Horizontal and Vertical Polarization: Task-Specific Technological Change in a Multi-Sector Economy*

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(Preliminary and Incomplete)

Abstract

We analyze the effect of technological change on inequality using a novel framework that integrates an economy's skill distribution with its occupational and industrial structure. Individuals become a manager or a worker based on their managerial vs. worker skills, and workers further sort into a continuum of tasks (occupations) ranked by skill content. Our theory dictates that faster technological progress for middle-skill tasks raises the employment shares and relative wages of lower- and higher-skill occupations (*horizontal* polarization), but also raises those of managers over workers as a whole (*vertical* polarization). Both dimensions of polarization are faster within sectors that depend more on middle-skill tasks and less on managers. This endogenously leads to faster TFP growth among such sectors, whose employment and value-added shares shrink if sectoral goods are complementarity to each other (structural change). In the limiting growth path, middle-skill occupations vanish but all sectors coexist. We present several novel facts that support our model, followed by a quantitative analysis that shows that task-specific technological progress—which was fastest for occupations embodying routine-manual tasks but not interpersonal skills—is important for understanding changes in the sectoral, occupational, and organizational structure of the U.S. economy since 1980. In contrast, skill-biased and/or sector-specific technological change played only a minor role.

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1 Introduction

We develop a novel framework that integrates an economy's distribution of individual skills with its occupation and industrial structure. This enables us to analyze how changes in wage and employment shares across occupations and across industrial sectors are interrelated and reinforce each other, providing a comprehensive view on the economic forces that shape the sectoral, occupational, and organizational structure of an economy.

In our model, individuals are heterogeneous in two-dimensional skills, managerial talent and worker human capital, based on which they become a manager or a worker. Workers then select themselves into a continuum of tasks (or occupations) based on their human capital.[1] Managers organize the workers' tasks, in addition to their own, to produce sector-level output. Sectors differ only in how intensively different tasks are used in production. Individual skills are sector-neutral, so they only care about their occupation and are indifferent about which sector they work in. The equilibrium assignment is fully characterized, which is a theoretical contribution given the new, multi-layered aspect of our model.

We also prove that if different tasks are complementary for production, faster technological progress for middle-skill tasks relative to the others leads to: (i) higher employment shares and wages for low- and high-skill occupations relative to middle-skill occupations, i.e. job and wage polarization; (ii) a higher employment share and wages for managers relative to all workers as a whole, which we call *vertical* polarization to distinguish from the *horizontal* polarization across workers; (iii) faster horizontal and vertical polarization within those sectors that depend more on middle-skill tasks and less on managers; and (iv) faster endogenous total factor productivity (TFP) growth of such sectors, shrinking their employment and value-added shares if sectoral goods are complementary (i.e., structural change).

The last prediction merits further discussion. First, because sector-level TFP is endogenously determined by equilibrium occupational choices, task-specific technological progress—which is sector-neutral—has differential impact across sectors, causing structural change. Second, as the employment share of sectors that rely less on middle-skill workers and more on managers rise, the overall degree of (both horizontal and vertical) polarization in aggregate is reinforced. Third, if all structural change is driven by task-specific technological progress, in the asymptotic balanced growth path (BGP) only occupations with slow technological progress remain and all others vanish, but all sectors exist. This is in contrast to many theories of structural change that rely on

[1]Technically, a task is the technology used by a certain occupation. Throughout the paper, we will use task and occupation interchangeably.

sector-specific forces: In those models, the shift of production factors from one sector to another continues as long as those forces exist, so that shrinking sectors vanish in the limit. In our model, task-specific technological progress is sector-neutral and only affects sectors indirectly through how they combine tasks. Once the employment shares of the occupations with faster progress become negligible, structural change ceases even as productivity continues to grow differentially across occupations.

Predictions (i) through (iv) are salient features observed in the U.S. since 1980: (i) job and wage polarization are well-documented in the literature, e.g., Autor and Dorn (2013), which we refer to as horizontal polarization; (ii) using the same data, we highlight that vertical polarization is also pronounced; and (iii) we verify that manufacturing is more reliant on middle-skill workers and less on managers than services, and provide suggestive evidence that both dimensions of polarization are indeed faster within manufacturing than in services.[2] Finally, (iv) it has long been understood that the faster growth of manufacturing TFP is an important driver of structural change from manufacturing to services. Consequently, our model shows that all these empirical facts have a common cause: faster technological progress for middle-skill tasks than the rest.

Our theoretical model is based on one managerial task and a continuum of worker tasks. To quantify the model, we discretize workers into 10 broadly defined occupation categories in the data. Our quantitative analysis confirms that task-specific technological progress alone—without any exogenous change to sector-specific TFP growth—can account for almost all of the observed growth in sector-level TFP's. More broadly, task-specific technological progress, especially for middle-skill tasks, is important for understanding the changes in the sectoral, occupational, and organizational structure in the United States over the last 35 years.

The natural next question is what can explain such differential productivity improvements across tasks. To explain what we call horizontal polarization, Autor and Dorn (2013), Goos et al. (2014) and others hypothesized that "routinization"—i.e., faster technological advancement for tasks that are more routine in nature (which tend to be middle-skill tasks in the data)—reduced the demand for middle-skill occupations. They test this empirically by constructing a "routinization index" (RTI) for each occupation, which is constructed by aggregating various information available from the Dictionary of Occupation Titles (DOT) or O*NET, the successor of DOT. But when we consider detailed characteristics of occupations, we find that the

[2] In addition, we provide evidence from establishment-level data that corroborates faster vertical polarization in manufacturing: manufacturing establishments shrank faster when measured by employment and grew faster by value-added than those in services, which is predicted by our model. Since the model assumes one manager per establishment, we need to assume that the number of managers per establishment was stable over time (we do not have data on this).

task-specific technological progress we quantify is much more strongly correlated with disaggregated measures—specifically the routine-manual index and the inverse of the manual-interpersonal index—than with RTI, which is by now commonly used in the polarization literature. In other words, technological progress in the last three or four decades heavily favored those manual tasks that are repetitive in nature and require few interpersonal skills.

Related Literature Our model is a first attempt to provide a framework that links the occupational structure of an economy to sectoral aggregates. In particular, we can use micro-estimates of occupational employment and wages, which have been studied extensively in labor economics, to study how occupational choices aggregate up to macro-level sectoral shifts.

This is of particular empirical relevance for the U.S. and other advanced economies. The 1980s marks a starting point of rising labor market inequality, of which polarization is a significant feature. This coincided with the rise of low-skill service jobs (Autor and Dorn, 2013) and also a clear rise in manufacturing productivity (Herrendorf et al., 2014). Our main finding in this regard is that task-specific productivity growth and micro-level elasticities in the labor market can be of first-order importance for understanding economic growth at the sector and even aggregate level.

Costinot and Vogel (2010) present a task-based model in which workers with a continuum of skills sort into a continuum of tasks. They present several comparative statics including a few that lead to polarization. We extend their analysis by including a manager and considering multiple sectors. Moreover, while their analysis was purely theoretical, we quantify our model to the data and empirically verify its mechanisms. Other models such as Acemoglu and Autor (2011) are variants of Costinot and Vogel (2010), but none relate polarization to structural change across macroeconomic sectors, nor treat managers as an occupation that is qualitatively different from workers.

Goos et al. (2014) construct an empirical task-based model which they use to decompose employment polarization in Europe to within- and between-industry components, but they do not consider macro-level implications. Based on an extensive analysis of occupations in the IPUMS International, Dürnecker and Herrendorf (2017) argue that most of the structural change across sectors can be accounted for by shifts at the occupation level. However, their conclusion is based on labeling some occupations in the data as manufacturing jobs and others as services. In contrast, we rank occupations based only on their mean wage, preventing any misclassification bias. Neither of these papers allow skill or wage heterogeneity.

The manager-level technology in our model is an extension of the span-of-control model of Lucas (1978), in which managers hire workers to produce output. However, unlike all existing variants of the span-of-control model, in our managers organize tasks instead of workers. That is, instead of deciding how many workers to hire, they decide on the quantities of each task to use in production, and for each task, how much skill to hire (rather than how many homogeneous workers). Moreover, rather than assuming a Cobb-Douglas technology between managerial talent and workers, we assume a CES technology between managerial talent and tasks.[3]

Our model is closely related to the rapidly growing literature in international trade that use assignment models to explain inequality between occupations and/or industries (Burstein et al., 2015; Lee, 2015). The majority of such models follow in the tradition of Roy: all workers have as many types of skills as there are available industry/occupation combinations, and select themselves into the job in which they have a comparative advantage. To make the model tractable, they typically employ a Fréchet distribution which collapses the model into an empirically testable set of equations for each industry and/or occupation pair. The manager-worker division in our model is also due to Roy-selection, but the horizontal sorting of workers into tasks is qualitatively different. Having only two skill dimensions facilitates mapping them to individual characteristics in the data; Moreover the endogenous skill formation of workers' human capital can be easily explored using traditional labor and macroeconomic tools.

Since Ngai and Pissarides (2007), most production-driven models of structural change rely on exogenously evolving sectoral productivities. Closer to our model is Acemoglu and Guerrieri (2008), in which the capital-intensive sector (in the sense of having a larger capital share in a Cobb-Douglas technology) vanishes in the limiting balanced growth path. While sectors in their model differ in how intensively they use capital and labor, in our model they differ in how intensively they use different tasks. By contrasting different types of labor, rather than capital and labor, we can connect structural change—which happens across sectors—to labor market inequality across occupations. As we briefly alluded to earlier, unlike any other existing explanation of structural change, our model implies that it is certain occupations, not broadly-defined sectors, that may vanish in the limit.

While we are the first to build a model in which individuals with different skills sort themselves into different occupations, which in turn are used as production inputs in multiple sectors, there have been recent attempts such as Buera and Kaboski (2012) and Buera et al. (2015), in which multiple sectors use different combinations of heterogeneous skills as production inputs. Similarly, Bárány and Siegel (2017) argue that

[3]While Lucas' original model is based on a generic HD1 technology, his empirical analysis and virtually all papers that followed assume a Cobb-Douglas technology. Starting from there, we incorporate (i) non-unitary elasticity between managers and workers, (ii) heterogeneity in worker productivity as well as in managerial productivity, (iii) multiple worker tasks (or occupations), and (iv) multiple sectors.

polarization can be explained by structural change, but they assume that skills are occupations-specific and that occupations are sector-specific, so task-specific change cannot be separated out independently.

An important distinction from these papers is that we separate worker human capital from the task or occupation in which it is used. Consequetly, we are able to model our driving force (i.e., routinization) as specific to a task or occupation, not to workers' human capital levels. This way, not only can we address broader dimensions of wage inequality and use micro-labor estimates to discipline our model,[4] but also represent sectoral TFPs endogenously by aggregating over equilibrium occupational choices, rather than relying on exogenously evolving worker-skill specific productivities.

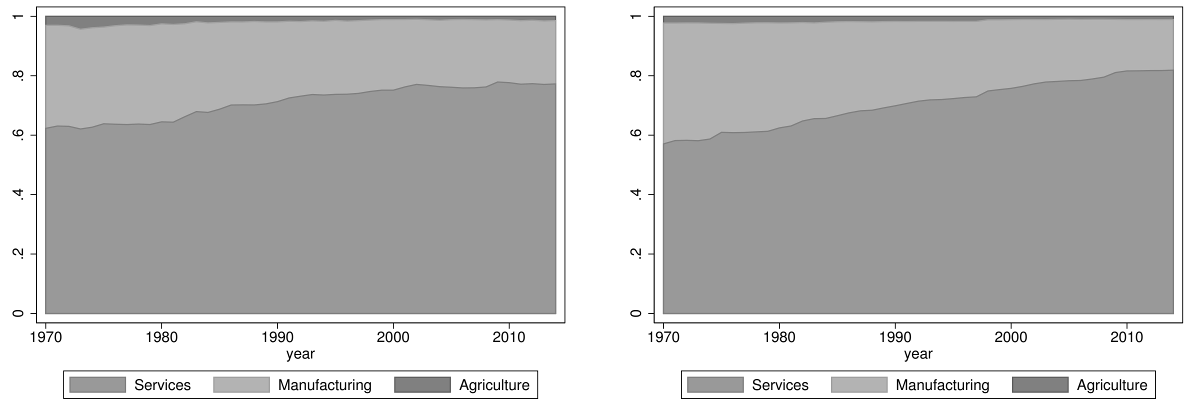
The rest of the paper is organized as follows. In section 2, we summarize the most relevant empirical facts: horizontal and vertical polarization in the overall economy, the faster speed of polarization within manufacturing than in services, and structural change. In section 3, we present the model and solve for its equilibrium. In section 4 we perform comparative statics demonstrating that faster technological progress for the middle-skill worker tasks leads to horizontal and vertical polarization, and ultimately to structural change. Section 5 characterizes the limiting behavior of the dynamic economy. Section 7 calibrates an expanded version of the theoretical model to data from 1980 to 2010, and quantifies the importance of task-specific technological progress. Section 9 concludes.

2 Facts

In this section we summarize known facts related to long-run trends in structural change and polarization, and present novel evidence on how the two may be related. We also provide a new way of looking at managerial occupations by considering them as qualtitatively different from all other occupations, while also relating them to establishments.

Structural change Figure 1 shows the (real) value-added output and employment share trends of three broadly defined sectors: agriculture, manufacturing and services, from 1970 to 2013. Following convention, e.g. Herrendorf et al. (2014), "manufacturing" is the aggregation of the manufacturing, mining and construction sectors and "services" the sum of all service and government sectors. The data are from the National Accounts published by the Bureau of Economic Analysis (BEA). In particular,

[4] Autor et al. (2006); Acemoglu and Autor (2011) show that residual wage inequality controlling for education groups is much larger than between-group inequality.



(a) GDP, Value-Added

(b) Aggregate Employment Shares

Fig. 1: Structural Change, 1970-2013.
From the BEA NIPA accounts. "Manufacturing" combines manufacturing, mining and construction, and services subsumes service and government. Employment is based on full-time equivalent number of persons in production in NIPA Table 6.

