ARUP

Oxera

How should the revised elasticity-based framework be implemented?

Guidance report

Prepared for
the Department for Transport,
Transport Scotland, and
the Passenger Demand Forecasting Council

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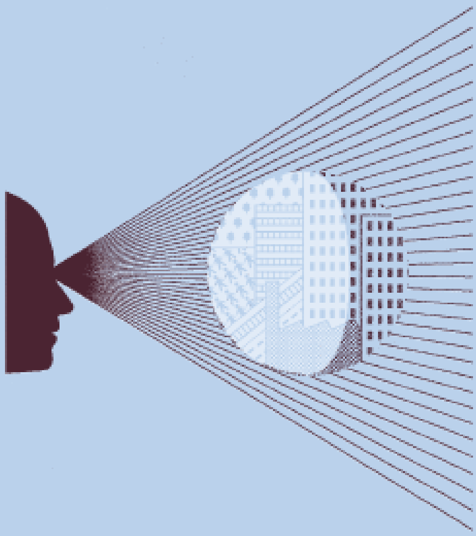

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1 Introduction

This report provides practical guidance on how to implement the revised elasticity-based framework produced by Oxera and Arup for the Department for Transport (DfT), Transport Scotland and the Passenger Demand Forecasting Council (PDFC). The report explains how to use the new framework to produce demand forecasts for specific rail passenger markets in Great Britain. It aims to be accessible to anyone in the industry and permit them to produce forecasts even if they have limited experience of demand forecasting; nevertheless, a familiarity with the existing Passenger Demand Forecasting Handbook (PDFH) approach will aid the reader.[1] Throughout the guidance, detailed worked examples are given, together with explanations using diagrams and charts.

In order to achieve the objective of the study—to improve the quality of rail demand forecasting—the enhancements recommended by the project team need to be implemented correctly by those producing forecasts.

The intention is that the framework can be used in any context, whether in strategic planning by the DfT or in calculating the impact of a limited change in a single market, such as an increase in fares by a train operating company (TOC). However, due to data restrictions or restrictions from the model, there are certain contexts where the framework cannot be applied—which include, for example, ticket-type choice below full fare, reduced fare and season tickets, impacts that last less than one year, the effect of changing restrictions on tickets, modelling disaggregate modal choice decisions, etc.

As part of this study, a number of reports have been produced, detailed below, which form key elements in the formulation of the overall final forecasting framework, and are referenced a number of times here. It is not necessary to read these in order to apply the framework, but they do provide an explanation of how the revised forecasting approach has been developed.

Reports prepared by Oxera and Arup for the 'Revisiting the Elasticity-Based Framework' study:

- 'What are the findings from the econometric analysis?' (the *Findings* report)
  - 'Is the data capable of meeting the study objectives?' (the *Data capability* report)
  - 'How has the preferred econometric model been derived?' (the *Econometric approach* report)
  - 'What are the key issue for model specification?' (the *Model specification* report)
  - 'How has the market for rail passenger demand been segmented?' (the *Market segmentation* report)
  - 'Does quality of service affect demand?' (the *Service quality* report)
- 'How should the revised elasticity-based forecasting framework be implemented?' (the *Guidance* report)

With forecasting, and modelling more generally, the quality of the outcome is determined in part by the availability and quality of the data used as an input. Section 2 indicates which variables are needed to produce forecasts and suggests some sources for obtaining them.

Section 3 then briefly reviews the existing PDFH approach to GB rail passenger demand forecasting, and its core structure, which forms the basis for the revised framework. This

[1] ATOC (2009), 'Passenger Demand Forecasting Handbook', version 5, August.

section should be familiar to experienced forecasters of rail passenger demand, but will be beneficial to those who are new to the subject.

Two main changes to the forecasting approach have been incorporated in this revisit of the elasticity-based framework. The first is the inclusion of dynamic effects, so that a richer pattern of lagged effects can be modelled. The most recent version of the PDFH explicitly discusses lags and recommends paths of percentage adjustments to the long-run effect.[2] However, the revised approach resulting from this current study allows more flexible dynamics, with the adjustment paths derived from the empirical findings of the study team. The second main enhancement to the existing PDFH approach is the inclusion of variable elasticities, which change depending on the base level of the demand driver in question.

These two revisions are covered in sections 4 and 5 respectively, and the combination of the two is addressed in section 5. Section 6 provides a summary of the guidance, and a number of technical appendices give further detail for the interested reader.

[2] See ATOC (2009), op. cit., Section B12.

2 Data

Before any forecasting can be undertaken, it is necessary to obtain the input data on which the forecasts will be based. This stage will be familiar to readers who are used to producing forecasts of passenger rail demand. However, in some aspects of the revised forecasting framework, the data requirements are different to those in the PDFH. This section explains the data that is required for producing forecasts, together with suggested sources for this data.

2.1 Demand data

The starting point for forecasting demand for rail services is base data. In an elasticity-based framework, which is designed to model relatively small changes in demand drivers,[3] knowing the base demand is an important starting point.

It is expected that an individual wishing to produce rail demand forecasts will have access to LENNON data. The LENNON database allows up-to-date rail ticket sales data to be downloaded via a secure web-based portal. Access to LENNON should allow a forecaster to obtain information on the current total journeys and revenues on the required flows. Alternative sources could include an organisation's own information, specialist rail software such as MOIRA, or passenger surveys. The use of LENNON data is recommended because it should capture most journeys made, and forms the basis on which the parameter values were estimated.[4]

On its own, data on historical demand is insufficient to produce elasticity-based forecasts of the demand for passenger rail travel.[5] Therefore, the forecasting approach discussed here incorporates projected or planned changes in demand *drivers*. Based on these drivers, factors that will increase or decrease demand can be applied within the forecasting framework to produce more informed forecasts. Hence, data on changes in the relevant demand drivers—at least for those whose effect is to be analysed—is also required.[6]

2.2 Demand drivers

Demand drivers are the variables that influence passenger behaviour and will result in a change in the number of passengers travelling by rail. There are two main types: *endogenous* drivers (ie, those determined or influenced by the rail industry itself, such as fares), and *exogenous* drivers (eg, socio-economic factors, such as population).

2.2.1 Endogenous drivers

Historical values and forecasts of endogenous variables that are determined by a TOC (such as fare) will often be available 'in house'. For those without access to such information, other industry sources will need to be consulted, or appropriate assumptions made. For the fare variable, franchise-wide base values are obtainable from industry sources, such as the Office of the Rail Regulator's (ORR) National Rail Trends. For flow-specific values, an estimate of

[3] A demand *driver* is a variable that affects the level of passenger demand by influencing passengers' travel and mode choice decisions.

[4] For further discussion of the sources of demand data, see ATOC (2009), op. cit., Section A3.5.

[5] There are relatively sophisticated techniques that can be used to model demand for passenger rail travel using previous data only. However, these models are not elasticity-based and hence have not been examined by this study.

[6] For some variables and specifications, it may be necessary to know the levels of the demand drivers in addition to the growth rates. These cases include variables such as car journey time, where the position on the speed–flow curve may be required in order to understand how it changes, and demand specifications with variable elasticities (see section 5).

fare could be obtained from LENNON data by dividing total revenues by total journeys.[7] As a further example, generalised journey times can be calculated using MOIRA (see Section B4 of the PDFH v5 for more details). Table 2.1 suggests potential sources of information for base values and forecast data on these endogenous variables.

Table 2.1 Possible base and forecast data sources

Variable	Source	Other potential source
Fare	Internal information, Internet	LENNON data can be used to calculate yield and approximation of fare
Performance	Internal information, Internet	National Rail Trends, Network Rail
Generalised journey time (GJT)	Internal information, Internet	MOIRA
Service quality index	DfT	TOCs may create their own from their surveys

Source: Oxera.

The objective of much forecasting work is to examine the effect of changes in endogenous variables in order to inform business and operational decisions. As a result, the forecast paths of endogenous variables can usually be chosen by the modeller since they are the inputs under analysis. Alternatively, known policy changes or regulatory targets can be assumed.

2.2.2 Exogenous drivers

For changes in the exogenous drivers, a range of third-party data sources is likely to be needed; socio-economic variables are usually forecast by specialists. Table 2.2 suggests various potential sources for the exogenous demand drivers.

Table 2.2 Possible base and forecast data sources

Variable	Source	Comments
Disposable income per capita at origin	Oxford Economics or another independent economic forecaster	Available to PDFC members in due course
GVA per employee at destination	Oxford Economics or another independent economic forecaster	Available to PDFC members in due course
Employment at destination	Oxford Economics or another independent economic forecaster/NTEM database	Forecasts available until 2041 from NTEM
Population	Oxford Economics or another independent economic forecaster/NTEM database	Forecasts available until 2041 from NTEM
Car ownership	NTEM database	Forecasts available until 2041 from NTEM
Car cost	DfT	Tables of vehicle running costs (eg, from the AA) are an alternative

Note: Not every segment requires the inclusion of all these variables when forecasting. Oxera understands that in future PDFC members should have access to forecasts from Oxford Economics for a range of variables, and at different levels of aggregation, through their membership.
Source: Oxera.

[7] Technically this gives yield, but it is an approximation of the average fare.

From these sources it will be important to collect both the base level of the variable (to match with base demand) and the forecast levels, so that the relative change (tomorrow's level relative to today's) can be determined.[8]

With quantitative work, data availability or quality can often be a problem. The sources in Table 2.2 should be of adequate quality; however, if they cannot be accessed by a forecaster, alternatives can be sought (although these may be less accurate for the purpose at hand). In some cases, consistency may need to be checked, particularly in terms of geographic regions corresponding to those on which the parameters were estimated.

2.2.3 Additional data requirements of the revised approach

All of the data described so far has been required to produce forecasts under the existing PDFH approach. Under the revised approach recommended by this project, there are some additional data requirements.

In geographic regions for which the approach recommends the use of variable elasticities[9] in the demand specification, the absolute level of each variable must also be known.[10] For example, it is not enough to know that fares are going to increase by 5% per annum; the actual level of the fare in pounds sterling at constant 2007 prices is also needed. (For more information on this, see Appendix 5, which presents average fares by segment.)

Another enhancement in the framework is a more formal incorporation of dynamic effects. For dynamics to be included in forecasting, it may be necessary to have historical as well as current data on demand and demand drivers. This depends on whether demand drivers have changed in preceding periods. If (as is likely) this is the case then, without the historical data, forecasts will be missing lagged effects in their first few periods. Crucially, it is necessary for at least as many periods of data to be available as the number of lags that are being included in the forecasting. In most cases, three years of data will be enough.

2.2.4 Variables that differ from the existing PDFH approach

The forecasting framework developed for the 'Revisiting the Elasticity-Based Framework' study uses both additional demand drivers and measures of existing demand drivers that are not used in the current PDFH approach. These measures include:

– a passenger performance measure (PPM), to measure performance;
– service quality indices;
– car cost;
– disposable income;
– employment at destination.

The sources for forecasts of these variables are detailed in Tables 2.1 and 2.2 above, but they are not the same demand drivers, or measures of demand drivers, as those contained in the PDFH. The use of PPM should be reasonably straightforward since TOCs already produce forecasts of this variable, or these are contained in other public sources, such as franchise specifications.

