Dear CMA DCT Study,

I have reviewed my responses to January 2017 in the light of research into actuarial maths. The key difference is my belated recognition of the use by providers and DCTs of nominal rates alongside supposed effective rates (AER, APR and APRC).

According to obscure tradition, both nominal rates and effective rates exist. Actuarial nominal rates come with a statement such as “converted 4 times annually”. Actuarial sources expend much effort to equip actuaries with the ability to convert their functions between the two rate bases.

For example, an effective rate of 5%AER has an equivalent nominal rate “converted 12 times annually”:-
((1+5/100)^(1/12)-1)x12x100%=4.8889...% nominal monthly.
5% nominal monthly becomes ((1+5/1200)^12-1)x100%=5.116...% effective annually.
=PMT() requires feeding with the nominal monthly rate divided by 12
=PMT() models initial monthly mortgage payments advertised by providers and DCTs

Thus providers and DCTs operate with nominal rates (without precisely naming them as such).
However, nominal rates only apply to equally spaced points on the curve of the effective rate.
Linear extrapolation or interpolation of a nominal rate for any other time step amounts to simple interest.

Equally spaced monthly payments means every 365/12=30.4166...days, which is impractical.
Using 5% nominal monthly in a straight line (simple interest) calculation for January will under estimate the effective rate curves of AER, APR and APRC. April will be over estimated.
Over a single 365 day account, the total deviation will be small (but still un-necessary).
Over a 25 year mortgage, 6 leap days will be lost.

This is just another part of why I continue to pursue the sole application of proper compound interest and the scrapping of all approximate methods, such as simple interest.

[✂] I still feel that since the definitions of APR and APRC include the fees, fees should form part of the monthly payments, but clearly that level of logic has yet to penetrate the credit market.
Please find attached:-
Update Paper response form (responses to Q7, Q21 & Q25 of 28-Mar-17)
MemberG.docx (consolidated revision of 3 earlier Word items)

[✂]

I am not a professional banker but I strive to improve my knowledge by fundamental research into the history of maths and the practices of banking. I am routinely dismayed by the lack of challenge offered to my work. The longer it goes un-challenged by the maths of others, the more secure my position becomes.

14 place accuracy or better has been available since 1624. How can competition be fair without accurate interest calculation?

I look forward to a mention of calculation accuracy in your final report.

Best Regards

Digital Comparison Tools Market Study:

Update Paper - Response form

1. Thank you for taking the time to respond to the questions in the Update Paper for our Market Study of Digital Comparison Tools (DCTs), published on our website on 28 March 2017.

2. Please download and save this form before completing it. Please submit your response by **5pm on Monday, 24 April 2017**, either by:

   - Email to: comparisontools@cma.gsi.gov.uk.
   - Or by post to: Digital Comparison Tools Market Study
     Competition and Markets Authority
     7th floor
     Victoria House
     37 Southampton Row
     London WC1B 4AD

3. Please note:

- You can choose which questions to respond to, but we ask all respondents to provide a small amount of background information at the start of this form. The boxes will 'expand' to accommodate long responses if required.

- We are particularly keen to receive evidence in support of responses. If you are able to supply evidence please attach this with your response.

- We intend to publish responses to our Update Paper in full. If you wish to submit information that you consider to be confidential, this should be indicated to us clearly and an explanation given as to why you consider it to be confidential.

- The CMA may use the information you provide for the purposes of facilitating the exercise of any of its statutory functions. This may include the publication or disclosure of the information. Prior to publication or disclosure, in accordance with its statutory duties under Part 9 of the Enterprise Act 2002, the CMA will have regard to (among other considerations) the need to exclude, so far as is practicable, any information relating to the private affairs of an individual or any commercial information relating to a business which, if disclosed, would or might, in our opinion, significantly harm the individual's

interests or, as the case may be, the legitimate business interests of that business (confidential information). Further information about how the CMA will use information submitted during the Market Study can be found on our website.

4. If you have any questions about our Market Study or this online form please contact the team at comparisontools@cma.gsi.gov.uk.

Your details

(Fields marked * are required)

Title*	[✂]

Forename	[✂]

Surname*	[✂]

Email*	

What is your role / profession*	Chemical Engineer

Are you representing yourself or an organisation?*	Myself

If you are representing yourself rather than an organisation would you be content for us to include your name when we publish your response?*	Yes

If you are representing an organisation:

(a) What is the organisation's name?*	
(b) Please could you briefly explain the role of your organisation, including the sectors in which it operates or has most interest?*	

Consumers

1. Should we focus our attention on the consumer groups we identify in Chapter 5 (see paragraphs 5.82 to 5.95) and if not, what groups should we focus on?

2. In which sectors do DCTs not currently play a major role but could in principle offer substantial benefits to consumers? Why have they not become established in these sectors?

3. How has the growing use of DCTs affected suppliers' offers to consumers who do not use DCTs in our case study sectors and more broadly? What impact have DCTs had on suppliers' ability to discriminate between active and inactive consumers? What are the implications for vulnerable consumers?

4. What factors, if any, have we missed that may be holding back consumers from using DCTs?

5. What, if anything, should be done about consumers' concerns about data sharing and the extent to which they feel in control?

6. What actions, if any, are needed to improve the way consumers use DCTs – including multi-homing and using DCTs' functionalities such as filtering and ranking?

Inputs to DCTs

7. Have we captured the range of issues that might prevent DCTs from operating effectively?

You have failed to mention that DCTs will never be able to compare financial products fairly until providers and DCTs are forced to advertise and use APR, APRC and AER according to their exponential definitions. Comparisons based on approximate calculations are just not acceptable.
You have not highlighted that the biggest financial decision in a DCT user's life might rely on un-representative representative APRC rates.

Readers of my original response and attachments should note that since then I have researched the actuarial view of interest and I have modified my analysis of mortgages.
Modification 1. - Rate basis
I had already noted the exact agreement between =PMT() and the advertised initial monthly payments. I have found an actuarial equation from 1915 that exactly models =PMT() when the rate used is the nominal rate convertible 12 times each year.
The nominal rate is the slope of a chord to the curve of the effective rate when the chord is drawn across a regular division of the effective rate determining step.
Consider the monthly payments on a 25 year mortgage of £200,000 at "5%":-

Typical online calculator	and	=PMT(5/1200, 25x12, £200,000)	= £1,169.18
Todhunter, 1915, Eqn 17, p35		£200,000x5/1200/(1-(1+5/1200)^(-25x12))	= £1,169.18

The advertised (non-APRC) rates all appear to be nominal rates convertible 12 times each year, but without using the word nominal.
For example, a nominal (but un-declared as such) 1.50% in the typical advert implies:-
((1+1.5/1200)^12-1)x100%=1.5104...% effective rate.
Therefore my earlier results that were termed "compound" were wrongly based on the nominal rates instead of the effective rates. My earlier results termed "simple" still stand, but should be described as "nominal, disturbed by simple", since linear extrapolation of a nominal monthly rate (based on 365/12=30.4166...days) to cover 31 days is simple interest. Likewise, with linear interpolation of a February.
Nominal rates are based on equal time steps. In the typical monthly payment plan, this means payment every 30.4166...days. This is impractical but it turns out to be acceptable to providers and DCTs. It gives reasonable results for a year but it becomes a problem over typical mortgage timescales.

