Rail Service Analysis

Comparing Rail Forecasting Approaches

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Contents

Chapter 1: Management Summary

This paper is part of a programme of work to evaluate the *'Revisiting'* study. Its intention is to investigate:

- How accurately the *'Revisiting'* framework is able to replicate actual rail demand, both using the *'Revisiting'* market segmentation and the market segmentation outlined in the Passenger Demand Forecasting Handbook (PDFH)

- How the accuracy of the *'Revisiting'* framework compares to the accuracy of the PDFH forecasting framework

The key findings are set out below.

The 'Revisiting' model with 'Revisiting' segmentation

The *'Revisiting'* back-casts were found to be reasonably accurate overall, for both the number of journeys and the compound annual growth rates.

The *'Revisiting'* framework was able to reasonably replicate actual journeys in a back-cast for all market segments. However, the framework performs less well at a more disaggregated level, for instance when splitting the results by ticket type or geographical segmentation.

In particular, the framework performed poorly for London and the South East and for season tickets.

The 'Revisiting' model with PDFH segmentation

The framework performed in a similar way when split by the geographical segmentation outlined in the PDFH. The back-casts were accurate for all but one of the segments, the London Ticketing Area, which was particularly inaccurate.

Again, the back-casts for season tickets were particularly inaccurate.

Comparing the 'Revisiting' framework with the PDFH.

Overall, the *'Revisiting'* framework was better able to replicate overall journeys compared to the PDFH. However, the PDFH was more accurate for certain subgroups, such as full price and season tickets.

The compound annual growth rates back-casted by the *'Revisiting'* model were much closer to the actual rates than those back-casted by the PDFH model.

The 'Revisiting' back-casts were largely more accurate for both market segmentations.

It is well documented that the framework contained within the PDFH has been under-forecasting rapid rail demand in recent years. The *'Revisiting'* framework has performed better in this area than the PDFH in recent years.

Chapter 2: Introduction

As part of the evaluation of the *'Revisiting'* study, IHAC have been asked to investigate the accuracy of the new forecasting method.

Back-casts for both the forecasting method described in the *'Revisiting'* study and that of the PDFH were provided. The accuracy of the *'Revisiting'* back-casts was investigated and compared against the accuracy of the PDFH back-casts.

The analysis was performed on the number of journeys taken between 2002/03 and 2009/10. Back-casts of revenue have not been considered in this report, but could be in a future draft.

2.1   Analytical Method

The analysis was performed on the data as a whole and also on key subgroups of the data. This included analysis by ticket type, analysis by year and analysis by market segmentation.

This report focuses on three areas of analysis;

- Accuracy of *'Revisiting'* back-casts using the *'Revisiting'* market segmentation;

- Accuracy of *'Revisiting'* back-casts using the PDFH market segmentation;

- Comparing the accuracy of the *'Revisiting'* model and the PDFH model (not splitting by segment).

Various tests have been performed, analysing back-casts for both the number of journeys and the growth rates. Annex A sets out key definitions and methods used.

This report is accompanied by several spreadsheets giving results in greater detail.

2.2   Scope of Analysis

This analysis is performed on back-casts A back-cast is an exercise to test model fit. It uses known data for past years (such as GDP, employment figures etc.) to work out what the predicted number of journeys would have been. This can be compared with the actual number of journeys to test the accuracy of the forecasting model.

The analysis here is based on two back-casts, the first using the *'Revisiting'* framework and the second using the forecasting framework outlined in the PDFH. Each back-cast was undertaken for the years 2002/03 to 2008/09. The back-casts were based on a consistent scenario, full details of which can be found in Steer Davies Gleave's report 'RAFF modelling assumptions v 0.82'

This method is useful in analysing model accuracy. However, it should be borne in mind that a model is only as good as its inputs. Regardless of how accurate the model is, forecasts will be flawed if the forecasted GDP (for example) that is fed into the model is later found to be inaccurate.

Chapter 3: The *'Revisiting'* back-cast of journeys with *'Revisiting'* segmentation

Overall, there were around 6.5 billion rail journeys taken nationally between 2002/03 and 2009/10.

The *'Revisiting'* back-cast modelled these journeys. These back-casts were tested against the actual number of journeys for each category to test the accuracy of the *'Revisiting'* model.

A variety of tests were performed. The results are presented in this chapter.

This chapter largely focuses on journeys split by market segment, using the segmentation set out in the *'Revisiting'* study.

The analysis uses several different statistical methods. These are briefly described in this section; more detail is given in Annex A.

Naïve Back-cast

In this report, the naïve back-cast assumes that the number of journeys taken in each year will be exactly the same as that of the base year. Whilst this back-cast is rarely useful in itself, its key use is as a base for comparison, testing whether a back-cast is better or worse than the naïve back-cast.

Root Mean Squared Error

RMSE is a measure of average difference between the actual number of journeys and the number back-casted. It measures difference irrespective of sign; the RMSE does not consider whether the back-cast is higher or lower than the

Key findings

- 6.5 billion passenger journeys between 2002/03 and 2009/10.

- Analysis performed on both number of journeys and growth rates.

- Overall, 'Revisiting' back-casts were accurate.

- Less accurate for lower level breakdowns.

- Particularly inaccurate for season tickets.

actual number of journeys, only the size of the difference.

U Comparison

The U comparison gives an indication of how accurate a back-cast is. A U figure of one shows that the back-cast is equally as accurate as the naïve back-cast, and a figure of less than one shows that the back-cast is more accurate than the naïve back-cast.

The U comparison also gives an indication of scale. A U figure of 0.5 means that the back-cast was twice as accurate as the naïve back-cast. A figure of 2 means that the back-cast was twice as inaccurate as the naïve back-cast.

Compound Annual Growth Rate

The CAGR gives an average compound growth rate over the time period. This only takes into account the number of journeys in the base year and the end year, and the number of years in the period. The actual growth rates for each year are not considered in the calculation.

3.1   Initial tests

In order to find patterns in the accuracy of back-casts, checks were performed to find whether the back-casts over- or under-estimated the actual number of passenger journeys.

The tables below give the back-casted value minus the actual value.

Ticket type/market segment combinations for which the back-casts were larger than the actual figures are given in blue, those for which the back-casts were smaller are given in purple.

