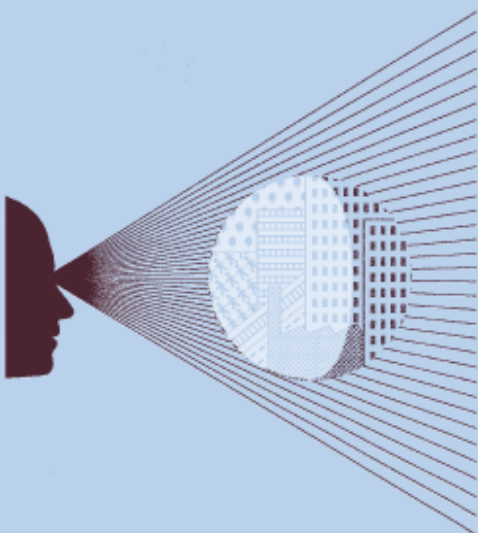


What are the key issues for model specification?

Model specification report

Prepared for
the Department for Transport,
Transport Scotland, and
the Passenger Demand Forecasting Council

March 2010



Oxera Consulting Ltd is registered in England No. 2589629 and in Belgium No. 0883.432.547. Registered offices at Park Central, 40/41 Park End Street, Oxford, OX1 1JD, UK, and Stephanie Square Centre, Avenue Louise 65, Box 11, 1050 Brussels, Belgium. Although every effort has been made to ensure the accuracy of the material and the integrity of the analysis presented herein, the Company accepts no liability for any actions taken on the basis of its contents.

Oxera Consulting Ltd is not licensed in the conduct of investment business as defined in the Financial Services and Markets Act 2000. Anyone considering a specific investment should consult their own broker or other investment adviser. The Company accepts no liability for any specific investment decision, which must be at the investor's own risk.

© Oxera, 2010. All rights reserved. Except for the quotation of short passages for the purposes of criticism or review, no part may be used or reproduced without permission.

Contents

| | | |
|----------|--|-----------|
| 1 | Introduction | 1 |
| 1.1 | Current PDFH formulation | 2 |
| 1.2 | Optimal model structure | 3 |
| 2 | Literature review | 4 |
| 2.1 | Economic demand drivers | 5 |
| 2.2 | Socio-demographic and -economic demand drivers | 5 |
| 2.3 | Modal competition | 6 |
| 2.4 | Fares | 7 |
| 2.5 | Summary | 7 |
| 3 | Economic model | 8 |
| 3.1 | Theoretical relationships | 9 |
| 3.2 | Functional forms | 13 |
| 4 | Conclusions | 16 |

List of tables

| | | |
|-----------|------------------------------------|----|
| Table 3.1 | Summary of combined data decisions | 11 |
|-----------|------------------------------------|----|

List of figures

| | | |
|------------|------------------------------------|----|
| Figure 3.1 | Summary of combined data decisions | 11 |
|------------|------------------------------------|----|

1 Introduction

Oxera and Arup have undertaken a study, 'Revisiting the Elasticity-Based Framework', by the Department for Transport (DfT), Transport Scotland, and the Passenger Demand Forecasting Council (PDFC). The primary aim of the study is to update and estimate the fares and background growth elasticities contained within the Passenger Demand Forecasting Handbook (PDFH).

The study has a number of secondary objectives, which include:¹

- exploring the use of innovative or alternative econometric techniques;
- re-specifying and extending the core elasticity-based framework;
- improving the underlying data.

As part of this study, a number of reports have been produced, detailed below, which form key elements in the formulation of the overall final forecasting framework, and are referenced a number of times here.

Reports prepared by Oxera and Arup for the 'Revisiting the Elasticity-Based Framework' study:

- 'What are the findings from the econometric analysis?' (the *Findings* report)
 - 'Is the data capable of meeting the study objectives?' (the *Data capability* report)
 - 'How has the preferred econometric model been derived?' (the *Econometric approach* report)
 - 'What are the key issue for model specification?' (the *Model specification* report)
 - 'How has the market for rail passenger demand been segmented?' (the *Market segmentation* report)
 - 'Does quality of service affect demand?' (the *Service quality* report)
- 'How should the revised elasticity-based forecasting framework be implemented?' (the *Guidance* report)

The purpose of this report is to set out a clear economic framework within which the econometric modelling has taken place. (The *Econometric approach* report considers the econometric methodologies that are available and their suitability for use in this study, together with the initial modelling undertaken by the study team.)

The report is structured as follows.

- The remainder of section 1 considers the demand forecasting model in the current PDFH and why it is being reviewed, together with a review of what properties an optimal econometric model would exhibit.
- Section 2 provides a brief literature review, drawing lessons from recent reviews of the PDFH and other recent studies relevant to the forecasting of passenger rail demand.
- Section 3 sets out the economic model on the basis of relationships derived from transport economics, and discusses some functional form issues.

¹ Department for Transport (2008), 'Rail Passenger Demand Forecasting: Revisiting the Elasticity-Based Framework Request for Proposal and Statement of Requirement', July, pp. 12–13.

- Section 4 presents some conclusions.

Each section begins with a non-technical summary of the section.

1.1 Current PDFH formulation

This section examines the current demand forecasting model set out in the PDFH. In this model, the demand for passenger rail travel changes in response to a change in a ‘demand driver’ (eg, fares, income, etc). In most cases, the model forecasts how demand changes by taking the ratio of the value of the explanatory variable after the change to its value before the change, and applying the relevant elasticity given in the PDFH.

