Low Flow Frequency Analysis

Guidelines for Best Practice

R&D Technical Report W6-064/TR1

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Tel: 01454 624400 Fax: 01454 624409
Website:www.environment-agency.gov.uk

ISBN: 1 85705 996 4

This report is the result of a collaborative study jointly funded by the Environment Agency and the Natural Environment Research Council.

Dissemination Status
Internal: Released to Regions
External: Released to Public Domain

Statement of Use
This report provides guidance for frequency analysis of annual minimum D-day flows. It will be of use to Water Resources staff involved in operational and planning tasks.

Key Words
Low flow, annual minima, flow-duration-frequency methods, probability distributions.

Research Contractor
This document was produced under R&D Project W6-064 by:
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Environment Agency Project Manager
The Agency's Project Manager for R&D Project W6-064 was:
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ACKNOWLEDGEMENTS

The authors would like to thank Andrew Wall (Environment Agency, Wales) and Daniel Cadman (Environment Agency, Anglian Region) for their helpful guidance in preparation of this guidance note.

EXECUTIVE SUMMARY

This document is a technical annex to R&D project W6-064, 'Probability distributions for x-day daily mean flow events'. It provides guidance on best practice for frequency analysis of annual minimum flows.

The general issues to be considered when contemplating a frequency analysis are discussed with a strong emphasis on likely sources of error and uncertainty. In order to encourage a consistent methodology to be adopted in the UK a single parametric approach for estimating the probability of occurrence of low flow events is recommended. This approach is based on the use of L-moments with a Pearson Type III probability distribution to estimate the flow - return period relationship for annual minima of different durations.

CONTENTS

LIST OF SYMBOLS

x	Observed annual minimum flow
x_i	Annual minimum flow of rank i (in ascending order)
n	Number of sampled values
X	Random variable corresponding to annual minimum flow
D	Duration of low flow period (in days)
T	Return period or recurrence interval
p	Probability of non-exceedance
F(x)	Cumulative distribution function
Q(p)	Quantile function
k	Shape parameter
α	Scale parameter
ξ	Location parameter
Γ	Gamma function
G	Incomplete Gamma Function
ρ_s	Spearman Rank correlation coefficient
χ^2	Chi-square goodness of fit parameter

LIST OF ABBREVIATIONS

p.d.f.	Cumulative probability distribution function
AM	Annual minima
MAM(D)	Mean annual minimum flow at duration D.
MF	Mean flow
LFFA	Low Flow Frequency Analysis
RMSE	Root mean square error
Q95	The one-day discharge exceeded 95% of the time

1. INTRODUCTION

1.1 Objectives and Applicability

This document provides guidance for frequency analysis of annual minimum flows. It is not intended as a 'rule book', rather its aim is to promote a consistent approach to low flow frequency analysis in the UK. Whilst a guideline methodology is described, no programs or software tools are provided for its implementation. However references to appropriate software packages are provided where required.

The guidelines given are specifically for good quality flow records longer than 25 years in length, and may give misleading and uncertain results if applied to shorter or incomplete flow records. For example, a semi-deterministic approach, relying on alternative sources information about the low flow regime, such as from an analogue catchment or rainfall-runoff model might be more suitable for sites with limited (less than 25 years) flow data. Furthermore, as the method has been derived and tested using measured flow series, it is not necessarily best practice for synthetic or modelled data, such as naturalised flow series. Similarly, it should be used with caution for series that are known to be subject to large hydrometric errors, especially where these are unquantifiable. The method is also generally inappropriate for frequency analysis of other hydrological variables, such as annual maxima, level, velocity and so on, or for annual minima from regimes unlike those found in the UK.

The document is pitched at those with a basic understanding of statistical principles, and a glossary is provided for clarification where necessary. For those unfamiliar with the subject, the main principles and assumptions of low flow frequency analysis are also summarised later in this chapter (Section 1.3). The main aim to increase awareness of some of the pitfalls and issues connected with frequency analysis, such as sources of uncertainty or bias. These points are discussed in Chapter 2, whilst a detailed step-by-step methodology is presented in Chapter 3. Finally a worked example is included in Chapter 4.

1.2 Project Details

The recommendations presented are based on the findings of R&D project (W6-064) entitled 'Probability Distributions for "x-day" Daily Mean Flow Events' (Zaidman *et al.*, 2002). The project reviewed the use of parametric estimation methods within low flow frequency analysis, and examined the ability of different candidate probability distributions to describe the occurrence of D-day annual minima flow events. The study was based on data for 25 UK rivers having long, stable and natural flow records. It should be noted that of these, 20 were located in upland areas, including three in Wales and 14 in Scotland, whilst only five were from the aquifer dominated regions to the south east of England. The range of durations examined included D=1, 7, 30, 60, 180 and 365 days. The study also examined different methods of deriving the minima and ensuring that these were both stationary and independent.

Three candidate distributions were considered in the study: the Generalised Extreme Value, the Pearson Type III and the Generalised Logistic. In each case the 3-parameter form was used. Extreme value theory suggests that the frequency behaviour of annual minima will follow that of a Generalised Extreme Value distribution, however in practice a number of other distributions have been shown to describe the observed equally well. Using the method of L-moments for parametric estimation, the parameters of each of the three candidate distributions were determined based on the flows and plotting positions probabilities of the observed data. A fourth distribution, the Generalised Pareto, was also considered. Although this distribution was not theoretically suitable for describing extreme events, it was included as a 'control', i.e. to see whether this expectation was born out in the results.

All of the four distributions were able to satisfactorily represent the form of the observed data points, to some degree or another. A method of ranking the candidates according to their goodness-of-fit criteria, and RMSE was used to identify which distribution best represented the observed frequency curve for each series. A number of trends became clear from this exercise. For annual minima of short duration the Generalised Extreme Value and Pearson Type III distributions performed well in responsive catchments found in upland areas. These distributions performed less well for series based on longer durations, where short-lived extremes become averaged out. The Generalised Logistic was best in the high-storage catchments that typify many lowland areas of south east England. In some cases the distributions were not able to provide physically reasonable estimates for annual minima-recurrence intervals much beyond the observed range. Where the prescribed flow was less than 10%MF 'sensible' estimates of recurrence interval were, in general, obtained only for annual minima of short duration for impermeable catchments. Where annual minima of longer duration or catchments of high permeability were considered recurrence interval estimates were realistic only for higher prescribed flows.

1.3 Low Flow Frequency Analysis

Low Flow Frequency Analysis (LFFA) is a stochastic approach for characterising low flow events. The pivotal aim is to quantify the likelihood that the flow at a particular site will persist below a particular level over a particular duration. LFFA is thus typically utilised where using a single statistic or index, such as the MAM7 or Q95, is insufficient to describe the low-flow regime. For example in water resource planning, where low flow events of different length and severity need to be considered within an historical context, LFFA provides a means to quantify the flow-duration-frequency behaviour of the site of interest. Individual low flow events can be delineated by considering periods where the flow falls below a threshold level (i.e. LFFA is applied to a partial duration series). However, unless the flow record is particularly short, a simplified approach, using some representative annual value to typify the overall character of the low flow season, is often favoured. Customarily the minimum D-day average discharge per year is considered.