A real 25 years	(15-Nov-41) minus (15-Nov-16) = 9,131 days
25 years of average months	(25x12)x(365/12) = 9,125 days

The average month nominal rate model doesn't consider the 6 leap days.
My revised spreadsheet still calculates compound interest balances with calendar dates but now uses the effective rates implied by the advertised nominal rates.
Modification 2. - Fees
While providers and DCTs have accepted adverts with claimed APRCs that are inaccurate (Chart A is not changed by rate basis issues), it seems clear that payment plans do not include the fees. Fees are up-front, not part of the payment calculation.
Therefore I used my revised spreadsheet to optimise (Goal Seek) the initial monthly payment to achieve a zero final balance by proper compound interest at the effective rate but not including the fees in the amount borrowed.
Revised Conclusions.
1. The difference in closing balance between a payment plan based on nominal rates with simple interest and that based on the effective rates (all with calendar dates) is not significant (below £1 in the 12 checked examples). However, to avoid confusion, it is a requirement of the use of nominal rates and effective rates that they be clearly identified. APRC is an effective rate. The typical advert uses nominal and effective side by side,

(even with the same font and font size in some cases) so providers and DCTs must be required to include text to explain how they are different for as long as they persist in using both types of rate.

2. Given that nominal monthly rates only apply to 30.4166...day payment plans, they should be scrapped in favour of sole use of effective rates that apply to any day count - notably to calendar month day counts and incidental counts due to weekends and public holidays.

3. The interpretation of non-APRC advertised rates as nominal rates results in all checked plans ending with a positive closing balance. The customer doesn't over-pay. However, in a credit market, how can it be fair to allow providers and DCTs to offer examples that don't pay off the borrowed amount? If they are not held to the requirement that their examples actually work then who knows how much final error they might absorb, in order to attract customers? It must be made clear that advertising a plan that doesn't give a zero final balance is mis-selling.

4. The BBC's response to the update report "Price comparison sites given thumbs up by regulator" (28-Mar-17, online business pages) shows how little regard is given to evidence only available in a long list of links. A survey of thousands of DCT users that don't appear to have been asked if they had checked the maths seems to count more than the evidence from one person who had checked the maths. Providers and DCTs must be made responsible for checking the maths of their offerings because it is clear that users have assumed the accuracy of displayed information. Providers and DCTs must be made to apply the equations of AER, APR and APRC, according to their proper compound interest definitions and without approximations.

8. Do the issues identified materially affect DCTs' ability to operate effectively and deliver good consumer outcomes?

9. Are current or planned initiatives sufficient to address the issues found?

Competition

DCTs' market position and barriers to entry and expansion

10. What explains the strong position of a specific DCT in each of our case study sectors? What do DCTs do to grow their business in sectors where they appear to be relatively small compared to the leading DCT of the sector?

11. What are the barriers, if any, for DCTs to enter or expand into sectors where they currently do not provide comparison services or where they are currently relatively small?

Agreements between DCTs and suppliers

12. What has been the impact of the removal of wide MFNs in the private motor insurance sector?

13. What has been the impact of narrow MFNs in the sectors where we have observed them (home insurance, private motor insurance, credit cards, broadband and flights)?

14. What is the commercial rationale for the non-brand bidding and negative matching agreements we have observed (in all of our case study sectors) and what is their commercial and competitive impact?

15. What is the commercial rationale for the non-resolicitation agreements we have observed (in home insurance and energy) and what is their commercial and competitive impact?

16. In which other sectors, if any, are (i) wide or narrow MFNs; (ii) non-brand bidding or negative matching; or (iii) non-resolicitation agreements in place? What impacts do they have in these sectors?

17. Are there any other agreements in place that may affect the effectiveness of DCTs and/or the effectiveness of competition between DCTs (and competition between DCTs and other sales channels)?

Unbundling and hollowing out

18. How has the growth of DCTs affected product features and/or the product mix in our case study sectors over time? What specific evidence/examples indicate these changes?

19. How widespread is the use of product reviews and ratings on DCTs and what has been the impact, if any, of the use of these tools?

20. What needs to be in place to prevent or mitigate any harmful impact of product unbundling or hollowing out and what can DCTs do about it?

Regulation

21. What are your views on the issues we list in Table 8.1 and at paragraphs 8.13 to 8.42 of Chapter 8 and how could they be addressed?
APR is defined by UK statute and EC directive. APRC is defined by FCA handbook. Both require fees to be included in the calculation. My checks show that advertised APR and APRC are not always accurate. Enforcement of the application of the definitions by regulators would prevent providers and DCTs getting away with misuse of these parameters.

22. What is the balance between potential benefits and risks in introducing a cross-sector approach? What would be the most effective approach(es), and why?

23. How could a cross-sector approach interact with existing regulatory frameworks?

The future of DCTs

24. What future developments outlined in Chapter 9 are likely to have the greatest impact in driving engagement? If there are any important developments we have missed, what are they and why are they important?

25. What future DCT-related technologies might affect or assist vulnerable consumers?
DCTs must be made responsible for checking the validity of the representative rates (APR, APRC and AER) of the products that they choose to list. This means educating DCTs in the use of proper compound interest and in the wording of rate definitions that include fees (APR and APRC). It also means that DCTs and providers must scrap reliance on approximate spreadsheet functions such as =PMT() that are based on imaginary 30.4166...day months and are inaccurate for real calendar months over mortgage timescales. Vulnerable consumers would include anyone without the time and skill to do the proper calculations (possibly most consumers of savings, mortgages and credit cards).

Other comments and further contact

We welcome submissions on any of the issues we address in our update paper from interested parties. We would particularly like to hear views, supported wherever possible by evidence, on the following themes if not already addressed above:

a) What DCTs do and the benefits they can offer.

b) Consumers' views on and use of DCTs.

c) Inputs to DCTs.

d) Competition between DCTs and between DCTs and the suppliers whose services they compare.

e) Regulation of DCTs.

f) The future of DCTs.

g) The focus of the second part of the market study.

Do you have any other comments you would like to add?

Would you be willing for us to contact you to discuss your response?*	Yes

Thank you for taking the time to complete this form.

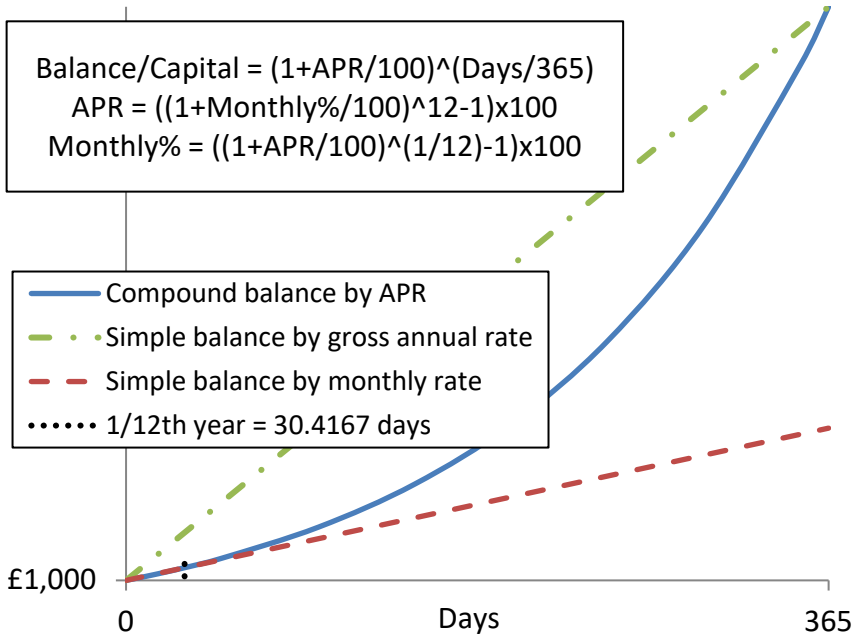
Please email it to: comparisontools@cma.gsi.gov.uk.