The PDFH is being reviewed for a number of reasons, including the following.

- While the PDFH forecasts perform well over the medium to long term, they perform less well for year-on-year changes and on longer-distance, non-London flows.
- In a review of the PDFH, the external environment and fares sections were rated among the highest priorities for improvement (see Steer Davies Gleave (2008), ‘PDFH Update—Phase 1’, June, p. 11).

Section 1.2 below provides a summary of the properties of an optimal econometric model.

The current version of the PDFH (version 5)² retains the same basic framework as the previous version (version 4.1), where the change in demand for rail travel in response to a change in a demand driver (such as income or fares) is calculated, in most cases, by taking the ratio of the value of the explanatory variable after the change to its value before the change, and applying the relevant elasticity given in the PDFH. Thus, for a fare increase of 10% from a starting value of £2, the model requires the user to divide the new fare (£2.20) by the old fare (£2), and raise the result (1.1) to the power of the elasticity (say –0.7), which implies that demand is around 6.5% lower than before the fare change.³

The current PDFH framework for demand drivers outside the influence of rail companies is as described in the equation below:

$$I_E = \left(\frac{\text{GDP per capita}_{\text{new}}}{\text{GDP per capita}_{\text{base}}} \right)^g \cdot \left(\frac{\text{POP}_{\text{new}}}{\text{POP}_{\text{base}}} \right)^p \cdot \exp(n(\text{NC}_{\text{new}} - \text{NC}_{\text{base}})) \cdot \left(\frac{\text{FUELCOST}_{\text{new}}}{\text{FUELCOST}_{\text{base}}} \right)^f \cdot \left(\frac{\text{CARTIME}_{\text{new}}}{\text{CARTIME}_{\text{base}}} \right)^c \cdot \left(\frac{\text{BUSCOST}_{\text{new}}}{\text{BUSCOST}_{\text{base}}} \right)^b \cdot \left(\frac{\text{BUSTIME}_{\text{new}}}{\text{BUSTIME}_{\text{base}}} \right)^t \cdot \left(\frac{\text{BUSHEAD}_{\text{new}}}{\text{BUSHEAD}_{\text{base}}} \right)^b \cdot \left(\frac{\text{AIRCOST}_{\text{base}}}{\text{AIRCOST}_{\text{new}}} \right)^a \cdot \left(\frac{\text{AIRHEAD}_{\text{new}}}{\text{AIRHEAD}_{\text{base}}} \right)^r$$

where I_E is the volume increase in demand from the change in the external factors, NC is the proportion of households without access to a car, n is ‘a parameter that drives the elasticity to car ownership’,⁴ and the superscripts are the elasticities given in the PDFH. (For more details, see section B1 of the PDFH v5.)

The PDFH is the key source for the generation of demand forecasts for passenger rail travel in Great Britain. However, the Request for Proposal⁵ and the review in 2008 of the PDFH

² Association of Train Operating Companies (ATOC) (2009), ‘Passenger Demand Forecasting Handbook Version 5’, August.

³ Specifically, according to the model, in this example a value of 0.93546 should be multiplied by the base level of demand to give the new level of demand, and with around 93% of previous customers paying 10% more for the product, the fare increase is likely to generate revenue.

⁴ ATOC (2009), op. cit.

⁵ Department for Transport (2008), op. cit.

v4.1 by Steer Davies Gleave set out a number of reasons why the PDFH is being updated, including the following.

- While the PDFH forecasts perform well over the medium to long term, they perform less well for year-on-year changes and longer, non-London flows.
- The external environment and fares sections were rated among the highest priorities for improvement.

1.2 Optimal model structure

An optimal econometric model will have a number of features, some theoretical and others of an applied nature. Features include being easy to use; able to identify the relationships of interest; the estimated relationships being consistent with economic theory and industry knowledge; having a good fit with the observed data and forecasting well.

An optimal econometric model will have a number of features, including:

- the ability to identify the effect of the variables of interest (income, fares, etc) on the demand for passenger rail travel (*identifiability*);
- the ability to produce robust results which can be used in a straightforward forecasting framework which is suitable for a wide range of uses (*usability*);
- the ability to provide unbiased estimates of the parameters, in an efficient way—ie, the model can estimate the ‘true’ elasticity, making the best use of the available data (*unbiased/efficient*);
- the estimated relationships between the variables of interest are consistent with economic theory, industry knowledge and intuition;
- the ability to produce accurate forecasts, while fitting historical data well (*forecasting/fit to data*).

These features are a combination of the theoretical and applied properties of the model. Identifiability, unbiased and efficiency are (mostly) theoretical properties of the model and/or the estimation technique used to estimate the model, while usability and forecasting/fit to data are applied properties of the model.

The theoretical factors have been considered at length in the econometrics literature, and many textbooks contain a discussion of some or all of these issues.⁶ Therefore, a detailed discussion of these issues is not undertaken here.

The applied factors are also important, but must be considered on a case-by-case basis when the modelling has been undertaken. The next section reviews the literature to consider some of the issues that have been covered in other studies.

⁶ See, for example, Greene, W.H. (2003), *Econometric Analysis*, 5th edn, Upper Saddle River: Prentice-Hall.

2 Literature review

This section reviews both the PDFH and a number of recent studies on the demand for passenger rail travel, to determine what lessons can be learnt from these studies and how they might be applicable to the 'Revisiting the Elasticity-based Framework' study. The review highlights the following lessons.