Given a set of observed annual minima $\{X_i, i=1..n\}$, the goal is to estimate the probability of occurrence of some minima, x. Presuming that the observed values are independent and identically distributed (i.e. that observed minima are random realisations of a single population of annual minima), this can be achieved by finding the cumulative distribution function, denoted by F(x), which represents the probability, *p*, of any previous or future minima, X, being less than or equal to some given value, x.

$$p = F(x) \qquad (1)$$

Similarly the probability of x being exceeded by X, termed F'(x), is given by 1-*p*

$$1-p = F'(x) \qquad (2)$$

The corresponding quantile function, denoted by Q(p), defines the value x associated with the p^{th} quantile:

$$x = Q(p) \qquad (3)$$

and can be used to define the flow x_T, associated with a recurrence interval T, where T is the reciprocal of *p* (and vice versa), as follows:

$$x_T = Q(\frac{1}{T}) \qquad (4)$$

As a single probability distribution function is assumed to describe all annual minima occurring at the site of interest during the lifetime of the river, including those within the observed series, the sample data are used to define the form of F(x). For instance the statistical characteristics of the observed data, particularly aspects of the shape and density (e.g. skewness and kurtosis), are assumed to be valid for the population as a whole. However, with few rivers having flow records longer than 50 years in length, most annual minima series represent a relatively small fraction of the possible range for the site of interest (i.e. there is a large sampling basis). Moreover, the observed data provides little detail regarding the shape of the distribution function in its upper and lower tails (at extremely high and low probabilities. Thus the problem of estimating F(x) reliably for all possible values of x is very difficult. Ironically the interest is usually in minima that lie well outside the range of observed values. As a result frequency analyses predominantly rely on parametric estimation procedures.

In the parametric approach, *a priori* assumptions about the shape of the cumulative distribution function are used to select an appropriate hypothetical distribution family as the basis of the mathematical expression describing the distribution. The observed data set are then used to constrain the parameters of this distribution (i.e. select the most relevant family member) usually using fitting techniques such as L-moments (Hosking & Wallis, 1997) or maximum likelihood estimation (Cox & Hinkley, 1974). The danger of this approach lies in choosing an inappropriate hypothetic distribution that will produce a misleading quantile estimates. To minimise the chance of using an unsuitable distribution a number of 'candidate' distributions are considered. The uncertainties associated with the resulting models are then assessed using goodness of fit tests or by applying re-sampling techniques to produce confidence intervals (e.g. Takara & Stedinger, 1994). Unfortunately, as most flow records are short, a number of different distribution types may all fit the observed annual minima reasonably well and it may not be possible to discriminate between them on an objective basis. Thus a particular model may be favoured for practical reasons, such as computational convenience, or because it exhibits certain characteristics that the user believes a low flow distribution should have. For example, a distribution having a finite lower limit equal to zero (to represent the possibility of recording a zero, but not a negative, flow) is often considered preferable to one that does not. This lack of objectivity in LFFA, coupled with the unparsimonious use of assumptions, has brought criticism from a number of authors including Klemes (2000), and facilitated wider use of non-parametric function estimation routines.

Candidate distributions are generally chosen from the extensive 'library' of established distribution families - some of the more common families are summarised by Evans *et al.* (1993). 'Bespoke' distribution functions are also sometimes used, such as those of Gottschalk *et al.* (1997) based on low flow recession behaviour. Extreme value distributions such as the EV1 (Gumbel) and EVIII (Weibull) have traditionally been considered the most applicable of the established distribution families. This goes back to the work of Fisher and Tippet (1928) who showed that the minima of a particular sample will theoretically tend to one of three extreme value forms, named the EVI, EVII and EVIII. The Weibull distribution, advocated by Gumbel (1963), has found particular favour in the UK (e.g. Institute of Hydrology, 1980). The Log Pearson Type III (Loganathan *et al*., 1986), the Log-Normal (Kroll & Vogel, 2002) and the Gamma (Bobée & Ashkar, 1991) are also commonly used. A summary of recent work is given by Tallaksen (1999) and by Zaidman *et al*. (2002).

2. GENERAL CONSIDERATIONS

2.1 Introduction

This chapter discusses the general considerations which must be made when undertaking a frequency analysis of annual minima data, outlining sources of error and uncertainty. The various considerations are presented in order of importance, with the need for a consistent and pragmatic approach outweighing some ‘scientific’ considerations.

Figure 1 shows the general stages that should be involved in a low flow analysis, from deriving and manipulating the annual minimum series, through parameterisation of a distribution selected *a-priori*, to the assessment of the resulting flow-recurrence relationship derived from its quantile function.

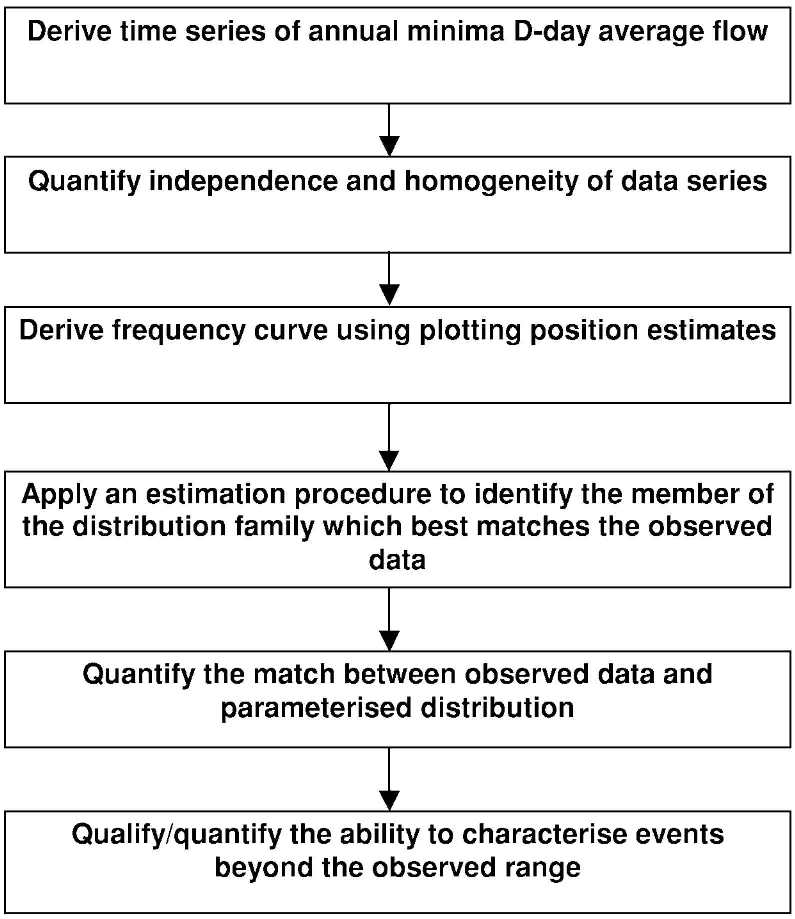

Figure 1: General stages of a low flow frequency analysis

2.2 Record Length

Record length is possibly the most influential factor in frequency analysis. Decreasing the sample size introduces sampling errors and increases the inherent uncertainty associated with the flow and recurrence interval relationship derived from the sampled data. This is because the assumption that the sampled minima are random representations of the true low-flow distribution breaks down where the record length is short. Whereas a large sample is likely to clearly exhibit the features (such as the level of skew and kurtosis) of the population of interest, a small sample is highly unlikely to be representative of the population. The level of sampling error is also influenced by the period over which the observations were made (e.g. a 30-year record from 1940 to 1970 may contain fewer extreme events than a 30-year record from 1970 to 2000). Furthermore, plotting position formulae are not robust for small sample sizes because they do not take into account the range and skew of the sampled flows, and thus cannot be thought of as unbiased estimators for short records, whilst many of the statistical tests used within the LFFA framework are unable to provide unequivocal answers where the sample size is small.