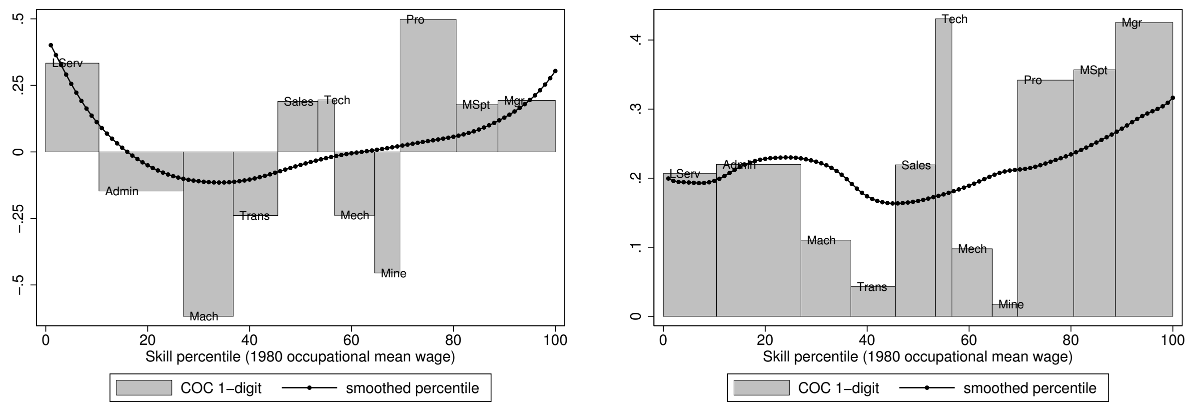
employment is based on National Income and Product (NIPA) Table 6 (persons involved in production), counted in terms of full-time equivalent workers.[5]

Two facts are well documented in the literature. First, starting from even before 1970, agriculture was already a negligible share of the U.S. economy. While it shrank from about 4% of GDP to 2% in the 1990s, its share has stayed at this level both in terms of output and employment. Thus for the remainder of this paper, we will drop the agricultural sector and only consider the manufacturing and service sectors, broadly defined. Accordingly, all moments will be computed as if the aggregate economy consists only of these two sectors (e.g. aggregate employment is the sum of manufacturing and service employment, the manufacturing and service shares sum up to unity, etc).

Second, structural change—the shifting of GDP and employment from manufacturing to services—exhibits a smooth trend starting from at least the 1970s. Moreover, GDP and employment shares are almost identical both in terms of levels and trends. This implies a close to constant input share of labor across the two sectors, which we will assume in our theoretical model.

Job and Wage Polarization Most of the rest of our empirical analysis is based on the decennial U.S. Censuses 1980–2010, for which we closely follow Autor and Dorn (2013). We restrict our sample to 16–65 year-old non-farm employment. Figure 2 plots employment and wage changes by occupation from 1980 to 2010, extending Figure 1 in

[5] Computing employment shares from the decennial censuses result in more or less the same trend.



(a) Employment Polarization (b) Wage Polarization

Fig. 2: Job and Wage Polarization, 30 years.

Autor and Dorn (2013) who considered changes up to 2005.[6] Occupations are sorted into employment share percentiles by skill along the x-axis, where skill is proxied by the mean (log) hourly wage of each occupation. We follow the `occ1990dd` occupation coding convention in Dorn (2009), which harmonizes the `occ1990` convention in Meyer and Osborne (2005). This results in 322 occupation categories for which employment is positive for all 4 censuses. Employment is defined as the product of weeks worked times usual weekly hours.[7]

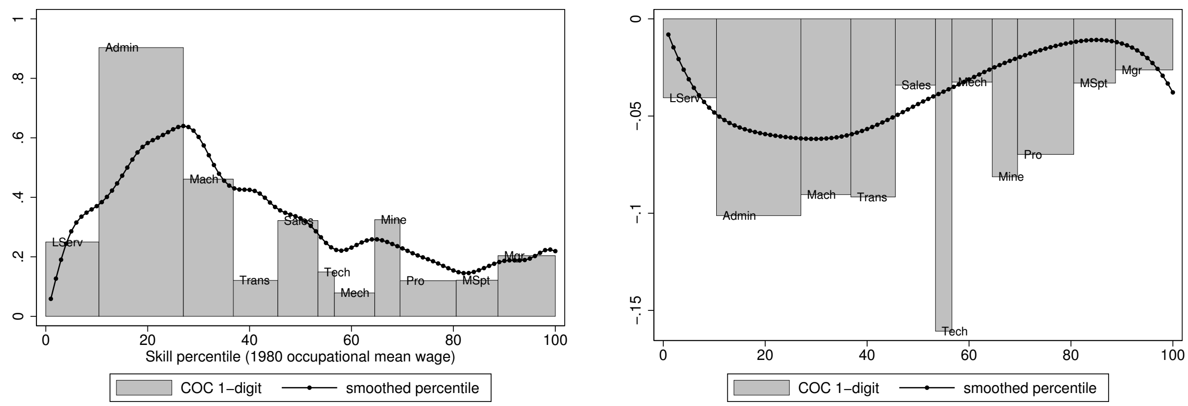
The data is presented in two ways. First, we follow Autor and Dorn (2013) and smooth changes across neighboring occupations. Each dot represents one percent of employment in 1980, and its height the percentage point change from 1980 to 2010. The changes are smoothed using a locally weighted smoothing regression. Second, we group

J obs and wages have polarized, figure 2 Source: U.S. Census and ACS 2010, replicates and extends Autor and Dorn (2013). Occupations are ranked by their 1980 mean wage.

R outinizable jobs correlate with structural change, figure 3. Source: U.S. Census and ACS 2010. Left panel replicated from Autor and Dorn (2013), right panel shows change in manufacturing employment by occupation cell.

[6]Despite the Great Recession happening between 2005 and 2010, the long-run patterns are virtually the same.

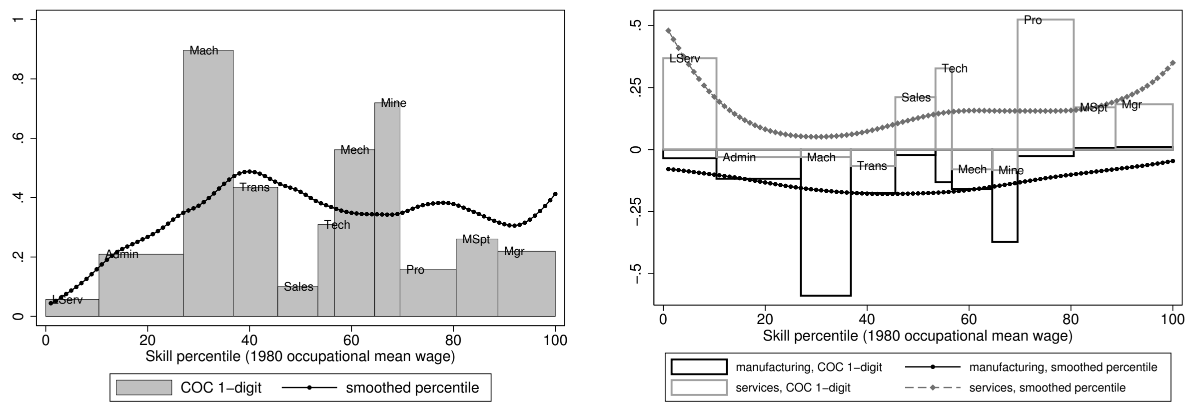
[7]See Appendix A for more details.



(a) Routinization

(b) Manufacturing Employment

Fig. 3: Routinization and Structural Change, 30 years.



(a) Manufacturing

(b) Employment Polarization by Sector

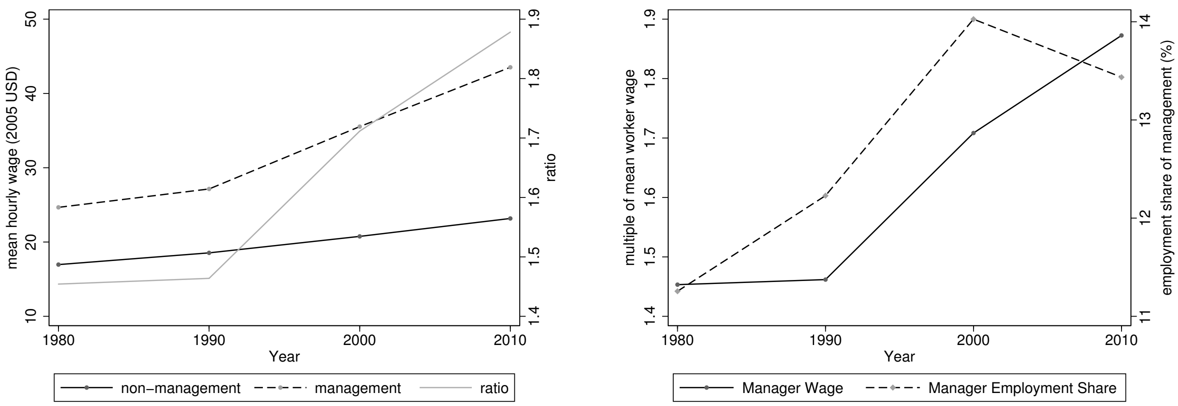
Fig. 4: Manufacturing employment shares across skill percentiles.

M anufacturing has relatively higher share of intermediate occupations, of which low-to-middle skill jobs have been relatively shrinking, figure 4. The right panel compares employment polarization by manufacturing and services, separately. Source: U.S. Census and ACS 2010.

B oth employment share and relative wages for managers are increasing: figure 5. See appendix for definition of managers in the census.

M oreover, this occurs faster in the manufacturing sector: figure 6.

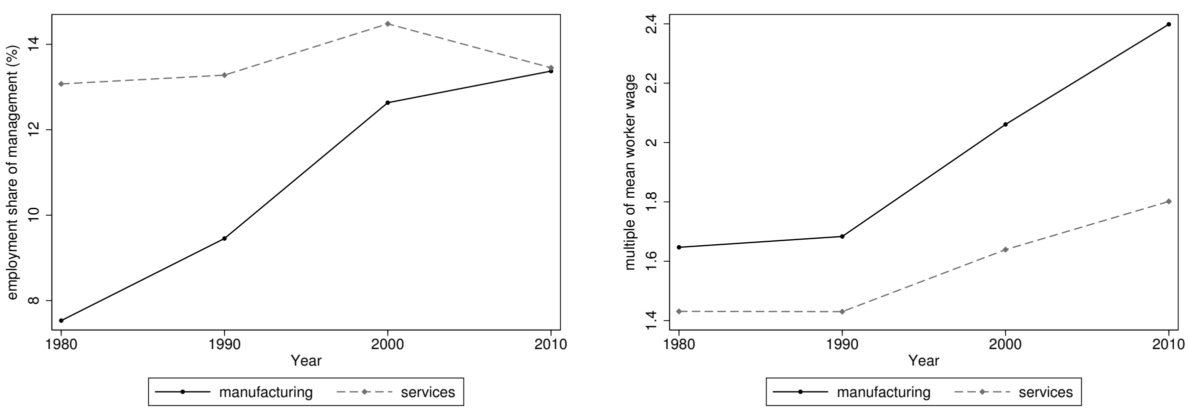
A verage size of establishments have been rising, but shrinking in the manufacturing sector: figure 7. Source: BDS 1980-2013.



(a) Wage Levels

(b) Relative Employment and Wages

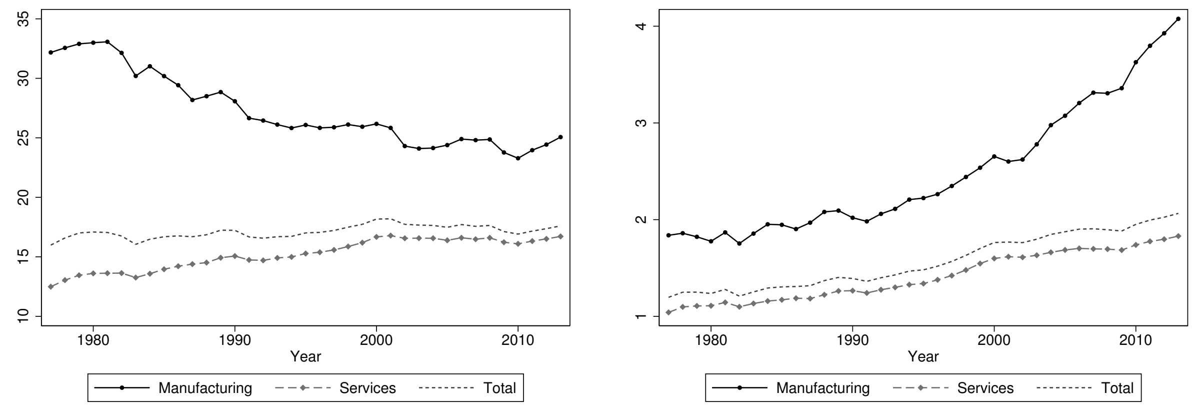
Fig. 5: Managers vs Workers



(a) Manager Employment Share

(b) Relative Manager Wage

Fig. 6: Managers by Sector



(a) Average No. of Workers (b) Average Value-added Output

Fig. 7: Establishment Size by Sector

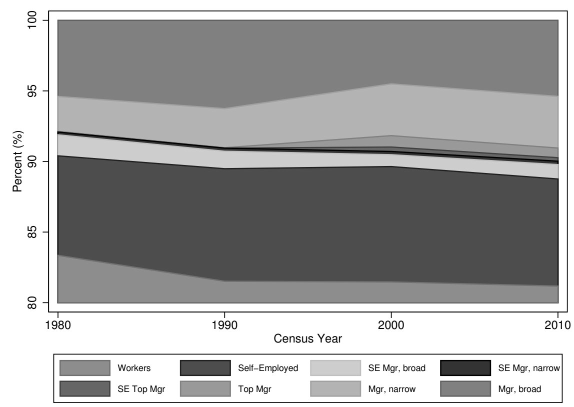


Fig. 8: Managers in the Data

M anager definition. Middle-management has been growing (relative to top management): figure 8

3 Model

There are a continuum of individuals each endowed with two types of skill, (h, z). Human capital, h, is used to produced tasks. Management, z, is a special skill for organizing tasks. WLOG we assume that the mass of individuals is 1, with associated distribution function μ.