Economic factors: there is evidence to suggest that different income measures are relevant for different journey purposes (eg, business and leisure). There is also evidence to suggest that distance may be an important aspect of model specification. Both these factors have been investigated in the econometric and market segmentation analysis.

Socio-demographic factors: the demand for passenger rail travel may be affected by the absolute size of the population, its composition, and its geographic dispersal around train stations. The degree to which these factors can be investigated in this study has been limited by the data.

Modal competition: rail faces several competitors, including private cars, bus/coach, and air travel. The way in which these other transport modes are modelled is clearly important in the production of accurate demand forecasts. The literature often uses fuel costs as a proxy for usage costs and car ownership to represent the long-run cost of owning a car. However, new variables have been developed for this study to represent the usage costs of a car.

Fares elasticities: this is a controversial area. One of the key aims for the guidance from this study is to provide a clear definition of the fare elasticity used in the study. Another important factor, which this study has not addressed due to data restrictions, is the impact of changing ticket restrictions on the demand for passenger rail travel.

There is a substantial volume of existing research on the modelling of demand for passenger rail travel, much of it summarised in the PDFH.⁷ This and other recent studies have been reviewed to gain an understanding of what has previously been investigated in the literature, and what lessons they may hold for this study.

The studies that have been reviewed in detail for this study have considered the following: economic and socio-demographic drivers of demand;⁸ patterns in rail demand over an extended period of time;⁹ regional flows;¹⁰ recent rapid growth in rail demand;¹¹ the current and previous versions of the PDFH;¹² travel to work;¹³ and the forecasting framework for aviation demand in the UK.¹⁴ This section does not consider these studies individually, but discusses the variables and the lessons for model specification that can be drawn from them. Issues relating to the functional form of the model—ie, how the variables enter the econometric model—are addressed separately in section 3.2.

⁷ ATOC (2009), op. cit.

⁸ Wardman, M, and Dargay, J. (2007), 'External Factors Data Extension and Modelling', Institute for Transport Studies, Leeds University, March.

⁹ MVA Consultancy (2008), 'Robust Foundations: Econometric Analysis of Long Time Series Rail Passenger Demand Aggregates', November.

¹⁰ MVA Consultancy (2009), 'Regional Flows: Regional Rail Demand Elasticities', September.

¹¹ Steer Davies Gleave (2008), 'Investigation into Recent Rapid Growth in Rail Industry Demand', March.

¹² ATOC (2009), op. cit, and Steer Davies Gleave (2008), op. cit.

¹³ AECOM (2009), 'The Impact of Changing Travel to Work Patterns on Rail Demand: Final Report'.

¹⁴ Department for Transport (2009), 'UK Air Passenger Demand and CO₂ Forecasts', January.

2.1 Economic demand drivers

The primary economic factors of interest to this study are income and employment, since these are the two principal variables believed to drive demand for rail travel.¹⁵

The PDFH v5 contains a substantial review of studies that have considered economic factors;¹⁶ hence this review considers only those studies not reviewed in the PDFH v5.

2.1.1 Income

One issue of concern for this study is to ensure that the income elasticity is as 'pure' as possible—ie, that what is labelled as the income elasticity reflects only changes in income, rather than other effects. A report into recent rapid growth in rail demand notes that: 'the GDP elasticity (in particular) is proxy for a wide and complex range of background growth effects.'¹⁷

The PDFH suggests that employment is an important driver of the demand for season tickets because of their use for commuting to work and education.¹⁸ However, it is also possible that the type of employment is important if, for example, workers in professional services industries make more rail trips than manual workers.¹⁹

This suggests that different income measures may be relevant for different journey purposes and/or ticket types.

2.1.2 Distance

One aspect of model specification which changed between publication of the PDFH v4.1 and the PDFH v5 is the distance effect on the income elasticity. Wardman and Dargay (2007) suggest that the positive distance effect found in the PDFH v4.1 is a function of the estimation procedure used.²⁰ Their study finds a negative distance effect, which they believe is unlikely to be an accurate reflection of the 'true' relationship. The existence (or otherwise) of a distance effect is important as it would (if present) amplify or diminish the effect of any changes in GDP on rail demand forecasts. The Steer Davies Gleave review of the PDFH v4.1 notes that:

it is unwise to assume a linear distance effect. Equally it is unwise to assume the same elasticities across all distances.²¹

This quote suggests that the effect of distance on the income elasticity is not yet comprehensively determined, and hence this investigation has been an important part of the econometric analysis used in this study.

Some of the socio-demographic issues identified in the literature, as they relate to the specification of the economic model, are considered next.

2.2 Socio-demographic and -economic demand drivers

A number of socio-demographic factors are identified in the literature as being of importance to the demand for rail travel. For example, the PDFH suggests that population is likely to affect demand for rail travel in three ways:²²

¹⁵ ATOC (2009), op. cit., Chapter B1, p. 3.

¹⁶ ATOC (2009), op. cit., Chapter C1, pp. 5–23.

¹⁷ Steer Davies Gleave (2008), op. cit.

¹⁸ ATOC (2009), op. cit., Chapter B1, p. 3.

¹⁹ MVA Consultancy (2009), op. cit.

²⁰ Wardman and Dargay (2007), op. cit. p. 4.

²¹ Steer Davies Gleave (2008), op. cit. p. 46, para 8.42.

- the absolute size of the population will affect the total number of trips made by the population;
- its geographical dispersion relative to the rail network is important since the closer a person lives to a station, the more likely they are to use rail services, all else being equal;
- the socio-economic characteristics of the population (eg, the type of job), are likely to affect their propensity to travel.