Ideally, to minimise sampling errors and increase the number of observed events in the extreme tails of the distribution, only records of several hundreds of years in length would be used for analysis. In practice, of course, most rivers in the UK have only been gauged since the 1960's, giving an average record length of around 40 years. Although it is difficult to quantify the length at which a record becomes 'short', an observed record length of 40 years is about the minimum record length that might give relatively reliable estimates for use in water resources application. **It is inadvisable to use LFFA with records less than 25 years in length.**

As a rule of thumb, a one in T year event requires a minimum record length of T/2 years, not counting missing or rejected years. If the aim is to characterise the 1 in 200 year annual minimum, then a longer record (100 years or more) is required than if the 1 in 50 year event was of interest (25 years or more). Similarly, a considerably higher uncertainty would be associated with a one in 100 year event based on a 30 year record, than a 1 in 100 year event estimated from a record of 60 years. Whilst uncertainties associated with individual components may be quantified (e.g. resampling methods may be used to provide some quantification of the sampling errors) the overall uncertainty associated with record length is not easily quantified.

2.3 Assumption of Statistical Independence and Stationarity

The assumptions that the data must be independent and originating from the same statistical population are central to the LFFA method. The latter assumption also implies that the data must be stationary (i.e. show no trends over time) and homogeneous, and there must be no outliers amongst the sample data. Short records are particularly vulnerable to the effects of short-term trends, which might average out over the long term.

Independence

Where sample data are not independent they cannot be thought as representing a population of random variables. Serial elements may manifest in annual minima due to catchment storage processes (e.g. carry-over of base flow conditions from one year to the next) and are more likely for longer duration minima, where two successive minima have a number of days in common. Statistical dependence has an important effect on the interpretation of the results from LFFA. Suppose for example that the analysis suggests that an annual minimum flow has a non-exceedance probability, p. The probability that annual minima in two consecutive years both do not exceed a particular flow will then be p^2, but if they are interdependent it would be closer to p. **Thus ignoring interdependence may lead to substantial underestimation of risks of sequences of years with low flows.** Non-independence can also influence the results of formal tests of fit and assessment of the uncertainty in parameter estimation used within the LFFA process.

Whilst a number of tests may be used to identify whether a data series is independent and originating from the same population (see Chapter 3), these tests are likely to perform poorly where the number of observations is small, and the user is advised to apply the most powerful tests as possible. It is possible to revise the LFFA procedure to take account of dependent data (e.g. Chung & Salas). However this will be beyond the capabilities of most non-statisticians! It is perhaps better to be aware of the effects of dependence on a qualitative level and take a view based on knowledge of the catchment and the drought events concerned. Generally, in impervious catchments where the soil moisture deficit is fully replenished during the winter, there is no serial correlation between annual minima induced by storage (indeed there may be independent events within the year). Storage from aquifers, reservoirs, soils that may not become saturated in winter and areas of persistent snow will all increase the potential for serial correlation.

Stationarity

Causes of non-stationarity include problems with the recording process (such as changes in rating equations, relocation of stations or changes in recording method), changes in the catchment (such as land use change), climatic variability or climate change. Trends are usually more evident in short records (the effects often average out over the longer term). If the user is confident that trend effects can be removed from the time series, the adjusted series may be used in LFFA.

Outliers

True outliers may be caused by 'one-off' artificial influences in the catchment or by measurement errors, and are less common for large durations, where the effects of abnormal daily flows are averaged out. However in a frequency analysis of extreme events it may be difficult to determine whether an outlying data point really is a true outlier or simply an extreme rare flow. In the latter case the data point is crucial as it will provide information that will help constrain the tail shape of the distribution function. It is therefore important not to remove apparent outliers arbitrarily. Where possible, outliers should be verified using circumstantial evidence, such as local knowledge, rainfall records, or by seeing if outlying events occurred on a regional basis. If an objective treatment is desired, a number of tests for outliers are well established in the statistical literature (e.g. see Barnett and Lewis, 1994).

2.4 Choice of Distribution

The aim of LFFA is to choose the theoretical distribution with the most appropriate shape for the data. However, the choice of distribution is rarely the major source of uncertainty in the end result. Whilst it is common practice to compare one or two different candidate distributions, there is little point in agonising over several alternatives giving similar results, particularly as record lengths for UK rivers are typically not long enough to ensure that a single 'best' distribution would be unequivocally identified in each case. We therefore **advocate the Pearson Type III distribution for use with all catchment types and all durations.** As well as giving the best performance in the study, the Pearson Type III has some physical attributes which make it a good choice for frequency analysis. In particular, the Pearson Type III is unlikely to predict negative flows for high recurrence intervals.

2.5 Fitting Technique

Within frequency analysis there is a profusion of competing estimation procedures that can give different results, and several studies have focussed on comparing the performance of these various techniques (e.g. Arora & Singh, 1987). However two parameter estimation methods dominate: likelihood based techniques (i.e. maximum likelihood), and moment based matching techniques, specifically the use of L-Moments. Both are well established techniques and there are many examples of their use in the frequency analysis literature.

L-Moments are linear combinations of the probability weighted moments and use the sample data to provide estimates of certain properties of the underlying population, which are then matched to a member of the chosen distribution family. A full description of the theory of the L-Moments method is beyond the scope of this document. Hosking and Wallis (1997) provide an extremely comprehensive review of the L-Moments methodology, although the same ideas are also discussed in a number of recent journal papers including Hosking (1990). The advantages of L-moments are that they have been shown to be unbiased, have relatively small sampling variance and are relatively insensitive to outliers (on the down side this latter point also means that large (or small) sample values reflecting important information in the tail of the parent distribution are given relatively little weight in the estimation procedure).

Likelihood based techniques are well established in statistical theory and practice. They are more versatile than L-moments and can be adapted to deal with a range of circumstances, such as dependence between observations. The estimators are known to have certain optimal properties in the sense that, one the sample size is large enough, no other estimators (including L-moments) have better properties. Unfortunately they do not perform as well where the sample size is small. A number of problems can arise in attempting to apply maximum likelihood in practise, such as the non-existence of a maxima, or existence of several local maxima, and so on.

Generally, the choice of fitting technique is irrelevant in the context of uncertainties generated by there being insufficient record length. Therefore from a pragmatic perspective, we suggest that **L-moments should be used for parameter estimation**. L-moments are widely applied in flood frequency estimation at present and are available in many statistical/ hydrological packages such as WINFAP and MIKE11.