Table 1: Back-cast Differences by Segment & Ticket Type

	Full	Seasons	Reduced	Total
LSEE to LSEE	134279	-151344	139766	122700
LSEE to Core	2294	-390	5843	7748
Core to LSEE	1999	-1626	-1101	-728
LSEE to Other	4711	855	-709	4857
Other to LSEE	4324	-7338	6436	3422
Core to Core	540	-5673	-14224	-19358
Core to Other	15494	-7576	16904	24823
Other to Core	-1474	-30299	37019	5246
Other to Other	-18304	-11770	-2916	-32990
To Airports	5336	-3272	-4243	-2179
Total	149199	-218434	182776	113541

Table 2: Back-cast Differences by Year & Ticket Type

	Full	Seasons	Reduced	All
2003/2004	1444	-25238	2406	-21389
2004/2005	10966	-22430	12197	733
2005/2006	23006	-41595	34607	16018
2006/2007	25215	-41930	26874	10159
2007/2008	28047	-37407	29091	19730
2008/2009	27569	-28022	30939	30486
2009/2010	32953	-21811	46662	57804
All	149198.98	-218434	182776	113541

The key trend shown in these tables is that the back-casts for season tickets tend to under-estimate the actual figures, whereas back-casts for full price tickets (and in some cases reduced tickets) tend to over-estimate.

3.2   Root Mean Squared Error

The RMSE was found for each combination of market segment and ticket type. A low RMSE indicates a good model.

For the majority (17 of 30, not including totals) of these combinations, the RMSE was smaller than that of the naïve model, although there was a large cluster of combinations (mainly involving full and season tickets for segments linked to LSEE) for which the RMSE was particularly large.

If the average error for a back-cast is larger than that for the naïve back-cast, this shows that the back-cast is very inaccurate. However, if the RMSE is smaller than that of the naïve, this does not necessarily imply that it is a very accurate back-cast, only that it is better than assuming the number of journeys remained constant. The scales of accuracy are investigated further in the next section.

The table below gives the RMSE for each ticket type/market segment combination. Those for which the RMSE is greater than the RMSE of the naïve back-cast are given in red.

Table 3: RMSE by Segment & Ticket Type

RAFF RMSE	Full	Seasons	Reduced	All
LSEE to LSEE	21,236	22,807	22,437	26,556
LSEE to Core	419	62	960	1,299
Core to LSEE	465	262	675	571
LSEE to Other	773	153	721	839
Other to LSEE	804	1,156	1,358	1,236
Core to Core	296	895	3,118	3,702
Core to Other	2 544	1 753	2 600	4,644
Other to Core	726	4,555	6,651	4,069
Other to Other	3,447	2,507	815	5,393
To Airports	857	483	679	706
All	23,664	32,268	29,411	27,974

How important these errors are depends largely on how many journeys were taken for each category. This is investigated further in section 3.4.

While there were many combinations for which the RMSE was large, the *'Revisiting'* method performed reasonably well overall for each segment. This was largely because errors for different ticket types would cancel each other out, to give a fairly accurate overall picture.

This was similar for full price tickets and reduced tickets. Whilst there were large errors for many of the segments, these cancelled themselves out to give a reasonably accurate picture overall.

The back-cast for season tickets gave larger errors that the naïve back-cast. This means that the *'Revisiting'* model is very inaccurate in forecasting the number of journeys taken with season tickets.

3.3   U Comparison

The U Comparison gives largely similar results to the RMSE tests, with the inaccurate combinations of segment and ticket type being clustered in the same areas (see the table below).

Table 4: U Comparison by Segment and Ticket Type

RAFF U Comparison	Full	Seasons	Reduced	All
LSEE to LSEE	2 8	6.1	2.0	0.6
LSEE to Core	6 3	1 5	0.3	0.6
Core to LSEE	4 0	6 2	0.2	0.1
LSEE to Other	49.2	1 2	0.1	0.1
Other to LSEE	8 8	1.4	0.3	0.1
Core to Core	0 0	1 5	0.9	0.5
Core to Other	0.7	0 8	0.8	0.3
Other to Core	0 0	0 6	0.3	0.0
Other to Other	0 2	0.7	0.0	0.1
To Airports	0 5	1.1	0.7	0.1
All	0.5	2.5	0.4	0.1

The U Comparison adds to the results of the RMSE tests, as it allows an element of scaling. For example, as full price tickets to airports have a U figure of 0.5, this means that the back-cast is twice as accurate as the naïve back-cast.

The U figure also enables us to find combinations where the back-cast is more accurate than the naïve back-cast, but is not much better. U figures between 0.8 and 1 have been highlighted in amber, to emphasise that whilst the back-casts for these combinations are better than not forecasting at all, they are not particularly accurate.

The overall U figures for each segment were all low (the maximum being 0.6). This means that for all segments, the

'Revisiting' back-cast error was at most 0.6 times the naïve back-cast error.

3.4   Accuracy by Market Share

The table below gives the number of journeys taken in each market segment/ticket type combination as a proportion of all journeys. The table excludes data from 2002/03 as this was the base year, i.e. all back-cast were the same as the actual figures.

The colours are taken from table 4, showing how accurate the back-cast was using the U test.

Table 5: Market Share by Segment and Ticket Type

	Full	Seasons	Reduced	All
LSEE to LSEE	13 2%	33 5%	19 0%	65.7%
LSEE to Core	0.2%	0.0%	1.0%	1.2%
Core to LSEE	0.2%	0.1%	1.0%	1.2%
LSEE to Other	0.2%	0.0%	1.1%	1.4%
Other to LSEE	0.4%	0.5%	1.5%	2.4%
Core to Core	0.5%	0.2%	1.3%	2.0%
Core to Other	1.6%	0.7%	1.8%	4.1%
Other to Core	3.8%	2.3%	5.9%	12.0%
Other to Other	3.7%	1.5%	3.8%	9.0%
To Airports	0.7%	0.1%	0.5%	1.2%
All	24.4%	38.9%	36.7%	100.0%

This shows that the combinations for which the back-casts were least accurate tended to be the ones with the most journeys taken.

The table below shows the proportion of journeys taken that fall into each of the categories (when disaggregated by market segment and ticket type).

Table 6: RAG Rating by Market Share

Red	Amber	Green
67.6%	3.8%	28.6%

This shows that disaggregating at the lowest level leads to inaccurate back-casts. However, disaggregating at higher levels, such as only by market segment or only by ticket type, the back-casts are more accurate.

3.5   CAGR Difference

In addition to the work on the number of journeys, the growth rates were also analysed.

The first statistic used to analyse this was the Compound Annual Growth Rate (CAGR), as defined in Annex A.

The table below gives the scale of CAGR error. The figures presented are the back-casted CAGR divided by the actual CAGR. Where the figures are close to 1, this implies rough equality between the back-cast and the actual figure.

A figure of 2 indicates that the back-casted CAGR was twice as large as the actual CAGR. A figure of 0.5 indicates that the back-casted CAGR was half the size of the actual CAGR. In this respect, figures of 2 and 0.5 are equivalent in saying that the back-casts were inaccurate by a factor of 2.