These all have implications for the specification of the economic model, although restrictions on the availability of data may limit how accurately these impacts can be modelled. However, it is clear that one of the important variables for inclusion in the general model of the forecasting framework is the population at the origin of the flow. The PDFH also suggests that in some cases (eg, commuting), the population impact which affects the demand for passenger rail travel is the change in population relative to the change in population on other relevant routes.²³

Socio-demographic factors link to both economic factors, and modal competition, which is the subject of the next section. For example, an increasing proportion of ‘white collar’ jobs in an area may raise personal disposable income, which may increase the proportion of car ownership/usage.

2.3 Modal competition

A rail demand forecasting framework will need to allow for a number of competing transport modes, including aviation, bus/coach, and car.

The analysis of modal competition is complicated by the fact that different modes can act as both complements and substitutes to rail travel; for example, buses can be used to access a train station, or as an alternative method of making the journey.

2.3.1 Car

The variable within the current version of the PDFH which is used to model the impact of changes in car ownership on rail demand is the proportion of households without access to a car.²⁴ Given that road congestion has increased significantly over the past 20 years, in both London and other cities, the study has investigated whether this is still the optimal measure of car ownership. This is essentially a measure of car *availability* as opposed to the cost or time effects of running a car. The impact of car ownership on the demand for passenger rail travel is uncertain since car availability may be a complement to rail travel (eg, by allowing easier access to train stations), as opposed to a substitute (eg, where owning a car allows the entire trip to be made by car rather than train).

The PDFH uses fuel cost to proxy for the short-run cost of running a car, separately from the car ownership variable. This suggests that there are two distinct aspects to the cost of owning a car: short-run marginal cost, currently modelled by fuel price, and long-run cost, currently modelled by car ownership. This approach is consistent with economic theory. However, to improve the representation of short-run marginal cost, the review of the PDFH v4.1 suggested that fuel costs be expanded to include:

all directly perceived motoring costs which we would suggest includes parking, tolls, congestion charges, and potentially, road pricing, as well as fuel costs.²⁵

²² ATOC (2009), op. cit. Chapter B1, p. 4.

²³ ATOC (2009), op. cit. Chapter B1, p. 8.

²⁴ ATOC (2005), op. cit. Chapter B1, p. 9.

²⁵ Steer Davies Gleave (2008), op. cit., p. 49, para 8.56.

The impact of car journey times on the demand for rail travel is currently imposed on the PDFH estimation and is based on an assumed trend in car journey times.²⁶ This may introduce inaccuracy into forecasts developed using this variable, as the trend may vary as congestion patterns alter.

Oxera and Arup have developed two new variables which capture increasing car journey times and car cost (see the accompanying *Data capability* report for more details).

2.3.2 Aviation

The PDFH uses two terms in the forecasting framework to explain the difference in rail demand resulting from changes in the characteristics of air services: monetary cost and headway.²⁷

The impact of airline competition is important but only for those flows for which air is an effective competitor. Therefore, any variable included in the forecasting framework will need to account for the flow-specific nature of air competition.

2.3.3 Bus/coach

The PDFH includes three variables to explain the impact of changes in bus characteristics on rail demand: cost, headway, and journey time.²⁸ However, discussion with the Project Steering Group suggests that these factors are not used in many practical applications. This finding is supported by the fact that only a relatively small percentage of commuters use the bus to travel to work in the areas considered—ie, Manchester and London.²⁹

This implies that bus and coach competition, while likely to be important in some local areas, may be of less importance for the overall forecasting framework than some other variables.

2.4 Fares

Fare elasticities are a controversial area, with some stakeholders continuing to use elasticities from the PDFH v4 rather than the PDFH v4.1.³⁰

Although an important issue, the impact of changes in ticket restrictions is difficult to investigate in econometric analysis of ticket sales data, which this study is pursuing. Where changing ticket restrictions have resulted in an important impact on the choice of product, this may show up as unexpected results from the econometric analysis. Hence, this restriction to the analysis will need to be considered when interpreting the results from the econometric analysis; as an example, the full and reduced fare ticket segments for the London, South East and East of England market have been combined due to the impact of fares regulation. (See the *Findings* report for more details.)

2.5 Summary

There is a substantial body of literature on forecasting the demand for passenger rail travel, which highlights a number of factors with important implications for the specification of the economic model used in this study.

This section has reviewed some of the literature, and has identified some issues which the study team has considered when conducting the econometric analysis. This literature review is supplemented by considering some relationships from economic theory in the next section.

²⁶ Steer Davies Gleave (2008), op. cit., p. 50, para 8.62.

²⁷ ATOC (2009), op. cit. Chapter B1, p. 6.

²⁸ Ibid.

²⁹ AECOM (2007), 'The impact of changing travel to work patterns on rail demand', July, p. 24.

³⁰ Steer Davies Gleave (2008), op. cit. p. 55, para 9.1.

3 Economic model

All models are simplifications of reality; the important issue is whether the model represents the behaviour of interest accurately enough for the model to be fit for purpose, which, in the case of this study, is to forecast the demand for passenger rail travel.

The relationships between the variables which are considered in this modelling framework can be assumed to be complex, and may change over time. Economic theory can be used to provide an expected direction of change of the relationships, and some of these relationships are considered in this section of the report.

The areas considered are summarised below.