There are 2 sectors $i \in \{m, s\}$.[8] In each sector, goods are produced in teams. A

[8]In our application, the two sectors stand in for "manufacturing" and "services," respectively. However, I analytical model can be extended to incorporate any countably finite number N of sectors; we use the subscripts m and s to avoid confusing them with the subscripts for tasks.

single manager uses her own skill and physical capital to organize three types of tasks $j \in \{0, 1, 2\}$ (e.g., low-, medium-, high-skill occupations; or manual, routine, abstract tasks). Each task requires both physical and human capital, and how much of each is allocated to each task is decided by the manager. Aggregating over the goods produced by all managers within a sector yields total sectoral output.

Within a sector, a better manager can produce more goods with the same amount of tasks, but task intensities may differ across sectors. Specifically, we assume that

$$y_i(z) = \left[\eta_i^{\frac{1}{\omega}} x_z(z)^{\frac{\omega-1}{\omega}} + (1-\eta_i)^{\frac{1}{\omega}} x_h(z)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}, \tag{1a}$$

$$x_z(z) = M_z k^\alpha z^{1-\alpha}, \qquad x_h(z) = \left[\sum_{j=0}^{2} \nu_{ij}^{\frac{1}{\sigma}} \tau_{ij}(z)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{1b}$$

$$\tau_{ij}(z) = M_j \int_{\mathrm{h_{ij}(z)}} t_j(k, h) d\mu, \tag{1c}$$

$$t_0(k, h) = k^\alpha \bar{h}^{1-\alpha}, \quad t_1(k, h) = k^\alpha h^{1-\alpha} \quad t_2(k, h) = k^\alpha (h-\chi)^{1-\alpha}, \tag{1d}$$

with $\sum_j \nu_{ij} = 1$. The $t_j(\cdot)$'s are the amounts of task output produced by an individual with human capital h and physical capital k, the latter of which is allocated by the manager. Integrating over individual task outputs over the set of workers hired by a manager of skill z for task j in sector i, $\mathrm{h_{ij}(z)}$, yields a task aggregate $\tau_{ij}(z)$. The substitutability between tasks is captured by the elasticity parameter σ, and ω captures the elasticity between all workers and managers.

For task (or occupation) 0, a worker's own human capital is irrelevant for production: all workers' effective skill input becomes $\bar{h}$. This is to capture manual jobs that do not depend on skills. For task 2, some of your skills become useless and effective skill input becomes $h - \chi$. This is to capture analytic jobs, for which lower levels of skill are redundant. We will refer to the managerial task as "task z," which is vertically differentiated from tasks $j \in \{0, 1, 2\}$, which are horizontally differentiated. The M_j's, $j \in \{0, 1, 2, z\}$, capture task-specific TFP's, which are sector-neutral.

Several points are in order. As is the case with most models of sorting workers into tasks, the worker side of our model can be viewed as a special case of Costinot and Vogel (2010). However, we model managers and have more than one sector. In contrast to Acemoglu and Autor (2011), we have a continuum of skills rather than tasks, and a discrete number of tasks rather than skills. While the implications are comparable, our formulation is more suitable for exploring employment shares across tasks (which are discrete in the data). The model is also comparable to Goos et al. (2014), who show (empirically) that relative price changes in task-specific capital, representing routinization, can drive employment polarization. However, they do not model skill and hence cannot explain wage polarization; nor do they consider macro-level implications.

Now let $\mathcal{H}_{ij}$ denote the set of individuals working in sector $i \in \{m, s\}$ on task $j \in \{0, 1, 2\}$. Also define

$$\mathcal{H}_i \equiv \cup_{j\in\{0,1,2\}}\mathcal{H}_{ij} \quad : \text{set of workers in sector } i, \tag{2a}$$

$$\mathcal{H}_j \equiv \cup_{i\in\{m,s\}}\mathcal{H}_{ij} \quad : \text{set of workers in task } j, \tag{2b}$$

$$\mathcal{H} \equiv \cup_{i\in\{m,s\}}\mathcal{H}_i = \cup_{j\in\{0,1,2\}}\mathcal{H}_j \quad : \text{set of all workers}, \tag{2c}$$

$$\mathcal{Z} \equiv \mathcal{Z}_m \cup \mathcal{Z}_s \quad : \text{set of managers in sector } i. \tag{2d}$$

Output in each sector is then

$$Y_i = A_i \int_{\mathcal{Z}_i} y_i(z) d\mu, \tag{3}$$

where A_i, $i \in \{m, s\}$ is an exogenous, sector-level productivity parameter. Final goods are produced by combining output from both sectors according to a CES aggregator:

$$Y = G(Y_m, Y_s) = \left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

where $\gamma_m + \gamma_s = 1$ and we will assume $\epsilon < 1$.[9]

3.1 Planner's Problem

We assume complete markets for solve a static planner's problem. A planner allocates aggregate capital K and all individuals into sectors $i \in \{m, s\}$ and tasks $j \in \{0, 1, 2, z\}$. The objective is to maximize current output (4) subject to (1)-(3) and

$$K = K_m + K_s \equiv \left\{ \int_{\mathcal{Z}_m} \left[\sum_{j\in\{0,1,2,z\}} k_{mj}(z) \right] + \int_{\mathcal{Z}_s} \left[\sum_{j\in\{0,1,2,z\}} k_{sj}(z) \right] \right\} d\mu$$

$$H_{ij} \equiv \int_{\mathcal{H}_{ij}} h d\mu = \int_{\mathcal{Z}_i} h_{ij}(z) d\mu, \quad j \in \{0, 1, 2\},$$

where K_i is the amount of capital allocated to sector i, H_{ij} the total amount of human capital allocated to task j in sector i, and $(k_{ij}(z), h_{ij}(z))$ is the amounts of physical and human capital allocated to task j in sector i under a manager with skill z, where $j \in \{0, 1, 2, z\}$ for k and $j \in \{0, 1, 2\}$ for h.

For existence of a solution, we assume that

[9] The estimated ϵ between the manufacturing and service sector (broadly defined) is close to zero, as we show in section 7.2.

Assumption 1 *The population means of both skills are finite, that is,*

$$\int z d\mu < \infty, \quad \int h d\mu < \infty.$$

and

Assumption 2 *There exists a strictly positive mass of individuals who do not lose all of their h-skill by working in task 2, i.e.* $\mu(h > \chi) > 0$.

The following assumption is needed for uniqueness:

Assumption 3 *The measure* $\mu(z, h)$ *is discontinuous at a finite number of points and has a connected support on* $(h, z) \in [0, h_u) \times [0, z_u)$*, where* $x_u \leq \infty$ *is the upperbound of skill* $x \in \{h, z\}$*; i.e.* $\mu(h, z) > 0$ *on* $(0, 0) \leq (h, z) < (h_u, z_u) \leq \infty$.

Along with assumption 2, this implies $\chi < h_u$. Before showing existence and uniqueness of the solution, we first characterize the solution in the following order:

1. Characterize optimal physical capital allocations across tasks within a sector.
2. Characterize optimal human capital (h) allocations across tasks within a sector.
3. Characterize optimal labor (manager-worker) allocations within a sector.
4. Solve for optimal capital and labor allocations across sectors.

Capital allocation within sectors Thanks to the HD1 assumptions, we can write sectoral technologies as

$$Y_i = A_i \left[\eta_i^{\frac{1}{\omega}} X_{iz}^{\frac{\omega-1}{\omega}} + (1 - \eta_i)^{\frac{1}{\omega}} X_{ih}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{6a}$$

$$X_{iz} = M_z K_{iz}^{\alpha} Z_i^{1-\alpha}, \qquad X_{ih} = \left(\sum_j \nu_{ij}^{\frac{1}{\sigma}} T_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{6b}$$

where $Z_i \equiv \int_{\mathcal{Z}_i} z d\mu$, X_{ih} is a sectoral task aggregate and

$$T_{i0} = M_0 K_{i0}^{\alpha} \left[\bar{h} \mu(\mathcal{H}_{i0}) \right]^{1-\alpha}, \quad T_{i1} = M_1 K_{i1}^{\alpha} H_{i1}^{1-\alpha}, \tag{7a}$$

$$T_{i2} = M_2 K_{i2}^{\alpha} \left[H_{i2} - \chi \mu(\mathcal{H}_{i2}) \right]^{1-\alpha}. \tag{7b}$$

Given sectoral capital K_i, the planner equalizes marginal product across tasks:

$$\begin{aligned} & MPK_{i0} = MPK_{i1} = MPK_{i2} \\ \Rightarrow \quad & \frac{MPT_{i0} \cdot \alpha T_{i0}}{K_{i0}} = \frac{MPT_{i1} \cdot \alpha T_{i1}}{K_{i1}} = \frac{MPT_{i2} \cdot \alpha T_{i2}}{K_{i2}} \\ \Rightarrow \quad & \frac{MPT_{i1} \cdot T_{i1}}{MPT_{i0} \cdot T_{i0}} = \frac{K_{i1}}{K_{i0}} \quad \equiv \quad \pi_{i1} = \left(\frac{\nu_{i1}}{\nu_{i0}} \right)^{\frac{1}{\sigma}} \cdot \left(\frac{T_{i1}}{T_{i0}} \right)^{\frac{\sigma-1}{\sigma}} \end{aligned} \tag{8a}$$

$$\frac{MPT_{i2}\cdot T_{i2}}{MPT_{i1}\cdot T_{i1}} = \frac{K_{i2}}{K_{i1}} \quad \equiv \quad \pi_{i2} = \left(\frac{\nu_{i2}}{\nu_{i1}}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{T_{i2}}{T_{i1}}\right)^{\frac{\sigma-1}{\sigma}}, \tag{8b}$$

where MPT_{ij} is the marginal product of T_{ij} w.r.t. X_i, and π_{ij} is the capital input ratio in tasks $j \in \{1,2\}$ and $j-1$. Due to the Cobb-Douglas assumption, π_{ij} divided by task output ratios is the marginal rate of technological substitution ($MRTS$) between tasks j and $j-1$; furthermore, π_{ij} divided by either factor input ratios in tasks j and $j-1$ measures the $MRTS$ of that factor between tasks j and $j-1$. (For capital, this is equal to 1.) Given (8) we can write

$$X_{ih} = \nu_{i0}^{\frac{1}{\sigma-1}} \underbrace{(1+\pi_{i1}+\pi_{i1}\pi_{i2})^{\frac{\sigma}{\sigma-1}}}_{\equiv \Pi_{ih}} T_{i0}. \tag{9}$$

Of course, MPK must also be equalized across the managerial task and the rest:

$$\begin{aligned}
& MPK_{iz} = MPK_{i0} \\
\Rightarrow \quad & \frac{MPX_{iz}\cdot \alpha X_{iz}}{K_{iz}} = \frac{MPX_{ih}\cdot MPT_{i0}\cdot \alpha T_{i0}}{K_{i0}} \\
\Rightarrow \quad & \frac{MPX_{iz}\cdot X_{iz}}{MPX_{ih}\cdot MPT_{i0}\cdot T_{i0}} = \frac{K_{iz}}{K_{i0}} \quad \equiv \quad \pi_{iz} = \left(\frac{\eta_i}{1-\eta_i}\right)^{\frac{1}{\omega}} \cdot \left(\frac{X_{iz}}{X_{ih}}\right)^{\frac{\omega-1}{\omega}} \cdot \Pi_{ih},
\end{aligned} \tag{10}$$

which then allows us to write, using (9),

$$\begin{aligned}
Y_i &= A_i(1-\eta_i)^{\frac{1}{\omega-1}}\left[1+\pi_{iz}/\Pi_{ih}\right]^{\frac{\omega}{\omega-1}} X_{ih} \\
&= A_i(1-\eta_i)^{\frac{1}{\omega-1}}\left[1+\pi_{iz}/\Pi_{ih}\right]^{\frac{\omega}{\omega-1}} \nu_{i0}^{\frac{1}{\sigma-1}} \Pi_{ih}^{\frac{\sigma}{\sigma-1}} T_{i0}
\end{aligned} \tag{11}$$

Sorting skills across worker tasks within sectors Given the sectoral production function (6), we can now decide how to allocate worker skills h to worker tasks $j \in \{0,1,2\}$. Since skill doesn't matter in task 0 and some becomes irrelevant in task 2, there is positive sorting of workers into tasks; i.e. there will be thresholds $(\hat{h}_1, \hat{h}_2)$ s.t. all workers with $h \leq \hat{h}_1$ work in task 0 and those with $h > \hat{h}_2$ work in task 2. Note that these thresholds must be equal across sectors, hence are not subscripted by i.