Where the figures are negative, this implies that the back-cast indicated that demand would increase, whereas that actual demand decreased (or vice versa).

Table 7: CAGR Error Proportions by Segment and Ticket Type

	Full	Seasons	Reduced	All
LSEE to LSEE	2 3	3 9	2.4	2.3
LSEE to Core	-1 3	-0 3	1.3	1.6
Core to LSEE	-0 8	-2 9	0.7	0.9
LSEE to Other	-6 9	1.1	0.8	1.0
Other to LSEE	-2 0	-0 3	1.5	1.5
Core to Core	1 2	-0 3	-0.1	0.3
Core to Other	1.7	1.7	1.7	1.7
Other to Core	1.1	0 5	1.7	1.2
Other to Other	0 5	0 0	1.1	0.6
To Airports	1 0	-0.1	0.0	0.7
All	1.5	-0.6	1.6	1.4

As the CAGRs are compounded, a relatively small difference in CAGR can result in a large difference in the number of journeys. For example, for LSEE to LSEE for full price tickets, the CAGRs for the actuals and the back-cast are 2.6% and 5.9% respectively (this was a difference of 2.7%). However, over the course of the seven-year period, these compound to a difference between 19% and 50% overall growth.

For the models to be accurate, the difference in back-casted CAGR and actual CAGR would need to be small, and this is not the case for all ticket type/segment combinations.

The model overall has a CAGR of 3.5%, compared with the actual figure of 2.5%, a difference of 1%. So whilst the model is inaccurate for many of the combinations, overall it is fairly accurate.

3.6   Annual Growth Rates

The Compound Annual Growth Rate is useful in finding the growth over the whole period, but is limited in that it only takes into account the first year and the last year, and takes no account of how accurate the intervening years are.

To investigate this further, IHAC calculated the growth rates for each year and for each ticket type. This is the percentage increase for each year.

There were 21 individual combinations (7 years for 3 ticket types). The *'Revisiting'* back-cast was within 1% of the actual figures for 8 of these, within 3% for 12 and within 5% for 16.

When split by year and market segment, the figures were 12 of the 70 combinations within 1%, 33 within 3% and 49 within 5%.

The tables below show the differences between the back-casted growth rates and the actual growth rates. Differences less than 3% are marked in green.

Table 8: Annual Growth Rate Difference by Ticket Type

	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
Full	0.9%	5.3%	6.0%	0.7%	0.7%	-0 8%	2.0%
Seasons	-8.4%	1.1%	-6.6%	0.3%	2.4%	3 2%	1.5%
Reduced	0.9%	3.6%	7.7%	-3.2%	0 2%	0 3%	4.4%
All	-2.9%	3.1%	2.0%	-0.8%	1.1%	1 2%	3.1%

Table 9: Annual Growth Rate Difference by Segment

	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
LSEE to LSEE	-3.2%	3.8%	2.3%	0.0%	1.0%	1.7%	3.6%
LSEE to Core	-2.4%	11.7%	3.9%	-1.6%	-1.5%	7.6%	0.6%
Core to LSEE	-4.4%	11.4%	2.5%	-12.0%	-4.2%	2.9%	0.1%
LSEE to Other	3.8%	8.0%	3.6%	-8.7%	-3.0%	-1.0%	-1.3%
Other to LSEE	-5.1%	3.1%	7.0%	-6.3%	2.7%	3.1%	8.9%
Core to Core	1.2%	-4.0%	-7.3%	-7.2%	-1.9%	-17.7%	-6.4%
Core to Other	0.6%	2.7%	5.2%	-1.0%	6.3%	-2.0%	13.6%
Other to Core	-2.1%	-0.2%	0.7%	-0.7%	3.4%	-0.1%	8.1%
Other to Other	-5.8%	0.8%	-0.6%	-0.4%	-1.8%	4.3%	-10.2%
To Airports	4.4%	-1.9%	3.1%	-9.7%	-1.9%	-4.5%	-2.1%
All	-2.9%	3.1%	2.0%	-0.8%	1.1%	1.2%	3.1%

Many of the inaccurate back-casts by ticket type are found in earlier years. After these years, the growth rates are closer to the actuals, suggesting that whilst the back-casts were particularly inaccurate for the early period, the growth rates forecasted were reasonably close to the actuals for the period afterward.

It is likely that forecasts for certain years are particularly inaccurate for understandable reasons. For example, 2005/06 actual rail journeys were low for several market segments after the London bombings.

If this is the case, then back-casted growth rates should be close to the actuals soon after the affected year. It does seem that this could be happening, particularly when split by ticket type, although more data would be needed before analysing this fully. When split by market segment, there is less evidence of this occurring, although again, more data would help in finding patterns in accuracy.

For each year overall, five of the seven years were within 3% of the actuals and all were within 5% of the actuals.

This shows that whilst there were problems with certain year/ticket type and year/market segment combinations, overall the yearly back-casts were reasonably accurate, and the framework has performed well in recent years.

3.7 Conclusions

The overall picture when investigating the *'Revisiting'* back-casts seems to be that at a high level, the back-casts are accurate.

This is apparent when considering both the overall U figure and the CAGR difference. A U figure of 0.1 demonstrates that the back-casts are far more (in fact 10 times more) accurate than the naïve method, and so the *'Revisiting'* model is useful in predicting the actual number of journeys. The back-casted CAGR and the actual CAGR were within 1% of each other, adding weight to the argument that at a high level, the *'Revisiting'* method is accurate.

Looking at a lower level, the *'Revisiting'* back-casts are less accurate in replicating actual rail journeys. Looking particularly at the U figures, we find that back-casts are accurate for each market segment, and for full price and reduced tickets, but not accurate for season tickets.

Disaggregating the results by geography and ticket type, the back-casts are not particularly accurate. The back-casts for 67.6% of journeys taken were less accurate than the naïve back-casts.

Disaggregating only by ticket type, this figure was 38.9%, and all were better than the naïve back-casts when disaggregating only by segment.

Annual growth rate accuracy varied over the years, with 2004/05 and 2009/10 being the least accurate.

There was little difference in accuracy between growth rates for each ticket type or each segment.

Chapter 4: The *'Revisiting'* back-cast of journeys with PDFH segmentation

Chapter 3 analysed the *'Revisiting'* back-casts split using the *'Revisiting'* market segmentation.

This chapter continues the analysis of the *'Revisiting'* framework, but splits the results using the geographical market segmentation outlined in the PDFH.