Constant and variable elasticities: tests can be applied to the data to ascertain whether elasticities are constant both over time and with the level of the variable of interest.

Model stability/market maturity: stability—ie, whether the model results depend on the time period used for the estimation—is important for any model, but especially so for one that is to be used for forecasting. This is a factor which has been tested in the analysis. Many demand forecasting models, such as the UK air passenger demand forecasts,³¹ impose an assumption of market saturation/maturity on demand forecasts. Whether there is evidence of market maturity is an empirical issue which has been an important part of the analysis in this study.

Intertemporal effects: these refer to whether the estimated relationships and parameters change over time. There are theoretical reasons why the estimated elasticities and relationships between the variables may change over time. This is a particular type of model instability, but if the parameters of the model do change over time, allowing for this should result in a more stable model for the other parameters.

Pooled versus corridor analysis: pooling flows in the econometric analysis imposes an assumption that the responses to changes in the explanatory variables are the same across the flows that are being pooled. Corridor analysis is a special case of pooling, whereby the pooling is conducted on the basis of geographic rail corridors—eg, the West Coast Main Line.

Generalised time/cost: the formulation of generalised time/cost includes assumptions placed on the relationship between time and cost.

Growth factors: given the importance of the income variable, discussed in section 2 above, it is important to account for this variable as accurately as possible, and hence to consider how the income variable should be deflated in order to reflect accurately the economic variable it is intended to.

Functional forms: functional forms are the assumptions about how the variables enter the econometric model, which are made to enable the estimation of the model. These assumptions can be tested using statistical techniques.

All models are simplifications of reality; the challenge is to use models which represent the behaviour of interest accurately enough for the purpose required—in this case, to forecast the demand for passenger rail travel for a variety of purposes. Section 1.2 above set out the properties of an ideal econometric model, while this section considers the issues which must be considered in the development process of any model:³²

- what is the structure of the model?
- what are the functional form issues which need to be considered?
- what techniques are available to estimate the model, allowing for the identified functional form issues?

³¹ Department for Transport (2009), 'UK Air Passenger Demand and CO₂ Forecasts', January.

³² De Dios Ortúzar, J, and Willumsen, L. G. (2008), *Modelling Transport*, Great Britain: John Wiley & Sons Ltd, July, p. 17.

- how are the variables used to populate the model specified?

The third question is discussed in detail in the *Econometric approach* report, while the fourth question is considered in the *Data capability* report.

The next section discusses some of the theoretical relationships which may be expected to arise in the demand for rail travel in Great Britain and which may affect the economic model.

3.1 Theoretical relationships

The relationships between those variables which drive the demand for passenger rail travel may be expected to be complex, and to change over time.

The Request for Proposal sets out a number of areas which require consideration of the relationships between variables.³³ In addition, the study team, in conjunction with Professor John Preston, has considered the theoretical relationships that might be expected to exist between the variables, based on transport economics.

The Request for Proposal asks for the following areas to be considered:

- constant versus variable elasticities;
- model stability/market maturity;
- intertemporal effects;
- pooled versus corridor analysis;
- generalised cost/generalised time framework;
- the relationship between monetised and non-monetised variables.

These factors are considered below, together with some issues identified by the study team.

3.1.1 Constant versus variable elasticities

Elasticities represent passengers' behavioural responses to changes in the drivers of demand, and hence it might be expected that they vary both between passengers and over time as passengers' preferences change. However, allowing elasticities to differ for every passenger in every time period would not result in a practical forecasting framework. This issue is considered in some detail in the *Market segmentation* report, as the market segmentation process is designed to group together flows which display similar responses to changes in demand drivers. The market segmentation work suggested that the behavioural response to a change in the demand driver may vary depending on the level of the variable. This, in turn, suggests that, as the level of the variable alters (either over time or across passengers), the elasticity may not be constant. This is an empirical and functional form issue, which is discussed further in section 3.2.1 below.

3.1.2 Model stability/market maturity

Given that the primary use of the models produced by this study is forecasting, the stability of the model—ie, whether the parameters remain constant over time or vary—is of considerable importance.

In addition, demand for transport is unlikely to continue increasing at its current rate indefinitely. This issue is dealt with in transport modelling in a variety of ways, as discussed in more detail in section 3.2.2 below.

3.1.3 Intertemporal effects

As stated above, elasticities are representations of behavioural responses and hence may be expected to change over time. Box 3.1 explores why this is the case.

³³ Department for Transport (2009), op. cit. pp. 27–28.

Box 3.1 Income elasticity over time

Dargay and Wardman (2007) note that, in certain circumstances, the rail income elasticities of passenger rail travel might be expected to increase over time. The aggregate travel elasticity can be expressed as:

$$I = S_r I_r + S_o I_o$$

where I = market income elasticity of travel, S_r = the share of rail travel in the total travel market, S_o = the share of other modes in the travel market, I_r = rail income elasticity and I_o = the income elasticity of other modes.³⁴ Then, for marginal changes, it can be shown that:

$$I_r/I = (\partial Q_r / \partial Q) / S_r$$

where Q_r = rail demand and Q = travel demand.

In other words, the ratio of the rail income elasticity to the market income elasticity is equal to the ratio of the rail sector's share of new and existing trips. Supposing that the market income elasticity is 1, and the rail sector's share of existing trips is 10% but its share of new trips is 20%, the rail income elasticity is then 2.