Or post it to:

Digital Comparison Tools Market Study
Competition and Markets Authority
7th floor
Victoria House
37 Southampton Row
London
WC1B 4AD

Figures 1 and 2 show the inaccuracy of simple interest in monthly accounts in general (applies to APR and AER). Figures 3 to 6 are about credit card inconsistencies. The credit card data was found on 30-Sep-16, using a DCT to direct me to providers' sites for credit cards all claiming to offer the same "Representative" Rate of 18.9%APR.

Figure 1 - Why simple interest fails in monthly accounts - overview.

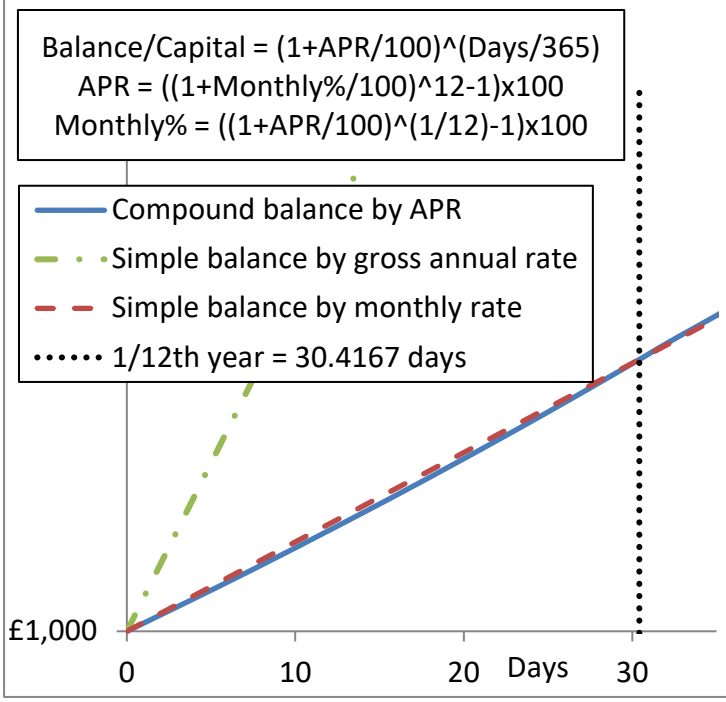


This chart uses a large APR, to amplify the curvature of proper compound growth.

Simple interest at intermediate points based on the annual gross rate is clearly an over-estimate.

The monthly rate looks close enough, until you zoom in to see the detail...

Figure 2 - Why simple interest fails in monthly accounts - detail



This chart uses a large APR, to amplify the curvature of proper compound growth

The average monthly rate is only applicable to a day count of 30.4166... days.

As with any simple interest calculation, intermediate points deviate from the curve of the advertised APR.

Only points on the blue curve fit the equation of the advertised APR.

Until simple interest is banned, monthly rates must be clearly displayed by DCTs because the so-called "Representative" APR is not currently reliable. Figure 3 shows the spread of APRs calculated from summary box monthly rates. 1.457% is 19.0%APR.

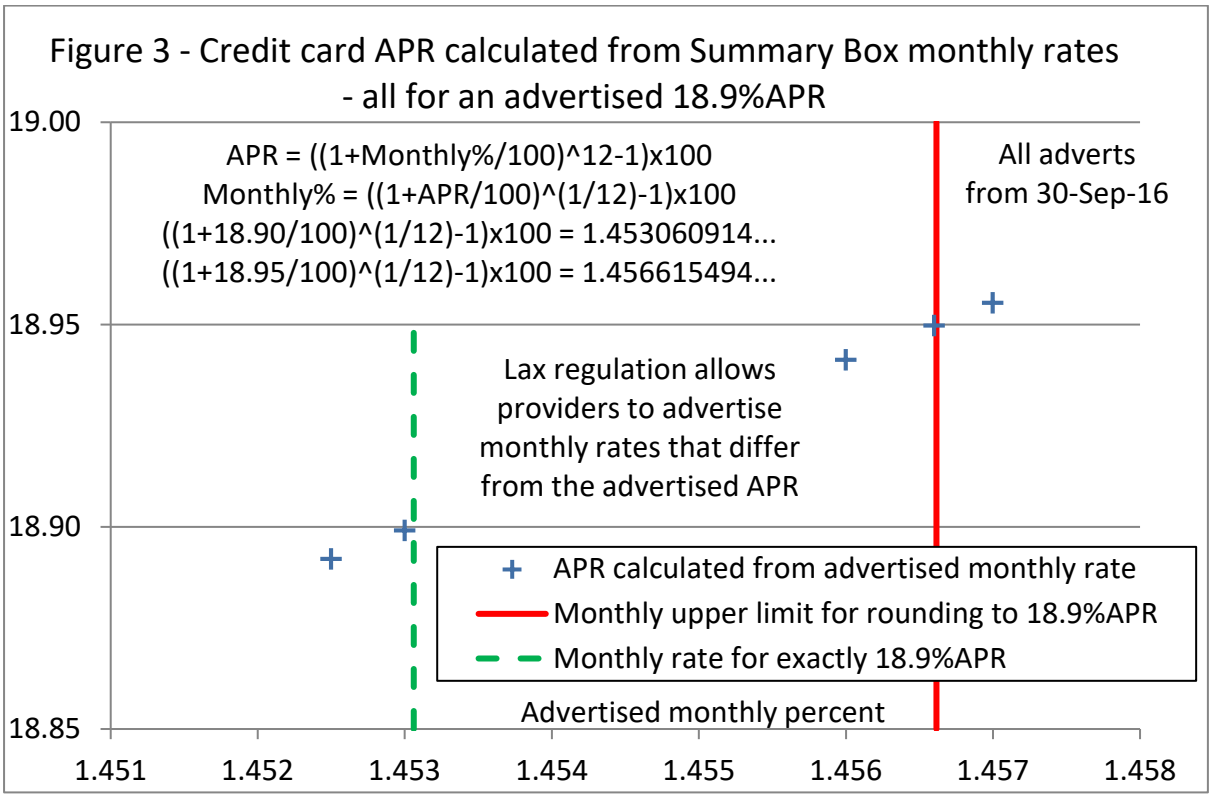
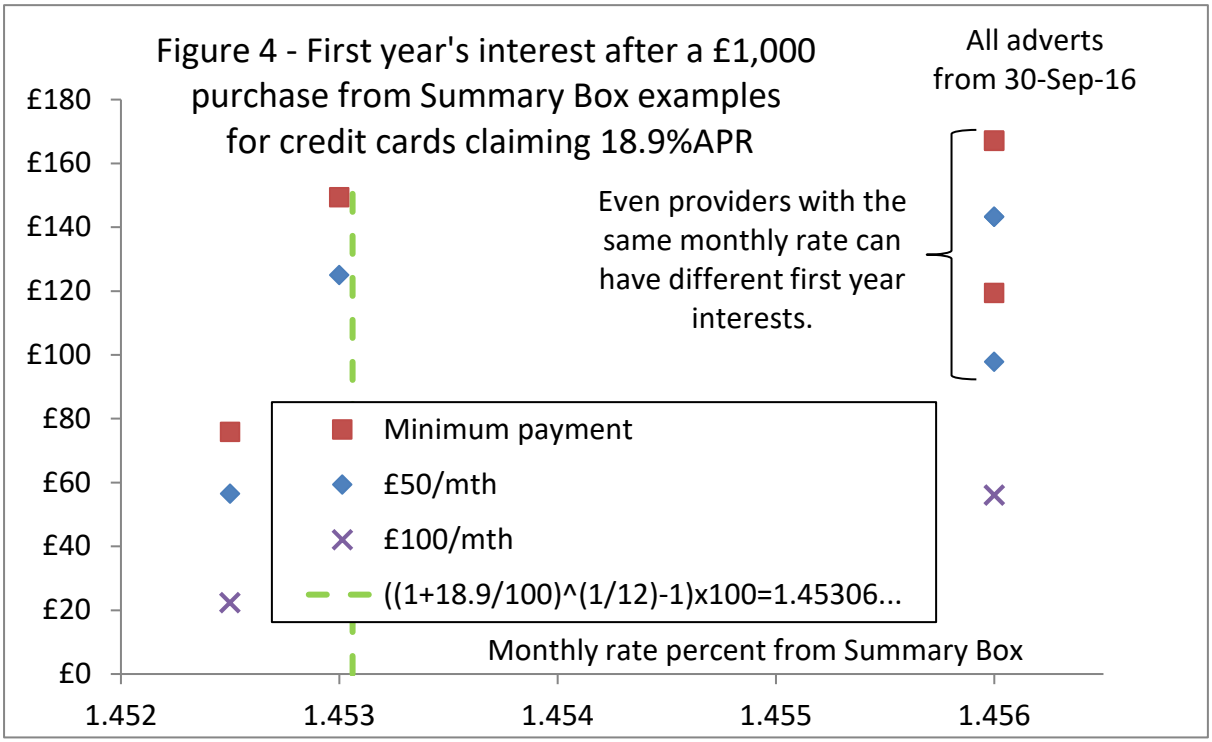