Chapter 5 compares the performance of the *'Revisiting'* framework against the forecasting framework contained within the PDFH.

Key findings

- Results were very similar to those for the RTEBF segmentation.

- Overall, the model was very accurate.

- However, less accurate for lower level breakdowns

- Particularly inaccurate for season tickets and the London Ticketing Area.

4.1   Initial Tests

As for the *'Revisiting'* segmentation, the back-casts were analysed to find which areas had under- or over-estimated the actual figures.

The differences between the back-casts and the actuals are given below.

Table 10: Back-cast Differences by Segment and Ticket Type

	Full	Seasons	Reduced	Total
LT area	81891	19536	56447	157874
RoSE to LT area	14736	-88278	48747	-24794
LT area to RoSE	12916	-9205	9045	12756
Non-London: Other flows in SE	16825	-69952	24255	-28873
RoC to LT area (<100 miles)	1417	-5356	1596	-2343
LT area to RoC (<100 miles)	1639	-39	-137	1463
RoC to LT area (>100 miles)	4456	-3486	1641	2612
LT area to RoC (>100 miles)	5553	18	4418	9989
Urban Areas (PTE)	1727	-27893	16523	-9643
Non-London: Outside SE (<20 miles)	-3092	-8820	12116	205
Non-London: Outside SE (20-100 miles)	-1326	-23893	2343	-22877
Non-London: Outside SE (>100 miles)	1057	-183	12033	12906
Airport	11400	-882	-6250	4267
Total	149199	-218434	182776	113541

Again, season tickets tend to under-estimate the actuals whereas full price and reduced tickets tend to over-estimate.

4.2   Root Mean Squared Error

The RMSE was found for all segments and all ticket types. These were compared against the RMSE of the naïve back-casts. The table below shows the RMSE for each combination. Those better than the naïve RMSE are marked in green, and those worse than naïve are marked in red.

Table 11: RMSE by Segment and Ticket Type

RAFF RMSE	Full	Seasons	Reduced	All
LT area	12,909	10,126	8,690	27,769
RoSE to LT area	2,396	13,283	7,821	4,624
LT area to RoSE	2,326	1,474	2,018	3,826
Non-London Other flows in SE	2,701	10,755	4,097	4,540
RoC to LT area (<100 miles)	279	833	356	487
LT area to RoC (<100 miles)	269	13	158	237
RoC to LT area (>100 miles)	871	569	536	815
LT area to RoC (>100 miles)	921	24	781	1,531
Urban Areas (PTE)	824	4,140	3,801	3,212
Non-London Outside SE (<20 miles)	1,205	1,419	1,925	1,133
Non-London Outside SE (20-100 miles)	486	3,561	833	3,412
Non-London Outside SE (>100 miles)	238	27	1,843	2,009
Airport	1,730	146	1,017	816
All	23,664	32,268	29,411	27,974

Overall, the back-cast is better than the naïve back-cast. This is also the case for all segments (other than LT area) and for full price and reduced tickets.

As with the *'Revisiting'* segmentation, the back-casts are largely inaccurate for season tickets.

4.3   U Comparison

The U comparison uses a similar approach to the RMSE, but also gives a scale of accuracy.

The table below shows the U figures for the ticket type/segment combinations. A figure of one implies very similar accuracy to the naïve back-cast, and a figure of 0.5 implies that the back-cast is twice as accurate as the naïve back-cast.

Figures between 0.8 and 1 are given in amber, as these are more accurate than the naïve back-cast but not much more accurate.

Table 12: U Comparison by Segment and Ticket Type

RAFF U Comparison	Full	Seasons	Reduced	All
LT area	253.9	0.4	21 3	2.6
RoSE to LT area	0 2	1.2	1.7	0.0
LT area to RoSE	0 6	1.0	0.3	0.2
Non-London Other flows in SE	0.7	1.7	0.9	0.1
RoC to LT area (<100 miles)	4.1	1.6	0.2	0.1
LT area to RoC (<100 miles)	60.1	0.1	0.1	0.2
RoC to LT area (>100 miles)	5 0	1.3	0.0	0.1
LT area to RoC (>100 miles)	13.0	0.4	0.1	0.3
Urban Areas (PTE)	0 0	0.7	0.1	0.0
Non-London Outside SE (<20 miles)	0.1	0.4	0.5	0.0
Non-London Outside SE (20-100 miles)	0 0	0.6	0.0	0.0
Non-London Outside SE (>100 miles)	0.1	0.5	1.2	0.6
Airport	0.7	1.5	0.3	0.0
All	0.5	2.5	0.4	0.1

This table shows that overall, the back-cast is very accurate (the U figure of 0.1 shows that it is 10 times more accurate than the naïve back-cast).

The back-casts for the market segments are also largely very accurate (except the LT Area), with only two of the13 being greater than 0.3.

The back-casts for full price and reduced tickets are not as accurate as these, but are still reasonably accurate, with U figures of 0.5 and 0.4 respectively. However, the back-casts for season tickets were inaccurate.

4.4   Accuracy by Market Share

The table below gives the number of journeys taken in each segment/ticket type combination as a proportion of the total number of journeys taken (excluding 2002/03). The colours are taken directly from table 12.

Table 13: Market Share by Segment and Ticket Type

	Full	Seasons	Reduced	All
LT area	4 6%	13.8%	5.2%	23.6%
RoSE to LT area	3.1%	14.3%	6.2%	23.6%
LT area to RoSE	1 8%	1.0%	2.2%	5.0%
Non-London Other flows in SE	2 8%	3.8%	4.1%	10.7%
RoC to LT area (<100 miles)	0 2%	0.4%	0.5%	1.0%
LT area to RoC (<100 miles)	0 0%	0.0%	0.2%	0.3%
RoC to LT area (>100 miles)	0.4%	0.2%	1.6%	2.2%
LT area to RoC (>100 miles)	0 2%	0.0%	1.4%	1.6%
Urban Areas (PTE)	4 2%	1.7%	4.6%	10.5%
Non-London Outside SE (<20 miles)	2 3%	1.3%	2.3%	5.9%
Non-London Outside SE (20-100 miles)	3 2%	2.1%	5.6%	10.9%
Non-London Outside SE (>100 miles)	0 3%	0.0%	1.8%	2.2%
Airport	1 3%	0.2%	0.9%	2.4%
All	24.4%	38.9%	36.7%	100.0%

The table below shows the proportion of journeys in each colour category (disaggregated by ticket type and market segmentation).

Table 14: RAG Rating by Market Share

Red	Amber	Green
38.6%	4.1%	57.3%

The majority of journeys fall into categories marked green (i.e. the back-casts are fairly accurate).