For each threshold, it must be that the marginal product of the threshold worker is equalized in either task:

$$\begin{aligned}
MPT_{i0}\cdot\frac{(1-\alpha)T_{i0}}{\bar{h}\mu(\mathcal{H}_{i0})}\cdot\bar{h} &= MPT_{i1}\cdot\frac{(1-\alpha)T_{i1}}{H_{i1}}\cdot\hat{h}_1, \\
MPT_{i1}\cdot\frac{(1-\alpha)T_{i1}}{H_{i1}}\cdot\hat{h}_2 &= MPT_{i2}\cdot\frac{(1-\alpha)T_{i2}}{H_{i2}-\chi\mu(\mathcal{H}_{i2})}\cdot(\hat{h}_2-\chi)
\end{aligned}$$

using assumption 3, so

$$\hat{h}_1 = \frac{\bar{h}_1 L_{i1}}{\pi_{i1}L_{i0}}, \qquad 1-\frac{\chi}{\hat{h}_2} = \frac{(\bar{h}_2-\chi)L_{i2}}{\pi_{i2}\bar{h}_1 L_{i1}}, \tag{12}$$

where $L_{ij} \equiv \mu(\mathcal{H}_{ij})$ and $\bar{h}_j \equiv H_{ij}/L_{ij}$; that is, we are assuming

Assumption 4 *The means of skills in tasks $j \in \{0, 1, 2, z\}$, that is, $(\bar{h}_j, \bar{z})$, are equal across sectors $i \in \{m, s\}$.*

This is an assumption needed because we assume discrete tasks; it can be thought of as the limit of vanishing supermodularity within segments of a continuum of tasks. Assumption 1 also guarantees that all objects are finite and well-defined. Using (12) we can reformulate (8) as

$$\pi_{i1} = \frac{\nu_{i1}}{\nu_{i0}} \cdot \left[\frac{M_1}{M_0} \left(\frac{\hat{h}_1}{\bar{h}} \right)^{1-\alpha} \right]^{\sigma-1}, \quad \pi_{i2} = \frac{\nu_{i2}}{\nu_{i1}} \cdot \left[\frac{M_2}{M_1} \left(1 - \frac{\chi}{\hat{h}_2} \right)^{1-\alpha} \right]^{\sigma-1}. \tag{13}$$

Sorting managers and workers within a sector Now we know how to allocate $K_i, \mathcal{H}_i$ within a sector, but we still need to know how to divide individuals into managers and workers; that is, determine $\mathcal{Z}_i \cup \mathcal{H}_i$ given a mass of individuals within a sector.

Since individuals are heterogeneous in 2 dimensions, the key is to get a cutoff rule $\tilde{z}_j(h)$ s.t. for every h, individuals with z above $\tilde{z}_j(h)$ become managers and below become workers. Since the h-skill is used differently across tasks, we need to get 3 such rules for each sector; however the rule must be identical across sectors.

For $h \leq \hat{h}_1$, this rule is simple. For these workers, h does not matter, so $\tilde{z}_0(h) = \hat{z}$, i.e., is constant. The constant is chosen so that the marginal product of the threshold manager is equalized in either task:

$$\begin{aligned} MPX_{iz} \cdot \frac{(1-\alpha)X_{iz}}{Z_i} \cdot \tilde{z}_0(h) &= MPX_{ih} \cdot MPT_{i0} \cdot \frac{(1-\alpha)T_{i0}}{\bar{h}\mu(\mathcal{H}_{i0})} \cdot \bar{h} \\ \Rightarrow \quad \tilde{z}_0(h) = \hat{z} &= \frac{Z_i}{\pi_{iz} L_{i0}} = \frac{\bar{z} L_{iz}}{\pi_{iz} L_{i0}}, \end{aligned} \tag{14}$$

where $L_{iz} = \mu(\mathcal{Z}_i)$ and $\bar{z} = Z_i / L_{iz}$ (which is equal across sectors by assumption 4). Then from (10) we can write

$$\pi_{iz} = \frac{\eta_i \nu_{i0}^{\frac{1-\omega}{\sigma-1}}}{1-\eta_i} \cdot \left[\frac{M_z}{M_0} \left(\frac{\hat{z}}{\bar{h}} \right)^{1-\alpha} \right]^{\omega-1} \cdot \Pi_{ih}^{\frac{\sigma-\omega}{\sigma-1}}. \tag{15}$$

For $h \in (\hat{h}_1, \hat{h}_2]$, the rule is linear:

$$\begin{aligned} MPX_{iz} \cdot \frac{(1-\alpha)X_{iz}}{Z_i} \cdot \tilde{z}_1(h) &= MPX_{ih} \cdot MPT_{i1} \cdot \frac{(1-\alpha)T_{i1}}{H_{i1}} \cdot h \\ \Rightarrow \quad \frac{\tilde{z}_1(h)}{h} = \phi_1 &= \frac{\pi_{i1} Z_i}{\pi_{iz} H_{i1}} = \frac{\pi_{i1} \bar{z} L_{iz}}{\pi_{iz} \bar{h}_1 L_{i1}}. \end{aligned}$$

and finally for $h > \hat{h}_2$, the rule is affine:

$$MPX_{iz} \cdot \frac{(1-\alpha)X_{iz}}{Z_i} \cdot \tilde{z}_2(h) = MPX_{ih} \cdot MPT_{i2} \cdot \frac{(1-\alpha)T_{i2}}{H_{i2} - \chi\mu(\mathcal{H}_{i2})} \cdot (h - \chi)$$

$$\Rightarrow \quad \frac{\tilde{z}_2(h)}{h-\chi} = \phi_2 = \frac{\pi_{i1}\pi_{i2} Z_i}{\pi_{iz}\left(H_{i2} - \chi L_{i2}\right)} = \frac{\pi_{i1}\pi_{i2}\bar{z} L_{iz}}{\pi_{iz}(\bar{h}_2 - \chi) L_{i2}}.$$

Observe that

$$\phi_1 = \hat{z}/\hat{h}_1, \qquad \phi_2 = \phi_1/\tilde{h}_2 = \hat{z}/\hat{h}_1\tilde{h}_2, \tag{16}$$

so $(\hat{h}_1, \hat{h}_2, \hat{z})$ completely determine the ϕ_j's, and all objects are well defined given assumption 2, since all tasks are essential.

Sectoral production function and allocation across sectors Equations (12)-(15) completely describe the task thresholds. What is important here is that all these thresholds are determined independently of the amount of physical capital. To see this more clearly, rewrite (11) to obtain

$$\begin{aligned} Y_i &= A_i \psi_i \cdot \left[1 + \pi_{iz}/\Pi_{ih}\right]^{\frac{\omega}{\omega-1}} \Pi_{ih}^{\frac{\sigma}{\sigma-1}} M_0 K_{i0}^{\alpha} L_{i0}^{1-\alpha} \\ \psi_i &\equiv (1-\eta_i)^{\frac{1}{\omega-1}} \nu_{i0}^{\frac{1}{\sigma-1}} \bar{h}^{1-\alpha} \end{aligned}$$

and furthermore since

$$K_i = K_{i0} \underbrace{(\Pi_{ih} + \pi_{iz})}_{\Pi_{K_i}} \tag{17}$$

$$L_i = L_{i0} \underbrace{\left[1 + (\hat{h}_1/\bar{h}_1)\pi_{i1} + \frac{1-\chi/\hat{h}_2}{(\bar{h}_2 - \chi)/\hat{h}_1} \cdot \pi_{i1}\pi_{i2} + (\hat{z}/\bar{z})\pi_{iz}\right]}_{\Pi_{L_i}}, \tag{18}$$

we obtain

$$Y_i = \underbrace{M_0 \cdot A_i\psi_i \cdot \Pi_{ih}^{\frac{\omega-\sigma}{(\omega-1)(\sigma-1)}} \Pi_{K_i}^{\frac{\omega}{\omega-1}-\alpha} \Pi_{L_i}^{\alpha-1}}_{\Phi_i\text{: TFP}} K_i^{\alpha} L_i^{1-\alpha}. \tag{19}$$

Note that sectoral TFP, Φ_i, can be decomposed into 3 parts: M_0, that is common across both sectors, $A_i\psi_i$, which is sector-specific but exogenous, and the parts determine by $(\Pi_{ih}, \Pi_{K_i}, \Pi_{L_i})$, which is sector-specific and endogenously determined by $(\hat{h}_1, \hat{h}_2, \hat{z})$. Furthermore, since the thresholds depend only on the relative masses of individuals across tasks *within* a sector, they do not depend on the employment size of the sector (nor capital). Hence even as K_i or L_i changes, these thresholds do not as long as the distribution of skills remains constant.

Sectors only differ in how intensely they use each task, i.e., the mass of individuals allocated to each task. As usual, these masses are determined so that the MPK and MPL are equalized across sectors:

$$\kappa \equiv \frac{K_s}{K_m} = \frac{L_s}{L_m} = \left(\frac{\gamma_s}{\gamma_m}\right)^{\frac{1}{\epsilon}} \left(\frac{Y_s}{Y_m}\right)^{\frac{\epsilon-1}{\epsilon}} = \frac{\gamma_s}{\gamma_m} \cdot \left(\frac{\Phi_s}{\Phi_m}\right)^{\epsilon-1} \tag{20}$$

where κ is capital input ratios between sectors m and s. Hence relative employment between sectors is completely determined by the relative endogenous TFP ratio between the two sectors. Since the Φ_i's are just functions of $(\hat{h}_1, \hat{h}_2, \hat{z})$ as seen in (19), so is κ and hence sectoral employment L_i since

$$L_m = 1/(1+\kappa),\ L_s = \kappa/(1+\kappa). \tag{21}$$

3.2 Equilibrium Existence and Uniqueness

As a final step, the planner needs to ensure that the within-sector allocations are consistent with the between sector allocations. That is, the total mass of individuals in occupation $j \in \{0,1,2,z\}$ must be consistent with (20). Formally, we define renormalizations of (Π_{ih}, Π_{L_i}):

$$\tilde{\Pi}_{ih} \equiv \nu_{i0} M_0^{\sigma-1} \bar{h}^{(\sigma-1)(1-\alpha)} \cdot \Pi_{ih}$$
$$\tilde{\Pi}_{L_i} \equiv (1-\eta_i)\nu_{i0} M_0^{\sigma-1} \bar{h}^{(\sigma-1)(1-\alpha)} \Pi_{L_i} = \sum_{j=0,1,2,z} V_{ij},$$

where the weights V_{ij} are

$$V_{i0} \equiv (1-\eta_i)\nu_{i0} M_0^{\sigma-1} \cdot \frac{\bar{h}^{\alpha+\sigma(1-\alpha)}}{\bar{h}}, \tag{22a}$$
$$V_{i1} \equiv (1-\eta_i)\nu_{i1} M_1^{\sigma-1} \cdot \frac{\hat{h}_1^{\alpha+\sigma(1-\alpha)}}{\bar{h}_1}, \tag{22b}$$
$$V_{i2} \equiv (1-\eta_i)\nu_{i2} M_2^{\sigma-1} \cdot \frac{\left(\hat{h}_1 \tilde{h}_2\right)^{\alpha+\sigma(1-\alpha)}}{\bar{h}_2 - \chi}, \tag{22c}$$
$$V_{iz} \equiv \eta_i \tilde{\Pi}_{ih}^{\frac{\sigma-\omega}{\sigma-1}} M_z^{\omega-1} \cdot \frac{\hat{z}^{\alpha+\omega(1-\alpha)}}{\bar{z}}. \tag{22d}$$

Note that the only differences in the V_{ij}'s across sectors i comes from the task intensity parameters ν_{ij}, η_i (since Π_{ih} is also a function only of the ν_{ij}'s in equilibrium). The total amount of labor in each task j can be expressed as

$$L_j = \sum_{i\in\{m,s\}} \frac{V_{ij}}{\tilde{\Pi}_{L_i}} \cdot L_i, \quad \text{where} \quad L_m = 1/(1+\kappa),\ L_s = \kappa/(1+\kappa) \tag{23}$$

for $j \in \{0,1,2,z\}$. This system of equations that solves the planner's problem are also the equilibrium market clearing conditions; the LHS is the labor supply and RHS demand for each task j. Since $\bar{h}_j L_j = H_j$, $\bar{z} L_z = Z$, $\sum_j L_j = 1$ and $\kappa = \kappa(\hat{h}_1, \hat{h}_2, \hat{z})$ is a function of $(\hat{h}_1, \hat{h}_2, \hat{z})$ from (20), the solution to $(\hat{h}_1, \hat{h}_2, \hat{z})$ is found from the system of three equations

$$\log \hat{z} = \frac{(1-\omega)\log M_z + \log Z - \log\left[\sum_i \frac{\eta_i \tilde{\Pi}_{ih}^{\frac{\sigma-\omega}{\sigma-1}}}{\tilde{\Pi}_{L_i}} \cdot L_i\right]}{\alpha + \omega(1-\alpha)} \tag{24a}$$

$$\log \hat{h}_1 = \frac{(1-\sigma)\log M_1 + \log H_1 - \log\left[\sum_i \frac{(1-\eta_i)\nu_{i1}}{\tilde{\Pi}_{L_i}} \cdot L_i\right]}{\alpha + \sigma(1-\alpha)} \tag{24b}$$

$$\log \tilde{h}_2 = \frac{(1-\sigma)\log M_2 + \log(H_2 - \chi L_2) - \log\left[\sum_i \frac{(1-\eta_i)\nu_{i2}}{\tilde{\Pi}_{L_i}} \cdot L_i\right]}{\alpha + \sigma(1-\alpha)} - \log \hat{h}_1, \tag{24c}$$

where

$$Z(\hat{h}_1, \hat{h}_2, \hat{z}) = \left[\int^{\hat{h}_1} \int_{\hat{z}} + \int_{\hat{h}_1}^{\hat{h}_2} \int_{\hat{z}h/\hat{h}_1} + \int_{\hat{h}_2} \int_{\hat{z}(h-\chi)/\hat{h}_1\tilde{h}_2}\right] z dF(z|h) dG(h) \tag{25a}$$

$$H_1(\hat{h}_1, \hat{h}_2, \hat{z}) = \int_{\hat{h}_1}^{\hat{h}_2} hF\left(\hat{z}h/\hat{h}_1|h\right) dG(h) \tag{25b}$$

$$H_2(\hat{h}_1, \hat{h}_2, \hat{z}) = \int_{\hat{h}_2} hF\left(\hat{z}(h-\chi)/\hat{h}_1\tilde{h}_2|h\right) dG(h), \tag{25c}$$

and $G(h)$ is the marginal distribution of h, and $F(z|h)$ the distribution of z conditional on h, so that

$$\mu(\tilde{h}, \tilde{z}) = \int^{\tilde{h}} \int^{\tilde{z}} dF(z|h) dG(h).$$

The mass $L_2(\hat{h}_1, \hat{h}_2, \hat{z})$ can be expressed likewise. Given the characterization of the solution, we can establish:

Theorem 1 *Under Assumptions 1-4,*

1. *The solution to the planner's problem exists.*
2. *For any $\mu_i \leq \mu$ that satisfies Assumptions 1-3, the solution to the within-sector problem is unique.*
3. *The two-sector equilibrium in which, for each occupation $j \in \{0, 1, 2, z\}$, the distribution function is split by a constant fraction $\kappa_j \in (0, 1)$ across sectors according to*

$$\begin{aligned}
\mu_m(z,h) &= (1-\kappa_0)\mu_(z,h) \quad \text{and} \quad \mu_s(z,h) = \kappa_0\mu_(z,h) \quad \text{for}(z,h) \in \mathcal{H}_0, \\
\mu_m(z,h) &= (1-\kappa_1)\mu_(z,h) \quad \text{and} \quad \mu_s(z,h) = \kappa_1\mu_(z,h) \quad \text{for}(z,h) \in \mathcal{H}_1, \\
\mu_m(z,h) &= (1-\kappa_2)\mu_(z,h) \quad \text{and} \quad \mu_s(z,h) = \kappa_2\mu_(z,h) \quad \text{for}(z,h) \in \mathcal{H}_2, \\
\mu_m(z,h) &= (1-\kappa_z)\mu_(z,h) \quad \text{and} \quad \mu_s(z,h) = \kappa_z\mu_(z,h) \quad \text{for}(z,h) \in \mathcal{Z},
\end{aligned}$$

   is unique. Furthermore, given such an equilibrium, any arbitrary split of the distribution function that satisfies Assumption 4 and preserves the mass of individuals allocated to each sector-occupation cell is also an equilibrium.