Figure 4 shows the disturbing range of interests reported by providers in their examples for their so-called 18.9%APR products. Where are the regulators and the watchdogs? Can even the account providers calculate proper compound interest?



However, the credit card comparison problem is not completely solved by banning simple interest.

One of the attractions of a credit card is the delay between taking an item home and actually paying for it. This variable delay makes analysis of the cash flow more complex.

Card providers generally allow a time between your bill date and the time before you have to make your payment to avoid interest. If you make a payment that does not clear the account but is within this time window then (non-penalty) interest is added to the next bill. If you manage to pay the bill in full within the allotted time window then you won't have any interest carried forward - I call this "interest waiving". This means that your final bill for an example £1,000 purchase paid in £50 monthly amounts will avoid the interest carried over from the previous statement. This final free gift lowers the true outcome APR. Different card users will have different approaches to when they actually pay each monthly bill (as soon as the bill arrives, to minimise total interest, or later, for any number of reasons) and so will have different timing-based effects on the interest eventually charged.

The only degree of freedom examined in the Summary Box examples that I found was the amount repaid each month - generally 1) minimum payment or 2) £50, but two cards also explored £100 monthly. Most examples were based on purchase and bill on the 1st of the month, payment on the 15th. One specified pay on due date (without giving the due date). Most provider examples gave the same 1st and 2nd year interests for each of their different free period (24 to 41 months) balance transfer cards. Only one attempted to give different examples for their different cards. 6 providers gave no example.

I made a spreadsheet to investigate the most commonly found provider example:- £1,000 purchase, £50 per month repaid. I investigated delays between purchase date and date of the first bill of zero to 30 days and the effect of delays between bill date and payment date of zero to 10 days. The proper compound interest chargeable for this model ranges from £175.17 to £195.96. This contrasts with the much wider range from the Summary Boxes of £101.59 to £200.46 for just the first 2 years (with timescales from 24 to 26 months).

Given that early payments of your card bills results in less interest, it is unfair for card providers (and the DCTs that list and sort them) to fail to advertise the benefit to the customer of the saving to be made for various payment strategies (various delays from bill date to payment date). A much more rigorous design and monitoring of the Summary Box is required, in order to give potential customers (and the DCTs that they might use) true information for comparison between products.

This may sound complicated, but then what are we trying to achieve by investigating financial product comparisons if not benefit to the customer?

Figure 5 shows the start of the model that I used to calculate the total interests at 18.9%APR for the various delays shown in Figure 6.

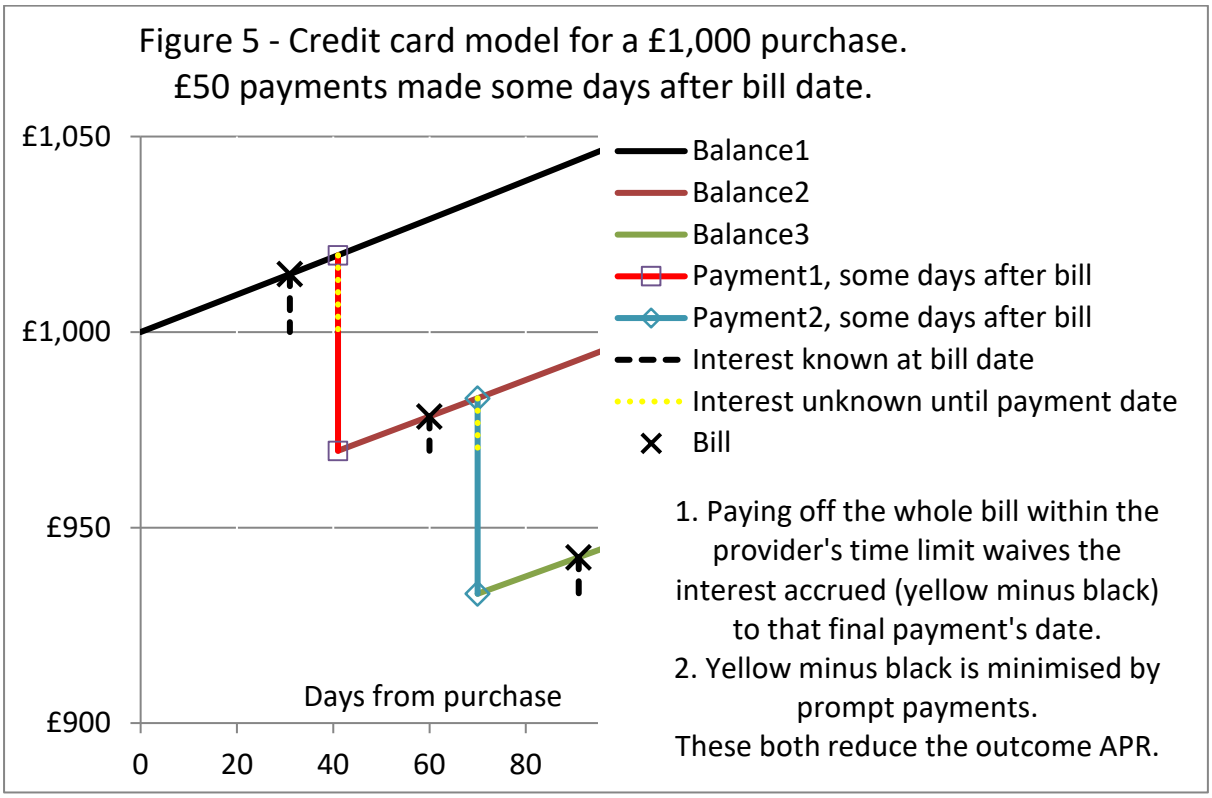
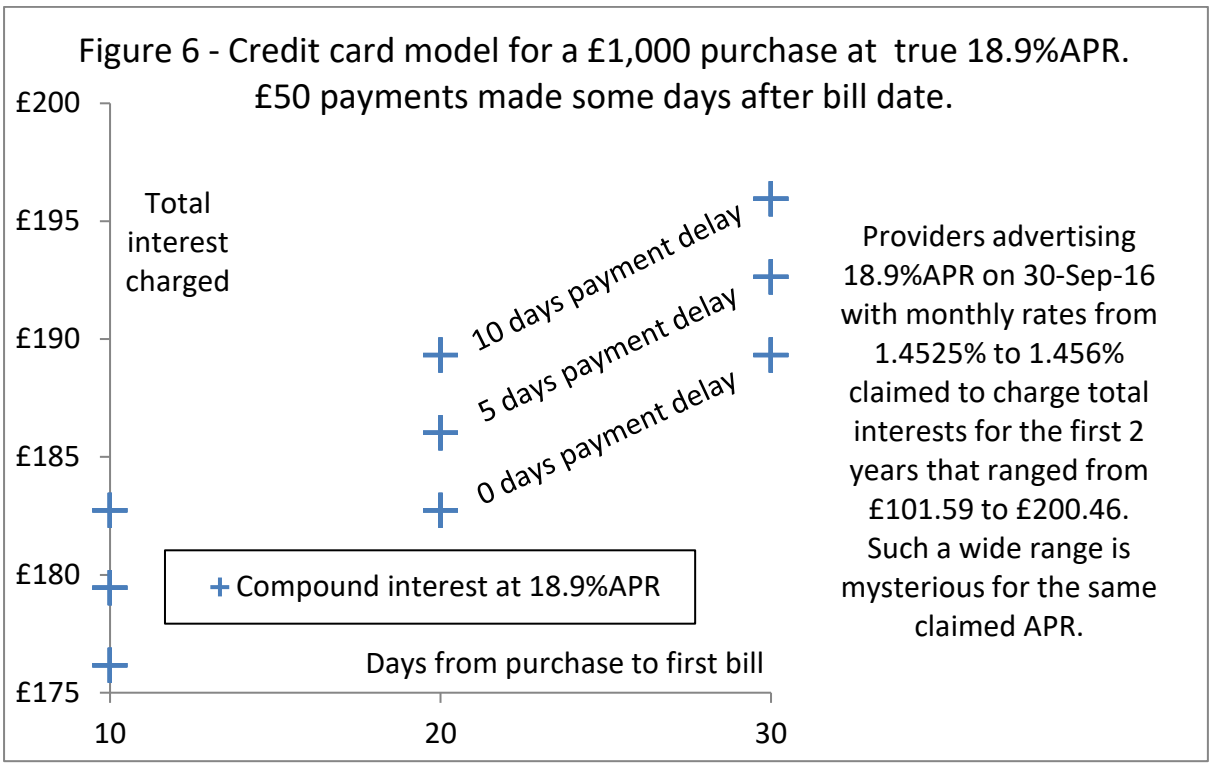