The use of service quality indices in the forecasting framework is more complex and requires the production of forecasts of how passengers will answer National Passenger Survey (NPS) questions in the future. These service quality indices could be forecast by considering, for example, station improvements; alternatively, where a franchise has explicit NPS targets, these should be forecastable. When the NPS scores are determined, they can be aggregated and weighted to provide forecasts of the service quality index. As explained in

[8] Again, note that absolute levels (both current and past) of some variables are required for forecasting when using certain variable elasticity demand specifications (see section 5).
[9] In particular, where demand responses vary by the level of the demand driver.
[10] For further explanation of this additional requirement, see section 5.

the *Service quality report*, the impact of performance is removed from the indices by adopting appropriate weights.

The car cost variable developed for this study is more sophisticated than the 'pump price' measure currently used in the PDFH. This variable can be forecast using data from the National Transport Model (NTM).[11]

In the absence of this measure, forecasts of pump prices or vehicle running costs could be used instead since the correlation between the aggregate growth rates is high (approximately 0.9) (see Figure 2.1). However, because the car cost variable has been constructed using a measure of car journey time based on NTEM data and speed–cost curves from the NTM, this variable is available at a far more disaggregate level than the pump price.

Figure 2.1 Pump price and car cost

Source: Oxera analysis.

The preferred measure of income is often disposable income per capita at the origin of the flow. This can be forecast by independent economic forecasters.

This has substantial implications for comparing elasticities between the new framework and the PDFH because it is well established that changes in employment often lag changes in GDP, and macroeconomic forecasts take this into account. Personal disposable income forms only part of GDP, and therefore the relationship between disposable income and employment is likely to differ from that between GDP/GVA and employment.

The relationship between levels of GDP, personal disposable income and employment is likely to be positive over time. However, around the turning points in the economic cycle, the lags in the relationships are likely to be important. For example, in the early stages of a recovery, personal disposable income and employment may continue to fall after GDP/GVA begins to rise, and conversely for a downturn. Therefore, it might be expected that the elasticities on personal disposable income and GDP/GVA are different, and that the dynamic adjustment of rail demand to changes in these variables may vary.

The relationship between these variables (particularly employment and GDP/GVA) may be changing over time as there is some evidence that employers have hoarded labour to a

[11] Readers wishing to use this variable in forecasts should discuss it with both the Integrated Transport Economics and Appraisal and Rail Service Analysis teams within the DfT (itea@dft.gsi.gov.uk, and railtag@dft.gsi.gov.uk).

greater degree in the recent recession than in previous recessions (see the 2009 Pre-Budget Report for more discussion).[12]

Finally, the precise measure of employment at destination is important, particularly around London. The study has used a measure of central London employment, provided by Oxford Economics, and this has been applied to those stations included in London BR or London Zone R1 in LENNON. This is conceptually consistent with the current PDFH approach and should be continued when producing forecasts using this framework. Outside of Central London, employment forecasts at the most disaggregate level available (eg, TEMPRO zone) are appropriate. If TEMPRO forecasts are not available or considered not to be appropriate, other forecasts may be used instead, such as regional employment.[13]

2.2.5 'Missing' demand drivers

There are two types of demand drivers which could be thought of as 'missing' from the revised forecasting framework. The first includes drivers that have been controlled for in the econometric analysis through the service quality index (eg, crowding, etc), which means that the presented elasticities are not affected by these changes. However, changes in these drivers from the level contained in the dataset will still have an effect on demand, and so should be modelled separately.

The other category of 'missing' demand driver is where there is little data available (eg, time/cost of bus/coach/aviation). The effect of these demand drivers will be included within the presented elasticities and hence any changes in these demand drivers will not be picked up—ie, forecasts produced using the presented elasticities will only be accurate if the relationship between the time/cost of these modes and the demand drivers contained in the forecasting framework remains the same.

2.3 Elasticities

In addition to the data described above, the other key pieces of information needed for the forecasting are the elasticities. These are values that describe mathematically how demand responds to changes in the demand drivers. These elasticities are provided by this project, and can usually be taken straight from Table A3.1 in Appendix 3, although in some cases a more complex process is needed to obtain them (see Appendix 2).

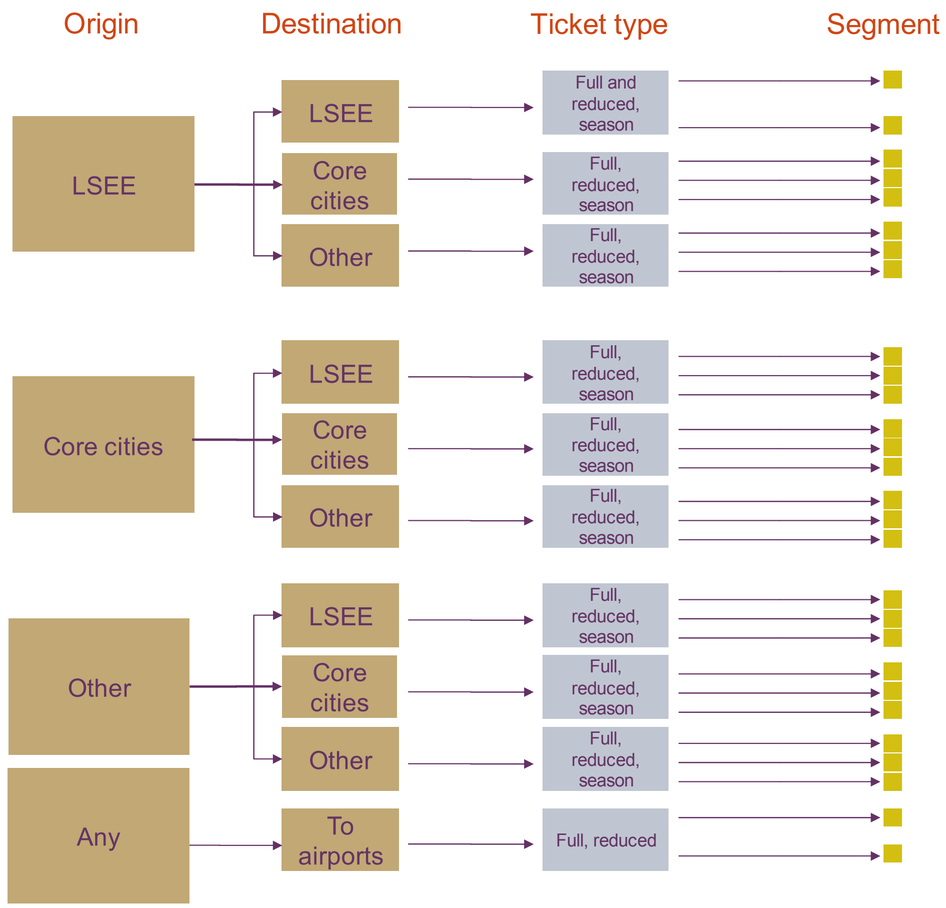
One reason why the elasticities cannot simply be 'looked up' is that they differ by geographic segment. The elasticities from the 'Revisiting the Elasticity-Based Framework' study are split into segments based on the locations of the origin and destination stations. Origins and destinations are classed as being in London, South-East and East of England (LSEE); non-London core cities; airport flows; or other (which covers all remaining flows).[14] Each combination of these origin and destination types (split further by ticket type) is classed as a separate segment. Figure 2.2 below shows the disaggregation of the total rail market in Great Britain into segments.

[12] HM Treasury (2009), 'Securing the recovery: growth and opportunity: Pre-Budget Report', December, Box A4.

[13] The exception is where the segment contains variable elasticities, in which case TEMPRO zone employment forecasts must be used to derive the elasticity (see section 5 for more details).

[14] Details of the airport flows and the non-London core cities are given in Appendix 6.

Figure 2.2 Segments



Source: Oxera.

Each of the segments shown in Figure 2.2 has a different recommended set of elasticities, which reflects the fact that demand drivers affect each of these segments in a distinct way.

It is possible to produce forecasts at any level of aggregation of demand data, whether it is flow-specific or for a whole region or franchise. It is acceptable to use the parameters from this study at any of these levels of aggregation because they represent average demand responses across the segment. The forecaster simply needs to understand into which segment the journeys they are forecasting fall so that the correct parameters/elasticities can be used. However, it is important to remember that the *average* effect and the *marginal* effect may differ.

Once all the core data referred to above is collected, and in a useable format, forecasting techniques can be applied. These methods are described in the remainder of this report.

3 Current PDFH approach

The PDFH, the first version of which dates from June 1986, includes what has become the industry standard method for producing forecasts of rail passenger demand in Great Britain. Experienced forecasters will be aware of the basic technique already and may choose to skip this section. For new forecasters, a very brief outline of the PDFH approach is given below; this is intended to provide sufficient detail so that the enhancements discussed later in this document can be understood and used. For a full exposition, the reader is referred to the current version of the PDFH (v5).

Note that all the elasticities used throughout this forecasting framework are with respect to real-terms changes in monetary demand drivers.[15] Forecasters should ensure that their data on the monetary demand drivers (such as fare or income) are in real (rather than nominal) terms (deflated using the RPI), before using them in a forecasting exercise. The examples used throughout are based on the examination of a (real-terms) fare change.[16]

3.1 Single variable changes

The basic approach taken to determine changes in demand is to calculate an index of the ratio of new demand to previous demand.[17] This index can then be applied to an absolute value of existing demand to determine the forecast level of new demand. Two main types of data are needed to calculate this index:

– the new value and base value[18] of the variable that is being changed—in this example, the variable is fare in real terms. The base fare corresponds to the base level of journeys (ie, it is taken from the same point in time). The new value is then divided by the old value to obtain a ratio;

– the relevant elasticity—an elasticity gives the mathematical relationship between two variables, showing how movements in one result in movements in the other. Both the PDFH and the ‘Revisiting the Elasticity-Based Framework’ study provide elasticity estimates. (See, for example, the values given in Appendix 4.)

When the forecaster has the above data (ie, the new and base values of the demand driver and the appropriate elasticity), creating a forecast is quite straightforward. The ratio of the new and old explanatory variables needs to be raised to the power of the elasticity, as shown algebraically below.

$$I=\left(\frac{Fare_{new}}{Fare_{old}}\right)^{elasticity}$$

By multiplying the index factor, I, by the base level of journeys, the forecast is derived. (For more detail on this approach, see, for example, Section B1.4 or B3.5 of the PDFH v5.) Without knowing the base level of demand, it is not possible to calculate forecast levels using this approach, although stand-alone growth factors can be derived from the changes in forecast demand drivers. This forecasting process is depicted in Figure 3.1.

[15] ‘Real terms’ refers to the value of a monetary variable after adjusting for any inflation, such that values are comparable over time.

[16] When analysing variables other than fare, the process is the same, although the formulae may differ.

[17] Forecasting for new services would require a different approach.

[18] The base value is usually the existing value, but it can be any past or future value, depending on when the forecaster wants to start forecasting from.

Figure 3.1 Schematic of basic PDFH-style forecast

Source: Oxera.

Table 3.1 shows some (hypothetical) numerical examples of this basic forecasting process. The first four columns contain the information that can be considered as 'raw data'; the last two columns show calculated values. Readers should be comfortable with how the calculated values were obtained before moving on to the later sections of this report.