Proof: See Appendix B. □

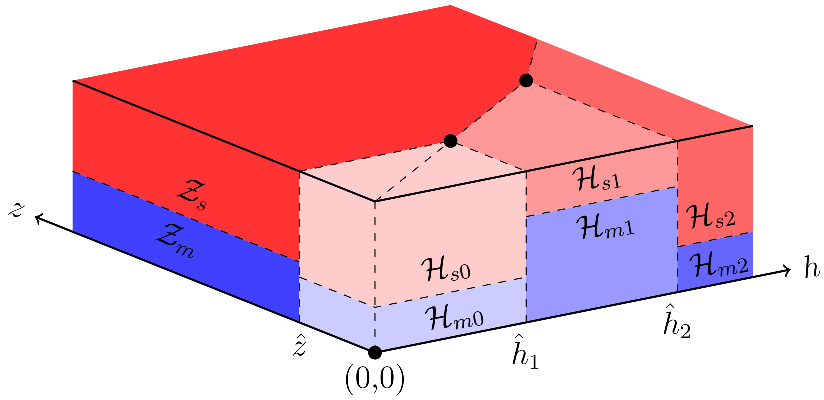


Fig. 9: Equilibrium

The resulting equilibrium skill allocation is depicted in Figure 10.[10] The thresholds determine the tasks, and employment is split across sectors while preserving the means for each task. The different masses of sectoral employment across tasks are due to the task intensity parameters ν_{ij}, η_i.

3.3 Equilibrium wages and prices

Since there are no frictions, the planner's allocation coincides with a competitive equilibrium. Hence, the solution $(\hat{h}_1, \hat{h}_2, \hat{z})$ gives all the information needed to derive equilibrium prices. The price of the final good can be normalized to 1:

$$P = 1 = \left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \qquad p_i = [Y_i/\gamma_i Y]^{-\frac{1}{\epsilon}}. \tag{26}$$

Sectoral output prices can be obtained by applying the sectoral output in (17)-(19) in (26). The interest rate R is given either by the dynamic law of motion for aggregate capital, or fixed in a small open economy. So w_0 is

$$w_0 = \frac{1-\alpha}{\alpha} \cdot \frac{K_{i0}}{L_{i0}} \cdot \frac{R}{\bar{h}}$$

where the capital-labor ratio can be found from (17)-(18).

Given w_0, indifference across tasks for threshold workers imply

$$w_0 \bar{h} = w_1 \hat{h}_1, \quad w_1 \hat{h}_2 = w_2(\hat{h}_2 - \chi) \tag{27a}$$

$$\Rightarrow \quad w_0/w_1 = \hat{h}_1/\bar{h}, \quad w_1/w_2 = 1 - \chi/\hat{h}_2. \tag{27b}$$

and likewise, the threshold manager implies a "managerial efficiency wage"

$$w_z \hat{z} = w_0 \bar{h} \quad \Rightarrow \quad w_0/w_z = \hat{z}/\bar{h}. \tag{27c}$$

Hence, relative wages for task j are simply the inverse of the thresholds.

[10] For illustrative purposes, the figure assumes that $\mu(z,h)$ is uniform.

4 Comparative Statics

The sectoral technology representation (19) implies that this model has similar implications as Ngai and Pissarides (2007): the sector with the larger TFP shrinks. The major difference is that these TFP's are endogenous.

What is more interesting is the implications of growth in task-specific TFP's—this is equivalent to the price of task-specific capital falling in Goos et al. (2014)—or changes in the distribution for skill. In particular, we are interested in the effect of routinization, which we model as an increase in the task 1's TFP, M_1. This is illustrated in a series of comparative statics, which is possible since the equilibrium is unique and skill distribution continuous (under assumption 3). To simplify notation, define the elasticities of the thresholds w.r.t. M_1:

$$\Delta_{h_1} \equiv \frac{d\log \hat{h}_1}{d\log M_1}, \quad \Delta_{h_2} \equiv \tilde{\chi} \cdot \frac{d\log \hat{h}_2}{d\log M_1}, \quad \Delta_z \equiv \frac{d\log \hat{z}}{d\log M_1}, \quad \text{where } \tilde{\chi} \equiv \frac{\chi}{\hat{h}_2 - \chi} > 0.$$

Similarly define Δ_x as the elasticity of any variable x with respect to M_1. Given $(\Delta_{h_1}, \Delta_{h_2}, \Delta_z)$ we know what happens to all the other variables of interest since

$$\begin{aligned}
&\Delta_{\phi_1} = \Delta_z - \Delta_{h_1}, \quad \Delta_{\phi_2} = \Delta_{\phi_1} - \Delta_{h_2}, \\
&\Delta_{W_1} = -\Delta_{h_1}, \qquad \Delta_{W_2} = -\Delta_{h_2}, \qquad \Delta_{W_z} = -\Delta_z.
\end{aligned}$$

where (ϕ_1, ϕ_2) are defined in (16) and W_j's are the wage ratios

$$W_1 = w_1/w_0, \quad W_2 = w_2/w_1, \quad W_z = w_z/w_0.$$

We proceed as follows:

1. approximate the change in thresholds $(\hat{h}_1, \hat{h}_2, \hat{z})$ within a sector, taking the distribution of skill in sector i, μ_i, as given;
2. given the comparative statics in the thresholds, approximate the change in employment shares across tasks within a sector, taking μ_i as given;
3. approximate the differences in polarization across sectors holding L_i constant;
4. approximate the change in employment shares across sectors.

4.1 Wage and Job Polarization

To approximate the change in thresholds, we will first focus on the within sector allocation of skill implied by (12) and (14):

$$\hat{h}_1 \cdot \pi_{i1}(\hat{h}_1) = H_{i1}(\hat{h}_1, \hat{h}_2, \hat{z}) \Big/ L_{i0}(\hat{z}, \hat{h}_1) \tag{28a}$$

$$\Big(1 - \chi_h/\hat{h}_2\Big) \cdot \pi_{i2}(\hat{h}_2) = \Big[H_{i2}(\hat{h}_1, \hat{h}_2, \hat{z}) - \chi L_{i2}(\hat{h}_1, \hat{h}_2, \hat{z})\Big] \Big/ H_{i1}(\hat{h}_1, \hat{h}_2, \hat{z}) \tag{28b}$$

$$\hat{z} \cdot \pi_{iz}(\hat{h}_1, \hat{h}_2, \hat{z}) = Z_i(\hat{h}_1, \hat{h}_2, \hat{z}) \Big/ L_{i0}(\hat{z}, \hat{h}_1) \tag{28c}$$

where (π_{ij}, π_{iz}) are defined in (13) and (15). The masses and skill aggregates are defined over a sector-specific distribution μ_i, which is taken as given.

For the approximation, we will assume that $\Delta_{L_{ij}} \to 0$ for $j \in \{0, 1, 2, z\}$. This implies that the density function is sufficiently small everywhere, which we assume to ignore the indirect effects of M_1 on (L_{ij}, H_{ij}, Z_i) that arise from changes in the thresholds. This can be thought of a limiting case of either when skills are discrete (Goos et al., 2014),[11] or when there are both a continuum of tasks and skills which are matched assortatively (Costinot and Vogel, 2010). Within the context of our model, it can be understood as approximating the equilibrium response using only the response of labor demand (the LHS's), while keeping labor supply (the RHS's) fixed. We can then show that

Proposition 1 (Routinization and Polarization) *Suppose there is an increase in M_1, and that $\Delta_{L_{ij}} \to 0$ for $j \in \{0, 1, 2, z\}$. Then*

1. *$\Delta_{h_1} \approx -\Delta_{h_2} > 0$ iff $\sigma < 1$, and*
2. *$\Delta_{\phi_1} < \{\Delta_{h_1}, \Delta_z \approx \Delta_{\phi_2}\} < 0$ if $\omega < \sigma < 1$.*

This implies that capital and labor flow out of task 1 (job polarization), relative wages decline in task 1 (wage polarization), and both the employment share and wages of managers increase (vertical polarization).

Proof: Under the assumption (or, holding labor supply fixed), the comparative static is identical across sectors. System (28) becomes

$$\Delta_{h_1} + \Delta_{\pi_{i1}} \approx 0, \quad \Delta_{h_2} + \Delta_{\pi_{i2}} \approx 0, \quad \Delta_z + \Delta_{\pi_{iz}} \approx 0,$$

where

$$\begin{aligned} \Delta_{\pi_{i1}} &= (\sigma - 1)\left[(1-\alpha)\Delta_{h_1} + 1\right], \quad \Delta_{\pi_{i2}} = (\sigma - 1)\left[(1-\alpha)\Delta_{h_2} - 1\right], \\ \Delta_{\pi_{iz}} &= (\omega - 1)(1-\alpha)\Delta_z + \frac{\sigma - \omega}{\sigma - 1} \cdot \frac{\pi_{i1}(1+\pi_{i2})\Delta_{\pi_{i1}} + \pi_{i1}\pi_{i2}\Delta_{\pi_{i2}}}{\Pi_{ih}} \end{aligned} \tag{29}$$

Hence we obtain that

$$\Delta_{h_1} \approx -\Delta_{h_2} \approx \frac{1-\sigma}{\alpha + \sigma(1-\alpha)} > 0, \quad \Delta_{W_1} < 0, \quad \Delta_{W_2} > 0 \quad \Leftrightarrow \quad \sigma < 1.$$

Furthermore if $\omega < \sigma < 1$,

$$\Delta_z \approx \frac{\sigma - \omega}{(\sigma - 1)\left[\alpha + \omega(1-\alpha)\right]} \cdot \frac{\pi_{i1}}{\Pi_{ih}} \cdot \Delta_{h_1} < 0, \tag{30}$$

[11] In fact, they assume that wages are fixed and labor is inelastically supplied.

$$\Delta_{\phi_1} < 0, \quad \Delta_{\phi_2} \approx \Delta_z < 0, \quad \text{and} \quad \Delta_{W_z} > 0.$$

$\square$

The change in thresholds makes it easier to analyze what happens to employment shares by task. If $\sigma < 1$, and holding management employment shares constant, employment and payroll in task 1 shrinks while they increase in tasks 0 and 2. Hence, similarly as in Goos et al. (2014), we get employment polarization only when tasks are complementary, i.e. $\sigma < 1$; we also get wage polarization even with endogenous choice of tasks. At the same time, capital flows out to the other tasks as well.

Furthermore if $\omega < \sigma$, we also find that the mass and wage of managers increase relative to all workers. But while it is clear that the thresholds move in a direction that continues to shrink L_{i1}, it is unclear what happens to L_{i0} and L_{i2}, since both tasks 0 and 2 gain employment from task 1 but lose employment to managers.

So let us think about the (supply side) changes in L_{ij}, $j \in \{0, 1, 2, z\}$, arising from the change in thresholds within a sector i, still taking the sectoral distribution μ_i as given. To sign the $\Delta_{L_{i0}}, \Delta_{L_{i2}}$, we need additional parametric restrictions for sufficiency:

Lemma 1 *Suppose the skill distribution in sector i is uniform and that $\omega < \sigma < 1$. A sufficient condition for employment in tasks 0 and 2 to rise is*

$$\sigma - \omega < (1 - \sigma)\left[\alpha + \omega(1 - \alpha)\right].$$

So if

$$\frac{\sigma - (1 - \sigma)\alpha}{1 + (1 - \sigma)(1 - \alpha)} < \omega < \sigma,$$

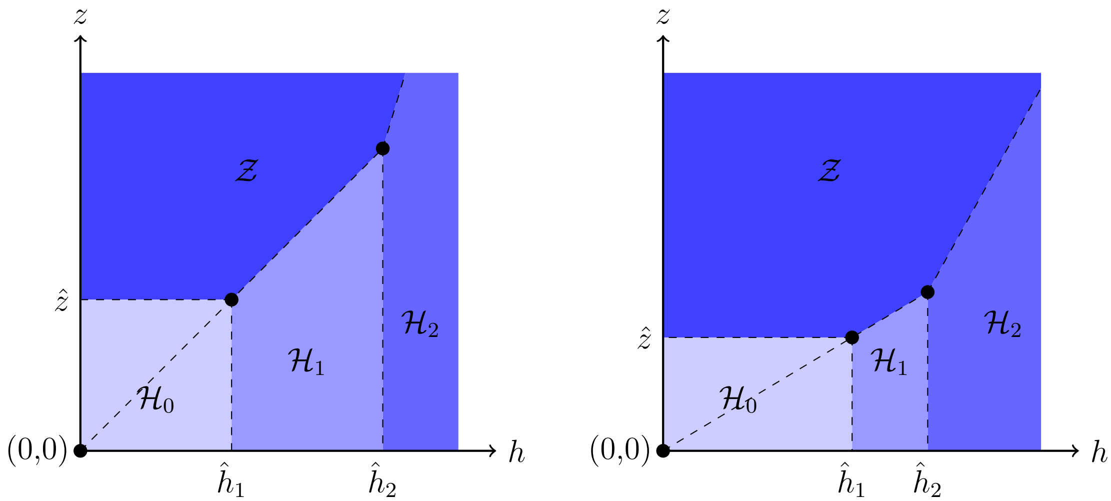
all employment shares except task 1's increase. This also implies that the average skill of task 1 workers rises.

