analysis appear in the “Precedent Significance” worksheet in the file “Tangent Analysis All Model Outputs.xlsx”. The ranking of variables arising from the radical tangential analysis appear in the “Precedent Significance” worksheet in the file “Tangent Analysis Main Model Outputs.xlsx”.



Terminal Precedents	Row	Max Dev	Log(Max Dev)
FOODDRTO__FIN_SHARE_COG	1506	0.315166	-0.50146
CHEM__FIN_SHARE_COG	225	0.305814	-0.51454
NMETMINP__FIN_SHARE_COG	2509	0.283474	-0.54749
ENG_VEH__FIN_SHARE_COG	1065	0.18095	-0.74244
CON_OMAN__FIN_SHARE_COG	510	0.162442	-0.7893
PUPAPRPU__FIN_SHARE_COG	2912	0.153782	-0.8131
NMETMINP__FIN_SHARE_OSF	2517	0.14547	-0.83723
NMETMINP__FIN_SHARE_COKE	2510	0.14547	-0.83723
IRNSTEEL__BOS_STEEL	1846	0.089984	-1.04584
Constant__TWH_MTH	718	0.087594	-1.05753
Constant__ELEC_ALL_UEF	657	0.083232	-1.07971
Constant__MTH_KTOE	676	0.082	-1.08618
TEXTPROD__FIN_SHARE_COG	3354	0.078986	-1.10245
MACRO__HOUSEHOLDS	2042	0.074122	-1.13006
DOMESTIC__GAS_HH_Share	865	0.074122	-1.13006
CHEM__FIN_SHARE_OSF	233	0.072917	-1.13717
CHEM__FIN_SHARE_COKE	226	0.072917	-1.13717
MACRO__GDP	2021	0.066087	-1.17989
DOMESTIC__WINT_DEG_GAS	943	0.056133	-1.25078
Constant__KTOE_MTH	671	0.055996	-1.25184
Constant__COKE_IS_TH_TON	635	0.048579	-1.31356
NMETMINP__FIN_SHARE_COAL	2508	0.041802	-1.37881
Constant__COAL_MAN_UEF	623	0.041077	-1.3864
Constant__GAS_DOM_UEF	661	0.040541	-1.39211
TRANSPOR__BIODERV_VOLSH	3696	0.036492	-1.4378
TRANSPOR__CAR_KM_LSTF_REDUCTION	3731	0.035341	-1.45172
TRANSPOR__CAR_EFF	3719	0.035341	-1.45172
Constant__GWH_KTOE	668	0.034224	-1.46567
FPRICES__COAL_VAT_IND	1662	0.02989	-1.52448
CON_OMAN__FIN_SHARE_OSF	518	0.027966	-1.55337
CON_OMAN__FIN_SHARE_COKE	511	0.027966	-1.55337
Constant__TJ_MTH	705	0.027352	-1.563
DOMESTIC__SOLID_HH_Share	930	0.026927	-1.56981
NFERRMET__FIN_SHARE_COG	2276	0.026426	-1.57797
TRANSPOR__DER_CO2_COEF	3765	0.025872	-1.58717
Constant__COAL_UPLIFT_IND	629	0.025747	-1.58927
FPRICES__Elec_Ind_VAT	1710	0.025421	-1.59481
Constant__TREND_1969	706	0.025405	-1.59508
FPRICES__CL_PRC_TON_FCST	1649	0.025067	-1.6009
Constant__Trend_1984	708	0.024519	-1.6105
MACRO__POPULATION	2054	0.024359	-1.61335
TRANSPOR__AV_FUEL_INTL_KTOE	3683	0.023016	-1.63797
Constant__GWH_MTH	669	0.02256	-1.64667
Constant__DUMpre96	656	0.022334	-1.65103

Screenshot of the Tangential Analysis ranking showing the "Precedent Significance" worksheet

ANNEX B. List of redundant variables

The list of redundant variables produced by the precedence analysis can be found in the worksheet “RedundantVariables” (see screenshot below). The file “EDM Conservative Precedents v1-0-0.xlsx” contains a list of the redundant variables for the full set of 970 MOVs and file “EDM Radical Precedents v1-0-1.xlsx” contains a list of the redundant variables for the main 3 MOVs.