Table 3.1 One variable, PDFH-style forecasts

Base demand (no. of journeys)	Elasticity	Base fare (£)	New fare (£)	Index (I)	Forecast demand (no. of journeys)
100	–1.0	10.0	11.0	0.91	91
250	–0.8	3.0	4.0	0.79	199
1,300	–1.2	6.0	8.0	0.71	920

Source: Oxera.

3.1.1 Multiple variable changes

Where multiple variables are changed, the approach is the same as above except there are now multiple components to the index. The components need to be multiplied together to create the index. An example of how this can be done for a 5% increase in real fares, GJT and real income is given in the equation below.

$$I=\left(\frac{\text{Fare}_{\text{new}}}{\text{Fare}_{\text{old}}}\right)^{\text{Fare elasticity}}\times\left(\frac{\text{GJT}_{\text{new}}}{\text{GJT}_{\text{old}}}\right)^{\text{GJT elasticity}}\times\left(\frac{\text{Income}_{\text{new}}}{\text{Income}_{\text{old}}}\right)^{\text{Income elasticity}}$$

Table 3.2 below provides a numerical version of the multiple variable example given in the above formula. It takes the market segmentation from the 'Revisiting the Elasticity-Based Framework' study, using elasticities from a specific segment; namely, Core cities to Core cities, reduced fare tickets.

Table 3.2 Multi-variable forecasting

Demand driver	Change in variable (expressed as an index)	Three-year elasticity	Index (I) formula		Index (I) value
Fare	1.05	–1.16	$(1.05)^{-1.16}$	=	0.94
Income	1.05	2.01	$(1.05)^{-2.01}$	=	1.10
GJT	1.05	–1.12	$(1.05)^{-1.12}$	=	0.95
Total effect					**0.99**

Note: Elasticities, obtained from Table A4.1, are for the Core cities to Core cities, reduced fare ticket segment.
Source: Oxera.

This implies that, after three years, the combined effect of a 5% increase in real fares, real income and GJT is a 1% decrease in passenger rail demand for this segment.

The main principle of this PDFH approach—using an index to calculate the effect of changes in demand drivers on the demand for passenger rail travel—is retained with the new framework. The additions are certain demand drivers and enhancements to the approach. These enhancements are explained in the following two sections.

4 Enhancement: dynamics

One of the main enhancements of the 'Revisiting the Elasticity-Based Framework' study is the introduction of the ability to include dynamics, in the form of lagged effects, via elasticities.[19] Such an approach allows a richer understanding of changes in demand over time in response to changes in the drivers of rail demand.

A change in a demand driver does not necessarily have an instant impact since it can take some time for passengers to realise that there has been a change and then adapt to it. This can occur for many reasons, including:

- **behavioural patterns**—people tend to form habits and then not to reconsider their decisions very often, such as their choice of transport mode;
- **informational changes**—it can take time for journey or fare change information to be distributed and acted on;
- **structural reasons**—individuals may be 'locked' into travelling by another mode for a while, for example, because they already have a season ticket or because of their ownership (or not) of a car.

Such factors can mean that it takes time for the impact of a change in a demand driver to be fully realised. There is likely to be some impact in the first year followed by further, usually smaller, adjustments as demand adjusts towards its long-run level associated with the levels of the new demand drivers.

There is no reason why lagged effects should have the same direction of impact on demand as the immediate impact. They are determined by the behaviour of passengers. For example, it may be natural to think that an increase in GJT will lead to an immediate decline in passengers followed by a further decline, as those passengers travelling on season tickets switch away and gradually try other transport modes. Alternatively, there may be an initial 'shock' effect—passengers choosing not to travel by rail in response to the extra journey time—but they may come back after a year or so due to the realisation that the initial shock was not as bad as expected (or because the alternative mode did not live up to expectations).

Regardless of the precise mechanisms in operation, there are theoretical reasons for why, and empirical evidence that, dynamic effects exist. This section now goes on to show a method for modelling their impact.

As with the PDFH approach, for simple examples it is possible to do the necessary calculations by hand or in a spreadsheet. However, for more detailed calculations, spreadsheet software or specialist transport software (such as EDGE) is recommended.

4.1 Structure of the calculations

The data requirements for producing forecasts with dynamics are very similar to those for basic PDFH calculations. The new and old levels of the variable(s) being studied are needed,

[19] Section B12 of the PDFH v5 does cover lagged effects, but in a simplified manner.

in real terms (if appropriate), together with the (one-year) *marginal elasticity*,[20] and the marginal elasticity for lagged changes.

Marginal elasticities show the effect of a change for one year at a time, independent of any previous effects. In most cases (ie, constant elasticities), these can be looked up from Appendix 4. The worked example in section 4.2 gives more detail on their derivation. The forecasting approach itself is as follows.

– In the first period of the change, time t=1,[21] the approach is identical to the PDFH approach described in section 3. This is because the dynamics have not had time to take effect.

– In subsequent periods the observed demand response will have two components:

  – the response to any additional change in that subsequent period;
  – the lagged response to the previous change. This is simply the lagged response of passengers to the initial change in demand drivers. Passengers may still be adjusting to that change;

– these two effects (the one-year demand response and the lagged response) can be combined to give the total demand response in a given year.

For demonstration, in the following sections, text and numbers relating to the change in the demand drivers are in red; text and numbers relating to one-year changes in demand are in purple; and text and numbers relating to lagged changes in demand are in green. The basic formulae needed for these calculations are shown in Table 4.1.

Table 4.1 Components of forecasting formulae

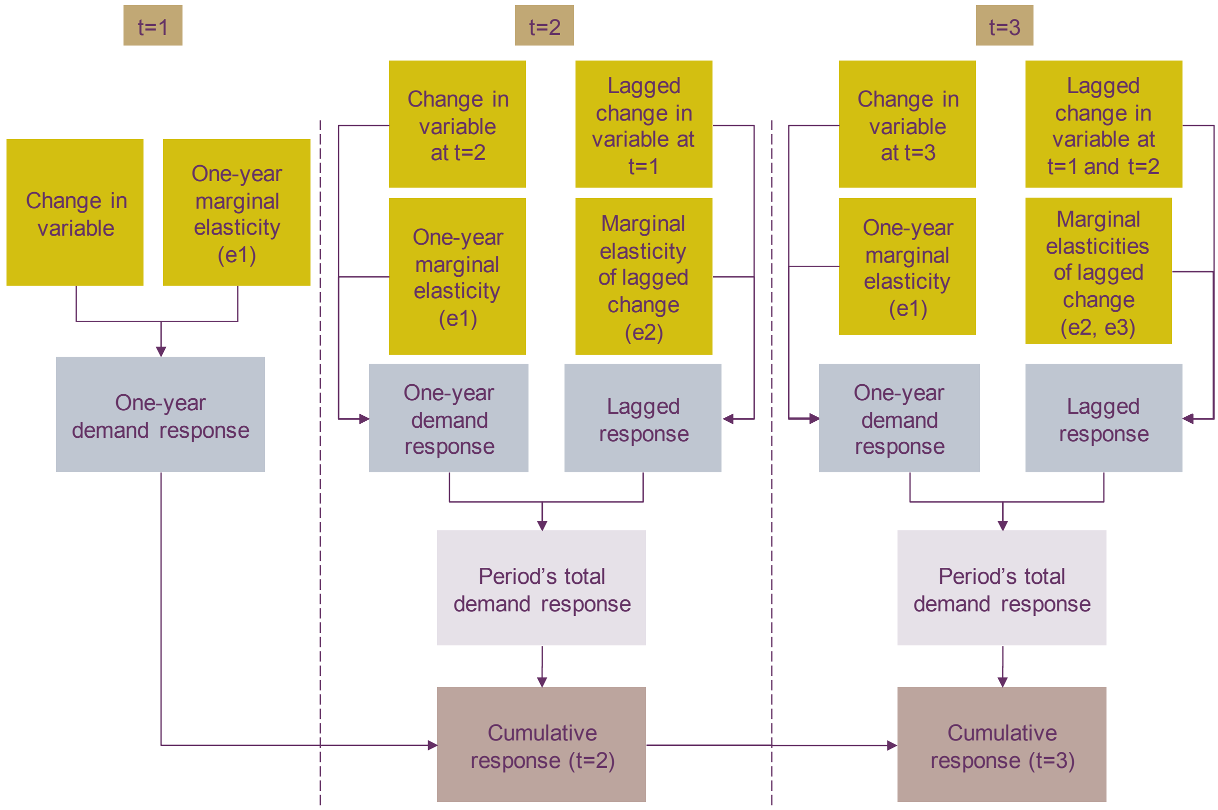
	Description of step	Formula
One-year demand response	**Standard PDFH calculation, using the t=1 marginal elasticity**	$\left[\frac{Fare_t}{Fare_{t-1}}\right]^{e1}$
Lagged response	**The past change in demand driver (fare), raised to the power of the elasticity for lagged changes**	$\left[\frac{Fare_{t-1}}{Fare_{t-2}}\right]^{e2}$
Total demand response	This period's one-year response multiplied by the lagged effect(s) of all past changes	$\left[\frac{Fare_t}{Fare_{t-1}}\right]^{e1} \times \left[\frac{Fare_{t-1}}{Fare_{t-2}}\right]^{e2}$
Cumulative demand response	This period's total response multiplied by all responses since the analysed change	Cumulative $response_{t-1}$ x total $response_t$

Note: e1 represents the marginal elasticity to a change in the current period and e1 represents the marginal elasticity to a change in the previous period. All variables should be in real terms (ie, adjusted for inflation). In year 1, the cumulative response is equal to the total response.
Source: Oxera.

Figure 4.1 below shows how this process works, with the flows indicating which input box is used in the calculation of the output box.

[20] Marginal elasticities are used to determine the demand response to a specific previous change. The 1-year marginal elasticity refers to the impact from a change in the current year.
[21] Throughout this report, 't' is used to refer to time periods (usually one year). Subscripts to variables indicate the period for which the variable is being considered; hence a subscript of t–1 means the value of that variable in the previous period/year.

Figure 4.1 Schematic of forecasting with dynamic effects



Source: Oxera.

For clarity (and to make the process more tangible), a detailed worked example is set out below.

4.2 Worked example: Dynamics

To provide a practical demonstration of the approach outlined, the fully worked example given below is for a non-London core city to London, the South East and East of England (LSEE) full fare ticket flow. In this example, the impact of a one-off fare increase of 5% (in real terms) is examined.

Data (such as the base and new fare levels) is taken as given and is shown in Table 4.3, where the first forecasting calculations are performed.

4.2.1 Calculating marginal elasticities

The main information that needs to be collected (and which practitioners may be unfamiliar with) is the marginal elasticities. These give the response in each period to a specific prior change. In general, they would expect to decrease in absolute magnitude over time, reflecting the fact that the initial impact tends to be the largest in absolute magnitude. Examples for a Core city to LSEE full fare flow are shown in Table 4.2.

The calculation of marginal elasticities can be complex. For the interested reader with a knowledge of basic calculus, Appendix 1 demonstrates how these elasticities are derived, and then shows how the estimated econometric parameters can be used to determine the marginal elasticities. For those who do not require such a detailed understanding of the approach, the relevant elasticities can be looked up from the table in Appendix 4, provided that the segment uses constant elasticities.[22] If the segment under examination requires a different specification for elasticities, see section 5 and Appendix 2.

The values listed in Table 4.2 have been taken straight from the table of marginal elasticities in Appendix 4; no further calculations were required.