Proof: Using the approximations from Proposition 1 and (28), we can approximate

$$\Delta_{L_{i0}} - \Delta_{L_{i1}} \approx \Delta_{\bar{h}_1}, \quad \Delta_{L_{i0}} - \Delta_{L_{i2}} \approx \Delta_{\bar{h}_2 - \chi}, \quad \Delta_{L_{i0}} - \Delta_{L_{i1}} \approx \Delta_{\bar{z}}.$$

Since $\{\Delta_z, \Delta_{h_2}\}$ <0, we know that $\{\Delta_{\bar{z}}, \Delta_{\bar{h}_2 - \chi}\}$ <0, that is, the average skill of managers, and workers in task 2, become diluted. We cannot sign $\Delta_{\bar{h}_1}$; however, under the uniform distribution assumption

$$\Delta_{L_{i0}} = \Delta_z + \Delta_{h_1} \approx \left[1 - \frac{\sigma - \omega}{(1 - \sigma)\left[\alpha + \omega(1 - \alpha)\right]} \cdot \frac{\pi_{i1}}{\Pi_{ih}}\right] \Delta_{h_1}$$



(a) Equilibrium wt low M_1

(b) Equilibrium wt high M_1

Fig. 10: Comparative Static, Within-Sector

using (30). Since π_{i1}/Π_{ih} is a fraction bounded above by 1, the condition in the lemma guarantees that $\Delta_{L_{i0}} > 0$, so

$$\Delta_{L_{i1}} < 0 < \Delta_{L_{i0}} < \{\Delta_{L_{i2}}, \Delta_{L_{iz}}\},$$

although we can still not order the last two. □

This is intuitive. If tasks were substitutes, task 1 would crowd out all other tasks, including managers. As task 1 becomes the dominant occupation, wages also increase. However, when tasks are complements, workers need to flow to the other tasks, and for this to happen relative wages must decline in task 1. Moreover, if management is more complementary with tasks than tasks are among themselves, more individuals must become managers—and in equilibrium, manager wages must increase. The within-sector comparative static is depicted in figure 10.

4.2 Structural change

Previous models of structural change either rely on a special non-homogeneous form of demand (rise in income shifting demand for service products) or relative technology differences across sectors (rise in manufacturing productivity relative to services, combined with complementarity between the two types of goods, shifting production to services). Our model is also technology driven, but transformation arises from a skill neutral increase in task productivities, or routinization. Most importantly, in contrast to recent papers arguing that sectoral productivity differences can explain the skill premia or polarization (?Buera et al., 2015), we argue exactly the opposite—that routinization can explain sectoral productivity differences and structural change.

To begin this analysis, note that from (29),

$$\Delta_{\Pi_{ih}} \equiv \frac{d\log \Pi_{ih}}{d\log M_1} = \frac{d\log \tilde{\Pi}_{ih}}{d\log M_1} \approx \frac{(1-\sigma)\left[\alpha + \omega(1-\alpha)\right]}{\sigma - \omega} \cdot \Delta_z, \tag{31}$$

and using Proposition 1, the $\Delta_{V_{ij}}$'s can be approximated from (22) as

$$\Delta_{V_{i0}} = 0, \qquad \Delta_{V_{i1}} \approx -\Delta_{\bar{h}_1} < 0 \quad \text{by Lemma 1}, \tag{32a}$$

$$\Delta_{V_{i2}} \approx -\Delta_{\bar{h}_2 - \chi} > 0, \qquad \Delta_{V_{iz}} \approx \Delta_{\bar{z}} > 0. \tag{32b}$$

So the $\Delta_{V_{ij}}$'s are sector-neutral and can be ordered as

$$\Delta_{V_1} < \{0 = \Delta_{V_0}\} < \{\Delta_{V_2}, \Delta_{V_z}\}.$$

Also note that

$$\Delta_{\Pi_{L_i}} \equiv \frac{d\log \Pi_{L_i}}{d\log M_1} = \frac{d\log \tilde{\Pi}_{L_i}}{d\log M_1} = \frac{\sum_{j=0,1,2,z} V_{ij}\Delta_{V_j}}{\tilde{\Pi}_{L_i}} = \sum_{j=0,1,2,z} \frac{L_{ij}}{L_i} \cdot \Delta_{V_j}. \tag{33}$$

Decomposing Polarization The change in the total amount of labor in each task, expressed in (23), can be decomposed similarly as in Goos et al. (2014):[12]

$$\begin{aligned} \Delta_{L_j} &= \sum_{i\in\{m,s\}} \frac{L_{ij}}{L_j} \cdot \left[\Delta_{V_j} - \Delta_{\Pi_{L_i}} + \Delta_{L_i}\right] \\ &= \sum_{i\in\{m,s\}} \frac{L_{ij}}{L_j} \cdot \Big[\underbrace{\Delta_{V_j} - \sum_{j'=0,1,2,z} \frac{L_{ij'}}{L_i} \cdot \Delta_{V_{j'}}}_{B_{ij}} + \Delta_{L_i}\Big] \quad \text{by (33)}, \quad j \in \{0,1,2,z\}. \end{aligned} \tag{34}$$

A change in the V_{ij}'s occurs even holding L_i's constant, shifting the term B_{ij}. This leads to "within-sector polarization," as we saw in the previous subsection. In particular, from (32), the Δ_{V_j}'s are sector-neutral and common across sectors. So any difference in how the share of task j employment evolves across sectors depends on the weighted average of the Δ_{V_j}'s by the employment shares of all tasks within a sector, L_{ij}/L_i.

Holding L_i's constant, we know from Lemma 1 that task 1 is shrinking and other tasks growing within-sectors. Now we can compare the $\Delta_{\Pi_{L_i}}$'s across sectors, which is the weighted average of within-sector employment shifts as seen in (33). Thus

[12] However, our decomposition differs from theirs. Their thought experiment is to separate the effects from keeping industry output fixed and when it is allowed to vary. Ours is to separate the effect from keeping sectoral employment fixed and when allowing it to vary.

Lemma 2 *The weighted average of within-sector employment share changes, $\Delta_{\Pi_{L_i}}$, is smaller in the sector with a larger within-sector employment share in task 1, and larger in the sector with larger shares in all other tasks. That is,*

$$L_{s1}/L_s < L_{m1}/L_m \quad \Rightarrow \quad \Delta_{\Pi_{L_s}} > \Delta_{\Pi_{L_m}}. \tag{35}$$

This implies that, holding sectoral employment shares constant, manufacturing polarizes more compared to services.[13]

The term Δ_{L_i} in (34) captures structural change. To compute Δ_{L_i}, plug in the expressions for the endogenous TFP's from (19) in (20):

$$\kappa = \left(\frac{\gamma_s}{\gamma_m}\right)\left[\left(\frac{1-\eta_s}{1-\eta_m}\right)^{\frac{1}{\omega-1}}\left(\frac{\nu_{s0}}{\nu_{m0}}\right)^{\frac{1}{\sigma-1}}\cdot\left(\frac{\Pi_{sh}}{\Pi_{mh}}\right)^{\frac{\omega-\sigma}{(\omega-1)(\sigma-1)}}\right.$$
$$\left.\times\left(\frac{\Pi_{K_s}}{\Pi_{K_m}}\right)^{\frac{\omega}{\omega-1}-\alpha}\left(\frac{\Pi_{L_s}}{\Pi_{L_m}}\right)^{\alpha-1}\right]^{\epsilon-1}. \tag{36}$$

Since the elasticities of Π_{ih} are sector-neutral (both change at the negative rate of $\hat{z}$), we obtain

$$\Delta_\kappa \approx (1-\epsilon)\left[\left(\alpha-\frac{\omega}{\omega-1}\right)\left(\Delta_{\Pi_{K_s}}-\Delta_{\Pi_{K_m}}\right)+(1-\alpha)\left(\Delta_{\Pi_{L_s}}-\Delta_{\Pi_{L_m}}\right)\right]. \tag{37}$$

So if $(\Delta_{\Pi_{K_i}}, \Delta_{\Pi_{L_i}})$ are larger in services, employment shifts to services; that is, routinization (a rise in M_1) leads to structural change. We have already seen that $\Delta_{\Pi_{L_i}}$ is smaller in the manufacturing when (35) holds. Now from (31),

$$\Delta_{\Pi_{K_i}} \approx \frac{\Delta_z}{\Pi_{K_i}}\cdot\left[\pi_{iz}+\Pi_{ih}\cdot\frac{(1-\sigma)[\alpha+\omega(1-\alpha)]}{\sigma-\omega}\right]<0,$$

so under the assumption in Lemma 1, $\Delta_{\Pi_{K_2}} > \Delta_{\Pi_{K_1}}$ if

$$\frac{\pi_{sz}}{\Pi_{sh}} > \frac{\pi_{mz}}{\Pi_{mh}} \quad \Leftrightarrow \quad \frac{\eta_s}{1-\eta_s}\cdot(\nu_{s0}\Pi_{sh})^{\frac{1-\omega}{\sigma-1}} > \frac{\eta_m}{1-\eta_m}\cdot(\nu_{m0}\Pi_{mh})^{\frac{1-\omega}{\sigma-1}},$$

which holds when the manager share of capital is larger in services, or $\eta_1 << \eta_2$. Hence, both because of shifts in labor *and* capital, structural change occurs toward services.

To understand why capital reallocation matters for structural change, note that we can write change in sectoral employment shares as

$$\Delta_{L_m} = -L_s\cdot\Delta_\kappa < 0, \qquad \Delta_{L_s} = L_m\cdot\Delta_\kappa > 0, \tag{38}$$

[13] Of course, the assumption in the lemma is a condition on employment shares, which are endogenous. However, the condition holds throughout our observation period in the data, so our analysis is valid. Alternatively, we could assume $\nu_{m1} >> \nu_{s1}$ and $\eta_m << \eta_s$. The astute reader would have already noticed that what the task-specific TFP's effectively do is shift the relative employment shares over time as if the parameters ν_{ij}, η_i were changing.

or plugging in Δ_κ from (37),

$$\Delta_{L_i} \approx (1-\epsilon)\Bigg[(1-\alpha)\underbrace{\Big(\Delta_{\Pi_{L_i}} - \sum_{i'} L_{i'}\Delta_{\Pi_{L_{i'}}}\Big)}_{C_{L_i}} + \left(\alpha + \frac{\omega}{1-\omega}\right)\underbrace{\Big(\Delta_{\Pi_{K_i}} - \sum_{i'} K_{i'}\Delta_{\Pi_{K_{i'}}}\Big)}_{C_{K_i}}\Bigg].$$

This makes clear that structural change in our model is due to a reallocation of both labor *and* capital, in contrast to Goos et al. (2014). The reason that capital matters in our model is because labor in our model is in skill units, which is different from employment shares. However, given sectoral capital K_i within a sector, physical capital does not affect employment shares as it is simply allocated to equalize its MRTS with the MRTS of skills across tasks; only when we let factors move across sectors does its effect appear in the model. Also note that the term C_{L_i} can be written as

$$C_{L_i} = \sum_j \frac{L_{ij}}{L_i}\cdot\Delta_{V_j} - \sum_{i'} L_i \cdot \left(\sum_{j'} \frac{L_{i'j'}}{L_{i'}}\Delta_{V_{j'}}\right),$$

which is the "between-sector" counterpart to the within-sector component B_{ij}: that is, C_{L_i} captures the average change in employment in sector i compared to the weighted average across sectors. The contribution from capital, C_{K_i}, is additional.

Of course, from (34), structural change also contributes to polarization. To see this, rewrite (34) using (38) as

$$\Delta_{L_j} = \Delta_{V_j} - \sum_{i\in\{m,s\}} \frac{L_{ij}}{L_j}\Delta_{\Pi_{L_i}} + \left[\frac{L_{sj}}{L_j}L_m - \frac{L_{mj}}{L_j}L_s\right]\Delta_\kappa \tag{39}$$

$$\Rightarrow \quad L_j\left(\Delta_{L_j} - \Delta_{V_j}\right) = -\sum_{i\in\{m,s\}} L_{ij}\Delta_{\Pi_{L_i}} + \left[\frac{L_{sj}}{L_s} - \frac{L_{mj}}{L_m}\right] L_m L_s \Delta_\kappa.$$

Thus,

Lemma 3 *Suppose lemma 2 holds. Then structural change also contributes to polarization.*

Proof: The term in the square brackets in (39) is negative for $j = 1$, and positive for all other tasks, under lemma 2. □

This is intuitive. Manufacturing has a larger within-sector employment share in task 1 (that is, if it is more routine-intense), employing more for that task. So in addition to task 1 shrinking in both sectors, if sector 1 also shrinks (structural change), there is even more polarization.