	A	B
1	RedundantVariables	Row
2	AGRIC__BOIL_KTOE	12
3	AGRIC__COAL_FIN_MTN	20
4	AGRIC__COKE_FIN_MTN	27
5	AGRIC__ELEC_PROD_RP	35
6	AGRIC__FOIL_KTOE	51
7	AGRIC__GAS_NOX	59
8	AGRIC__GAS_NOX_COEF	60
9	AGRIC__GOIL_KTOE	67
10	AGRIC__LPG_KTOE	72
11	AGRIC__OIL_FIN_DEM	77
12	AGRIC__OIL_FIN_KTOE	78
13	AGRIC__OSF_FIN_MTN	87
14	AGRIC__RENS_FIN_MTN	94
15	AGRIC__TOTAL_MTH	101
16	AGRIC__TOTAL_TOT_DEM	102
17	CHEM__BOIL_FIN_CON	105
18	CHEM__BOIL_FIN_MTONNE	108
19	CHEM__BOIL_PROD_RP	109
20	CHEM__BOIL_PROD_RPU	110
21	CHEM__CCA_CO2	120
22	CHEM__CCA_CO2_SH	121
23	CHEM__CHP_COAL_FFH	125
24	CHEM__CHP_COALE_CO2	127
25	CHEM__CHP_COG_FFH	129
26	CHEM__CHP_COG_HEAT	130
27	CHEM__CHP_COGE_CO2	131
28	CHEM__CHP_ELEC_OTHR	132
29	CHEM__CHP_ELECOTH_GWH	134
30	CHEM__CHP_ESIGAS_GWH	135
31	CHEM__CHP_FOIL_FFH	138
32	CHEM__CHP_FOILE_CO2	140
33	CHEM__CHP_GAS_FFH	142
34	CHEM__CHP_GOIL_FFH	145
35	CHEM__CHP_GOILE_CO2	146
36	CHEM__CHP_HEATGEN	147
37	CHEM__CHP_OTHERE_CO2	150
38	CHEM__CHP_OTHR_FFH	152
39	CHEM__CHP_OTHR_HEAT	153
40	CHEM__CHP_RENS_FFH	158
41	CHEM__CHP_RENS_HEAT	159
42	CHEM__CHP_RENSE_CO2	161
43	CHEM__CHP_TOTAL_GWH	162
44	CHEM__COAL_FIN_CON	171
45	CHEM__COAL_FIN_MTONNE	174
46	CHEM__COG_PROD_RP	182

AllVars / ModelOutputs / AllPrecedents / TerminalPrecedents / RedundantVariables

Screenshot of the “RedundantVariables” worksheet

ANNEX C. Global sensitivity analysis plots

There are three subfolders within the *morris_results* folder:

- results - ranges from scenarios - 89 inputs
- results - 10% - 89 inputs
- results - 5% - 140 inputs

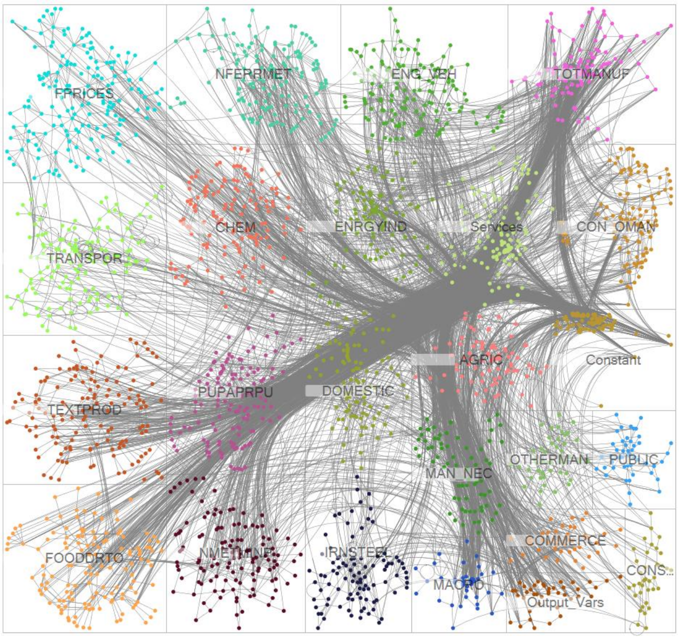
Each contains:

- *clustermaps* folder and
- *headline scatter plots* folder and
- Excel workbooks containing data for the headline output variables