2.6 Hydrometric Errors

It is assumed that the data sets to be used will be free from hydrometric errors. An analysis of errors carried out as part of the project showed that where the daily flow series are subject to random hydrometric errors of less than ± 10%, there will generally be little effect on the results of the frequency analysis, especially where longer duration are considered (the errors will tend to average out). However random hydrometric errors of up to ± 20% can result in more than a 20 year uncertainty in recurrence interval. Systematic hydrometric errors (bias) are likely to impose a systematic bias on the flow-recurrence interval curve.

3. RECOMMENDED METHOD

3.1 Introduction

Specific guidelines for low flow frequency analysis are presented in this chapter. As discussed earlier, the guidelines are based on gauged flows in catchments with minimal artificial influence and more than thirty years of record. The flow chart shown in Figure 2 gives an overview of the various stages in the recommended approach. The stages are discussed further in Sections 3.2 to 3.6.

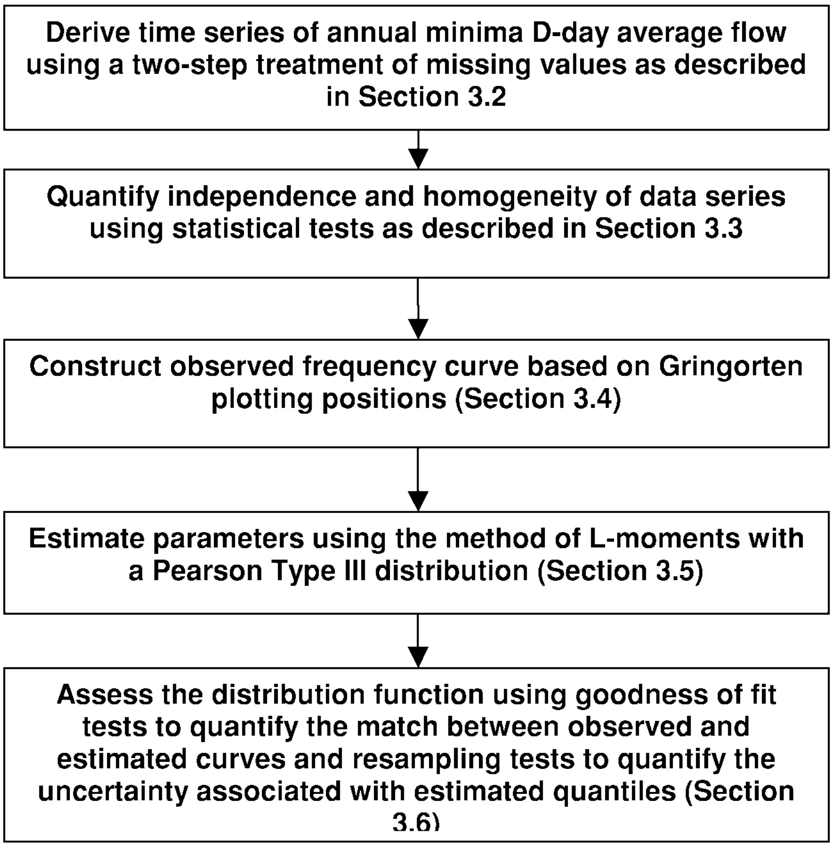

Figure 2: Overview of guidelines for best practice in LFFA

3.2 Computation of Minimum D-day Flow per Calendar Year

Here D-day average discharge is calculated on a running average basis for the entire period of record. This means that a window of duration D days is moved sequentially through the record using an increment of one-day, the mean flow over the duration being calculated at step. Each D-day mean is indexed to the middle day of the interval. For any interval of length D, the index day with be the m^{th} day where m is defined as:

$$m = \begin{cases} \left(\frac{D}{2}+1\right) & \text{, where D is even} \\ \left(\frac{D+1}{2}\right) & \text{, where D is odd} \end{cases} \tag{5}$$

For example the 7-day average determined using flow data from 3rd September 1983 to 9th September 1983 inclusive would be indexed to the 6th September 1983. As the running average method is utilised with an increment of one day, 365 running average values are determined for each calendar year, the smallest being used as the annual minimum.

If some daily flows are missing a particular year within the flow record, the level of uncertainty associated with the minimum D-day flow for that year increases, particularly where the missing values occur within the low flow period or where the number of missing values is relatively large. However, data missing from periods of relatively high-flow are unlikely to have much influence on the annual minima calculated for a particular year. Although missing values may be filled in by interpolation, this is inappropriate where several consecutive values are missing and an objective and consistent treatment of missing data is therefore required. To avoid filling in large gaps by interpolation whilst also avoiding rejecting years unnecessarily, the annual minima from data-poor years are rejected as a first step, whilst years with too much missing data within the low flow period are excluded in a second step as follows:

Step 1. The number of missing data allowed per calendar year is constrained to 30 days, i.e. an annual minimum is rejected outright if the year it represents contains 30 or more days with missing values.

Step 2. A 'low flow period' is delineated for each remaining year based on the shortest continuous period in which the lowest 20% (73 days) of daily flows are represented. For example if the 60 smallest flows occurred between 15th July and 15th September, and the next 13 lowest flows occurred between 10th and 22nd June the 'low flow period' is assumed to span the period from 10th June to 15th September. The year is discarded if the maximum number of consecutive missing values within the 'low flow period' exceeds 7 days. However if there are less than seven consecutive missing days, the year is only discarded if the aggregate number of missing data during the 'low flow period' is greater than 10 days, otherwise missing data can be filled by interpolation.

Although strictly not missing data, particular problems occur at the beginning and end of the flow record. The first (D-m) days at the start of a record and the last (m-1) values at the end of the flow record cannot be used as index days. For instance to derive a value for 1st January 1968 requires (m-1) extra days from the previous year 1967, yet if 1st January 1968 is the first day in the record, these extra data do not exist. A similar situation arises at the end of the flow record and results in a reduced number of running-averages being computed for years at the end and beginning of the record. Where these years are acceptable in Step 2, they can be included in the analysis provided that the value of m does not exceed 75 (tests showed that the value of the annual minima was changed only if more than 75 days were missing).

3.3 Validation of Independence and Stationarity

Sample data possessing serial elements cannot be thought of representing a population of random variables, therefore the independence, stationarity and homogeneity of the annual minimum series should be verified prior to beginning the frequency analysis procedures. In particular, non-independence may manifest as larger durations are considered.

The statistical literature gives examples of appropriate tests that may be applied to the data. For instance independence may be assessed using Anderson's Test, the Wald-Wolfowitz Test, the Durbin-Watson method or by determining the auto-correlation function of the data. Similarly the level of stationarity can be quantified by using tests such as the Spearman Rank Test and the Mann-Kendall Test, as well as via a number of non-linear methods. Tests for homogeneity include the Mann-Whitney Test. However it is important to note, that most of these tests do not perform well when the sample size is small (i.e. for many stations the record length will be insufficient to ensure the tests perform reliably).

3.4 Construction of the Observed Frequency Curve

Low flow frequency curves (probability plots) are used to depict the variation in observed annual minima with exceedance probability. The plotting positions along the probability axis are estimated using empirical formulae that evaluate the probability associated with a particular observation from its rank in the sample set (i.e. the plotting position is a distribution-free estimator of the probability). A number of plotting position formulae have been suggested over the years. Some are said to be optimised for particular distribution types, although these differences are really only evident for data points of the very lowest and highest ranks. Reviews of plotting position formulae are given by Cunnane (1978) and again by Cunnane (1989). Based on his recommendations, this guidance document advocates the use of the Gringorten formula for estimating plotting positions. The Gringorten formula is given by

$$p_i = (i-0.44) / (n +0.12) \tag{6}$$

where p_i is the estimated **exceedence** probability for the data point of rank i, and n is the total number of observations. The rank, i, is calculated by reordering the set of data-values according to size, the largest value being assigned a rank of 1, and the smallest value a rank of n.