Figure 6 shows the benefits of paying the card bill earlier (under the customer's control) and the effect of delaying the first bill (under the card provider's control). At present, this detail is not formally in the Summary Box and so cannot be used by DCTs.



Mortgage finding also suffers from un-necessary approximation.

DCT's may offer a monthly repayment calculator. The idea is to enter the amount of your required loan (based on house cost and deposit), the number of years and “the annual rate” to receive a monthly repayment estimate (perhaps only to the nearest whole pound). I investigated several such calculators. I found that they agree with the Excel spreadsheet function =PMT() to within pence when the advertised annual rate (not the APRC) is entered. This is **a disaster for accuracy** because =PMT() is a nominal rate payment calculator.

A nominal rate is the slope of a chord to the curve of the effective rate of growth and only applies to regular time steps that are equal to the length of the defining chord. For example, a nominal rate of 1.50% convertible 12 times a year (but un-defined as such in the typical advert) implies ((1+1.5/1200)^12-1)x100%=1.5104...% effective rate.

The =PMT() function for monthly repayments advises entering annual rate divided by 12 and time as years x 12, all without using the word “nominal”. 365/12=30.4166...days. Given the close agreement of =PMT() results with adverts, this unrealistic time step turns out to be acceptable to providers and DCTs. The application of a nominal rate to any other time step by linear extrapolation or interpolation (simple interest) deviates from the curve of the annual effective rate for the period in question. A calendar January will be underestimated. A calendar February will be overestimated. Over a non-leap year of 12 such steps, the error will be small, but still un-necessary. Things go badly wrong over the timescale of a typical mortgage.

A real 25 years	(15-Nov-41) minus (15-Nov-16) = 9,131 days
25 years of average months	(25x12)x(365/12) = 9,125 days

The average month nominal rate model doesn’t consider the 6 leap days.

APRC may include various improvement additions to the main house loan but includes fees by definition. The APRC never comes with its equation of growth in the advert. All this is before the complication of fixed rate introductory offers and early repayment.

Having examined introductory rate mortgages, I am even more convinced that DCTs are unreliable, and will be until approximation is removed from banking.

I found a DCT that listed over 500 mortgages using £250,000 purchase, £200,000 borrowed (£50,000 deposit) and 25 years as search values. I analysed 9 with fixed rate periods and 3 with lifetime single variable rates. In all 12 cases, the reported initial monthly repayments were modelled exactly by the Excel spreadsheet function =PMT() when excluding fees from the amount borrowed. =PMT() also models the advertised post-introductory repayments within a few pence. Advertised repayments are therefore approximations to proper compound interest. Sheet A summarises my results. My full spreadsheet runs to over 2,200 rows, to cope with up to 30 year mortgages.

Excluding fees from the calculation of APRC is contrary to the definition of APRC. See FCA Handbook MCOB 10A.2 below. Clearly, the two imaginary 2-stage plans in this table will have different APRCs:-

Date	Cash flows including £1,000 fee	Cash flows not including £1,000 fee
1-Dec-16	£201,000	£200,000
1-Jan-17	-£900	-£900
1-Feb-17	-£900	-£900
...		
1-Jan-19	-£1,100	-£1,100
1-Feb-19	-£1,100	-£1,100
...		

I used =XIRR() to find the actual APRCs. =XIRR() demands the actual date of each cash flow and is equal to the method based on summing the present values when only whole day steps are involved. =XIRR() doesn't allow for part days but is easier to apply than the manually iterated (Goal Seek) Present Value method for rate finding.

While =PMT() and =IRR() are approximate in average monthly calculations, =XIRR() is proper compound interest provided (as assumed here) the dates represent whole day steps.

I found that the true APRCs (cash flow including fees) were mostly smaller than the advertised APRCs, and most could have claimed to be at least 0.1%APRC less than advertised. Chart A indicates that DCTs, watchdogs, regulators and at least some providers don't know how to calculate an APRC (or at least fail to check what is advertised).

One DCT's repayment calculator didn't mention fees and only offered to calculate single rate mortgages, with rates limited to 1 dp (2.09% rounds to 2.1% before calculating) and started at a 2.0% minimum (I was trying to check their own reported rates down to 1.44%). Another DCT's calculator also gave initial repayments modelled by =PMT() but gave fee inclusion as an option and only reported initial monthly payments, the later monthly repayments were not detailed.

Of my 12 examples, only 2 were easy to find on the original providers' web sites. Mostly I failed due to reluctance to put my life story online just to obtain their version of the numbers reported by others. If such details are really required to get to the mortgage numbers then how can DCTs provide reliable data without those details?

Not all mortgages come with fees. The ones that I studied imply fees are paid before lending begins.

Calculating by nominal rates and simple interest (the approximate method, implied by providers that agree with =PMT()) or by effective rates based on those nominal rates gives positive closing balances of up to about £190 after 25 years, with fees not included in the advertised repayments and both methods using calendar dates (not 30.4166...day steps). A positive closing balance means under-repayment of the mortgage.