4.5   Compound Annual Growth Rate

Similar CAGR analysis was performed as in Chapter 3. The table below gives the back-casted CAGR divided by the actual CAGR.

Table 15: CAGR Error Proportions by Segment and Ticket Type

	Full	Seasons	Reduced	All
LT area	-56.6	0 3	-4 0	-0.6
RoSE to LT area	1.4	-0 3	2 0	1.1
LT area to RoSE	1.7	0 8	1 6	1.6
Non-London Other flows in SE	1.6	-0 5	2.1	0.8
RoC to LT area (<100 miles)	-2.7	-0.4	1 3	1.1
LT area to RoC (<100 miles)	-3 3	0 6	0 8	1.1
RoC to LT area (>100 miles)	-0 8	-0 3	1 0	1.2
LT area to RoC (>100 miles)	-1 3	0.4	1 0	1.2
Urban Areas (PTE)	1.1	0 3	1 6	1.1
Non-London Outside SE (<20 miles)	0.7	0 3	1 6	0.9
Non-London Outside SE (20-100 miles)	1.0	0.7	0 9	0.9
Non-London Outside SE (>100 miles)	1.5	0 5	1.7	1.6
Airport	1.2	-0.4	0.7	1.0
All	1.5	-0.6	1.6	1.4

The overall figures for the segments were largely fairly accurate, with 9 of the 13 being within 1% of the actual CAGR and 12 of the 13 being within 3%.

As with the *'Revisiting'* segmentation, the overall figure was only 0.95% different from the actual.

4.6   Annual Growth Rates

The results by ticket type for the PDFH segmentation are exactly the same as for the *'Revisiting'* segmentation.

The table below gives the differences between the back-casted values and the actual values.

Table 16: Annual Growth Rate Difference by Segment

	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
LT area	-2.3%	7.8%	2.1%	3.1%	4.0%	4.3%	3.8%
RoSE to LT area	-3.0%	1.2%	1.1%	-1.7%	-1.3%	0.7%	3.9%
LT area to RoSE	-5.6%	0.4%	7.2%	-1.8%	5.8%	6.8%	4.0%
Non-London: Other flows in SE	-5.8%	1.2%	2.9%	-1.7%	-2.2%	-3.3%	4.4%
RoC to LT area (<100 miles)	-5.9%	-0.9%	9.2%	-10.1%	0.0%	1.4%	9.1%
LT area to RoC (<100 miles)	2.6%	8.4%	8.9%	-11.3%	-3.0%	1.0%	0.0%
RoC to LT area (>100 miles)	-5.3%	8.0%	5.1%	-7.2%	-1.8%	4.4%	3.9%
LT area to RoC (>100 miles)	2.2%	12.2%	2.8%	-6.7%	-2.0%	3.6%	-2.2%
Urban Areas (PTE)	-3.7%	-1.0%	-0.4%	0.8%	3.3%	-0.6%	6.9%
Non-London: Outside SE (<20 miles)	-2.5%	2.5%	2.7%	-0.9%	0.0%	-0.7%	-5.1%
Non-London: Outside SE (20-100 miles)	-3.1%	-0.2%	0.5%	-2.6%	0.7%	-0.3%	1.6%
Non-London: Outside SE (>100 miles)	1.1%	8.1%	4.0%	-3.8%	2.2%	-0.3%	3.1%
Airport	5.0%	0.4%	3.8%	-7.3%	0.1%	-1.4%	-1.6%
All	-2.9%	3.1%	2.0%	-0.8%	1.1%	1.2%	3.1%

Of the 91 market segment/year combinations, the back-casted growth rates for 18 were within 1% of the actuals. 48 were within 3% and 68 were within 5% of the actuals.

As for the *'Revisiting'* framework, it is possible that unpredicatble events such as rail accidents or terrorist attacks could affect growth rates for particular years, but that the growth rates afterwards would return to normal.

There is insufficient evidence to suggest that this is happening here, although looking across more years could be useful in investigating this further.

4.7   Conclusions

The *'Revisiting'* framework performs in a similar way whether it is split by PDFH or *'Revisiting'* market segmentation. It is reasonably accurate overall, but has areas of inaccuracy, particularly for London and for season tickets.

Looking only at the market segments, all segments are more accurate than the naïve back-cast (other than the London Ticketing Area for the PDFH segmentation). The overall segment U figures suggest that for both segmentations, the back-casts for each segment (excluding LT Area) are fairly accurate, with all the U figures being 0.6 or lower (implying that the *'Revisiting'* back-casts are always at least 1.6 times as accurate as the naïve back-cast).

The CAGRs for the PDFH segmentation were slightly better than those for the *'Revisiting'* segmentation, with 12 of 13 segment CAGRs being within 3% of the actuals, compared with 8 of 10 for the *'Revisiting'* segmentation.

Disaggregating by both market segment and ticket type, the back-casts for 38.6% for journeys were less accurate than the naïve back-casts. The equivalent figures when disaggregating only by ticket type and only by market segment were 38.9% and 23.6% respectively.

Chapter 5: Comparing *'Revisiting'* and PDFH back-casts of journeys

The previous chapters have focussed on how accurate the *'Revisiting'* model is. The analysis has shown that the *'Revisiting'* back-casts have been accurate at a high level, but become less accurate when split into the various categories.

This chapter compares the accuracy of the *'Revisiting'* back-casts with the accuracy of the PDFH back-casts to find which is the most accurate.

Key findings

- The 'Revisiting' model was compared with the PDFH model.

- Overall, the 'Revisiting' model was found to be more accurate.

- However, PDFH model was more accurate for full price and season tickets.

- PDFH was more accurate for earlier years while 'Revisiting' model tended to be more accurate for later years.

5.1   Root Mean Squared Error

The RMSE was found for each ticket type and for each segment for both models. The smallest error for each is shaded in the tables below.