Table 4.2 Marginal elasticities: Core–LSEE full fare tickets

Time since change	Notation	Marginal elasticity
1	e1	–1.78
2	e2	0.30
3	e3	0.08

Source: Oxera.

In this example, the marginal elasticities indicate a moderating of the initial negative demand shock in subsequent periods; this is known as ‘overshooting’. This is apparent from the negative one-year marginal elasticity, followed by positive marginal elasticities for subsequent periods. For further discussion on overshooting, see section 2 of the *Findings* report. However, for the purposes of this *Guidance* report, overshooting has no practical implications for producing forecasts.

4.2.2 Forecasting

In order to undertake the forecasting, it is sensible to break the process down into steps, beginning by writing out the variable change that is being analysed numerically. The first row in Table 4.3 shows the change in real fares for this example. The fare increases by 5% in period 1 and is then held constant. The resultant annual change is shown in the second row as a ratio (ie, a value of 1 implies ‘no change’). Rows three and four of Table 4.3 show the components of the impact on demand: the one-year demand response and the lagged effect. Rows five and six show the total and cumulative forecast effect.

Tables 4.3–4.6 show the formulae and values, respectively, that give the relevant forecasts in this example.

[22] The segments without constant elasticities are: LSEE–Other, full, reduced and season tickets; core cities–core cities, full price tickets; core cities to other, reduced price tickets; other to core cities, full price tickets; to airports, full and reduced price tickets.

Table 4.3 Dynamics: change at time t=1, text

t=	0	1	2	3
$Fare_t$	1.000	1.050	1.050	1.050
$Fare_t/fare_{t-1}$	–	1.050	1.000	1.000
One-year demand response	–	First-year response to change at t=0	First-year response to change at t=1	First-year response to change at t=2
Lagged response	–	–	First-year lagged response to change at t=0	First-year lagged response to change at t=1, and second-year lagged response to change at t=0
Total demand response in each year	–	First-year response to change at t=0	First-year response to change at t=1 and first-year lagged response to change at t=0	First-year response to change at t=1, first-year lagged response to change at t=1, and second-year lagged response to change at t=0
Cumulative response		First-year response to change at t=0	Cumulative response at t=1 and first-year response to change at t=1 and first-year lagged response to change at t=0	Cumulative response at t=2 and first-year response to change at t=2, first-year lagged response to change at t=1, and second-year lagged response to change at t=0

Source: Oxera.

Table 4.4 Dynamics: change at time t=1, formulae

t=	0	1	2	3
$Fare_t$	1.000	1.050	1.050	1.050
$Fare_t/fare_{t-1}$	–	1.050	1.000	1.000
One-year demand response	–	$\left[\frac{Fare_1}{Fare_0}\right]^{e1}$	$\left[\frac{Fare_2}{Fare_1}\right]^{e1}$	$\left[\frac{Fare_3}{Fare_2}\right]^{e1}$
Lagged response	–	–	$\left[\frac{Fare_1}{Fare_0}\right]^{e1}$	$\left[\frac{Fare_1}{Fare_0}\right]^{e3} \times \left[\frac{Fare_2}{Fare_1}\right]^{e2}$
Total demand response in each year	–	$\left[\frac{Fare_1}{Fare_0}\right]^{e1}$	$\left[\frac{Fare_2}{Fare_1}\right]^{e1} \times \left[\frac{Fare_1}{Fare_0}\right]^{e2}$	$\left[\frac{Fare_3}{Fare_2}\right]^{e1} \times \left[\frac{Fare_1}{Fare_0}\right]^{e3} \times \left[\frac{Fare_2}{Fare_1}\right]^{e2}$
Cumulative response		$\left[\frac{Fare_1}{Fare_0}\right]^{e1}$	$\left[\frac{Fare_1}{Fare_0}\right]^{e1} \times \left[\frac{Fare_2}{Fare_1}\right]^{e1} \times \left[\frac{Fare_1}{Fare_0}\right]^{e2}$	$\left[\frac{Fare_1}{Fare_0}\right]^{e1} \times \left[\frac{Fare_2}{Fare_1}\right]^{e1} \times \left[\frac{Fare_1}{Fare_0}\right]^{e2} \times \left[\frac{Fare_3}{Fare_2}\right]^{e1} \times \left[\frac{Fare_1}{Fare_0}\right]^{e3} \times \left[\frac{Fare_2}{Fare_1}\right]^{e2}$

Source: Oxera.

Table 4.5 Dynamics: change at time t=1, formulae and numbers

t=	0	1	2	3
Fare$_t$	**1.000**	**1.050**	**1.050**	**1.050**
Fare$_t$/fare$_{t-1}$	**–**	**1.050**	**1.000**	**1.000**
One-year demand response	**–**	$\left[\frac{1.050}{1.000}\right]^{-1.784}$	$\left[\frac{1.050}{1.050}\right]^{-1.784}$	$\left[\frac{1.050}{1.050}\right]^{-1.784}$
Lagged response	**–**	**-**	$\left[\frac{1.050}{1.000}\right]^{0.297}$	$\left[\frac{1.050}{1.000}\right]^{0.077} \times \left[\frac{1.050}{1.050}\right]^{0.297}$
Total demand response in each year	–	$\left[\frac{1.050}{1.000}\right]^{-1.784}$	$\left[\frac{1.050}{1.050}\right]^{-1.784} \times \left[\frac{1.050}{1.000}\right]^{0.297}$	$\left[\frac{1.050}{1.050}\right]^{-1.784} \times \left[\frac{1.050}{1.000}\right]^{0.077} \times \left[\frac{1.050}{1.050}\right]^{0.297}$
Cumulative response	–	$\left[\frac{1.050}{1.000}\right]^{-1.784}$	$\left[\frac{1.050}{1.000}\right]^{-1.784} \times \left[\frac{1.050}{1.050}\right]^{-1.784} \times \left[\frac{1.050}{1.000}\right]^{0.297}$	$\left[\frac{1.050}{1.000}\right]^{-1.784} \times \left[\frac{1.050}{1.050}\right]^{-1.784} \times \left[\frac{1.050}{1.000}\right]^{0.297} \times \left[\frac{1.050}{1.050}\right]^{-1.784} \times \left[\frac{1.050}{1.000}\right]^{0.077} \times \left[\frac{1.050}{1.050}\right]^{0.297}$

Source: Oxera.

Table 4.6 Dynamics: change at time t=1, values

t=	0	1	2	3
Fare$_t$	**1.000**	**1.050**	**1.050**	**1.050**
Fare$_t$/fare$_{t-1}$	**–**	**1.050**	**1.000**	**1.000**
One-year demand response	**–**	**0.917**	**1.000**	**1.000**
Lagged response	**–**	**–**	**1.015**	**1.004**
Total demand response in each year	–	0.917	1.015	1.004
Cumulative demand response	–	0.917	0.931	0.934

Source: Oxera.

The third line of Table 4.4 calculates the one-year demand response using the marginal elasticity of –1.784 from Table 4.2 and the standard PDFH approach. Because there is only a one-off fare change, there is no 'one-year demand response' in the later periods 2 and 3.

In this example, the only demand effect in periods 2 and 3 is that associated with a lagged response to the initial fare change. This is shown in the fourth row of Table 4.4. For the sake of clarity, it is worth explaining precisely how the values were derived. For example, the value 1.004 for the t=3 lagged response is calculated by the fare change in year 1, 1.050, raised to the power of the 2-year marginal elasticity, 0.077. If there had been a change in real fare at t=2, the fourth line at t=3 would reflect this change, as well as the lagged response to the change at t=1.

The total demand response (in the fifth row) is the product of the lagged response (the fourth row) and that year's one-year demand response (the third row).

The cumulative demand response is the product of all total demand responses since the change occurred. The cumulative demand response is an index that can be applied to the level of base demand to obtain the forecast absolute demand in that year. In this way, the forecasts have been obtained.

The above example is highly simplified. Not only does it contain only one variable but that variable only has a one-off change. This is plausible for factors (such as changes in GJT) that might arise from a structural timetable change, but less likely for variables such as population or fare.

Calculations become slightly more complicated when variables change in more than one time period. See the example given in Table 4.7, which shows (in the first row) a real-fares increase by 5% per annum in each time period.

Table 4.7 Dynamics: compounding the change

t=	0	1	2	3
$Fare_t$	**1.000**	**1.050**	**1.103**	**1.158**
$Fare_t/fare_{t-1}$	**–**	**1.050**	**1.050**	**1.050**
One-year demand response		**0.917**	**0.917**	**0.917**
Lagged response (I): to first change			**1.015**	**1.004**
Lagged response (II): to second change				**1.015**
Total demand response		0.917	0.931	0.934
Cumulative demand response		0.917	0.854	0.798

Source: Oxera.

Because there is now a fare increase in each year, there is an associated one-year demand response for each year. Compare Tables 4.4 and 4.5 (row 3) to see that there is now a one-year demand response in time periods t=2 and t=3.

Lagged response (I), as shown in Table 4.6, represents the lagged response to the period 1 change, calculated as before. An additional line, however, now shows lagged response (II), which corresponds to the response in time t=3 to the fare change at t=2. Therefore, there are three separate parts to the total demand response at time t=3: a two-year lagged response; a one-year lagged response; and the response to that year's fare change. With many variables and with more lags, this becomes increasingly complicated, but the process remains the same.

Having examined dynamics in this section, the report now turns to the second main enhancement—that of variable elasticities.

5 Enhancement: variable elasticities

The second major enhancement contained in the 'Revisiting the Elasticity-Based Framework' study is the introduction of variable elasticities. Variable elasticities offer a different structure than constant elasticities to the way in which demand responds to changes in demand drivers. In some cases, they can therefore be appropriate as a method of capturing individuals' actual behavioural patterns. However, they are significantly more complex to implement in a forecasting framework than constant elasticities. The purpose of this section is to demonstrate how this can be done.

The section also contains the guidance for model specifications involving squared terms, which are a special case of variable elasticities. Implementing these follows the same process as implementing variable elasticities, but section 5.4 covers these specifications specifically.

Knowledge of dynamic effects is assumed from section 4, as both variable elasticities and dynamic effects are shown below.

5.1 What is a variable elasticity?

Elasticities indicate how consumers respond to changes in demand drivers. A constant elasticity suggests that, for a given *percentage* change, there is a fixed *percentage* demand change, regardless of the base level of the demand driver. With a variable elasticity, for a given *percentage* fare change, the *percentage* demand change depends on the base level of the demand driver.[23]

See the *Findings report* for discussion of the reasons for why one specification has been chosen rather than another.

Basic examples of how variable elasticities may differ in a one-period setting are shown in Table 5.1. In the other examples given below, however, dynamic effects are included too.