The probability plot is then constructed by plotting X_i (vertically) against p_i. The plotting position is often expressed as a recurrence interval (i.e. reciprocally) such that the curve describes the average interval between years in which the annual minimum D-day falls below a given discharge. If the user does not have the facility to plot on a probability scale a reduced variate scale may be used to 'linearise' the probability axis. In this approach the variate value corresponding to each plotting position or recurrence interval is determined, making use of the assumption that the data is likely to conform to an extreme-value type distribution. For ease, the Weibull reduced variate given by the following formula is recommended:

$$V = 4\left(1 - e^{-0.25(-\ln(-\ln p))}\right) \tag{7}$$

where V is the reduced variate value, and p is the exceedance probability.

It is important to note that whilst plotting positions are fairly accurate estimators of the probability where n is very large (and the sampling errors are small), they are predominantly influenced by the number of observations in the sample set where n is small. This means that plotting positions will be equally spaced along the probability axis, regardless of how the magnitudes of the observed flows are distributed. For instance for a sample size of 35 years a recurrence interval of 62 years is always assigned to the smallest flow, regardless of its size or relation to the other data.

3.5 Parameter Estimation

As discussed earlier, the method of L-moments is suggested as the preferred estimation method. Parameter estimation is best attempted using existing software, rather than from a first principle approach; commercial software packages providing L-moments include MIKE11, S-PLUS and WINFAP, the latter being specifically designed for regional flood estimation procedures (Institute of Hydrology, 1999). Fortran subroutines for L-moments are also available as freeware e.g. Hosking (2000).

The Pearson Type III distribution family is advocated as the best all-round choice of distribution for use in low flow frequency analysis. However where procedures for estimation based on the Pearson Type III are not available, the three parameter Generalised Extreme Value distribution is the next most preferred option.

When the three parameters have been determined, they can be used to define the form of F(x). Here we present the Pearson Type III formulation given by Hosking and Wallis (1997). The three parameters are shape, k, scale, α, and location, ξ. Where $k>0$ the appropriate cumulative distribution function is

$$F(x) = G\left(\mu, \frac{x-\gamma}{\beta}\right) / \Gamma(\mu) \qquad (8)$$

where $\mu = \frac{4}{k^2}$, $\beta = \frac{1}{2}\alpha|k|$ and $\gamma = \xi - \frac{2\alpha}{k}$ (these do not hold true in where k=0, as in that case the distribution reduces to the normal distribution). The range of x is $\gamma \le x \le \infty$.

Similarly where $k<0$ the appropriate cumulative distribution function is

$$F(x) = G\left(\mu, \frac{\gamma-x}{\beta}\right) / \Gamma(\mu) \qquad (9)$$

and the range of x is $\gamma \le x \le \infty$.

G and Γ represent the incomplete (integrated between 0 and x) and complete (integrated between 0 and infinity) Gamma Functions respectively, and may be estimated according to the following relationships:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \qquad (10)$$

$$G(\mu, x) = \int_0^x t^{\mu-1} e^{-t} dt \qquad (11)$$

3.6 Assessing the Estimation of F(x)

In assessing whether the fitted distribution is acceptable, the bias, variability and accuracy of the parameter estimates are taken into account. As the 'true' form of the distribution function for annual minima at the site of interest is always unknown, the estimation is usually assessed in relation to the observed data points. This usually involved some kind of comparison between the plotting position estimators of the observed minima and those inferred from the estimated form of the distribution function. Various goodness-of-fit tests may be used to quantify how well the distribution fits the observed data. Equally by-eye fitting tests are often used, but should be backed up with some quantitative justification. Resampling methods such as the jacknife or bootstrap may be used to quantify the uncertain associated with quantile estimates or to define confidence limits. These procedures are standard statistical methods, and are detailed in many different statistical texts. However a comprehensive description may be found in volumes 9 and 12 of the Handbook of Statistics (Rao, 1993; Patil & Rao, 1994).

4. WORKED EXAMPLE

This example is based on the North Esk gauged at Dalmore Weir (station 19004), located in the SEPA-East region. The annual minima of interest are those for durations of 1 day and 180 days. The upstream catchment area is 81.6 km^2 and the flow regime is moderately responsive - annual average rainfall is 951 mm and the Base Flow Index is 0.54. This record is of high quality and received a grade 'A' rating for hydrometric quality in the study by Gustard *et al*. (1992).

Daily Flow Record and Mean Flow

The daily flow record is available from 01/01/1960 to 31/12/1999. The period of record Mean Flow (MF) is 1.53 m^3s^{-1}.

Derivation of Annual Minima Series

Table 1 gives the minimum D-day flow derived for each year where D=1 and D=180, expressed both in absolute terms and as a percentage of the Mean Flow (%MF). For 1960, the annual minima where D=180 is rejected as no running average values could be calculated for the first 90 days of that year (i.e. there is no record for the latter months of 1959). For D=1 two or more years have the same annual minimum flow (e.g. 1961 and 1962 and 1977 and 1978). This is a feature of the data, but the same effect may occur due to rounding up errors and so on.

Validation tests for independence and homogeneity

Figure 3 shows the annual minima series for the North Esk derived for durations of D=1 and D=180. In both cases the annual minima varies from year to year, and on visual inspection no long term (linear) trends are apparent.

Table 1: Annual Minima (AM) Series for the North Esk at Dalmore Weir

	D=1		D=180	
Year	AM (m^3s^{-1})	AM (%MF)	AM (m^3s^{-1})	AM (%MF)
1960	0.323	20.85	N/A	N/A
1961	0.391	25.41	76.86	49.59
1962	0.391	25.41	113.82	73.43
1963	0.459	29.97	136.46	88.04
1964	0.416	27.36	89.2	57.55
1965	0.297	19.55	124.79	80.51
1966	0.388	25.41	111.54	71.96
1967	0.391	25.42	107.51	69.36
1968	0.549	35.83	153.74	99.19
1969	0.305	20.2	75.67	48.82
1970	0.315	20.3	93.93	60.6
1971	0.316	20.85	80.83	52.15
1972	0.233	14.98	45.34	29.25
1973	0.214	13.68	50.51	32.59
1974	0.236	15.63	51.82	33.43
1975	0.143	9.121	58.37	37.66
1976	0.24	15.64	58.64	37.83
1977	0.355	22.8	109.99	70.96
1978	0.355	22.8	92.26	59.52
1979	0.317	20.85	69.75	45
1980	0.361	23.45	84.09	54.25
1981	0.392	25.41	75.25	48.55
1982	0.298	19.55	71.63	46.21
1983	0.222	14.33	83.17	53.66
1984	0.138	9.12	49.72	32.08
1985	0.382	24.76	117.41	75.75
1986	0.29	18.89	103.2	66.58
1987	0.369	24.11	103.4	66.71
1988	0.234	14.98	106.07	68.43
1989	0.263	16.94	50.76	32.75
1990	0.276	18.24	85	54.84
1991	0.234	14.98	60.43	38.99
1992	0.1	6.515	93.16	60.1
1993	0.408	26.71	124.85	80.55
1994	0.27	17.59	57.02	36.79
1995	0.258	16.93	66.25	42.74
1996	0.263	16.94	53.38	34.44
1997	0.336	22.15	95.34	61.51
1998	0.611	39.74	159.03	102.6
1999	0.403	26.06	92.6	59.74