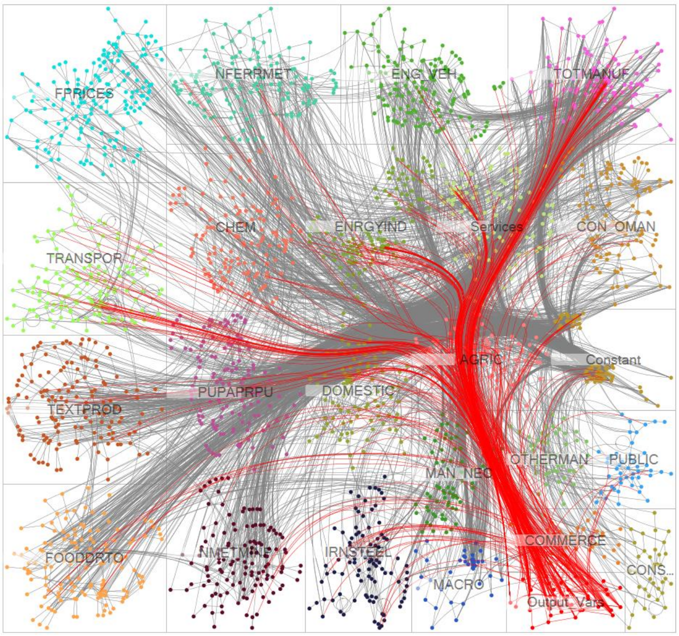
See ANNEX E for further details on these plots.

ANNEX D. Logic maps

Logic maps for the equations in the model where produced using NodeXL, a free, open-source template for Excel that makes it easy to explore network graphs. Once installed, the tool facilitates investigation by allowing a user to zoom in and select an individual variable and trace its links; hence the power of NodeXL is in its interactive nature. As an example, the graphics below show what happens when you select the output variables; one can see the red highlighted edges that join them to the rest of the model variables. Further details on the tool, edge sets and graphs can be found in ANNEX E.



Complete map of equations linked to MOVs



Output variables links to different sectors

ANNEX E. Description of directory structure and documents

Graphs	
NodeXLExcelTemplate2014Setup.exe	The NodeXL installer. See http://nodexl.codeplex.com/ for more details.
Graphs/Edges	
Precedent Graph Edges v1-0-0.xlsx	This file contains the graphs describing the entire EDM model (connected to all the ModelOutputs) in directed edge form (i.e. Column B is precedent of Column A) as well as the intra-sector graphs.
Graphs/NodeXL	
24 NodeXL templates.	Derived from the edge sets in Precedent Graph Edges v1-0-0.xlsx. One for each sector (except Constants whose graph is trivial) and one for the entire EDM.
Graphs/Pictures	
[Sector Name].png	A picture of the intra-sector relationships between variables.
[Sector Name] link.png	A picture of the direct links between the sector variables and other model variables
Complete Map macro_households selected.png	A picture of the direct links between the DOMESTIC__HOUSEHOLDS variable and other model variables
Complete map of equations linked to MOVs.png	A picture of all the links between the Model Output variables and their precedents.
Precedent Analysis	
Complete dependent and precedent graph edges for conservative output.xlsx	Contains worksheets with the entire precedent and dependent graphs in edge format for all the variables connected to the full set of 970 MOVs.
EDM Conservative Precedents v1-0-0.xlsx	List of all precedents for the full set of 970 MOVs together with the list of redundant variables etc.
EDM Radical Precedents v1-0-1.xlsx	List of all precedents for the 3 main MOVs together with the list of redundant variables etc.
NodeXLGraphPrecedents v1-0-0.xlsm	Identical to NodeXLGraphPrecedentsALL in Graphs/NodeXL
Precedent and Dependent Matrices.xlsb	Contains worksheets with the complete set of