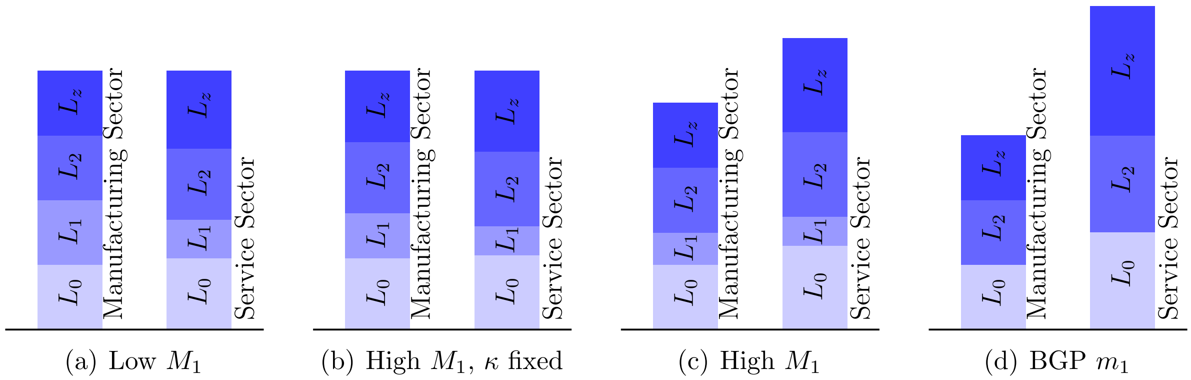


Fig. 11: Comparative Statics, Across-Sectors

Lemmas 2 and 3 are depicted in the first 3 subplots in figure 11. In figure (a), manufacturing is depicted as having a higher share in task 1, and services in task z. As we move from (a) to (b), sectoral employment shares are held fixed, and task 1 shrinks in both sectors. The change in employment shares is larger in manufacturing due to lemma 2. This leads to structural change in (c), according to lemma 3. Because manufacturing has a higher share in task 1, shrinking its size contributes to polarization.

4.3 Polarization or Structural Change?

One may argue that it is not task productivities that lead to structural change, but advances in sector-specific productivities that lead to polarization. While it is most likely in reality that both forces are in play, in the context of our model, as long as technologies are either task- or sector-specific (that is, there are no task- *and* sector-specific technologies), sector-specific productivity shifts does not lead to polarization within sectors.

To see this, consider an exogenous change in the manufacturing sector's exogenous productivity, A_m. As in Ngai and Pissarides (2007), a rise in A_m changes κ at a rate of $1-\epsilon$, that is, manufacturing shrinks. But it is easily seen that none of the thresholds change, and hence neither do the Φ_i's (the endogenous sectoral TFP's). So polarization can only arise by the reallocation of labor across sectors that use different mixes of tasks. To be precise, from (34),

$$\frac{d\log L_j}{d\log A_m} = (1-\epsilon)\cdot\frac{d\log L_j}{d\log\kappa} = (1-\epsilon)\left[\frac{L_{sj}}{L_j}L_m - \frac{L_{mj}}{L_j}L_s\right] < 0. \tag{40}$$

Note that $d\log L_j/d\log\kappa$ is equal to the term in square brackets in (39), and negative under assumption (35). Hence, polarization only occurs because manufacturing shrinks. The reason is that In our micro-founded model, tasks are aggregated up into sectoral output, not the other way around.

Equation (40) also puts a bound on how much sectoral shifts alone can account for job polarization. For example, in the data, manufacturing employment fell from approximately 33% to 19% from 1980 to 2010. If this were solely due to a change in A_m, this means that (denoting empirical values with hats):

$$\frac{d\log\hat{\kappa}}{d\log A_m} \approx 14/67 + 14/33 \approx 0.63$$

which means that

$$\frac{d\hat{L}_j}{d\log A_m} \approx 0.63\left[\hat{L}_{sj}\hat{L}_m - \hat{L}_{mj}\hat{L}_s\right] = 0.63\left[\frac{\hat{L}_{sj}}{\hat{L}_j}0.33 - \frac{\hat{L}_{mj}}{\hat{L}_j}0.67\right].$$

In Section 7, we measure the employment share of routine, manufacturing jobs and routine, service jobs (as a share of total employment; that is, L_{m1} and L_{s1}) in 1980 were 26% and 33%, respectively (refer to Table 1). So

$$\frac{d\hat{L}_j}{d\log A_m} = 0.63\left[0.33\cdot 0.33 - 0.26\cdot 0.67\right] = -0.04,$$

that is, a change in A_m alone would imply an approximately 4 percentage point drop in routine jobs from 1980 to 2010. As shown in Table 1, the actual drop was 13 percentage points.

5 Dynamics

The above result implies that on a dynamic path in which M_1 grows at a constant rate, polarization happens faster than structural change. This implies that in the limit, task 1 vanishes, structural change ceases, but both sectors still employ non-trivial amounts of labor, unlike previous models of structural change.

Such a dynamic version of the model is a straightforward extension of the neoclassical growth model. Assume that aggregate labor L grows at rate n, and a representative household with CRRA preferences

$$\int_0^\infty \exp(-\rho t)\cdot\frac{c(t)^{1-\theta}-1}{1-\theta}dt$$

where $c_t = C_t/L_t$, and a law of motion for aggregate capital

$$\dot{K}_t = Y_t - \delta K_t - C_t,$$

and for simplicity let us assume that $\dot{M}_1/M_1 = m_1$ and $M_0 = M_2 = M_z$, $\dot{M}_0/M_0 = m$. Then from (20), we can also write the aggregate production function as

$$Y_t = Y_{st}\cdot\left[\gamma_m^{\frac{1}{\epsilon}}\left(\frac{Y_{mt}}{Y_{st}}\right)^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} = \Phi_{st}\cdot L_{st}K_t^{\alpha}L_t^{1-\alpha}\cdot\left(\gamma_s^{\frac{1}{\epsilon}}L_{st}^{-1}\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$= \Phi_{st} \cdot (L_{st}/\gamma_s)^{\frac{1}{1-\epsilon}} \cdot K_t^{\alpha} L_t^{1-\alpha}$$

where Φ_{st}, L_{st}, the endogenous sectoral TFP and employment share of services at time t, are functions of $(\hat{z}_t, \hat{h}_{1t}, \hat{h}_{2t})$.

Now define

$$\Phi_t^{1-\alpha} \equiv \Phi_{st} \cdot (L_{st}/\gamma_s)^{\frac{1}{1-\epsilon}}$$

the endogenous aggregate (Harrod-neutral) TFP and its growth rate $g_t \equiv \dot{\Phi}_t/\Phi_t$. As in the RCK model, define the normalized consumption and capital per efficiency unit of labor

$$\hat{c}_t \equiv C_t/\Phi_t L_t \qquad \hat{k}_t \equiv K_t/\Phi_t L_t,$$

so output per efficiency unit of labor is

$$\hat{y}_t \equiv Y_t/\Phi_t L_t = f(\hat{k}_t) \equiv \hat{k}_t^{\alpha}.$$

The dynamic equilibrium is characterized by

$$\begin{aligned}
\dot{\hat{c}}_t &= \frac{1}{\theta} \cdot \left[f'(\hat{k}_t) - (n + \delta + \rho + g_t \theta) \right] \cdot \hat{c}_t \\
\dot{\hat{k}}_t &= f(\hat{k}_t) - (n + \delta + g_t)\hat{k}_t - \hat{c}_t \\
g_t &\equiv \frac{\dot{\Phi}_t}{\Phi_t} = g\left(\hat{h}_{1t}, \hat{h}_{2t}, \hat{z}_t\right).
\end{aligned}$$

So instead of having sectoral shares as in Acemoglu and Guerrieri (2008), we have endogenously evolving TFP which pins down the sectoral shares at every instant. Using (19) and (37), the endogenous growth rate g_t becomes

$$(1-\alpha) g_t = m + \sum_i L_{it} \cdot \left[\frac{\omega - \sigma}{(\omega - 1)(\sigma - 1)} \cdot \frac{\dot{\Pi}_{ih}}{\Pi_{ih}} + \left(\frac{\omega}{\omega - 1} - \alpha \right) \cdot \frac{\dot{\Pi}_{K_i}}{\Pi_{K_i}} + (\alpha - 1) \cdot \frac{\dot{\Pi}_{L_i}}{\Pi_{L_i}} \right].$$

On a BGP, g_t must be constant. Hence it must be that $(\hat{h}_1, \hat{h}_2, \hat{z})$ no longer evolve: Clearly this happens when $\hat{h}_1 = \hat{h}_2$, or from (12)-(13),

$$\begin{aligned}
\hat{h}_2 - \chi &= \frac{(\bar{h}_2 - \chi) L_{i2}}{L_{i0}} \cdot \left[\frac{\nu_{i0}}{\nu_{i2}} \cdot \left(\frac{\hat{h}_2 - \chi}{\bar{h}} \right)^{(1-\alpha)(1-\sigma)} \right] \\
\Rightarrow \qquad \frac{\nu_{i0} L_{i2}}{\nu_{i2} L_{i0}} &= \frac{(\hat{h}_2 - \chi)^{\sigma + \alpha(1-\sigma)}}{\bar{h}_2 - \chi},
\end{aligned}$$

assuming $\bar{h} = 1$. Then $\hat{z}$ is determined by (14), sectoral-task employment masses are determined by (Π_{K_i}, Π_{L_i}) according to (36), and on a BGP

$$g^* = \frac{m}{1-\alpha}.$$

The long-run dynamics is depicted in figure 11(d), where both polarization and structural change continue until task 1 vanishes.

6 Quantitative Analysis

The goal of our quantitative analysis is to quantify how much of the observed changes in employment and wage shares from 1980 to 2010 can be explained by task-level productivity growth, and relate such productivity growth to empiricaly measurable sources. Whenever possible, we fix parameters to their empirical counterparts, and separately estimate the aggregate technology (4) from time series data on sectoral price and output ratios. Then we choose most model parameters to fit the 1980 data exactly, inlcuding a parametric skill distribution of (h, z). The rest of the model parameters, which includes the elasticity parameters (σ, ω), are calibrated to empirical time trends from 1980 to 2010.

7 Quantitative Analysis

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7.1 Occupations and Skills

In the quantitative analysis, we assume that there are 10, rather than 3, (horizontally differentiated) worker task/occupations. (There is still only one management task.) Recall that in task 0, workers can only utilize $\bar{h}$ regardless of their own level of human capital, and in task 1 they simply use all of their human capital. Each of the additional worker tasks is characterized by a skill-loss parameter $\{\chi_j\}_{j=2}^{9}$.[14]

We assume each of these 11 occupations (10 worker + 1 manager occupation) in the model broadly correspond to the 11 one-digit occupation categories in the census, discussed in Section 2 and summarized in Table 1. The 10 worker occupation groups can further be broadly grouped into low/medium/high skill tasks, or manual/routine/abstract jobs, according to the mean wages of each occupation group and routinization indices.

[14] The characterization of the equilibrium is exactly the same as before.

Ranked by mean wage (except management)	SOC Code	Employment Shares 1980	Employment Shares 2010	Employment Shares Manufacturing		Total Wage Shares 1980	Total Wage Shares 2010	Total Wage Shares Manufacturing	
Low Skill Services	400	**10.44**	**13.92**	**0.59**	**0.23**	**6.75**	**7.60**	**0.52**	**0.16**
Middle Skill		**59.09**	**46.48**	**25.86**	**12.93**	**53.43**	**35.90**	**24.76**	**10.02**
Administrative Support	300	16.57	14.13	3.47	1.53	12.90	9.60	2.90	1.15
Machine Operators	700	9.81	3.75	8.79	3.02	8.21	2.39	7.37	1.91
Transportation	800	8.73	6.64	3.80	2.28	7.73	4.15	3.37	1.46
Sales	240	7.87	9.37	0.79	0.62	7.40	8.45	1.06	0.85
Technicians	200	3.23	3.86	1.00	0.57	3.35	4.33	1.13	0.66
Mechanics & Construction	500	7.91	6.02	4.44	3.19	8.40	4.88	4.91	2.61
Miners & Precision Workers	600	4.97	2.71	3.58	1.73	5.43	2.10	4.03	1.38
High Skill		**19.22**	**26.16**	**3.87**	**3.64**	**24.20**	**33.98**	**6.07**	**5.51**
Professionals	40	11.02	16.51	1.73	1.45	13.36	20.78	2.59	2.12
Management Support	20	8.20	9.65	2.14	2.20	10.84	13.20	3.48	3.39
Management	1	**11.26**	**13.44**	**2.47**	**2.59**	**15.62**	**22.52**	**4.22**	**5.81**

Table 1: Occupation Groups used for Calibration

Source: US Decennial Census (1%), 1980-2010.

In Table 1, the left panel shows the SOC of each occupation with a short job description, and the middle and right panels their employment and total wage shares in 1980 and 2010, respectively. For the employment and wage share panels, the first two columns shows the size of each occupation in 1980 and 2010 as a fraction of total employment/wages. The next two columns show the size of each occupation within manufacturing as a fraction of total employment/wages.

These were already depicted graphically in Section 2, and will be the bulk of our target moments in the calibration. The only other target moment is the growth rate of aggregate output.

Parametric Skill Distribution For the quantitative analysis, we assume a parametric skill distribution that is type IV bivariate Pareto (Arnold, 2014). Specifically, the c.d.f. we assume is

$$\mu(h, z) = 1 - \left[1 + h^{1/\gamma_h} + z^{1/\gamma_z}\right]^{-a}.$$

We normalize $\gamma_z = 1$, since we cannot separately identify both skills from the skill-specific TFP's. This is consistent with an establishment size distribution that is Pareto, and a wage distribution that is hump-shaped with a thin tail, as depicted in Figure 12.

7.2 Aggregate Production Function

The aggregate production function is estimated outside of the model. For the estimation, we only look at the manufacturing and service sectors, where manufacturing includes mining and construction, and government is included in services. We estimate

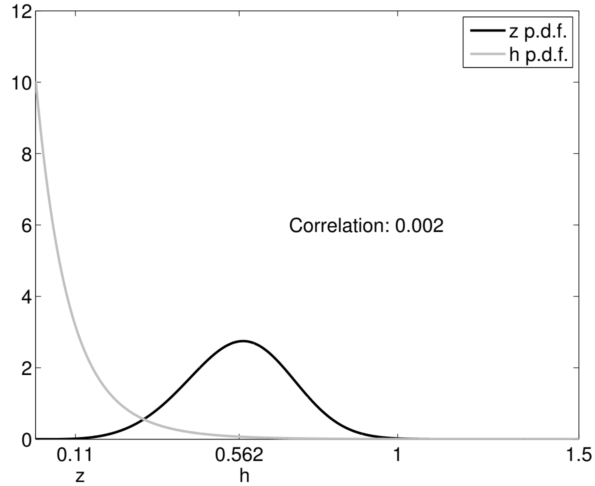


Fig. 12: Calibrated Skill Distribution

We use a type IV bivariate Pareto distribution to model the distribution over worker and manager skills (h, z). The figure depicts the marginal distributions each skill, and also their mean values below the x-axis. The implied Pearson correlation coefficient between the two skills is low, at 0.002.

the parameters (γ_m, ϵ) from the system of equations

$$\begin{aligned}\log\left(\frac{p_m Y_m}{PY}\right) &= \log\gamma_m + (1-\epsilon)\log p_m - \log\left[\gamma_m p_m^{1-\epsilon} + \gamma_s p_s^{1-\epsilon}\right] + u_1 \\ \log(Y) &= c + \frac{\epsilon}{\epsilon-1}\log\left[\gamma_m^{\frac{1}{\epsilon}} Y_m^{\frac{\epsilon-1}{\epsilon}} + \gamma_s^{\frac{1}{\epsilon}} Y_s^{\frac{\epsilon-1}{\epsilon}}\right] + u_2\end{aligned}$$

where $\gamma_s \equiv 1-\gamma_m$, using non-linear SUR (seemingly unrelated regression), on all years of real and nominal sectoral output observed in the BEA accounts.