These results are shown on Charts B and C.

Of course, mortgages with a variable rate period are likely to have different rates from the advert in the years following a fixed rate period. However, if the advertised plan doesn't even add up, what confidence can we have in the adverts or in those that display them?

By experimenting with Goal Seek, I found that to achieve a zero closing balance by proper compound interest (fees not included in the repayments) would need introductory monthly repayments to be increased by up to £2.65. This is significant when you consider that DCTs' results may be sorted by "Initial monthly payments" by default.

With such anomalies in the mortgage industry, how can DCTs offer a reliable service?

In a free market, providers are allowed to offer what rates and repayments they choose, but surely they must be restricted to advertising rates and repayments that at least give a zero closing balance by proper compound interest - the true time value of money!

Only basic exponential algebra is needed to handle effective rates and find the required comparator (AER, APR or APRC). Nominal rates only lead to approximation.

For improvement of banking accuracy, statutory effective rates (revised to exclude approximation by mythical 30.4166... day months, 52 week years and division by 366) must be imposed on savings, mortgages and other loans, including credit cards. When nominal rates and their misuse through simple interest are removed, providers will have to report truly comparable effective rates. This would result in different advertised rate numbers in the market but at least savers and borrowers would be able to trust DCT comparisons and original adverts based on one annual effective rate.

The difference between proper compound interest and linear approximation increases with rate. Rates are currently at historic lows. Now is a good time to minimise the magnitude of change.

Nominal rates and simple interest must be retired. Proper compound interest must be demanded in all banking. Until then, DCTs will struggle to compare reliably.

Below:-
Extract from FCA Handbook MCOB 10A.2
Chart A - Calculated APRC vs Advertised APRC
Chart B - Balance by proper compound interest using advertised payments
Chart C - The last days of Chart B
Sheet A - The first few rows of my 2,212 row spreadsheet

Extract from FCA Handbook

Repetition of "repayment or repayment of charges" emphasises that APRC calculation (as for statutory APR) includes fees. Note (3) includes appalling invitations to use un-necessary approximation.

MCOB 10A.2 APRC: mathematical formula and assumptions

MCOB 10A.2.1G21/03/2016
The mathematical formula for calculating the *APRC* in *MCOB 10A.2.2 R* is a basic equation for establishing the *APRC*. This equates, on an annual basis, the total present value of drawdowns on the one hand and the total present value of repayments and payments of charges on the other.
[**Note:** Annex I, Part I of the *MCD*]
MCOB 10A.2.2R21/03/2016
The equation referred to in *MCOB 10A.2.1 G* is:

$$\sum_{k=1}^{m} C_k (1+X)^{-t_k} = \sum_{l=1}^{m'} D_l (1+X)^{-s_l}$$

where:

- X is the *APRC*
- m is the number of the last drawdown
- k is the number of a drawdown, thus $1 \leq k \leq m$
- C_k is the amount of drawdown k
- t_k is the interval, expressed in years and fractions of a year, between the date of the first drawdown and the date of each subsequent drawdown, thus $t_1 = 0$
- m' is the number of the last repayment or payment of charges
- l is the number of a repayment or payment of charges
- D_l is the amount of a repayment or payment of charges
- s_l is the interval, expressed in years and fractions of a year, between the date of the first drawdown and the date of each repayment or payment of charges.

[**Note:** Annex I, Part I of the *MCD*]
MCOB 10A.2.3R21/03/2016
The following matters must be applied when calculating the *APRC*.
(1) The amounts paid by both parties at different times must not necessarily be equal and must not necessarily be paid at equal intervals.
(2) The starting date must be that of the first drawdown.
(3) (a) Intervals between dates used in the calculations must be expressed in years or in fractions of a year. A year is presumed to have 365 days (or 366 days for leap years), 52 weeks or 12 equal months. An equal month is presumed to have 30.41666 days (ie, 365/12), regardless of whether or not it is a leap year.
(b) Where intervals between dates used in the calculations cannot be expressed as a whole number of weeks, months or years, the intervals must be expressed as a whole number of one of those periods in combination with a number of days. Where using days:
(i) every day must be counted, including weekends and holidays;
(ii) equal periods and then days must be counted backwards to the date of the initial drawdown;
(iii) the length of the period of days must be obtained excluding the first day and including the last day and must be expressed in years by dividing this period by the number of days (365 or 366 days) of the complete year counted backwards from the last day to the same day of the previous year.

(4) The result of the calculation must be expressed with an accuracy of at least one decimal place. If the figure at the following decimal place is greater than or equal to 5, the figure at the preceding decimal place must be increased by one.
(5) The equation can be rewritten using a single sum and the concept of flows (Ak), which will be positive or negative, in other words either paid or received during periods 1 to n, expressed in years, using the following formula:

$$s = \sum_{k=1}^{n} A_k (1+X)^{-t_k}$$

where s is the present balance of flows. If the aim is to maintain the equivalence of flows, the value of s will be zero.

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My further comments on the FCA's document:-

The equations are true enough, if rather too small and fuzzy. Enlargement is not the answer. They are images pasted by the FCA from an original source. This is a pity since the equations are the most important parts. Hindrances to the use of proper compound interest must be removed.

$$\sum_{k=1}^{m} C_k (1+X)^{-t_k} = \sum_{i=1}^{m'} D_1 (1+X)^{-s_i}$$

$$s = \sum_{k=1}^{n} A_k (1+X)^{-t_k}$$

According to the Time Value of Money:- Present Value = Future Value / $(1+i)^n$ where

i = period growth factor, n = number of periods.

APRC merely values all money flows on a common time basis (the not-necessarily-whole number of 365 day periods from the start, with the rate based on 365 days).

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Chart A - Calculated APRC vs Advertised APRC