Table 5.1 Comparison of constant and variable elasticities to a 5% real fare increase

	Base fare (£)	Base passengers	Elasticity	Change in passengers (%)
Constant elasticity	1	100	–1	–5
	1.5	100	–1	–5
	2	100	–1	–5
Variable elasticity	1	100	–1	–5
	1.5	100	–1.5	–7.1
	2	100	–2	–9.3

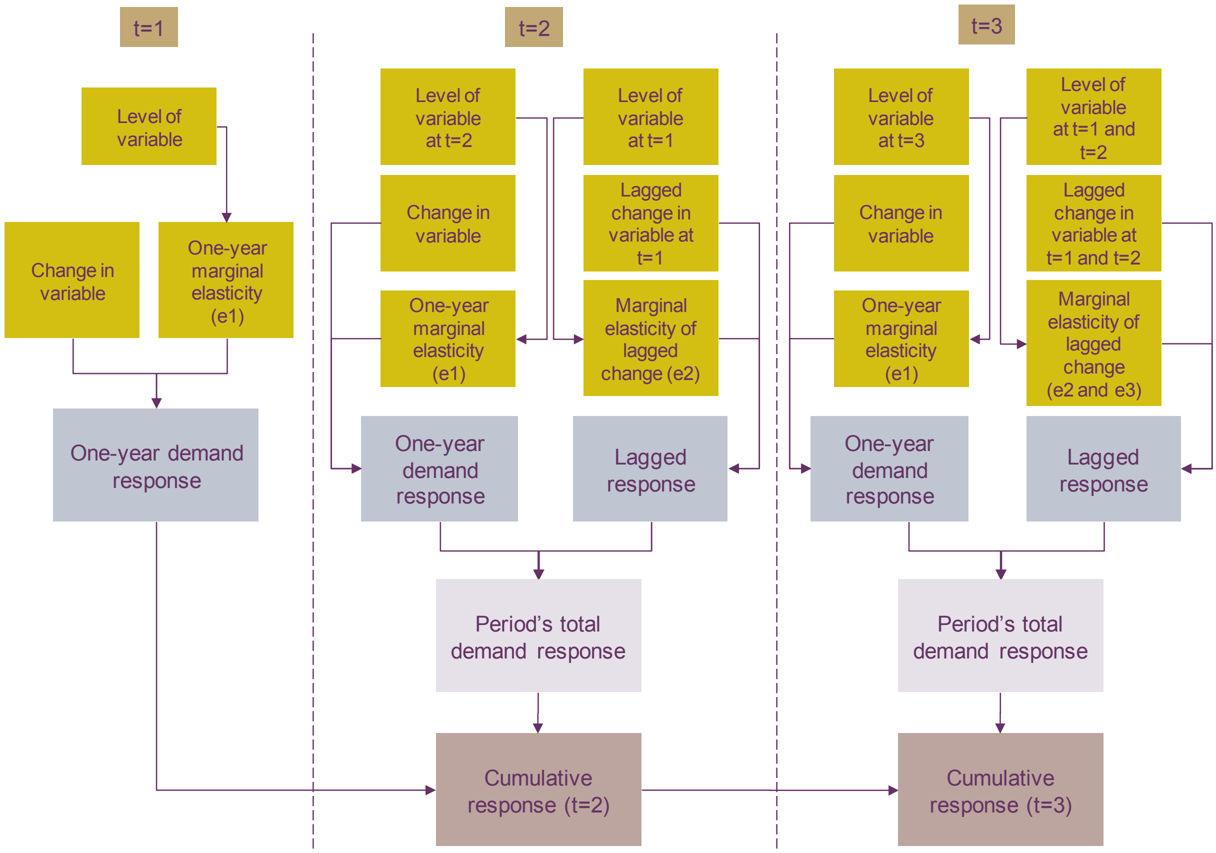
Source: Oxera.

[23] Economically, the different types of elasticity simply reflect the curvature of the demand curve. A linear demand curve has a variable elasticity; specific shapes of demand curves have constant elasticities.

5.2 Structure of the calculations

A written description of the process of forecasting would follow closely the approach outlined previously in section 4.2. The main difference comes in the determination of the marginal elasticities. Their calculation now requires the use of the absolute level of the demand driver being analysed. Figure 5.1 shows that the process of forecasting is the same as in Figure 4.1 but with the additional aspect of calculating the marginal elasticities. The detail on how to do this in practice is given in the worked example in the next section.

Figure 5.1 Schematic of forecasting with variable elasticities and dynamic effects



Source: Oxera.

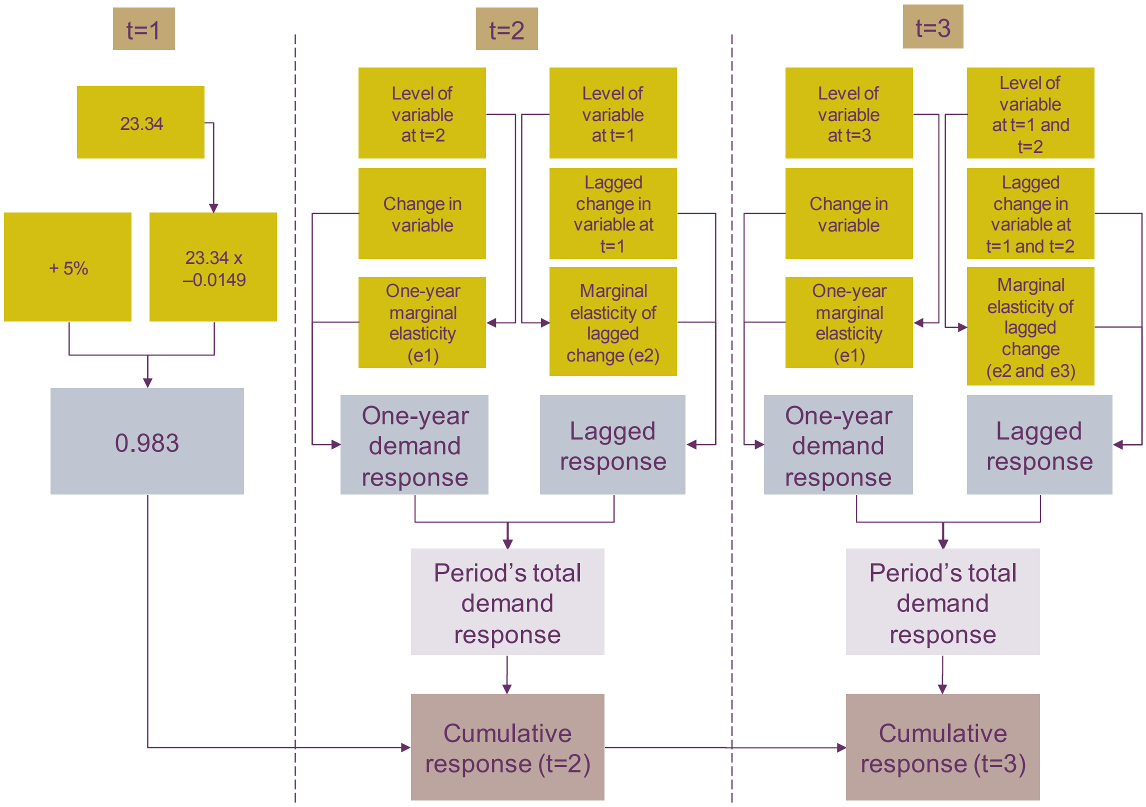
5.3 Worked example: Variable elasticities

This worked example is for a LSEE to other, reduced fare ticket-type flow.

5.3.1 Calculating marginal elasticities

As in section 4, the first step is to calculate the marginal elasticities for the flow(s) being analysed. It is at this first step that the first difference to previous approaches appears, as the marginal elasticities cannot be given in a 'look-up' table. They will depend on the average fare in the base year. Determining the marginal elasticities under a variable specification inevitably involves a calculation by the forecaster (as opposed to the constant elasticities, which can be looked up from the table in Appendix 4), since these elasticities will depend on the level of the demand driver under analysis. Appendix 2 explains how these marginal elasticities should be calculated, and the parameters which can be used to calculate them are provided in Appendix 3. The marginal elasticity for a one-year change is the appropriate parameter multiplied by the value of the variable in that period (eg, the fare elasticity for LSEE–Other in 2007 is –0.0149 x 23.34 = –0.348) (see Figure 5.2).

Figure 5.2 Illustrative diagram for the first-period change



Source: Oxera.

Table 5.2 shows the values obtained from this example that can be used in the subsequent example.

Table 5.2 Marginal elasticities: LSEE–Other, reduced fare tickets

Time since change	Notation	Marginal elasticity
1	e1	–0.348
2	e2	0.255
3	e3	0.039

Note: Values for LSEE–Other, reduced fare ticket-type flow, based on an average fare of £23.34 in constant 2007 prices.
Source: Oxera.

5.3.2 Forecasting

In order to produce forecasts, the process can be broken down into steps, as per section 4. Table 5.3 shows the components of the calculation. The change in the real fare is shown in the first row. The fare increases by 5% in real terms in period 1 and is then held constant. The resultant annual change is shown as a ratio in the second row. The third row calculates the one-year demand response using the marginal elasticity of –0.348 (e1) from Table 5.2 and the standard PDFH approach. Because the fare change is only one-off, there is no 'one-year demand response' in the later periods 2 and 3.

The only demand effect in periods 2 and 3 is that associated with a lagged response to the initial fare change. This is shown in the fourth row of Table 5.3. For the sake of clarity, it is worth explaining precisely how the values were derived. For example, the value 1.002 for the t=3 lagged response is calculated by the fare change in year 1 (1.05), raised to the power of the 2-year marginal elasticity, 0.039. As can be seen, this is very similar to the example with constant elasticities given in section 4.2 above.

Table 5.3 Variable elasticities and dynamics: change at time t=1

t=	0	1	2	3
$Fare_t$	23.340	24.507	24.507	24.507
$Fare_t/Fare_{t-1}$	–	1.050	1.000	1.000
One-year demand response	–	0.983	1.000	1.000
Lagged response	–	–	1.012	1.002
Total demand response in each year	–	0.983	1.012	1.002
Cumulative demand response	–	0.983	0.995	0.997

Source: Oxera.

As before, the total demand response is the product of the lagged response and that year's one-year demand response.

The cumulative demand response is the product of all total demand responses since the change occurred. The cumulative demand response is an index that can be applied, by multiplication, to the level of base demand to obtain the forecast absolute demand in that year.

If the change to be examined is not a one-off then a compounding effect is introduced in the same way as for the example given in section 4.2, but with the additional requirement for the forecaster to calculate the marginal elasticities (see Table 5.4). Using the same example and parameters as above, the effect of an annual 5% fare rise is shown in Table 5.4.

Table 5.4 Variable elasticities and dynamics: compounding the change

t=	0	1	2	3
$Fare_t$	23.340	24.507	25.732	27.019
$Fare_t/Fare_{t-1}$	–	1.050	1.050	1.050
One-year demand response	–	0.983	0.982	0.981
Lagged response (I): to first change	–	–	1.012	1.002
Lagged response (II): to second change	–	–	–	1.013
Total demand response	–	0.983	0.995	0.996
Cumulative demand response	–	0.983	0.978	0.974

Source: Oxera.

Because there is now a fare increase in each year, there is an associated one-year demand response for each year.

Lagged response (I), as shown in Table 5.4, represents the lagged response to the period 1 change, calculated as before. An additional line, however, now shows lagged response (II), which corresponds to the response in time t=3 to the fare change at t=2. Therefore, there are three separate parts to the total demand response at time t=3: a two-year lagged response; a one-year lagged response; and the response to that year's fare change.

5.4 Squared terms

A specific case of variable elasticities occurs when there are squared terms in the specified demand equation. In this case, the calculation of marginal elasticities is more complex (see section A2.1.2). However, their implementation in forecasting follows precisely the same approach as that given above.