However, it may also be expected that if demand for rail travel reaches saturation point, the income elasticity may decrease as income continues to increase but the demand for rail travel remains constant. Hence, it is not possible to specify, a priori, what the expected direction of change (if any) in the income elasticity is.

Nevertheless, the study team is conscious that, in the long run, the market for rail travel may be expected to reach saturation point, and hence the relationship between the income elasticity and the level of income is likely to be negative at that point for the reason set out above.

In addition to elasticities changing over time, the relationship between monetary variables and non-monetary variables may change over time, as the average level of income increases. This issue is discussed further in section 3.2.1 below.

3.1.4 Aggregation and corridor analysis

In the presence of many cross-sectional units of data (such as rail flows), it might be feasible in some cases to treat several units as a single, larger unit. This is referred to as 'aggregating' the data, and might be undertaken, for example, on a regional basis, according to an urban/rural split, on the basis of demographics such as income, or using a mix of several criteria. An alternative would be to group the data by rail network routes—known specifically as 'corridor' data.

Flows differ by characteristics such as length, journey time, and journey purpose. These characteristics will influence the demand for travel on a particular flow. However, the relationships between these characteristics and demand may differ between flows. Accurate identification of such relationships for each flow is important if forecasts are to be reliable.

A higher volume of data provides a more reliable picture, since individual passengers will have less influence on the total. For example, revenue on a flow with two season ticket holders will be greatly affected if one passenger switches to travelling by car. Such influence would be much less important on a flow with 100 season ticket holders. For this reason, relationships estimated with more data can be considered more robust.

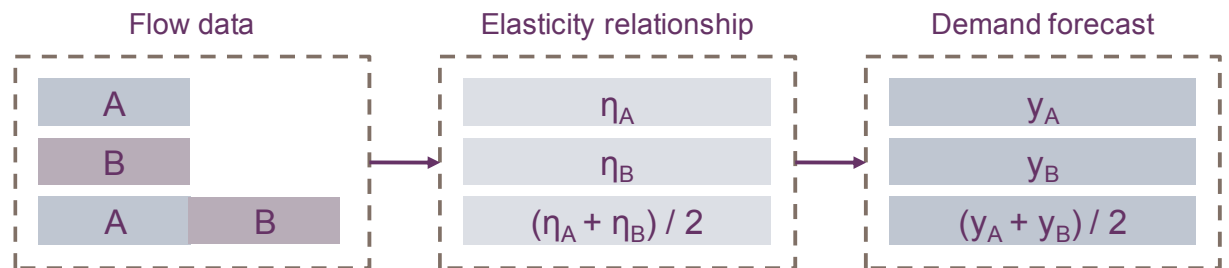
This holds only if the relationship between demand and the drivers of demand is sufficiently similar across the combined flows. Relationships estimated using combined data are a weighted average of those obtained from individual flows. Consequently, if the relationships vary significantly between the combined flows, the average estimate may not represent the true relationships for any of the flows, and hence may result in inaccurate forecasts.

³⁴ See Oum, T.H., Waters, W.G. and Yong, J.S. (1992), 'Concepts of Price Elasticities of Transport Demand and Recent Empirical Estimates: An Interpretative Survey', *Journal of Transport Economics and Policy*, 26:2, pp. 139–154.

As an example, consider two flows *with an equal level of demand*. Assume that Flow A consists primarily of leisure travellers who care relatively little about journey time, while Flow B consists mainly of business travellers who are sensitive to journey time. Flow A has a low volume of passengers travelling on full-fare tickets, as it is less common for leisure travellers to use a full-fare ticket. Combining A and B to estimate the response to changes in journey time for full-fare ticket holders may suggest that both flows are somewhat responsive to journey times. Subsequent forecasts would overestimate the impact of an increase in journey time for flow A, and would underestimate the impact for flow B.

Figure 3.1 illustrates how the flow, or pooled flows, will influence the demand forecasts through the estimated relationships.

Figure 3.1 Summary of combined data decisions



Source: Oxera.

The final row represents the combined data of A and B. The average elasticity relationship feeds through to average demand forecasts. If the relationships in flow A and flow B differ strongly, the combined forecast will be inaccurate for both. The aim of the market segmentation is to group together flows which have similar demand responses. (See the *Market segmentation* report for details.)

Table 3.1 summarises suitable approaches in different data contexts.

Table 3.1 Summary of combined data decisions

| | Low volume of data | High volume of data |
|----------------------------|-------------------------------------|---------------------|
| Homogeneous flows | Combined estimates | Flow estimates |
| Heterogeneous flows | Compare combined and flow estimates | Flow estimates |

Source: Oxera.

In the presence of high volumes of data there is no direct need to combine the data. However, if the flows are highly homogeneous, combined estimates will be more accurate, given the increased data and the reduced number of parameters that need to be estimated.

In the presence of low volumes of data, homogeneous flows can be combined without necessarily adversely affecting parameter estimates and forecasts. Where there are low volumes of data and reasons to expect differences in demand relationships, neither approach is obviously preferable. Parameter estimates for individual flows may be influenced by the actions of individual passengers, and thus may potentially be misleading, while the combined estimate may be misleading since it may mask different relationships. In extreme cases:

- where the number of flow-level observations is very small (< 30 observations per year), combined data is arguably more reliable, due to the increased volume of data;
- where demand relationships are expected to be highly heterogeneous, individual flow estimates may be preferable, as an average could mask important differences.

The issue of how to combine data—for example, by region, demographics or rail corridor—is therefore a question of heterogeneity. Combining may offer benefits if the data is sufficiently homogeneous, in that the behavioural responses influencing demand are symmetric across the passengers in the segment.