Table 17: RMSE Comparison by Ticket Type

	'Revisiting'	PDFH
Full	23,664	19,639
Seasons	32,268	12,734
Reduced	29,411	32,806
All	**27,974**	**49,358**

Table 18: RMSE Comparison by 'Revisiting' Segment

	'Revisiting'	PDFH
LSEE to LSEE	26,556	36,274
LSEE to Core	1,299	1,697
Core to LSEE	571	1,770
LSEE to Other	839	2,768
Other to LSEE	1,236	3,590
Core to Core	3,702	4,838
Core to Other	4,644	6,397
Other to Core	4,069	21,033
Other to Other	5,393	15,563
To Airports	706	1,218
All	**27,974**	**49,358**

Table 19: RMSE Comparison by PDFH Segment

	'Revisiting'	PDFH
LT area	27,769	53,439
RoSE to LT area	4,624	9,997
LT area to RoSE	3,826	5,066
Non-London: Other flows in SE	4,540	13,669
RoC to LT area (<100 miles)	487	1,473
LT area to RoC (<100 miles)	237	663
RoC to LT area (>100 miles)	815	3,439
LT area to RoC (>100 miles)	1,531	3,275
Urban Areas (PTE)	3,212	19,990
Non-London: Outside SE (<20 miles)	1,133	6,382
Non-London: Outside SE (20-100 miles)	3,412	22,137
Non-London: Outside SE (>100 miles)	2,009	2,469
Airport	816	2,885
All	**27,974**	**49,358**

Overall, the *'Revisiting'* back-cast was the more accurate, with a much smaller average error, almost half that of the PDFH. However, the PDFH model was most accurate for two of the three ticket types.

For both the *'Revisiting'* and the PDFH segmentations, the RMSE was smaller

for each segment for the *'Revisiting'* framework. This is unexpected, as the frameworks were designed alongside their respective segmentations, and so we would expect the PDFH framework to perform better for the PDFH segmentation. Instead, the *'Revisiting'* framework performs better for both.

5.2   U Comparison

The U comparison gives a very similar picture, with the *'Revisiting'* model being more accurate overall but the PDFH model being more accurate for full price and season tickets.

For both segmentations, the *'Revisiting'* framework was universally more accurate. This depends on the needs of the user.

Table 20: U Comparison by Ticket Type

	'Revisiting'	PDFH
Full	0.5	0.3
Seasons	2.5	0.4
Reduced	0.4	0.5
All	**0.1**	**0.2**

Table 21: U Comparison by *'Revisiting'* Segment

	'Revisiting'	PDFH
LSEE to LSEE	0.6	1.1
LSEE to Core	0.6	0.9
Core to LSEE	0.1	1.1
LSEE to Other	0.1	1.1
Other to LSEE	0.1	1.1
Core to Core	0.5	0.8
Core to Other	0.3	0.7
Other to Core	0.0	0.6
Other to Other	0.1	0.8
To Airports	0.1	0.2
All	**0.1**	**0.2**

Table 22: U Comparison by PDFH Segment

	'Revisiting'	PDFH
LT area	2.6	9.4
RoSE to LT area	0.0	0.2
LT area to RoSE	0.2	0.4
Non-London: Other flows in SE	0.1	0.7
RoC to LT area (<100 miles)	0.1	0.9
LT area to RoC (<100 miles)	0.2	1.0
RoC to LT area (>100 miles)	0.1	1.2
LT area to RoC (>100 miles)	0.3	1.1
Urban Areas (PTE)	0.0	0.6
Non-London: Outside SE (<20 miles)	0.0	0.4
Non-London: Outside SE (20-100 miles)	0.0	0.9
Non-London: Outside SE (>100 miles)	0.6	0.9
Airport	0.0	0.5
All	0.1	0.2

Together, the RMSE and U figures confirm what previous analysis has shown, that the *'Revisiting'* back-casts are very accurate overall, but are less accurate for the low level breakdowns of ticket type. For these, PDFH is often more useful.

However, each of the U figures for PDFH ticket types is 0.5 or less, whereas for *'Revisiting'* it reaches 2.5 for season tickets.

For each market segment, the *'Revisiting'* back-cast is more accurate than the PDFH back-cast, no matter which segmentation is used.

Whilst the *'Revisiting'* model is more accurate overall, the PDFH model is by no means inaccurate. The U figure of 0.2 means that the PDFH model is still very accurate.

5.3   Compound Annual Growth Rate

The CAGR was compared for the two back-casting methods. The results are given below.

Table 23: CAGR by Ticket Type

	Actual	'Revisiting'	PDFH
Full	4.0%	6.1%	1.0%
Seasons	0.7%	-0.4%	1.2%
Reduced	3.5%	5.5%	0.3%
All	2.5%	3.5%	0.8%

The table shows again that the *'Revisiting'* model is more accurate overall than the PDFH model. The overall difference between back-casted CAGR and actual CAGR for the PDFH model is 1.7%, larger than the 1.0% of the *'Revisiting'* model.

Table 24: CAGR by *'Revisiting'* Segment

	Actual	'Revisiting'	PDFH
LSEE to LSEE	1.0%	2.3%	1.2%
LSEE to Core	4.5%	7.1%	-0.8%
Core to LSEE	4.0%	3.4%	-1.3%
LSEE to Other	5.7%	5.9%	-1.3%
Other to LSEE	4.0%	5.9%	-1.3%
Core to Core	8.9%	2.7%	-0.2%
Core to Other	5.3%	8.9%	0.2%
Other to Core	6.0%	7.3%	0.7%
Other to Other	5.5%	3.4%	-0.2%
To Airports	5.4%	3.6%	1.9%
All	2.5%	3.5%	0.8%

Table 25: CAGR by PDFH Segment

	Actual	'Revisiting'	PDFH
LT area	-2.1%	1.2%	2.0%
RoSE to LT area	2.1%	2.2%	1.0%
LT area to RoSE	4.0%	6.4%	0.6%
Non-London: Other flows in SE	3.7%	3.1%	-0.3%
RoC to LT area (<100 miles)	3.7%	4.1%	-1.0%
LT area to RoC (<100 miles)	6.5%	7.5%	-0.7%
RoC to LT area (>100 miles)	4.2%	5.2%	-1.7%
LT area to RoC (>100 miles)	6.0%	7.5%	-1.4%
Urban Areas (PTE)	6.1%	6.9%	0.6%
Non-London: Outside SE (<20 miles)	5.1%	4.5%	1.6%
Non-London: Outside SE (20-100 miles)	6.2%	5.7%	-0.8%
Non-London: Outside SE (>100 miles)	3.3%	5.3%	-0.9%
Airport	4.8%	4.7%	0.9%
All	2.5%	3.5%	0.8%

As with the U comparisons, the *'Revisiting'* back-casts are largely more accurate than the PDFH back-casts, no matter which segmentation is used.

The only exception to this is for the LSEE to LSEE segment.

5.4   Annual Growth Rates

The annual growth rates for both back-casts were analysed to find which were closest to the actual growth rates.

The tables below show the difference between the back-casted growth rate and the actual growth rate. The smallest absolute difference is shaded.