6 Summary and non-modelling guidance

Having read through this guidance, a forecaster should be able use the new forecasting framework developed for the 'Revisiting the Elasticity-Based Framework'. In particular, they should be able to examine numerically the impact of:

– dynamics;
– variable elasticities; and
– squared terms.

6.1 Further issues

It is important to be aware that the framework has some limitations. Elasticity-based forecasts are suitable for forecasting demand effects from incremental changes (up to approximately 10%) in demand drivers. They cannot, however, determine the effect of large changes or that of entirely new services which lie outside the range of changes in the demand drivers. In latter case, there is no base data from which to start; and an entirely different approach is therefore needed in these circumstances.

In addition, the forecasts produced under this framework are purely statistical. In many cases users may wish to apply a degree of specific and or local knowledge to amend forecasts, in light of factors that are not captured in the modelling. These could be specific construction or development schemes, or the impact of constraints (such as overcrowding).

Although the elasticities presented control for the impact of crowding, and the other components of the service quality index, future changes in these variables will still affect the demand for rail travel. Therefore, a separate crowding model would be an appropriate addition to the forecasting framework presented in this report (as would changes in rolling stock, stations and train cleanliness, for example). See the *Service quality* report for more details.

The elasticities presented here have been derived using relatively coarse market segmentation. This should be borne in mind when applying the elasticities—in particular, that the more disaggregated the application, the more likely local factors are to affect the variables (eg, local housing developments or initiatives to increase economic growth in a certain area); or the elasticities (eg, due to differing modal competition through an established tram network).

This forecasting framework has been based on economic theory and estimated on a maximum of 18 years of annual data. As with any forecasts, the longer the forecast, the greater the range of uncertainty surrounding the forecasts. The framework can be used to produce scenarios for up to 30 years, on the assumption that the elasticities remain constant and will be dependent on the assumed scenarios for the drivers. Note that the impact of the recent recession has not been captured in the data used in this study and therefore it might be necessary to build scenarios where the parameters have changed.

Where the effect of omitted variables is significant (most likely to be in the case of aviation or coach competition), it is suggested that specialist advice be sought to ensure that the elasticities used are not conflated with the effects of these variables.

6.1.1 Ticket type to journey purpose conversion

This study has estimated elasticities on the basis of ticket types, rather than journey purpose, due to data restrictions. However, for some forecasting cases the use of journey purpose-based forecasts may be more useful than those by ticket type.

The conversion from ticket type to journey purpose is typically done using survey data, predominantly derived from the National Rail Travel Survey (NRTS). In order to convert the elasticities presented in this report to journey purpose, it is recommended that the elasticities are weighted by the proportion of journeys within that segment, for the year 2004 (to enhance consistency with the NRTS results). Depending on the format of the base data, an approximation may need to be made to calculate the weights for each of the market segments.

– Journey purpose elasticities can be derived from ticket-type elasticities by using a weighted average of the ticket-type elasticities, where the weights are derived from survey data, such as the NRTS. For example, the one-year marginal fare elasticity for business travel between non-London core cities and Other could be calculated as follows: calculate weights for the conversion from the NRTS. For journeys not in London and the South East,[24] the weights are 47% on full fare tickets, 42% on reduced fare tickets, and 11% on season tickets;
– derive the appropriate ticket-type elasticities. In this case, these are –1.85 for full fare tickets, –1.48 for reduced fare tickets, and –0.12 for season tickets;
– the business elasticity can then be calculated as:

$$(0.47 \times -1.85) + (0.42 \times -1.48) + (0.11 \times -0.12),$$

– which gives the result of –1.50.

Where the elasticities are variable, there is an added complication, which can be dealt with by calculating the ticket-type elasticities as usual and then applying the standard weighting procedure as above.

The relationship between journey purpose and ticket type is likely to change over time as, for example, business users purchase more reduced fare tickets in response to economic downturns.

[24] The market segmentation used to derive the NRTS journey purpose ticket type conversions is different to that of the 'Revisiting the Elasticity-Based Framework' study, and therefore this example should be considered as being indicative only.

A1 Calculus of elasticities

This appendix shows how the estimated econometric parameters relate to the specified demand equations. It also shows the mathematical derivation of marginal and 'total' elasticities from these parameters. For further details on obtaining the marginal elasticities, together with actual examples, see Appendix 2.

It is worth noting that dynamic effects occur even in specifications where only the current value of the demand driver, and not its value in previous periods, appears in the final equation.[25]

A1.1 Constant elasticities

The equation below shows the estimated demand equation. The dependent variable y represents the number of rail journeys (passenger demand) and the explanatory variable x represents the demand driver being analysed. There is also an error term (ε_t) in the equation, which occurs because the demand equation is not a perfect relationship. The t subscripts to each element of the equation indicate whether the value for that variable comes from this period, t, or a previous one (eg, t–1).

$$y_t=\beta_o+\beta_1 y_{t-1}+\beta_2 x_t+\beta_3 x_{t-1}+\beta_4 x_{t-2}+\varepsilon_t$$

Here, y and x are both in logs[26] and therefore the elasticity of y with respect to x is: $\frac{dy_t}{dx_t}$

Marginal elasticity at time t: $\frac{dy_t}{dx_t}=\beta_2$

Marginal elasticity at time t+1: $\frac{dy_{t+1}}{dx_t}=\beta_1\frac{dy_t}{dx_t}+\beta_3$
$=\beta_1\beta_2+\beta_3$

Marginal elasticity at time t+2: $\frac{dy_{t+2}}{dx_t}=\beta_1\frac{dy_{t+1}}{dx_t}+\beta_4$
$=\beta_1^2\beta_2+\beta_1\beta_3+\beta_4$

and so on.

Therefore, the one-year elasticity is: $\vartheta_1=\beta_2$.

The three-year elasticity is equal to the sum of the elasticities at time t, t+1 and t+2:

$$\vartheta_3=\beta_2+\beta_1\beta_2+\beta_1^2\beta_2+\beta_1+\beta_1\beta_3+\beta_4$$
$$=\beta_2\left(1+\beta_1+\beta_1^2\right)+\beta_3(1+\beta_1)+\beta_4$$

T-year elasticity (T very large):

$$\vartheta_{l.r}=\frac{(\beta_2+\beta_3+\beta_4)}{1-\beta_1}$$

[25] The technical reason for this is the presence of the lagged dependent variable. This variable means that changes in demand drivers have dynamic effects on demand, even if there are no lagged explanatory variables in the demand equation.
[26] For instance, the y value used here is the natural logarithm of the actual number of passenger journeys.

A1.2 Variable elasticities

$$\ln(y_t) = \beta_o + \beta_1 \ln(y_{t-1}) + \beta_2 x_t + \beta_3 x_{t-1} + \beta_4 x_{t-2} + \varepsilon_t$$

This version of the demand equation is the same as in A1.1, except y_t and x_t are levels (ie, are not logged).

$$\text{Elasticity} = \frac{dy_t}{dx_t} \cdot \frac{x_t}{y_t}$$

$$= \left(\frac{dy_t}{y_t}\right) / \left(\frac{dx_t}{x_t}\right)$$

$$= d\log y_t / \left(\frac{dx_t}{x_t}\right)$$

$$= \frac{d\log y_t}{dx_t} . x_t$$

$$\frac{d\log y_t}{dx_t} = \beta_2$$

Therefore, the one-year elasticity: $\vartheta_1 = \beta_2 x_t$. From this point, the derivation is similar to the constant elasticities:

$$\vartheta_3 = [\beta_2 (1 + \beta_1 + \beta_1^2) + \beta_3 (1 + \beta_1) + \beta_4] x_t$$

$$\vartheta_{l.r} = \frac{(\beta_2 + \beta_3 + \beta_4) x_t}{(1 - \beta_1)}$$

A1.3 Squared terms

$$y_t = \beta_o + \beta_1 y_{t-1} + \beta_2 x_t + \beta_3 x_t^2 + \beta_4 x_{t-1} + \beta_5 x_{t-2} + \varepsilon_t$$

where y_t and x_t are both in logs, and therefore the elasticity of y with respect to x is $\frac{dy_t}{dx_t}$.

Marginal elasticity at time t: $\frac{dy_t}{dx_t} = \beta_2 + 2\beta_3 x_t$

Marginal elasticity at time t+1: $\frac{dy_{t+1}}{dx_t} = \beta_1 \frac{dy_t}{dx_t} + \beta_4$

$$= \beta_1 (\beta_2 + 2\beta_3 x_t) + \beta_4$$

Marginal elasticity at time t+2: $\frac{dy_{t2}}{dx_t} = \beta_1 \frac{dy_{t+1}}{dx_t} + \beta_5$

$$= \beta_1 [\beta_1 (\beta_2 + 2\beta_3 x_t) + \beta_4] + \beta_5$$

Therefore, the one-year elasticity is: $\vartheta_1 = \beta_2 + 2\beta_3 x_t$

$$\vartheta_3 = (\beta_2 + 2\beta_3 x_t)(1 + \beta_1 + \beta_1^2) + \beta_4 (1 + \beta_1) + \beta_5$$

$$\vartheta_{l.r} = \frac{\beta_2 + 2\beta_3 x_t + \beta_4 + \beta_5}{(1 - \beta_1)}$$

A2 Calculation of marginal elasticities from parameters

Following on from the Appendix 1, this appendix now shows how the marginal elasticities can be calculated from the parameter estimates obtained from the 'Revisiting the Elasticity-Based Framework' study.

Table A2.1 collects all the formulae, as derived in the previous appendix, for calculating marginal elasticities.

Table A2.1 Marginal elasticity formulae

Demand equations			
Constant elasticities	$y_t=\beta_o+\beta_1 y_{t-1}+\beta_2 x_t+\beta_3 x_{t-1}+\beta_4 x_{t-2}+\varepsilon_t$		
Variable elasticities	$\ln(y_t)=\beta_o+\beta_1\ln(y_{t-1})+\beta_2 x_t+\beta_3 x_{t-1}+\beta_4 x_{t-2}+\varepsilon_t$		
Squared terms	$y_t=\beta_o+\beta_1 y_{t-1}+\beta_2 x_t+\beta_3 x_t^2+\beta_4 x_{t-1}+\beta_5 x_{t-2}+\varepsilon_t$		
Marginal elasticity	**Constant elasticities**	**Variable elasticities**	**Squared terms**
One-year	β_2	$\beta_2 x_t$	$\beta_2+2\beta_3 x_t$
Two-year	$\beta_1\,\beta_2+\beta_3$	$(\beta_1\,\beta_2+\beta_3)x_t$	$\beta_1[\beta_1(\beta_2+2\beta_3 x_t)+\beta_4$
Three-year	$\beta_1^2\beta_2+\beta_1\,\beta_3+\beta_4$	$(\beta_1^2\beta_2+\beta_1\,\beta_3+\beta_4)x_t$	$\beta_1[\beta_1(\beta_2+2\beta_3 x_t)+\beta_4]+\beta_5$

Note: The βs in the table are the coefficient on various demand drivers from the econometric estimation. Each one with a subscript corresponds to a different demand driver. For the full model in each case, see Appendix 1. Due to the addition of squared terms the parameters in the squared terms specification represent coefficients on different variables than the parameters in the other two specifications.
Source: Oxera.