It is also possible for certain relationships to be similar across segments, and for others to differ—for example, responses to changes in income may vary geographically, while attitudes to reliability may be similar geographically, but may vary by journey purpose. The potential for bias through combining therefore depends on the relationships of interest.

In practice, economic reasoning and an empirical assessment of the flow characteristics should guide combining decisions. Comparisons of the combined estimate to the underlying individual flow estimates provide an important cross-check of the rationale for combining. The market segmentation work, detailed in the *Market segmentation* report, aims to group together flows which display similar responses to changes in demand drivers, and hence to produce more homogeneous groups of flows.

The use of corridors is a special case of combining data.

3.1.5 Generalised time/cost

Generalised journey time (GJT) is defined as:³⁵

$$\text{GJT} = \text{journey time} + \text{service interval penalty} + \text{interchange penalties.}$$

This imposes a particular assumption on the relationship between journey time, service interval penalties and interchange penalties—ie, that an increase in one can be offset by a decrease in another at a linear rate. Given the correct data, this could be tested empirically. However, given the data available for the purposes of this study, it is not possible to test this assumption and hence the study has continued to use GJT.

Generalised cost (GC) is defined as:

$$\text{GC} = \beta\text{GJT} + F$$

where β is the value of time and F is the fare. As in the discussion above, this imposes certain assumptions on the relationship between GJT and fare. This relationship is also testable, if the data is suitable. However, unless there is accurate data available for the value of time (eg, different journey purposes and mappings from journey purpose to ticket type) then it is likely to be difficult to draw firm conclusions from any analysis. Given that the requirement for this study is to estimate directly as many elasticities as is feasible within the econometric framework, the study team has not estimated generalised cost elasticities, but has estimated separate GJT and fare elasticities.

3.1.6 Non-monetised variables

The relationship between fares and GJT is complex, and is best illustrated by algebra. Assume that there is a relationship of the form:

$$Q = a \text{GC}^b Y^c$$

where $\text{GC} = F + \beta.\text{GJT}$, β = value of time, Q is the demand for travel by rail, F is the fare and Y is income. With this formulation, the fare elasticity = $b(F/\text{GC})$ and the GJT elasticity = $b(\beta\text{GJT}/\text{GC})$. Assuming $d\beta/dY$ is positive then, as real income increases, the GJT elasticity will increase and the fare elasticity will decrease, all else being equal. *This implies that over time, the GJT elasticity may be expected to increase, while the income elasticity may be expected to decrease.* However, as noted in Box 3.1, there may be factors which tend to increase the income elasticity over time.

³⁵ ATOC (2009), op. cit. Chapter B4, p. 2.

If it is further assumed that the demand for passenger rail travel can be expressed as $Q = a F^b G^c J^d Y^e$ (where Q , F , and Y are defined as above, and a , b , c and d are parameters to be estimated) and if the value of time is a function of income, the formulation of the income elasticity will become more complicated. Suppose that $\beta = dY$, the income elasticity becomes $bdGJT/[(F/Y) + dGJT] + c$. Given that the expected signs are $b < 0$ and $d > 0$, as income increases the income elasticity will decrease. In fact, the income elasticity will change with income as follows: $(bd GJT/ (F/Y + dGJT)^2) F/Y^2$.

This implies that a constant elasticity formulation (see section 3.1.1 above) may not be appropriate, given that there are theoretical considerations which suggest that the elasticities may change with the level of the variable. Whether there is evidence for this is an empirical issue which has been addressed in the econometric analysis.

3.1.7 Growth factors

Income is likely to be an important driver of the demand for rail travel, and hence it is important that the measure of income used in the econometrics reflects the income measure of interest as accurately as possible.

The optimal theoretical outcome is to have separate income variables for business and leisure journey purposes to model the different income drivers which affect rail demand, discussed in section 2.1 above:

- for business, the optimal income variable is gross value added (GVA) on an employment basis, deflated by an industry-weighted GDP deflator;
- for leisure, the optimal income variable is personal disposable income on a residence basis, deflated by a consumer price index.

Income may also affect demand for season tickets since, if incomes increase, passengers may be more inclined to use rail as their mode of transport. Therefore, it is important to allow for this factor in the analysis.

To change these into appropriate per-person variables, the business variable should be on a per-worker basis—ie, the most appropriate business income variable is productivity (GVA per worker), while, for leisure, the income variable should be personal disposable income per capita at the origin (or household).

However, these theoretical variables do not necessarily have practical counterparts. The available income variables are discussed in the accompanying *Data capability* report.

3.2 Functional forms

A functional form is a restriction placed on the form of a model to allow the behavioural relationships of interest to be estimated, such as constant elasticities or linear relationships between variables. These assumptions can be tested using statistical techniques to determine whether the restriction is supported by the data.

There are a number of functional form issues considered in this section:

- constant or variable elasticities;
- changing relationships between variables over time;
- market saturation/maturity;
- dynamics;
- structural breaks.

3.2.1 Constant or variable elasticities

The current version of the PDFH assumes that all elasticities are constant, both over time and with the level of the variable.³⁶ Whether elasticities vary over time has been considered in previous research, with the broad conclusion reached that there is limited evidence of elasticities varying over time. However, there is some evidence that elasticities may also vary depending on the level of the variable.³⁷

The most common approach to estimating constant elasticity models is the double log formulation.

$$\ln(y_t) = \beta_0 + \beta_1 \ln(x_t) + \varepsilon_t$$

where y_t and x_t are variables, ε_t is an error term, and β_0 , and β_1 are parameters to be estimated.