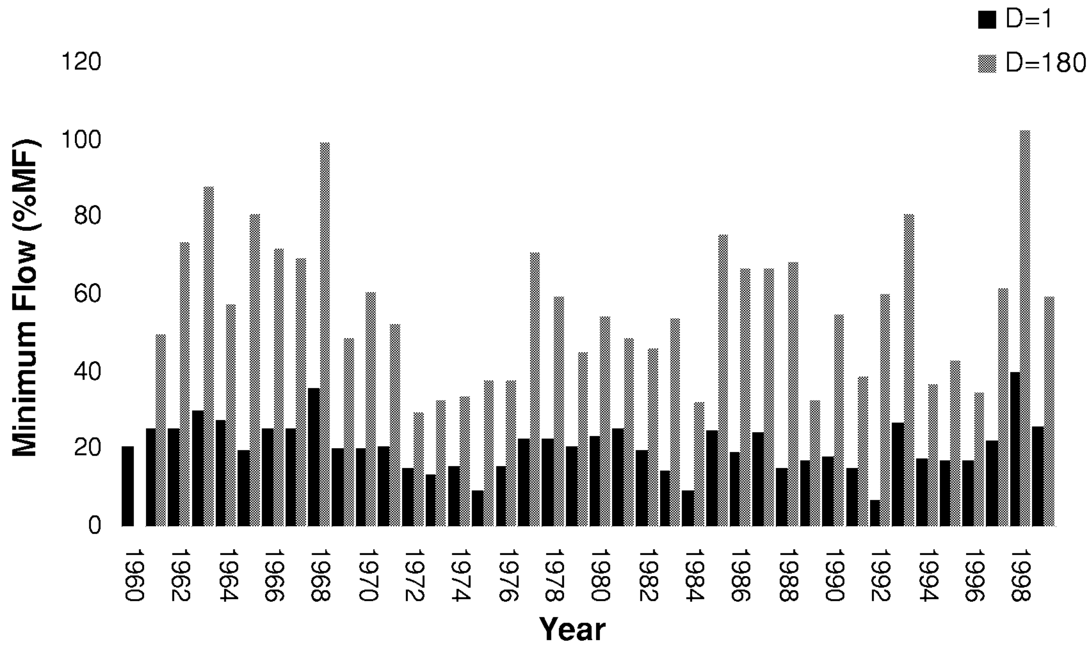

Figure 3: Annual Minimum Series for the North Esk at Dalmore Weir.

A number of validation test for independence, stationarity and homogeneity of the series are applied, in this case using the software package Systat 9 (©SPSS, 1998).

- Linear Regression Test

This shows that there is a poor linear relationship between the chronology and the annual minimum, giving a coefficient of (R^2) of 0.028 where D=1, and a value of 0.04 where D=180.

- Spearman Rank Correlation Test

The Spearman correlation coefficient between year and annual minima for D= 1 is equal to -0.236, and for D=180 is 0.127. As these values are much lower than the critical values for the samples, the hypothesis of rank order relationship between year and flow must be rejected.

- Autocorrelation test for independence

Fig. 4 shows partial autocorrelation plots for a) D=1 and b) D=180. Autocorrelation measures the correlation of the series with itself shifted by a time lag. Autocorrelation can be calculated for a lag of any length, and if autocorrelation is present at one or more lags then the data is not independent. Partial autocorrelation plots show the relationship of points in a series to preceding points after 'partialing' out the influence of intervening points, and thus give a more conservative/ better perspective of autocorrelation.

a) b)

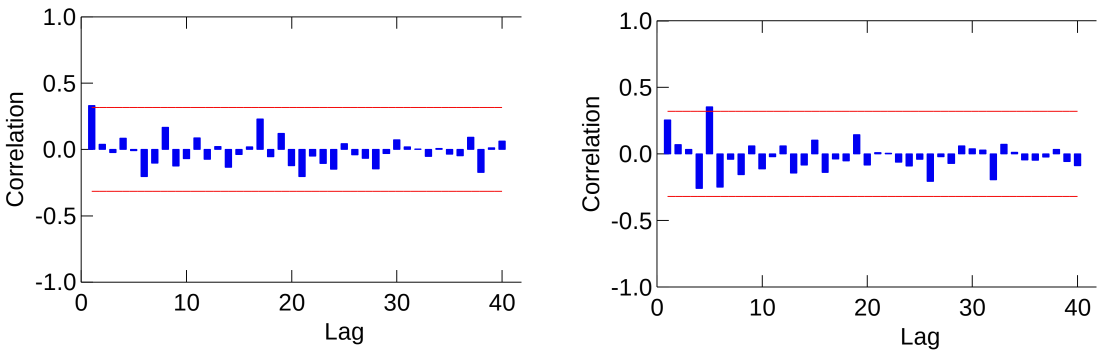

Figure 4: Partial autocorrelation plots for a) D=1 and b) D=180 for different lags (in days)

Values that lie outside the tramlines indicate that there is a significant correlation in the data at the lag (in years) indicated. Figure 4 shows that for D=1 there is little or no dependence within the data series, whilst for D=180 there seems to be some autocorrelation in the data at a lag of five years.

Construction of Observed Frequency Curve

The Gringorten plotting position is used to estimated the exceedance probability, p, using n = 40 where D = 1, and n = 39 where D = 180. Table 2 gives details of the values used, whilst Figure 5, shows the resulting probability plot.

Table 2: Derivation of plotting positions for D=1 and D= 180

	D=1, N= 40		D=180, N= 39	
I	**AM (%MF)**	**p**	**AM (%MF)**	**p**
1	6.515	0.014	29.25	0.014
2	9.12	0.039	32.08	0.040
3	9.121	0.064	32.59	0.065
4	13.68	0.089	32.75	0.091
5	14.33	0.114	33.43	0.117
:	:	:	:	:
35	26.06	0.861	80.51	0.883
36	26.71	0.886	80.55	0.909
37	27.36	0.911	88.04	0.935
38	29.97	0.936	99.19	0.960
39	35.83	0.961	102.6	0.986
40	39.74	0.986	N/A	N/A