Table A2.2 shows an example calculation of marginal elasticities for the Core cities to LSEE full fare segment. The three parameters in the top rows have been looked up directly from Appendix 3. The calculations themselves are then a straightforward application of the formulae from the constant elasticities section of Table A2.1.[27] As this segment uses constant elasticities, the values presented in Table A2.2 will always be the marginal elasticities. However, understanding how they have been derived is a useful starting point for the more complicated variable elasticity specifications.

[27] Whether the segment should use constant, variable or squared elasticities is apparent from Table 3.1 of the *Findings* report.

Table A2.2 Marginal elasticity calculation: example I

	Parameters		
Lagged demand parameter (β_1)	0.259		
One-year fare parameter (β_2)	–1.784		
Lagged fare parameter (β_3)	0.759		
Time since change		**Calculation**	**Marginal elasticity**
1		–1.784	
2		(0.259 x –1.784) + 0.759	0.297
3	-	(–1.784 x $(0.259)^2$) + (0.259 x 0.759)	0.077

Note: Based on Core cities to LSEE, full fare tickets segment.
Source: Oxera.

A2.1 Variable elasticities

The formulae for the derivation of marginal variable elasticities (given in Table A2.1) show that multiplication by the absolute level of the variable (x_t)—the average real fare in this case—is needed for variable elasticities.

Tables A2.3 and A2.4 give two example calculations. Table A2.4 shows a simple example where there are no dynamic effects; Table A2.5 shows a more complex example which includes dynamics effects. The resulting elasticities should be implemented as shown in section 5 of the main report.

Table A2.3 Marginal elasticity calculation: example II

	Parameters		
Lagged demand parameter (β_1)	0.117		
One-year fare parameter (β_2)	–0.072		
Average fare (£)	17.16		
Time since change		**Calculation**	**Marginal elasticity**
1		(17.16 x –0.072)	–1.236

Note: Based on LSEE to Other full fare ticket segment.
Source: Oxera.

Table A2.4 Marginal elasticity calculation: example III

	Parameters	
Lagged demand parameter (β_1)	0.154	
One-year fare parameter (β_2)	–0.0149	
Lagged fare parameter (β_3)	0.0132	
Average fare (£) at t=0	23.34	
Time since change	**Calculation**	**Marginal elasticity at fare value above**
1	(23.34 x –0.0149)	–0.349
2	23.34 x (0.0132 + (0.154 x 0.0149)	0.255
3	23.34 x (0.0132 x 0.154 + (0.154^2 x 0.0149)	0.039

Note: Based on LSEE to Other reduced fare ticket segment.
Source: Oxera.

A2.2 Squared terms

As noted in section 5.4 of the main report, the calculation of marginal elasticities can be complex, especially when the demand equation contains squared terms. The formulae required were given in Table A2.1, but Table A2.5 provides a specific example. Only two segments from the approach recommend the use of a squared terms demand equation.

Table A2.5 Marginal elasticity calculation: example IV

	Parameters	
Lagged demand parameter (β_1)	0.29	
One-year fare parameter (β_2)	0.0013	
Lagged fare parameter (β_3)	–0.285	
One period lagged fare parameter (β_4)	0.301	
Two period lagged fare parameter (β_5)	0.094	
Natural log of average fare at t=0	2.15	
Time since change	**Calculation**	**Marginal elasticity at fare value above**
1	(0.0013 + (2 x –0.285 x 2.15))	–1.226
2	0.29 x (0.0013 + (2 x –0.285 x 2.15) + 0.301	–0.055
3	0.29 x (0.29 x (0.0013 + (2 x 0.285 x 2.15))+0.301)+0.094	0.078

Note: Based on Core cities to Other, reduced fare ticket segment.
Source: Oxera.

For the sake of clarity, Table A2.6 gives a forecasting example using the elasticities from Table A2.5.

Table A2.6 Squared terms: compounding the change

t=	0	1	2	3
Natural log of $fare_t$	2.154	2.203	2.251	2.300
$Fare_t$	8.617	9.048	9.500	9.975
$Fare_t/fare_{t-1}$	–	1.050	1.050	1.050
One-year demand response	–	0.942	0.941	0.939
Lagged response (I)	–	–	0.997	1.004
Lagged response (II)	–	–	–	0.997
Total demand response	–	0.942	0.938	0.940
Cumulative demand response	–	0.942	0.884	0.831

Source: Oxera.

A3 Parameters

Table A3.1 Parameter estimates from ‘Revisiting the Elasticity-Based Framework’

	LSEE to LSEE, full/ reduced	LSEE to LSEE, season	LSEE to Core, reduced	LSEE to Core, full	LSEE to Core, season	LSEE to Other, reduced	LSEE to Other, full	LSEE to Other, season	Core to LSEE, reduced	Core to LSEE, full	Core to LSEE, season	Core to Core, reduced	Core to Core, full	Core to Core, season
Lag of journeys	0.0325	0.280	0.276	0.167	0.300	0.154	0.215	0.206	0.194	0.259	0.414	0.323	0.280	0.379
Fare (no lag)	–0.787	–2.081	0.391	–1.508	–0.568	–0.0149	–0.0235	–0.04	–0.556	–1.784	-0.94	–2.051	–0.0481	–1.704
Fare (one lag)	–0.1	1.642	–0.611	0.454		0.0132		–0.000952		0.759	0.813	1.336	0.0202	1.133
Fare (two lags)	–0.00347							–0.0278						
Fare (squared term)														
Cross-price (no lag)										0.183				
Cross-price (one lag)										0.285				
Cross-price (two lags)														
Income (no lag)	0.742		1.281	0.883		4.48×10^{-5}	0.0000425		–0.546	0.578		1.405	0.000134	
Income (one lag)	0.791						0.0000361		1.736					
Income (two lags)														
Employment (no lag)	0.476	1.036						0.000014						
Employment (one lag)														
Employment (two lags)														
Population (no lag)														
Population (one lag)														

	LSEE to LSEE, full/ reduced	LSEE to LSEE, season	LSEE to Core, reduced	LSEE to Core, full	LSEE to Core, season	LSEE to Other, reduced	LSEE to Other, full	LSEE to Other, season	Core to LSEE, reduced	Core to LSEE, full	Core to LSEE, season	Core to Core, reduced	Core to Core, full	Core to Core, season
Population (two lags)														
Car ownership (no lag)														
Car ownership (one lag)														
Car ownership (two lags)														
Car cost (no lag)	0.817		0.0533	1.327		0.174	0.761		0.426	1.363			1.41	
Car cost (one lag)	0.37		0.547				0.535							
Car cost (two lags)	0.209													
Performance (no lag)	0.425		0.215				0.624	1.614	0.311		–0.269		0.458	
Performance (one lag)	0.682		0.762								–2.887			
Performance (two lags)														
GJT (no lag)	–0.393	–1.423	–0.969	–1.126		–0.0316	–0.325	–1.858	–0.936	–0.725		–0.785	–0.929	–2.568
GJT (one lag)	–0.45	–1.828		–1.349		–0.433	–0.335			–0.326			–2.242	
GJT (two lags)	–0.731	–0.0819					–1.209			–1.445				
SQI no lag)				1.301										
SQI (one lag)														
SQI (two lags)														

	Core to Other, reduced	Core to Other, full	Core to Other, season	Other to LSEE, reduced	Other to LSEE, full	Other to LSEE, season	Other to Core, reduced	Other to Core, full	Other to Core, season	Other to Other, reduced	Other to Other, full	Other to Other, seasons	To airports, reduced	To airports, full
Lag of journeys	0.290	0.187	0.568	0.133	0.273	0.373	0.235	0.259	0.421	0.792	0.375	0.883	0.296	
Fare (no lag)	0.00131	–1.85	–1.476	–0.118	–1.814	–1.401	–1.071	–0.026	–1.361	–0.24	–1.363	–0.983	0.893	–0.0579
Fare (one lag)	0.301	0.464		–0.256	0.743			0.011	0.676		0.624	0.638		0.0294
Fare (two lags)	0.0942			–0.208										
Fare (squared term)	–0.285												–0.255	
Cross-price (no lag)					0.375									
Cross-price (one lag)														
Cross-price (two lags)														
Income (no lag)	0.526	1.33		1.253	0.692		1.77	4.39×10^{-5}		0.929	0.771	1.297	0.562	0.583
Income (one lag)	0.713										1.084			
Income (two lags)	1.46													
Employment (no lag)						1.411								
Employment (one lag)														
Employment (two lags)														
Population (no lag)					1.875									
Population (one lag)														
Population (two lags)														
Car ownership (no lag)		–4.217					–6.723	–5.249					3.288	

	Core to Other, reduced	Core to Other, full	Core to Other, season	Other to LSEE, reduced	Other to LSEE, full	Other to LSEE, season	Other to Core, reduced	Other to Core, full	Other to Core, season	Other to Other, reduced	Other to Other, full	Other to Other, seasons	To airports, reduced	To airports, full
Car ownership (one lag)														
Car ownership (two lags)														
Car cost (no lag)	0.41	1.161	1.089	0.913	0.679		0.567	0.36	0.656		1.061	–0.344	0.382	0.845
Car cost (one lag)				–0.221							–0.584	0.435		
Car cost (two lags)				0.766										
Performance (no lag)	0.248	0.394		0.796			0.533	0.335	0.333	–0.0907	0.235	–0.056		0.711
Performance (one lag)	0.679	0.621		0.669			0.686	0.666	0.501	0.649	0.688	0.683		
Performance (two lags)												0.902		
GJT no lag)	–0.0267	–0.253	–1.391	–0.327	–0.160	–0.336		–0.493	–1.266		–0.251	–0.019	–0.497	–1.637
GJT (one lag)		–0.634		–0.678	–0.759	–1.582		–0.675	–0.884		–0.369	–0.305		
GJT (two lags)		–0.693		–0.863				–0.688	–1.007		–0.301			
SQI no lag)					1.149									
SQI (one lag)														
SQI (two lags)														

Source: Oxera analysis.