This model specification has the advantage that it is easy to estimate and interpret, but the disadvantage that it cannot be derived from aggregation over individuals, and therefore restrictions from consumer choice theory cannot be readily imposed on the model. This may result in models that are not internally consistent; therefore, when interpreting the results of models of this type, it is important to be aware of this limitation. This means that the results may not satisfy restrictions from consumer choice theory. Specifications of this type are commonly used in empirical studies (eg, UK air passenger demand forecasts³⁸).

Constant elasticities impose an assumption on the data that the elasticities do not change over time. This has been tested in this study.

3.2.2 Market saturation/maturity

Some demand forecasting methodologies in the UK assume that the market will become saturated in the future. The methods for imposing this saturation effect include adjusting the demand forecast by an 'x' factor—eg, the Department for Transport's UK air passenger demand model assumes that market saturation will arise between 2010 and 2020 in different markets.³⁹

An alternative to imposing an external saturation effect is to test whether a saturation hypothesis is supported by the data, by including a squared term of the variable of interest in the model to be estimated—for example:

$$\ln(y_t) = \beta_0 + \beta_1 \ln(x_t) + \beta_2 \ln(x_t)^2 + \varepsilon_t$$

where y_t and x_t are variables, ε_t is an error term, and β_0 , β_1 and β_2 are parameters to be estimated.

If the estimated coefficient on this squared term (β_2) is negative, this provides support for the hypothesis that there is a market saturation effect because of the U-shape of the quadratic function—ie, as x_t increases, so does the elasticity, up to a point. After that point, as x_t increases further, the elasticity decreases. One potential issue to consider when forecasting is that there may be a saturation effect in the future which is not present in the data available for model calibration. Under this scenario, it may be necessary to impose assumptions on the forecasts relating to market saturation.

A different approach, which is adopted in the PDFH, is to use an exponential function to allow the elasticity to decrease as the level of the variable increases. For example, as the

³⁶ The exception is the car ownership elasticity, which is specified such that it varies with the level of the car ownership variable.

³⁷ See, for example, the accompanying *Market segmentation* report.

³⁸ Department for Transport (2009), op. cit. p. 98.

³⁹ Ibid, p. 114.

proportion of households with access to a car tends towards one, the effect of a marginal change decreases.

3.2.3 **Dynamics and structural breaks**

As stated above, all models are simplifications of reality. Dynamics are a way of relaxing some of the assumptions imposed on the models by allowing adjustment to changes in the explanatory variables to occur over a period of time, rather than within the same period as the change occurred. These types of models are discussed in more detail in the *Econometric approach* report.

Structural breaks allow the relationships between the variables within the model to change over time, either abruptly or over a period of time, and can be tested for using a variety of techniques.

The introduction of dynamics and structural breaks can introduce greater complication into the models, making the interpretation of the coefficients more challenging. However, they also allow for a much richer model specification and for more realistic relationships between the explanatory and dependent variable, and hence the production of more accurate forecasts.

This section has considered some of the functional form issues which the econometric analysis has accounted for. The next section concludes this report.

4 Conclusions

In conclusion, there are many complex relationships between the variables included in the forecasting framework. This report has considered some of the functional form and model specification issues that have been accounted for in the econometric modelling.

As can be seen from the above sections, the specification of the form of the economic model is a complex exercise. Following the specification of the economic model, further assumptions need to be made in order to specify the econometric model. It is important that as many of these assumptions as possible are tested using the data in order to determine whether the data supports the assumptions.

The general models, derived from economic theory and industry knowledge, are as follows:

$$\text{Journeys} = \text{Journeys}_{t-1} + \text{fare} + \text{population} + \text{income} + \text{employment} + \text{prop. no car} + \text{car cost} + \text{car journey time} + \text{GJT} + \text{performance} + \text{SQI}$$

For season and full fare tickets, the income measure is GVA per employee at destination, while for reduced fare tickets, it is personal disposable income per capita at origin.

The study team has investigated four separate functional forms for each of the market segments, in addition to the basic double-log formulation:

- to allow for elasticities which alter with the level of the variable, a specification is run where income, population and employment enter the specification as levels, not logs;
- to allow for elasticities which vary with time, time dummies are interacted with fares, income, GJT, and car journey time, along with the level of the variable;⁴⁰
- market saturation is tested by including squared terms of the variables in the model;
- the impact of distance is tested by interacting the distance of the flow with the variable (income, population, employment) and testing whether the interaction term is significantly different from zero.

For details of the econometric techniques considered, and the process for the econometric modelling, see the accompanying *Econometric approach* report. The data collected for use in this study is discussed in the accompanying *Data capability* report.

⁴⁰ An F-test is then conducted on the interactions to test whether they are jointly different from zero.

Park Central
40/41 Park End Street
Oxford OX1 1JD
United Kingdom

Tel: +44 (0) 1865 253 000
Fax: +44 (0) 1865 251 172

Stephanie Square Centre
Avenue Louise 65, Box 11
1050 Brussels
Belgium

Tel: +32 (0) 2 535 7878
Fax: +32 (0) 2 535 7770

Thavies Inn House
7th Floor
3/4 Holborn Circus
London EC1N 2HA
United Kingdom

Tel: +44 (0) 20 7822 2650
Fax: +44 (0) 20 7822 2651