Table 26: Annual Growth Rates by Ticket Type – *'Revisiting'* Framework

'Revisiting'	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
Full	0.9%	5.3%	6.0%	0.7%	0.7%	-0.8%	2 0%
Seasons	-8.4%	1.1%	-6.6%	0.3%	2.4%	3.2%	1 5%
Reduced	0.9%	3.6%	7.7%	-3.2%	0.2%	0.3%	4.4%
All	**-2.9%**	**3.1%**	**2.0%**	**-0.8%**	**1.1%**	**1.2%**	**3.1%**

Table 27: Annual Growth Rates by Ticket Type – PDFH Framework

PDFH	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
Full	-2.2%	1.1%	1.6%	-2.2%	-4.0%	-8.6%	-6.3%
Seasons	0.2%	4.0%	2.5%	-1.3%	-3.4%	-0.8%	2.4%
Reduced	1.5%	1.6%	2.7%	-7.5%	-5.7%	-9.1%	-5.5%
All	**0.1%**	**2.5%**	**2.4%**	**-3.7%**	**-4.4%**	**-5.7%**	**-2.7%**

Table 28: Annual Growth Rates by *'Revisiting'* Segment – *'Revisiting'* Framework

'Revisiting'	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
LSEE to LSEE	-3.2%	3.8%	2.3%	0.0%	1.0%	1.7%	3.6%
LSEE to Core	-2.4%	11.7%	3.9%	-1.6%	-1.5%	7.6%	0.6%
Core to LSEE	-4.4%	11.4%	2.5%	-12.0%	-4.2%	2.9%	0.1%
LSEE to Other	3.8%	8.0%	3.6%	-8.7%	-3.0%	-1.0%	-1.3%
Other to LSEE	-5.1%	3.1%	7.0%	-6.3%	2.7%	3.1%	8.9%
Core to Core	1.2%	-4.0%	-7.3%	-7.2%	-1.9%	-17.7%	-6.4%
Core to Other	0.6%	2.7%	5.2%	-1.0%	6.3%	-2.0%	13.6%
Other to Core	-2.1%	-0.2%	0.7%	-0.7%	3.4%	-0.1%	8.1%
Other to Other	-5.8%	0.8%	-0.6%	-0.4%	-1.8%	4.3%	-10.2%
To Airports	4.4%	-1.9%	3.1%	-9.7%	-1.9%	-4.5%	-2.1%
All	**-2.9%**	**3.1%**	**2.0%**	**-0.8%**	**1.1%**	**1.2%**	**3.1%**

Table 29: Annual Growth Rates by *'Revisiting'* Segment – PDFH Framework

PDFH	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
LSEE to LSEE	1.5%	5.1%	5.1%	-1.9%	-4.0%	-3.9%	-0.6%
LSEE to Core	-1.0%	7.0%	-1.7%	-11.0%	-11.5%	-8.7%	-9.5%
Core to LSEE	-2.2%	9.0%	-1.7%	-12.5%	-9.9%	-6.5%	-12.3%
LSEE to Other	-3.7%	4.9%	-3.9%	-15.0%	-10.7%	-9.3%	-11.3%
Other to LSEE	-1.8%	2.1%	-0.6%	-11.4%	-9.8%	-7.1%	-8.0%
Core to Core	-3.7%	-5.2%	-4.8%	-8.1%	-4.4%	-20.4%	-16.8%
Core to Other	-2.9%	-3.1%	-3.5%	-7.9%	-2.6%	-9.9%	-5.9%
Other to Core	-3.8%	-7.3%	-4.4%	-6.7%	-3.1%	-8.6%	-2.8%
Other to Other	-4.6%	-3.6%	-3.8%	-5.8%	-6.2%	-8.8%	-6.4%
To Airports	8.5%	-2.7%	1.2%	-11.6%	-4.8%	-8.4%	-5.7%
All	**0.1%**	**2.5%**	**2.4%**	**-3.7%**	**-4.4%**	**-5.7%**	**-2.7%**

Table 30: Annual Growth Rates by PDFH Segment – *'Revisiting'* Framework

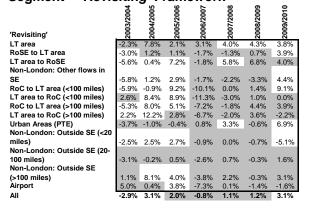| 'Revisiting'	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
LT area	-2.3%	7.8%	2.1%	3.1%	4.0%	4.3%	3.8%
RoSE to LT area	-3.0%	1.2%	1.1%	-1.7%	-1.3%	0.7%	3.9%
LT area to RoSE	-5.6%	0.4%	7.2%	-1.8%	5.8%	6.8%	4.0%
Non-London: Other flows in SE	-5.8%	1.2%	2.9%	-1.7%	-2.2%	-3.3%	4.4%
RoC to LT area (<100 miles)	-5.9%	-0.9%	9.2%	-10.1%	0.0%	1.4%	9.1%
LT area to RoC (<100 miles)	2.6%	8.4%	8.9%	-11.3%	-3.0%	1.0%	0.0%
RoC to LT area (>100 miles)	-5.3%	8.0%	5.1%	-7.2%	-1.8%	4.4%	3.9%
LT area to RoC (>100 miles)	2.2%	12.2%	2.8%	-6.7%	-2.0%	3.6%	-2.2%
Urban Areas (PTE)	-3.7%	-1.0%	-0.4%	0.8%	3.3%	-0.6%	6.9%
Non-London: Outside SE (<20 miles)	-2.5%	2.5%	2.7%	-0.9%	0.0%	-0.7%	-5.1%
Non-London: Outside SE (20-100 miles)	-3.1%	-0.2%	0.5%	-2.6%	0.7%	-0.3%	1.6%
Non-London: Outside SE (>100 miles)	1.1%	8.1%	4.0%	-3.8%	2.2%	-0.3%	3.1%
Airport	5.0%	0.4%	3.8%	-7.3%	0.1%	-1.4%	-1.6%
All	**-2.9%**	**3.1%**	**2.0%**	**-0.8%**	**1.1%**	**1.2%**	**3.1%**