	precedence and dependent matrices. Direct matrices just show directly linked/dependent variables. The other matrices show all linked/dependent variables.
EDM v15.1.Ref.1-Restricted Named Ranges.xlsb	Copy of the EDM with named ranges in the Values sheet restricted to 52 columns.
EDM All Outputs Precedents Marked.xlsb	Copy of EDM derived from above file will all precedents for the full set of 970 MOVs marked, terminal precedents in red, others in green.
EDM Main Outputs Precedents Marked.xlsb	As previous file, but for the main 3 outputs.
Tangent Analysis	
Analysis of main outputs versus precedents with estimable CVs.xlsx	Presents the analysis of all 9 “main” output variables and compares them to the Method of Morris analysis. Note All Output_Var appears to be corrupted.
Tangent Analysis All Model Outputs.xlsx	Tangent analysis arising from all 970 MOVs. Important worksheet is “Precedent Significance” which give a ranking of all variables as described in the report.
Tangent Analysis Main Model Outputs.xlsx	As above, but for the output variables restricted to the 3 main variables.
VBA	
Formula Analysis Tools-0-2-0.xlsm	All the VBA code used to analyse the data. This version of the code is not in final form. A final version of the code will be delivered at the end of April. This will have the Morris Method implemented in VBA as well as some more user friendly versions of the current macros. In particular, there will be a universal macro that will generate an entire analysis of an EDM style model with one click.
Formula Analysis Tools v0-2-2 Procedure Documentation.pdf	A hyperlinked pdf file documenting the procedures and functions in the VBA code.
Formula Analysis Tools v0-2-0 Source Code.pdf	A hyperlinked version of the VBA code including a variable index etc.
Morris Method Analysis	
Clustermaps folder	A clustermap pdf *[sector name].pdf* is drawn for each sector, where the analysis has been performed over all sectors. There are also clustermaps drawn which exclude output variables with CCP in the name. These are identified by the filename *[sector*

	name]noCCP.pdf
Headline scatter plots folder	One scatter plot named by the variable name for each of the eight OUTPUT_VARS sectors.
Results – range from scenarios – 89 inputs	The results from the initial analysis in which 89 varying inputs were derived from the six scenario files provided. Clustermaps are provided for each sector both including and excluding CCP output variables
Results – 10% - 89 inputs	Clustermaps and scatter plots are provided. There are two Excel spreadsheets. One contains the headline figures for the eight output variables. The other contains a comparison between the values of a blanket ±10% range around the reference scenario value with the ranges derived from the six scenarios.
Results – 5% - 140 inputs	Only full clustermaps are provided as just 8 variables from the OUTPUT_VARS sector were included in the analysis. Scatter plots covering each of the output variables are only plotted for non-CCP variables (as CCP variables have been pre-screened from the input variables).

ANNEX F. Glossary

Energy Demand Model (EDM)

The EDM is an Excel based tool run by DECC to calculate energy demand on a final fuel use basis and all non-electricity generation transformation processes.

Method of Morris Analysis

A global sensitivity analysis technique used to quantify the degree to which model inputs influence model outputs.

Tangent Space Analysis

This methodology identifies the effects of internal variables and inputs deterministically by investigating the strength and form of their individual marginal effects on output variables.

Terminal precedents

When tracing the variables in equations defining the model output variables, those variables which are not defined by formulae

Dependent variables

When tracing the variables in equations defining the model output variables, those variables which are defined by other variables. In EDM this is usually a subset of the terminal precedents.

Model Output Variables (MOVs)

The set of variables that are deemed to be outputs of the model, i.e. the variables that are required by external sources and not just used in intermediate calculations.

Mu (μ)

In the Method of Morris sensitivity analysis, μ is the mean of the effect of each variable

Mu_star (μ^*)

In the Method of Morris sensitivity analysis, μ^* is the mean of the absolute effect of each variable

Sigma (σ^2)

In the Method of Morris sensitivity analysis, σ^2 is the standard deviation of the mean effect of each variable

Marginal Effect Analysis Tool (MEAT)

This is a VBA tool developed to perform the tangent space analysis. It detects the effect of small (in percentage terms) perturbations of inputs and internal variables on the output variables by automatically working through the EDM altering a single variable at a time and letting the model update the relevant output cells, before resetting and moving onto the next input cell.

Aggregate deviation

Defined as the sum of absolute deviations.

Mean deviation

Defined as the ratio of the aggregate deviation over the number of MOVs dependent upon the perturbed PV.

Maximum deviation

Defined as the maximum absolute deviation.

Pseudo Coefficient of Variation (CV)

The CV is the product of the absolute deviation recorded for an input variables percentage variation and a measure of the input variables natural relative variability. Defined to be the half width of the maximum and minimum range in the variable observed, divided by the average of the maximum and minimum.