Department for Education

Mathematics key stage 4

Introduction

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programme of study for key stage 4 is organised into apparently distinct domains, but pupils should develop and consolidate connections across mathematical ideas. They should build on learning from key stage 3 to develop greater fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge wherever relevant in other subjects and in financial contexts.

This programme of study specifies:

- the mathematical content that should be taught to all pupils, in standard type; and
- additional mathematical content to be taught to more highly attaining pupils, in **bold** type.

Together, the mathematical content set out in the key stage 3 and key stage 4 programmes of study covers the full range of material contained in the GCSE Mathematics qualification. Wherever it is appropriate, given pupils' security of understanding and readiness to progress, pupils should be taught the full content set out in this programme of study.

Working mathematically

Through the mathematics content pupils should be taught to:

Develop fluency

- consolidate their numerical and mathematical capability from key stage 3 and extend their understanding of the number system to include powers, roots **and fractional indices**
- select and use appropriate calculation strategies to solve increasingly complex problems, including exact calculations involving multiples of π **and surds**, use of standard form and application and interpretation of limits of accuracy
- consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions**, and expressions involving surds and algebraic fractions**

- extend fluency with expressions and equations from key stage 3, to include quadratic equations, simultaneous equations and inequalities
- move freely between different numerical, algebraic, graphical and diagrammatic representations, including of linear, quadratic, reciprocal, **exponential and trigonometric functions**
- use mathematical language and properties precisely.

Reason mathematically

- extend and formalise their knowledge of ratio and proportion, including trigonometric ratios, in working with measures and geometry, and in working with proportional relations algebraically and graphically
- extend their ability to identify variables and express relations between variables algebraically and graphically
- make and test conjectures about the generalisations that underlie patterns and relationships; look for proofs or counter-examples; begin to use algebra to support and construct arguments **and proofs**
- reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and express their arguments formally
- assess the validity of an argument and the accuracy of a given way of presenting information.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial contexts
- make and use connections between different parts of mathematics to solve problems
- model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem.

Subject content

Number

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- apply systematic listing strategies**, including use of the product rule for counting**
- **estimate powers and roots of any given positive number**
- calculate with roots, and with integer **and fractional** indices
- calculate exactly with fractions, **surds** and multiples of π; **simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators**
- calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer
- **change recurring decimals into their corresponding fractions and vice versa**
- identify and work with fractions in ratio problems
- apply and interpret limits of accuracy**, including upper and lower bounds**.

Algebra

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- simplify and manipulate algebraic expressions (including those involving surds **and algebraic fractions**) by:
    - factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares**; factorising quadratic expressions of the form $ax^2 + bx + c$**
    - simplifying expressions involving sums, products and powers, including the laws of indices
- know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**
- where appropriate, interpret simple expressions as functions with inputs and outputs; **interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function'**
- use the form $y = mx + c$ to identify parallel **and perpendicular** lines; find the equation of the line through two given points, or through one point with a given gradient
- identify and interpret roots, intercepts and turning points of quadratic functions graphically; deduce roots algebraically **and turning points by completing the square**

- recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **the exponential function $y = k^x$ for positive integer values of *k*, and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size**
- **sketch translations and reflections of a given function**
- plot and interpret graphs (including reciprocal graphs **and exponential graphs**) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts**
- **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**
- solve quadratic equations **(including those that require rearrangement)** algebraically by factorising**, by completing the square and by using the quadratic formula**; find approximate solutions using a graph
- solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph
- **find approximate solutions to equations numerically using iteration**
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- solve linear inequalities in one **or two** variable**(s), and quadratic inequalities in one variable**; represent the solution set on a number line**, using set notation and on a graph**
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0 **or a surd**) **and other sequences**
- deduce expressions to calculate the n^{th} term of linear **and quadratic** sequences.

Ratio, proportion and rates of change

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity (including trigonometric ratios)

- understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; **construct and** interpret equations that describe direct and inverse proportion
- interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- **interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of instantaneous and average rate of change (gradients of tangents and chords) in numerical, algebraic and graphical contexts**
- set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes**.

Geometry and measures

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- interpret and use fractional **and negative** scale factors
- **describe the changes and invariance achieved by combinations of rotations, reflections and translations**
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- **apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**
- construct and interpret plans and elevations of 3D shapes
- interpret and use bearings
- calculate arc lengths, angles and areas of sectors of circles
- apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two **and three** dimensional figures
- know the exact values of $\sin\theta$ and $\cos\theta$ for $\theta = 0^0,\ 30^0,\ 45^0,\ 60^0$ and 90^0; know the exact value of $\tan\theta$ for $\theta = 0^0,\ 30^0,\ 45^0,\ 60^0$
- **know and apply the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule, $a^2 = b^2 + c^2 - 2bc\cos A$, to find unknown lengths and angles**
- **know and apply $\text{Area} = \frac{1}{2}ab\sin C$ to calculate the area, sides or angles of any triangle**
- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors**; use vectors to construct geometric arguments and proofs**.

Probability

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams**.

Statistics

In addition to consolidating subject content from key stage 3, pupils should be taught to:

- infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- interpret and construct tables and line graphs for time series data
- **construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:
    - appropriate graphical representation involving discrete, continuous and grouped data**, including box plots**
    - appropriate measures of central tendency, including modal class and spread**, quartiles and inter-quartile range**
- apply statistics to describe a population
- use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.