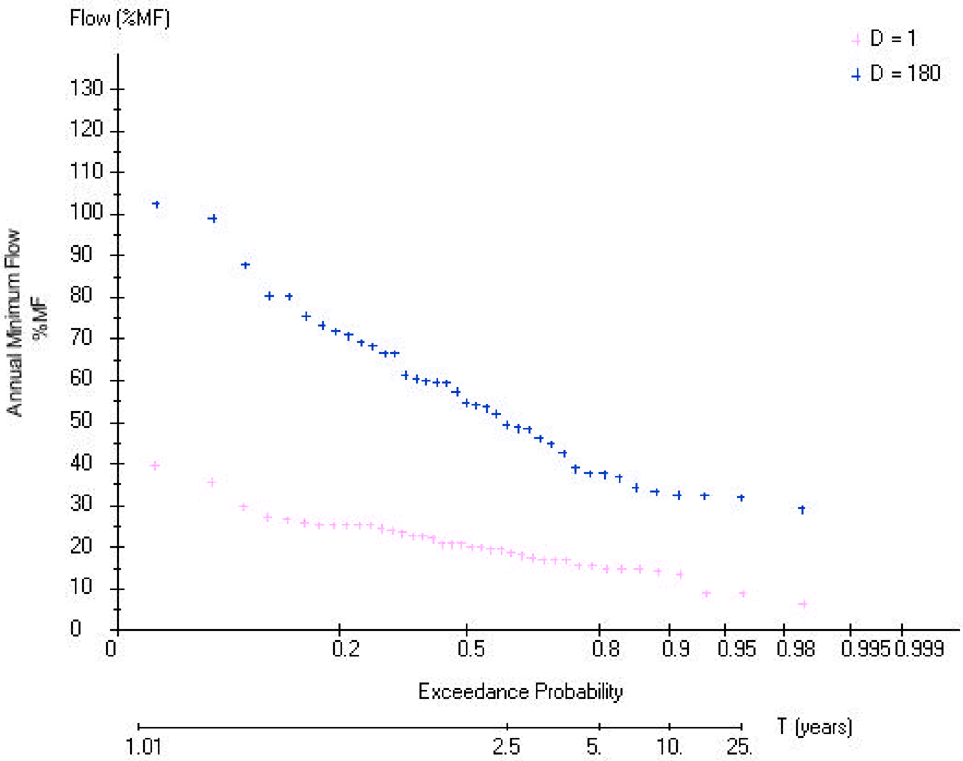

Figure 5: Frequency curves for D=1 and D=180

Fitting the Pearson Type III distribution

Parameter estimation via L-Moments is best attempted using existing software, rather than from a first principle approach. Several commercial software packages can provide the L-Moments estimation procedure with the Pearson Type III distribution, including MIKE11, S-PLUS, WINFAP. In this example the Fortran subroutines of Hosking (2000) were implemented yielding the parameters shown in Table 3.

Table 3: Parameters obtained via L-Moments for the Pearson Type III distribution

Parameter		D=1	D=180
a	Scale	0.1	0.29
?	Location	0.32	0.87
K	Shape	0.13	0.53

As *k*>0 in both cases, the appropriate cumulative distribution function is

$$F(x) = G\left(\mu, \frac{x-\gamma}{\beta}\right) / \Gamma(\mu) \quad (12)$$

For the case where D =1, the parameters are given by

$$\mu = \frac{4}{k^2} = 236.69 \quad (13)$$

$$\beta = \frac{1}{2}\alpha|k| = -0.0065 \quad (14)$$

$$\gamma = \xi - \frac{2\alpha}{k} = -1.22 \quad (15)$$

The range of x is $-1.22 \leq x \leq \infty$. Substituting these parameters gives the following analytical solution for F(x):

$$F(x) = \frac{G\left(236.69, \frac{x+1.22}{0.0065}\right)}{\Gamma(236.69)} \quad (16)$$

F(x) can be evaluated by solving the incomplete Gamma functions for different values of x. Table 4 gives values of F(x) for different values of x, with corresponding recurrence intervals (T). The fitted curves are shown in Figure 6.

Table 4: Estimation of F(x) for D =1

AM (%MF)	F(x)	T (years)
6.52	0.014	71.64
9.12	0.039	25.72
9.12	0.064	15.67
13.68	0.089	11.27
14.33	0.114	8.80
:	:	:
26.71	0.886	28.44
27.36	0.911	29.39
29.97	0.936	30.56
35.83	0.961	32.20
39.74	0.986	35.19

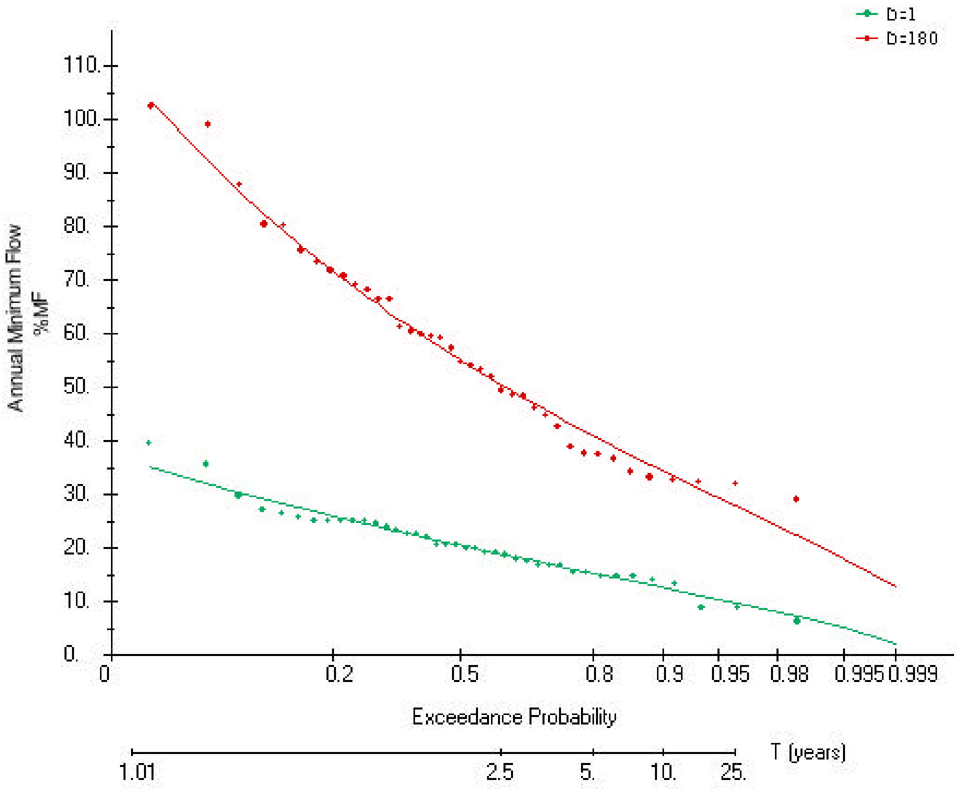

Figure 6: Fitted Curves for the North Esk for D=1 and D=180

Goodness of Fit Tests

The match between observed and predicted annual minima is quantified using root mean square error (RMSE) and chi square statistics, based on the residuals (Table 5).

Table 5: Comparison of observed and predicted flows, D= 180

T (years)	p	Observed AM (%MF)	Predicted AM (%MF)	residuals
69.86	0.014	29.25	22.51	-6.74
25.08	0.040	32.08	27.90	-4.18
15.28	0.065	32.59	31.11	-1.48
10.99	0.091	32.75	33.56	0.81
8.58	0.117	33.43	35.60	2.17
:	:	:	:	:
1.13	0.883	80.51	79.35	-1.16
1.10	0.909	80.55	82.56	2.01
1.07	0.935	88.04	86.64	-1.40
1.04	0.960	99.19	92.40	-6.79
1.02	0.986	102.60	103.40	0.80

Root mean square errors are 0.019 and 0.036 for D=1 and D=180 respectively, whilst the corresponding chi-square values are 1.23 and 2.31. At this point the user should refer to tables of critical values of the chi-square statistic, and based on the significance level of interest, determine whether they are exceeded by the observed values. The observed values must not exceed the critical values if the estimated curve is to be found acceptable.