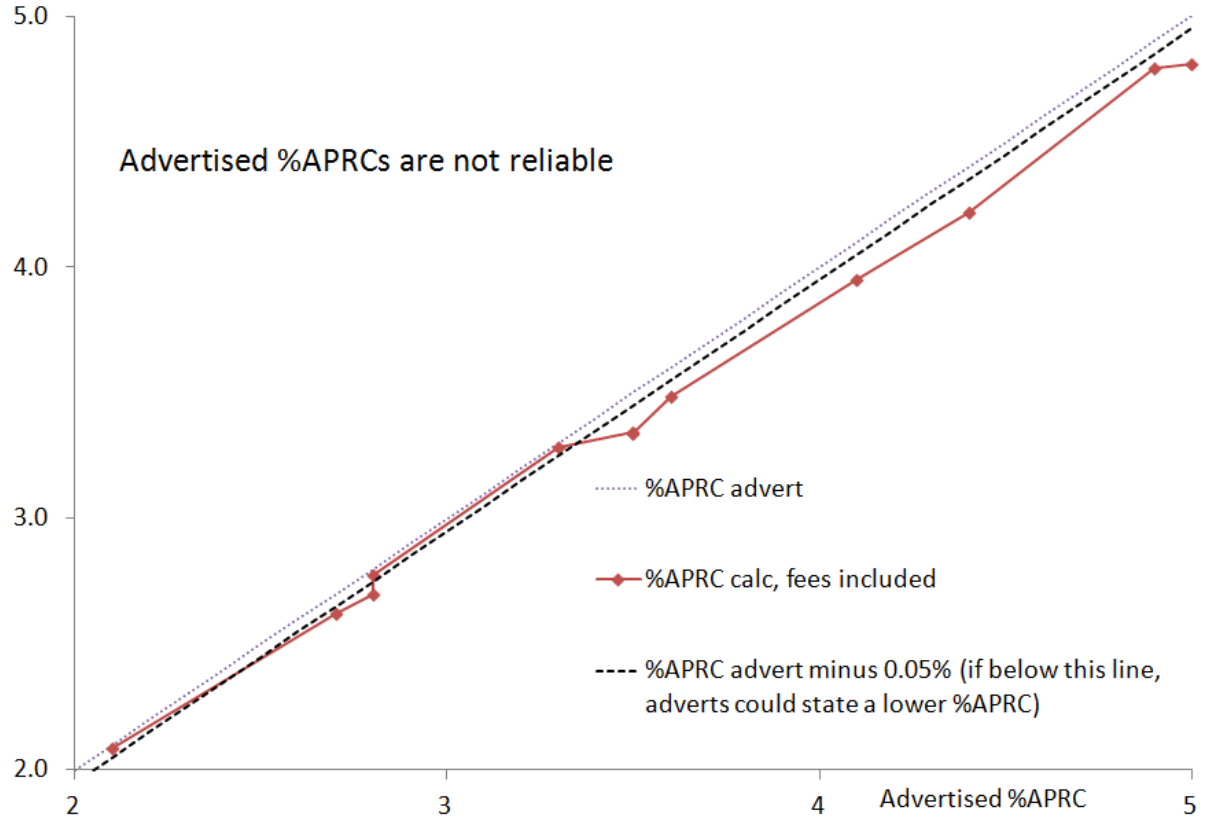


Chart B - Balance by proper compound interest using advertised payments

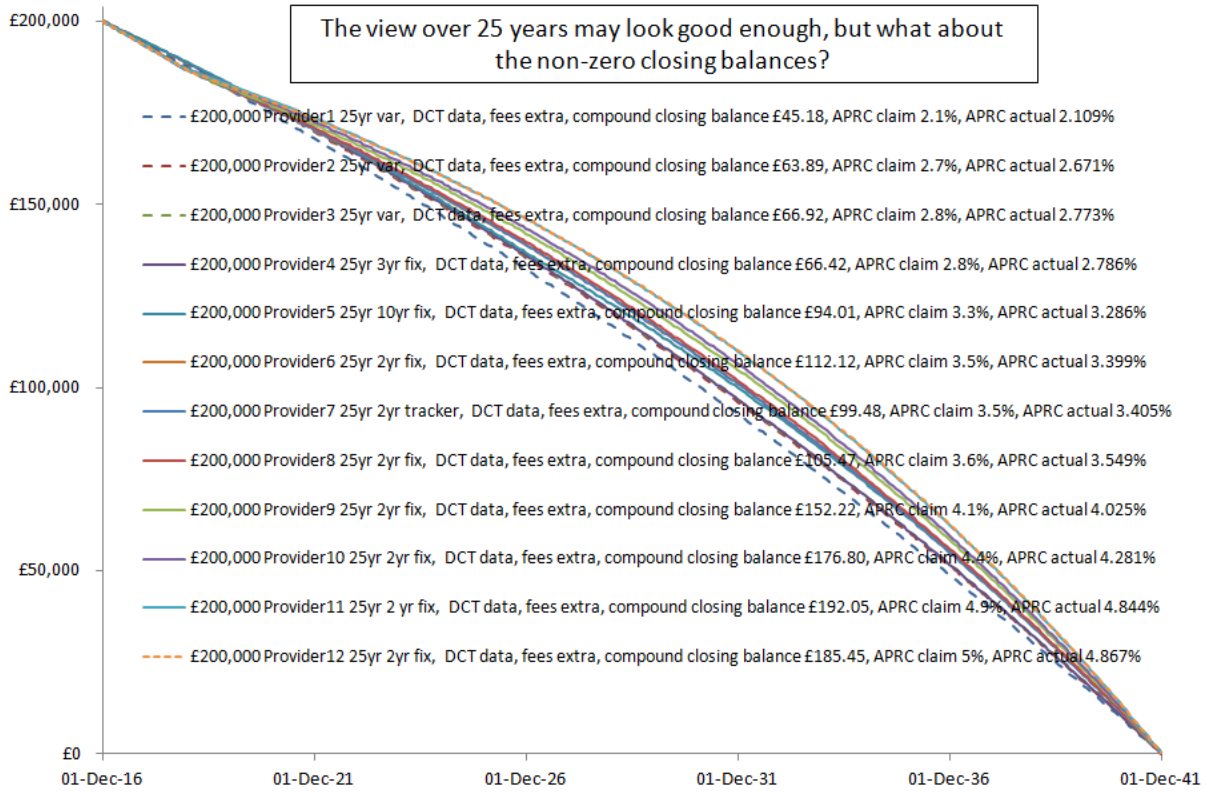
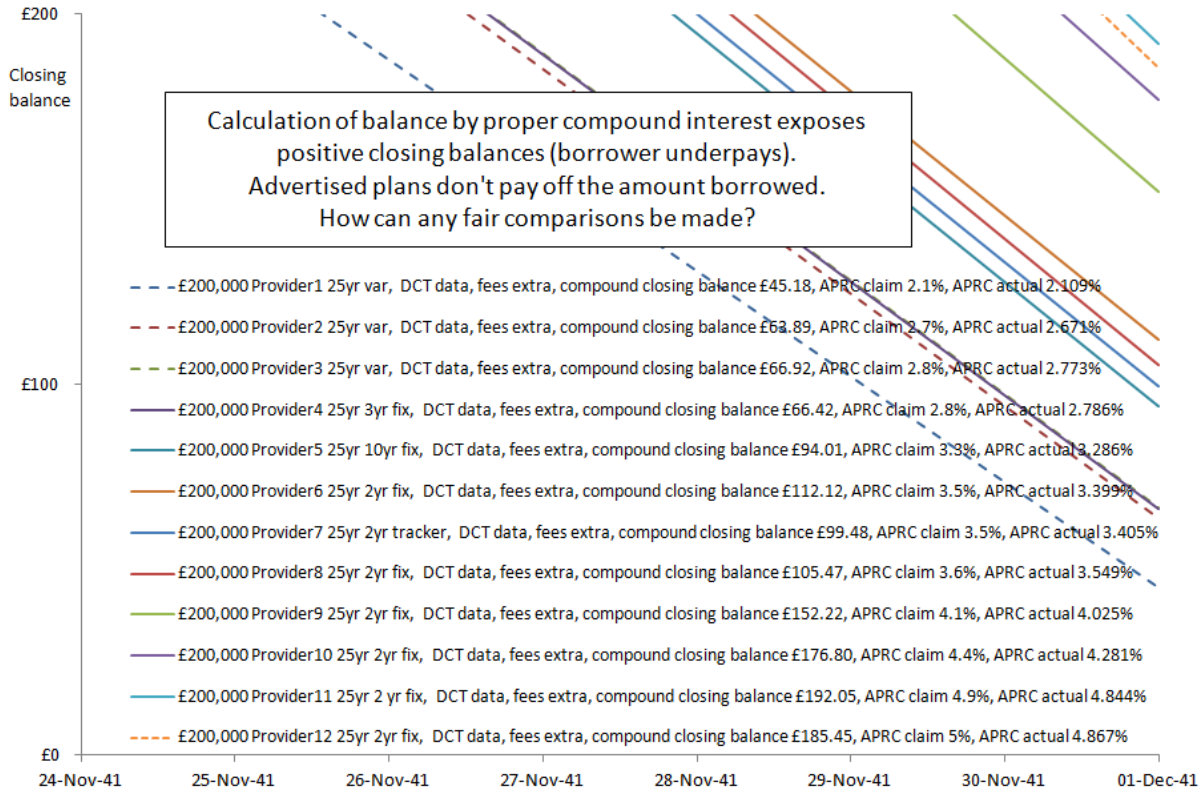


Chart C - The last days of Chart B



Sheet A The first few rows of my 2,214 row spreadsheet (sized for analysing up to a 30 year mortgage with 2 applied rates).