Table 31: Annual Growth Rates by PDFH Segment – PDFH Framework

PDFH	2003/2004	2004/2005	2005/2006	2006/2007	2007/2008	2008/2009	2009/2010
LT area	4.8%	9.9%	11.1%	4.4%	-0.4%	-1.9%	1.3%
RoSE to LT area	1.1%	2.7%	2.2%	-5.7%	-5.6%	-2.3%	0.2%
LT area to RoSE	-4.6%	-0.4%	1.3%	-3.3%	-4.2%	-7.8%	-4.7%
Non-London: Other flows in SE	-3.3%	1.0%	-0.8%	-5.9%	-8.5%	-8.6%	-2.4%
RoC to LT area (<100 miles)	-2.4%	0.8%	3.3%	-13.0%	-10.6%	-5.9%	-4.5%
LT area to RoC (<100 miles)	-4.2%	5.7%	-3.4%	-19.4%	-10.8%	-10.2%	-8.0%
RoC to LT area (>100 miles)	-2.4%	6.2%	-2.4%	-11.0%	-10.3%	-7.7%	-13.0%
LT area to RoC (>100 miles)	-2.2%	6.3%	-4.4%	-14.4%	-13.0%	-10.1%	-13.9%
Urban Areas (PTE)	-5.8%	-8.2%	-4.8%	-6.7%	-2.2%	-9.6%	-0.9%
Non-London: Outside SE (<20 miles)	-1.1%	0.6%	0.0%	-6.6%	-4.9%	-8.4%	-4.4%
Non-London: Outside SE (20-100 miles)	-4.0%	-5.3%	-5.3%	-6.9%	-5.8%	-11.1%	-10.2%
Non-London: Outside SE (>100 miles)	-1.8%	4.0%	-1.7%	-7.9%	-5.2%	-8.7%	-7.3%
Airport	5.4%	-2.9%	-0.4%	-12.0%	-5.3%	-6.6%	-5.9%
All	**0.1%**	**2.5%**	**2.4%**	**-3.7%**	**-4.4%**	**-5.7%**	**-2.7%**

These tables show that overall, the PDFH back-cast is more accurate for earlier years, but the *'Revisiting'* back-cast is more accurate for many of the later years (other than 2009/10).

For both segmentations, the *'Revisiting'* back-casts are more accurate than the PDFH back-casts for the majority of year/segment combinations.

5.5 Conclusions

The analysis shows that the different back-casting methods are more accurate for different areas. Consequently, it is impossible to state categorically which is superior.

As an overall back-casting system, the *'Revisiting'* method is more accurate using each of these measures. It has a lower RMSE, a lower U score and the

CAGR is much closer to the actual CAGR.

However, the *'Revisiting'* back-casts have some limitations, particularly when splitting the journey numbers into smaller groups. For ticket type, the *'Revisiting'* back-casts are often less accurate than the PDFH back-casts. In fact, for season tickets, the back-casts are worse even than the naïve forecast, meaning they are very inaccurate.

The rough trend appears to be that for annual growth rates, the *'Revisiting'* back-casts are more accurate than the PDFH back-casts for later years, although the PDFH back-casts are more accurate for earlier years. This is true (to some extent) for each ticket type.

The *'Revisiting'* back-casts are more accurate than the PDFH back-casts for the majority of ticket type/year and ticket type/segment combinations.

Annex A: Glossary

This section gives details on the various technical terms and techniques used in the report.

Back-cast: A back-cast is an exercise to test model fit. It uses known data for past years (such as GDP, employment figures etc.) to work out what the predicted number of journeys would have been. This can be compared with the actual number of journeys to test the accuracy of the forecasting model.

Ticket Type: Three ticket types are analysed in this report; full, season and reduced.

Market Segment: Market segments are different types of journey. Many market segments are defined by their end points (such as London Ticketing Area to rest of South East), whereas others are defined by area and length of journey (such as Non-London: Outside of South East (<20 miles)).

The *'Revisiting'* study used a different market segmentation to the PDFH (referred to in this report as the *'Revisiting'* segmentation). These market segmentations are not directly comparable, so separate analyses have been performed on these.

Naïve Back-cast: The naïve back-cast is a very basic back-cast, used mainly for comparison. For the purposes of this report, the naïve back-cast assumes that each year after 2002/03 will have exactly the same number of journeys as in 2002/03.

The naïve back-cast represents not forecasting at all. This gives a good base for comparison for other back-casts, as if a back-cast is worse than the naïve back-cast, this shows that it is very inaccurate.

Root Mean Squared Error: The RMSE is an average error for a model, calculated by:

Equation 1

$$RMSE = \sqrt{\frac{\sum_{i=1}^{NYears}(Backcast_i - Actuals_i)^2}{NYears}}$$

Where NYears is the number of years being investigated, Naive$_i$ is the naïve forecast for year i and Back-cast$_i$ is the back-cast for year i.

A large RMSE implies that the model is not particularly accurate.

U Comparison: The U comparison compares a back-cast against the naïve back-cast, and gives a figure to indicate which is better. The U figure is given by:

Equation 2

$$u = \frac{\sum_{i=1}^{NYears}\frac{(Backcast_i - Actuals_i)^2}{Actuals_i}}{\sum_{i=1}^{NYears}\frac{(Naive_i - Actuals_i)^2}{Actuals_i}}$$

Where NYears is the number of years, Back-cast$_i$ is the back-cast for year i, Naïve$_i$ is the naïve back-cast for year i and Actuals$_i$ is the actual number of journeys for year i.

A U figure of 1 implies that the model back-cast is equally as accurate as the naïve back-cast. A figure of less than one shows that it is more accurate than the naïve back-cast.

It should be noted that a U figure of less than one does not automatically mean that a model is accurate. Rather, it only means that it is better than not forecasting at all. If a U figure is less than one but still close (for example 0.95), this means that whilst it is superior to the naïve back-cast, it is not particularly accurate.

Compound Annual Growth Rate: CAGR is a measure of average compound growth over the years. It is given by:

Equation 3

$$CAGR = \left(\frac{Final}{Base} \right)^{\frac{1}{NYears}} - 1$$

Where final is the number of journeys performed in the last year, base is the number of journeys performed in 2002/03 and NYears is the number of years.

This gives a good measure of overall growth, but does not take into account the fluctuations of individual years.

Annex B: Key Factors Affecting Error

Additional analyses were performed to find which of the factors were most closely linked to particularly large or small errors in the back-casted values. These analyses involved performing a multiple linear regression on the error (difference between the back-casted value and the actual value) for each year/ticket type/market segment combination and finding which of these factors were most significant in predicting the size of the error.

The results are presented below:

Table 32: Key Factors Affecting Error

Model	Segmentation	Factor	Comment
'Revisiting'	RTEBF	'LSEE to LSEE'	Large errors
		Year	Later years have larger errors
'Revisiting'	PDFH	'LT Area'	Large errors
		'RoSE to LT Area'	Large errors
		'Non-London: Other flows in SE'	Large errors
		Year	Later years have larger errors
PDFH	RTEBF	'LSEE to LSEE'	Large errors
		'Other to Core'	Large errors
		Year	Later years have larger errors
		'Other to Other'	Large errors
PDFH	PDFH	'LT Area'	Large errors
		Year	Later years have larger errors
		'Non-London: Outside SE (20-100 miles)'	Large errors
		'Urban Areas (PTE)'	Large errors