Resampling methods would also be attempted, if desired, at this stage. A full description of resampling methods is beyond the scope of this guidance note. However there are plenty of introductory texts on the subject such as Good (1999).

Prediction of the Annual Minima – Recurrence Interval relationship

As the goodness-of-fit tests suggest that the estimated curve is satisfactory the distribution function, F(x), may be used to predict the probability (and thus the recurrence interval) associated with a particular annual minimum flow, x. Similarly if the annual minima associated with a particular recurrence interval was of interest, this could also be determined by calculating x based on F(x).

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GLOSSARY

Accuracy – In statistical estimation, accuracy refers to the deviation of an estimate from the true parameter value. In general, the term is used for the quality of a measurement that is both correct and precise.

Anderson–Darling test – A test procedure for testing the hypothesis that a given sample of observations comes from some specified theoretical population. It is particularly sensitive to deviations in the tails of the distribution. The combination of ease of computation and good power makes it an attractive procedure for a goodness-of-fit test.

Probability scale – Where a graph has uniform subdivisions for the x axis but the y axis is subdivided in such a way that a plot of the cumulative distribution appears as a straight line, it is said to be plotted using a probability scale. For example, the arithmetic probability scale describes a cumulative normal distribution.

Autocorrelation – In a time-series analysis autocorrelation is the internal correlation between observations often expressed as a function of the lag time between them.

Bias – A systematic error that may distort a statistical result in one direction. A biased estimator is one whose expected value does not equal the true value of the parameter being estimated.

Bootstrap – A nonparametric technique for estimating standard error of a statistic by repeated resampling (with replacement) from a sample. The technique treats a random sample of data as a substitute for the population and resamples from it a large number of times to produce sample bootstrap estimates and standard errors.

Chi-square test – A test of statistical significance based on the chi-square distribution. The chi-square statistic is obtained as the sum of all the quantities obtained by taking the difference between each observed and expected frequency, squaring the difference, and dividing this squared deviation by the expected frequency.

Correlation coefficient – An index used to measure correlation. It is also known as the Pearson product moment correlation coefficient. It is denoted by the letter r and its value ranges from -1 to +1. A value of +1 denotes that two sets are perfectly related in a positive sense and a value of -1 indicates that two sets are perfectly related in a negative sense. A value close to zero indicates that they are not linearly related.

Critical value – The theoretical value of a test statistic that leads to rejection of the null hypothesis at a given level of significance. Thus, in a statistical test, the critical value divides the rejection and the acceptance regions.

Distribution function – For any **random variable** X, the distribution function of X, denoted by $F(x)$, is defined by $F(x) = P(X \leq x)$; that is, the distribution function is equal to the **probability** that a random variable assumes a value less than or equal to x for $-\infty < x < \infty$.

Durbin–Watson test – A procedure for testing independence of error terms in least squares regression against the alternative of autocorrelation or serial correlation. The test statistic d is a simple linear function of residual autocorrelations, and its value decreases as the autocorrelation increases.

Estimation – The process of using information from sample data in order to estimate the numerical values of unknown parameters in a population.

Gamma Function – A function generalizing the factorial expression for natural numbers, also known as Euler's Second Integral.

Goodness-of-fit test – A statistical procedure performed to test whether to accept or reject a hypothesized probability distribution describing the characteristics of a population. It is designed to ascertain how well the sample data conform to expected theoretical values. It involves testing the fit between an observed distribution of events and a hypothetical distribution based on a theoretical principle, research findings, or other evidence by means of a Pearson chi-square statistic or any other test statistic.

Independence – In probability theory, two events or observations are said to be independent when the occurrence of one event has no effect on the probability of occurrence of another event. Thus, two events are independent if the probability of occurrence of one is the same whether or not the other event has occurred.

Jackknife – A nonparametric technique for estimating standard error of a statistic. The procedure consists of taking repeated subsamples of the original sample of n independent observations by omitting a single observation at a time.

Kolmogorov–Smirnov tests – Nonparametric tests for testing significant differences between two cumulative distribution functions. The one-sample test is used to test whether the data are consistent with a given distribution function and the two-sample test is used to test the agreement between two observed cumulative distributions. The test is based on the maximum absolute difference between the two cumulative distribution functions.

Mann–Whitney *U* test – A nonparametric test for detecting differences between two location parameters based on the analysis of two independent samples. The test statistic is formed by counting all the bivariate pairs from the two samples in which one sample value is smaller than the other. It is equivalent to the Wilcoxon rank-sum test.

Maximum likelihood estimation – A method of estimation of one or more parameters of a population by maximizing the likelihood or log-likelihood function of the sample with respect to the parameter(s). The maximum likelihood estimators are functions of the sample observations that make the likelihood function greatest. The procedure consists of computing the probability that the particular sample statistic would have occurred if it were the true value of the parameter. Then for the estimate, we select the particular value for which the probability of the actual observed value is greatest. Maximum likelihood estimates are determined by using methods of calculus for maximization and minimization of a function.

Method of moments – A method of estimation of parameters by equating the sample moments to their respective population values. It is generally applicable and provides a fairly simple method for obtaining estimates in most cases. The method, however, yields estimators that, in certain cases, are less efficient than those obtained by the method of maximum likelihood.

Parametric methods – These are statistical procedures that are based on estimates of one or more population parameters obtained from the sample data. Parametric methods are used for estimating parameters or testing hypotheses about population parameters.

Partial autocorrelation – An autocorrelation between the two observations of a time series after controlling for the effects of intermediate observations.

Quantiles – A general term for the $(n - 1)$ partitions that divide a frequency or probability distribution into n equal parts. In a probability distribution, the term is also used to indicate the value of the random variable that yields a particular probability.

Rank correlation – A nonparametric method for assessing association between two quantitative variables. A rank correlation is interpreted the same way as the Pearson product moment correlation coefficient. However, a rank correlation measures the association between the ranks rather than the original values. Two of the most commonly used methods of rank correlation are Kendall's tau and Spearman's rho.

Recurrence Interval (Return Period) – The average interval in years between two events of equal magnitude.

Resampling – The technique of selecting a sample many times and computing the statistic of interest with reweighted sample observations. Some commonly used resampling techniques include bootstrap, jackknife, and their variants.

Spearman's rho (ρ) – A correlation coefficient between two random variables whose paired values have been replaced by their ranks within their respective samples or which are based on rank order measured on an ordinal scale. It provides a measure of the linear relationship between two variables. This measure is usually used for correlating variable(s) measured with rank-order scores.

Stochastic model – A mathematical model containing random or probabilistic elements.

Unbiased estimator – An estimator whose expected value or mean equals the true value of the parameter being estimated. Thus, an unbiased estimator on the average assumes a value equal to the true population parameter.

Uncertainty – A term denoting the lack of certainty inherent in a random phenomenon.

Wald–Wolfowitz run test – A nonparametric test for testing the null hypothesis that the distribution functions of two continuous populations are the same.