This extract shows the closing balances using the advertised monthly repayments and rates (fees not included in repayments). It also shows the corrections to the initial repayment (Monthly A) needed to bring the proper compound interest closing balance to zero. In my current opinion, a multi-rate repayment mortgage has too many degrees of freedom for there to be a single equation linking all parameters and iteration is therefore required to achieve a zero closing balance. Calc1 and Calc2 investigate inconsistencies in reported data sets. =PMT() reports a negative repayment for a positive loan amount. APRC by =XIRR() requires loan (including fees) and repayments to have opposite signs. Spreadsheet length doubles due to considering both with and without fee inclusion in the opening balance.
Balances by effective rates are calculated by Time Value of Money (proper compound interest) over calendar date steps.
Balances by nominal rates are calculated by linear extrapolation/interpolation (simple interest) over calendar date (not 30.4166...) steps.

	Count	Date	Provider1 25yr var	Provider2 25yr var	Provider3 25yr var	Provider4 25yr 3yr fix	Provider5 25yr 10yr fix	Provider6 25yr 2yr fix	Provider7 25yr 2yr tracker	Provider8 25yr 2yr fix	Provider9 25yr 2yr fix	Provider10 25yr 2yr fix	Provider11 25yr 2 yr fix	Provider12 25yr 2yr fix	input
Source			DCT data	DCT data	DCT data	DCT data	DCT data	DCT data	DCT data	DCT data	DCT data	DCT data	DCT data	DCT data	input
Mortgage (Purchase Value - Deposit)			£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	£200,000.00	input
Int% A nominal monthly			2.09	2.64	2.74	2.79	2.89	1.45	1.44	1.54	1.49	1.50	1.50	1.49	input
Int% B nominal monthly			2.09	2.64	2.74	2.74	3.99	3.75	3.74	3.89	4.49	4.74	5.44	5.54	input
Int% A effective			2.1101	2.6722	2.7747	2.8260	2.9286	1.4597	1.4495	1.5509	1.5002	1.5104	1.5104	1.5002	calc
Int% B effective			2.1101	2.6722	2.7747	2.7747	4.0638	3.8151	3.8048	3.9601	4.5836	4.8443	5.5777	5.6829	calc
Stated %APRC			2.1	2.7	2.8	2.8	3.3	3.5	3.5	3.6	4.1	4.4	4.9	5.0	input
%APRC calc, fees not included			2.10894	2.67066	2.77312	2.78555	3.28620	3.39938	3.40517	3.54896	4.02503	4.28129	4.84430	4.86749	calc
%APRC calc, fees included			2.08676	2.62498	2.70107	2.77585	3.28620	3.34506	3.34028	3.48710	3.94921	4.21955	4.79474	4.80667	calc
%APRC calc, fee effect			0.02218	0.04568	0.07204	0.00970	0.00000	0.05432	0.06489	0.06186	0.07581	0.06174	0.04956	0.06083	calc
Months A			300	300	300	35	121	24	23	23	24	22	23	25	input
Monthly A advert			£856.50	£911.40	£921.60	£926.72	£937.02	£795.18	£794.25	£803.64	£798.93	£799.87	£799.87	£798.93	input
Monthly A for proper zero closing balance			£856.61	£911.55	£921.76	£927.72	£937.39	£797.13	£796.05	£805.48	£801.16	£802.52	£802.22	£800.99	Goal Seek
Monthly A correction for proper zero closing balance			£0.11	£0.15	£0.16	£1.00	£0.37	£1.95	£1.80	£1.84	£2.23	£2.65	£2.35	£2.06	calc
Months B			0	0	0	265	179	276	277	277	276	278	277	275	input
Monthly B			£0.00	£0.00	£0.00	£922.13	£1,010.31	£1,009.66	£1,009.35	£1,025.29	£1,085.36	£1,113.87	£1,187.89	£1,195.76	input
Stated Int			£56,949.75	£73,419.95	£76,479.47	£76,798.55	£94,225.08	£97,750.96	£97,857.87	£102,488.52	£118,733.21	£127,252.35	£147,442.66	£148,806.98	input
Admin/Product fee			£499.00	£800.00	£1,295.00	£0.00	£0.00	£995.00	£995.00	£995.00	£1,499.00	£895.00	£999.00	£995.00	input
Valuation fee			£0.00	£199.00	£275.00	£210.00	£0.00	£173.00	£400.00	£325.00	£0.00	£380.00	£0.00	£235.00	input
Booking fee			£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	input
CHAPS fee			£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£90.00	£0.00	£0.00	£0.00	input
Total fees			£499.00	£999.00	£1,570.00	£210.00	£0.00	£1,168.00	£1,395.00	£1,320.00	£1,589.00	£1,275.00	£999.00	£1,230.00	calc
Stated total payable			£257,448.75	£274,418.95	£278,049.47	£277,008.55	£294,225.08	£298,918.96	£299,252.87	£303,808.52	£320,322.21	£328,527.35	£348,441.66	£350,036.98	input
Calc1 = Monthlies + fees			£257,449.00	£274,419.00	£278,050.00	£277,009.65	£294,224.91	£298,918.48	£299,252.70	£303,809.05	£320,322.68	£328,528.00	£348,441.54	£350,037.25	calc
Calc2 = Mortgage + fees + interest			£257,448.75	£274,418.95	£278,049.47	£277,008.55	£294,225.08	£298,918.96	£299,252.87	£303,808.52	£320,322.21	£328,527.35	£348,441.66	£350,036.98	calc
Calc1 minus Stated			£0.25	£0.05	£0.53	£1.10	-£0.17	-£0.48	-£0.17	£0.53	£0.47	£0.65	-£0.12	£0.27	calc
Calc2 minus Stated			£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	calc
minus PMT() A			£856.50	£911.40	£921.60	£926.72	£937.02	£795.18	£794.25	£803.64	£798.93	£799.87	£799.87	£798.93	calc
Monthly A minus minus PMT()			£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	£0.00	calc
End A nominal balance			£45.09	£63.73	£66.75	£183,189.68	£136,157.93	£186,529.27	£187,085.47	£187,243.53	£186,595.09	£187,741.29	£187,180.59	£186,032.30	calc
minus PMT() B						£922.15	£1,010.57	£1,009.66	£1,009.37	£1,025.31	£1,085.36	£1,113.86	£1,187.91	£1,195.79	calc
Monthly B minus minus PMT()						-£0.02	-£0.26	£0.00	-£0.02	-£0.02	£0.00	£0.01	-£0.02	-£0.03	calc
Closing, nominal rates, fees not included in balance			£45.09	£63.73	£66.75	£66.24	£93.70	£111.76	£99.14	£105.09	£151.62	£176.08	£191.09	£184.48	calc
Closing, effective rates, fees not included in balance			£45.18	£63.89	£66.92	£66.42	£94.01	£112.12	£99.48	£105.47	£152.22	£176.80	£192.05	£185.45	calc
Opening balance, fees not included		01-Dec-16	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	-£200,000.00	
	1	01-Jan-17	£856.50	£911.40	£921.60	£926.72	£937.02	£795.18	£794.25	£803.64	£798.93	£799.87	£799.87	£798.93	