A4 Marginal elasticities

Table A4.1 Segments with constant marginal estimates from ‘Revisiting the Elasticity-Based Framework’

	LSEE to LSEE, full/ reduced	LSEE to LSEE, season	LSEE to Core, reduced	LSEE to Core, full	LSEE to Core, season	Core to LSEE, reduced	Core to LSEE, full	Core to LSEE, season	Core to Core, reduced	Core to Core, season
Fare (no lag)	–0.79	–2.08	0.39	–1.51	–0.57	–0.56	–1.78	–0.94	–2.05	–1.70
Fare (one lag)	–0.13	1.06	–0.50	0.20	–0.17	–0.11	0.30	0.42	0.67	0.49
Fare (two lags)	–0.01	0.30	–0.14	0.03	–0.05	–0.02	0.08	0.18	0.22	0.18
Cross-price (no lag)							0.18			
Cross-price (one lag)							0.33			
Cross-price (two lags)							0.09			
Income (no lag)	0.74		1.28	0.88		–0.55	0.58		1.41	
Income (one lag)	0.82		0.35	0.15		1.63	0.15		0.45	
Income (two lags)	0.03		0.10	0.02		0.32	0.04		0.15	
Employment (no lag)	0.48	1.04								
Employment (one lag)	0.02	0.29								
Employment (two lags)	0.00	0.08								
Population (no lag)										
Population (one lag)										
Population (two lags)										
Car ownership (no lag)										
Car ownership (one lag)										
Car ownership (two lags)										
Car cost (no lag)	0.82		0.05	1.33		0.43	1.36			
Car cost (one lag)	0.40		0.56	0.22		0.08	0.35			
Car cost (two lags)	0.22		0.16	0.04		0.02	0.09			

	LSEE to LSEE, full/ reduced	LSEE to LSEE, season	LSEE to Core, reduced	LSEE to Core, full	LSEE to Core, season	Core to LSEE, reduced	Core to LSEE, full	Core to LSEE, season	Core to Core, reduced	Core to Core, season
Performance (no lag)	0.43		0.22			0.31		–0.27		
Performance (one lag)	0.70		0.82			0.06		–3.00		
Performance (two lags)	0.02		0.23			0.01		–1.24		
GJT (no lag)	–0.39	–1.42	–0.97	–1.13		–0.94	–0.73		–0.79	–2.57
GJT (one lag)	–0.46	–2.23	–0.27	–1.54		–0.18	–0.51		–0.25	–0.97
GJT (two lags)	–0.75	–0.71	–0.07	–0.26		–0.04	–1.58		–0.08	–0.37
SQI no lag)				1.30						
SQI (one lag)				0.22						
SQI (two lags)				0.04						

	Core to Other, full	Core to Other, season	Other to LSEE, reduced	Other to LSEE, full	Other to LSEE, season	Other to Core, reduced	Other to Core, season	Other to Other, reduced	Other to Other, full	Other to Other, season
Fare (no lag)	–1.85	–1.48	–0.12	–1.81	–1.40	–1.07	–1.36	–0.24	–1.36	–0.98
Fare (one lag)	0.12	–0.84	–0.27	0.25	–0.52	–0.25	0.10	–0.19	0.11	–0.23
Fare (two lags)	0.02	–0.48	–0.24	0.07	–0.19	–0.06	0.04	–0.15	0.04	–0.20
Cross-price (no lag)				0.38						
Cross-price (one lag)				0.10						
Cross-price (two lags)				0.03						
Income (no lag)	1.33		1.25	0.69		1.77		0.93	0.77	1.30
Income (one lag	0.25		0.17	0.19		0.42		0.74	1.37	1.15
Income (two lags)	0.05		0.02	0.05		0.10		0.58	0.51	1.01
Employment (no lag)					1.41					
Employment (one lag)					0.53					
Employment (two lags)					0.20					
Population (no lag)				1.88						
Population (one lag)				0.51						
Population (two lags)				0.14						
Car ownership (no lag)	–4.22					–6.72				
Car ownership (one lag)	–0.79					–1.58				
Car ownership (two lags)	–0.15					–0.37				
Car cost (no lag)	1.16	1.09	0.91	0.68		0.57	0.66		1.06	–0.34
Car cost (one lag)	0.22	0.62	–0.10	0.19		0.13	0.28		–0.19	0.13
Car cost (two lags)	0.04	0.35	0.75	0.05		0.03	0.12		–0.07	0.12
Performance (no lag)	0.39		0.80			0.53	0.33	–0.09	0.24	–0.06
Performance (one lag)	0.69		0.77			0.81	0.64	0.58	0.78	0.63
Performance (two lags)	0.13		0.10			0.19	0.27	0.46	0.29	1.46
GJT (no lag)	–0.25	–1.39	–0.33	–0.16	–0.34		–1.27		–0.25	–0.02
GJT (one lag)	–0.68	–0.79	–0.72	–0.80	–1.71		–1.42		–0.46	–0.32

	Core to Other, full	Core to Other, season	Other to LSEE, reduced	Other to LSEE, full	Other to LSEE, season	Other to Core, reduced	Other to Core, season	Other to Other, reduced	Other to Other, full	Other to Other, season
GJT (two lags)	–0.82	–0.45	–0.96	–0.22	–0.64		–1.60		–0.47	–0.28
SQI (no lag)				1.15						
SQI (one lag)				0.31						
SQI (two lags)				0.1						

Note: The omitted segments are those with variable elasticities. These segments consist of all three LSEE to Other segments; Core cities to Core cities full fare tickets; Core cities to Other reduced fare tickets; Other to Core cities full fare tickets; and both ‘to airports’ flows.
Source: Oxera analysis.

A5 Average fares by segment

Table A5.1 Average fare by segment in 2007

Core	To	Ticket type	Average fare (£)
LSEE	LSEE	Combined full and reduced	6.60
		Season	3.90
LSEE	Core cities	Full	68.60
		Reduced	26.10
		Season	14.30
LSEE	Other	Full	58.65
		Reduced	23.30
		Season	11.50
Core cities	LSEE	Full	65.40
		Reduced	27.10
		Season	15.40
Core cities	Core cities	Full	41.80
		Reduced	19.40
		Season	8.50
Core cities	Other	Full	26.60
		Reduced	13.60
		Season	4.10
Other	LSEE	Full	61.70
		Reduced	23.60
		Season	13.90
Other	Core cities	Full	26.00
		Reduced	13.10
		Season	4.60
Other	Other	Full	18.00
		Reduced	9.60
		Season	3.00

Note: At constant 2007 prices.
Source: Oxera.

A6 Geographic detail of segments

This appendix clarifies the geographic split of the segments.

A6.1 London, South East and East

This segment contains all stations in Greater London, the East of England Government Office Region (GOR) and the South East of England GOR.

Figure A6.1 below gives a visual outline of the extent of this segment. The last stations within the segment on each line are indicated near the boundary. For a full list of all stations included in this category, see the accompanying Excel file.

A6.2 Non-London core cities

This segment includes stations from the following cities, with the stations in brackets.

– Birmingham (Birmingham BR);
– Manchester (Manchester BR);
– Liverpool (Liverpool);
– Nottingham (Nottingham);
– Bristol (Bristol Temple Meads, Bristol Parkway);
– Sheffield (Sheffield);
– Cardiff (Cardiff BR);
– Edinburgh (Edinburgh);
– Glasgow (Glasgow BR);
– Newcastle (Newcastle);
– Leeds (Leeds);
– Leicester (Leicester);
– York (York);
– Hull (Hull).

A6.3 To airports

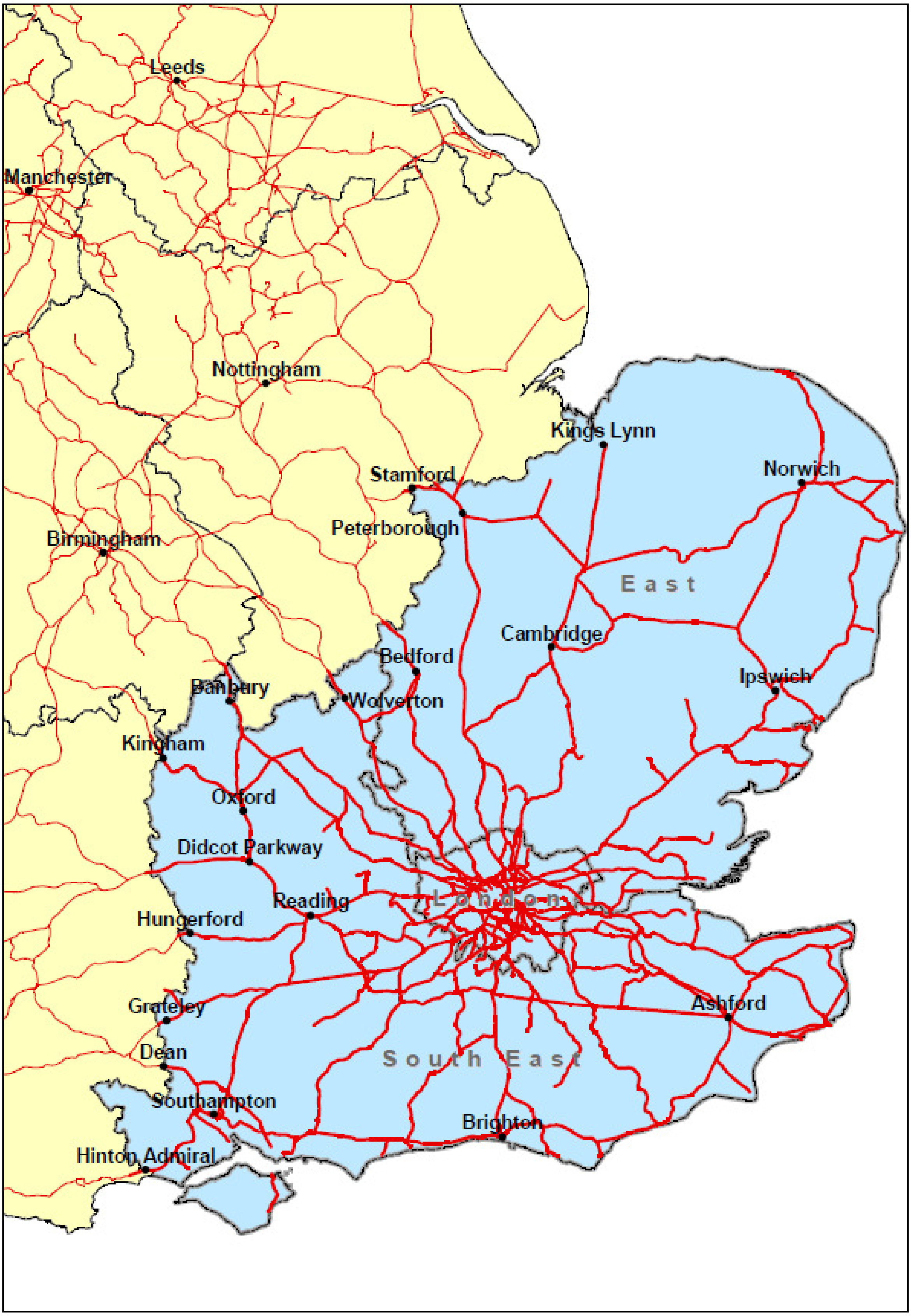
The following airport stations are included in this segment:

– Gatwick Airport;
– Stansted Airport;
– Luton Airport Parkway;
– Manchester Airport;
– Birmingham Airport;
– Glasgow Airport (Prestwick);
– Edinburgh Airport (Inverkeithing).

A6.4 Other

This includes all stations that do not fit into the above categories.

Figure A6.1 London, South East and East segment



Source: Oxera.

A7 Glossary

Base data	the data on which all forecasts are based. In most cases, this will be the data at the start of the forecast period
Constant elasticity	an elasticity which does not change with the level of the demand driver
Cumulative effect	the effect of a change in a demand driver, over a period of years
Demand drivers	Any variable that influences the level of passenger demand (eg, car cost)
LENNON	Latest Earnings Networked Nationally Over Night, the rail industry's central ticketing system
LSEE	London, South East of England, and East of England
Marginal elasticities	Elasticities giving the impact in a given year to changes in demand, without the cumulative effect of previous changes
MOIRA	industry software to forecast the effect of timetable changes
National Rail Trends	Office for Rail Regulation publication providing an overview of rail industry data
NTEM	National Trip End Model (the data accessible through TEMPRO, see below)
PDFH	Passenger Demand Forecasting Handbook
Real terms	applies to monetary variables (eg, fares and income) and refers to variables with the impact of inflation removed
Squared term	a specific type of variable elasticity
TEMPRO	the DfT's socio-demographic planning tool
Total demand response in a year	the effect of a change in a demand driver, within a year
Variable elasticity	an elasticity which does vary with the level of the demand driver

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