Real production by sector is computed by a cyclical expansion procedure as in (Herrendorf et al., 2014) using production value-added to merge lower level industries (as opposed to consumption value-added, as in their analysis).[15] The price indices are implied from nominal versus real sectoral quantities. We try different base years as well: 1947, 1980, and 2005, corresponding to columns (1)-(3) in Table 2. This is to check the robustness of the choice of base years: 1947 is the first year the required data is available, 1980 is the first year in our model, and 2005 is chosen as a year close to present but before the crisis.

As shown there, the values are in a similar range as in Herrendorf et al. (2014), although ϵ is not significant with 2005 as a base year. For the calibration, we will use take the values of (γ_m, ϵ) in column (1) values as a benchmark.

The capital income share α is computed as 1-(labor income/total income), and fixed at 0.360. Since we do not model investment, for the calibration we also need the level of total capital stock (for manufacturing and services) for each decade, which we

[15] The constant c is included since it is not levels, but relative changes that identify ϵ.

	(1)	(2)	(3)
γ_m	0.371^{**}	0.346^{**}	0.258^{**}
	(0.003)	(0.005)	(0.004)
ϵ	0.003^{**}	0.002^{**}	0.003
	(0.000)	(0.000)	(0.004)
AIC	-550.175	-551.264	-550.866
RMSE_1	0.106	0.106	0.106
RMSE_2	0.039	0.039	0.039

Standard errors in parentheses
† $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Table 2: Aggregate Production Function

The manufacturing share parameter γ_m and elasticity parameter between manufacturing and services, ϵ, are estimated off the time series of output and price ratios from 1947 to 2013, which are available from the BEA accounts. The service share parameter γ_s is assumed to equal $1 - \gamma_m$. For details of the estimation, we closely follow Herrendorf et al. (2014).

take from the Fixed Assets Table and directly plugged into the model. Since we do not model population growth, in practice we we normalize output per worker in 1980, y_{1980} to unity, and plug in $K_t = k_t/y_{1980}$ for $t \in \{1980, 1990, 2000, 2010\}$, where k_t is capital per worker in year t.

7.3 Calibration

Given the aggregate production function, we simulate two equilibria: one each for 1980 and 2010. Except for capital per worker, which we take exogenously from the data, any change between 1980 to 2010 is only due to exogenous task-TFP growth.

Most of the parameters are calibrated so that our our model equilibrium fits the 1980 data moments in Table 1 exactly. We then calibrate the elasticity parameters (σ, ω) and the 11 constant TFP growth rates $\{m_j\}_{j=z,0}^{9}$ to fit the time trends in aggregate output and employment shares from 1980 to 2010.

All model parameters are summarized in Table 3, except for the skill loss parameters χ_j, task intensity parameters (η_i, ν_{ij}), and task-TFP growth rates m_j, which are tabulated in Table 4.

Below, we explain in detail how these parameters are recovered.

Calibrating the distribution For any guess of $(\gamma_h, a, \{\chi_j\}_{j=2}^{9})$, we can find $(\hat{z}, \{\hat{h}_j\}_{j=1}^{9})$ that exactly match observed employment shares by occupation in 1980, by integrating over the guessed skill distribution. Given the thresholds, we then compute

	Parameter	Value	Target
Fixed from data	K_{1980}	2.895	Computed from BEA/NIPA data
	K_{2010}	4.235	
	α	0.361	
	γ	0.371	Estimated in section 7.2
	ϵ	0.003	
Fit to 1980	$M_j \equiv M$	0.985	Output per worker, normalization
	A_m	1.112	Manufacturing employment share
	A_s	1.000	Normalization
	ν_{ij} (18)	Table 4	Witin-sector employment shares by occupation
	$\eta_i(2)$		Within-sector manager share
	χ_j (8)	Table 4	Relative wages by occupation
	a	10.087	
	γ_h	0.216	
	γ_z	1.000	Normalizations;
	$\bar{h}$	1.000	Not separately identified from M_j
Fit to 2010	σ	0.704	Witin-sector employment shares by occupation
	ω	0.341	
	m_j	Table 4	Output per worker growth and employment shares by occupation

Table 3: Parameters

All parameters valued 1 are normalizations.

the model-implied relative wages using (27),

$$\frac{w_1\bar{h}_1}{w_0\bar{h}} = \frac{\bar{h}_1}{\hat{h}_1}, \quad \frac{w_2(\bar{h}_2 - \chi_2)}{w_1\bar{h}_1} = \frac{\bar{h}_2 - \chi_2}{\bar{h}_1(1 - \chi_2/\hat{h}_2)}, \quad \frac{w_z\bar{z}}{w_0\bar{h}} = \frac{\bar{z}}{\hat{z}},$$

and similarly for $j \in \{3, \dots 9\}$. The LHS is the ratio of mean wages by occupation, which we observe from the data. The RHS is a function only of the thresholds, which themselves are functions of (γ_h, a, χ_j). Hence, we iterate over $(\gamma_h, a, , \{\chi_j\}_{j=2}^{9})$ so that the model-implied ratios match observed mean wage ratios exactly.

For the rest of the calibration, we fix these three parameters that govern the skill distribution. By construction, we already know the thresholds $(\hat{z}, \{\hat{h}_j\}_{j=1}^{9})$ that fit 1980 employment shares by occupation exactly. And since the skill distribution is fixed by the data, we can similarly compute the thresholds that fit 2010 employment shares by occupation. Denote these two sets of thresholds as $\mathbf{x}_{1980}$ and $\mathbf{x}_{2010}$, respectively. Note that these thresholds are determined solely by the exogenously assumed skill distribution and the data, independently of our model equilibria.

Ranked by mean wage (except management)	χ_j	Emp Wgts (ν_{ij}, η_i) Manu.	Serv.	m_j	RTI
Low Skill Services	**-**	**0.016**	**0.136**	**-0.731**	**-0.211**
Middle Skill		**0.816**	**0.524**		
Administrative Support	-	0.088	0.173	2.930	2.445
Machine Operators	0.001	0.256	0.015	9.122	0.602
Transportation	0.002	0.119	0.081	4.348	-0.576
Sales	0.003	0.026	0.123	0.012	0.483
Technicians	0.005	0.034	0.040	-1.144	-0.269
Mechanics & Construction	0.006	0.159	0.065	2.315	-0.630
Miners & Precision Workers	0.007	0.134	0.027	6.328	0.639
High Skill		**0.168**	**0.340**		
Professionals	0.009	0.070	0.195	-2.248	-0.725
Management Support	0.010	0.098	0.146	-0.489	-0.655
Management	**-**	**0.076**	**0.130**	**-0.017**	**-1.103**

Table 4: Calibrated Employment Weights and Growth Rates

Normalizations Before we simulate the model equilibrium and calibrated the remaining parameters, some normalizations are in order. We have already normalized $\gamma_z = 1$. For notational convenience, we will denote the 1980 levels of the TFP's by (M, A_i) and denote their 2010 levels by multiplying them by their respective growth rates. For example, the manager-task TFP in 2010 is $M(1+m_z)^{30}$.

1. We normalize $\bar{h} = 1$ since it is not separately identified from M_0. This can be seen in (13).
2. We also normalize $M_j \equiv M$ for all $j \in \{0, 1, \ldots, z\}$ for 1980, since it is not separately identified from (η_i, ν_{ij}) in a static equilibrium. This is implied by the production technology we assume in (6)-(7).

The rest of the parameters are calibrated so that simulated moments from the model's 1980 and 2010 equilibria match the data.

Calibrated within the model There are 35 parameters that remain to be calibrated: the elasticity parameters (σ, ω), TFP parameters (M, A_m), task intensities $(\eta_i, \{\nu_{ij}\}_{j=1}^{9})$ for $i \in \{m, s\}$, and the task-TFP growth rates $\{m_j\}_{j=z,0}^{9}$.[16]

Given the aggregate production function, skill distribution, normalizations and thresholds $(\mathbf{x}_{1980}$, we can recover the remaining parameters as follows.

(A) Guess (σ, ω).

[16] There are only 9 intensity parameters to calibrate per sector, since we assume $\sum_{j=0}^{9} \nu_{ij} = 1$.

(B) Given the guess, first fit 1980 moments:

  (a) Guess (M, A_m).

  (b) Plug in the threshold values $\mathbf{x}_{1980}$, and the empirical values of $(L_{iz}, L_{i0}, \ldots, L_{i9})$—the employment shares of each occupation in sector $i \in \{m, s\}$—from Table 1, into (12) and (14). Then we recover all the ν_{ij}'s from (12)-(13), and the η_i's from (14)-(15) in closed form (since all M_j's are assumed to be equal). This ensures that the 1980 equilibrium exactly fits within-sector employment shares by occupation (20 parameters, 20 moments).

  (c) Repeat from (a) until we exactly fit the manufacturing employment share in 1980, and output per worker of 1.[17] Since (19)-(20) are monotone in (M, A_m), the solution is unique (2 parameters, 2 moments).

(C) Given the parameters recovered from the 1980 equilibrium, plug in threshold values $\mathbf{x}_{2010}$ into (24). Then find $\{m_j\}_{j=z,0}^{9}$ so that the 2010 equilibrium exactly fits employment shares by occupation (but not necessarily by sector), and also output per worker, in 2010 (11 parameters, 11 moments).

(D) Repeat from (1) to minimize the distance between the within-sector employment shares by occupation implied by the 2010 model equilibrium and the data (2 parameters, 11 moments).

Note that since all other moments are fit exactly, in essence we are only calibrating the two parameters (σ, ω) in (A) to match the 11 moments in (D).[18]

The resulting parameters are tabulated in Tables 3-4. The last column of Table 4 shows the empirical RTI indices constructed from Autor and Dorn (2013), which are also visualized in Figure 19.

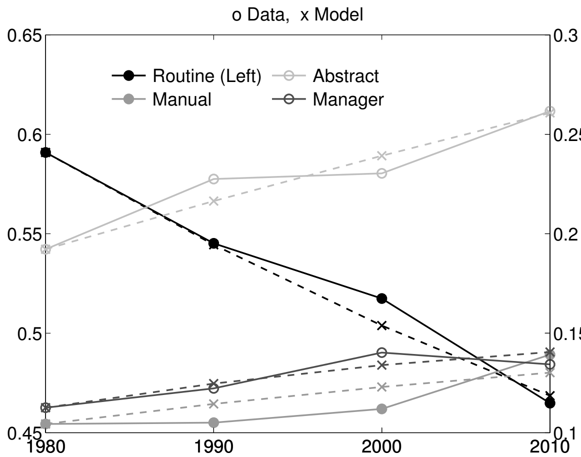
7.4 Model Fit

Figure 13 plots the model implied trends in employment shares across tasks, in aggregate and by sector, against the data. When computing the simulated paths for 1990 and 2000, we plug in the empirical values of $K_t = k_t/y_{1980}$ and the level of task-TFP's implied by the calibrated growth rates, and compute the respective equilbria allocations.

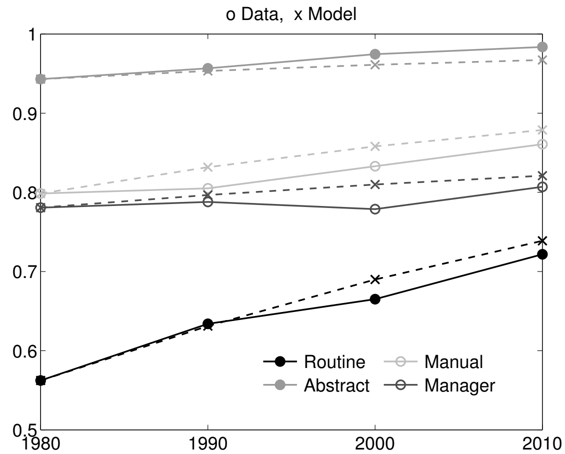
As we explained above, the aggregate trend can be solved in closed form using the same number of parameter and moments, so it is not surprising that we obtain a more or

[17] The latter must be matched since the value of K_{1980} we plug in from the data was normalized by 1980's output.

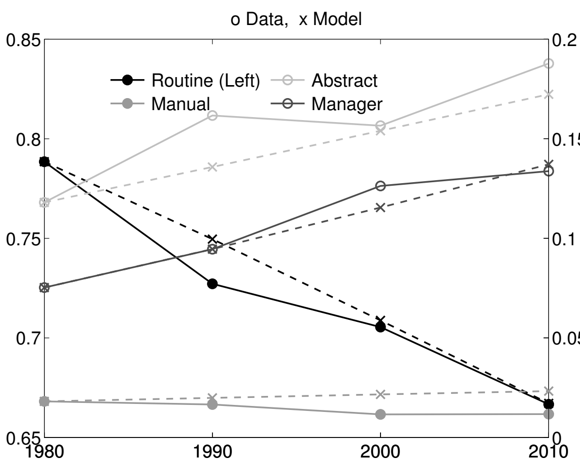
[18] When targeting 2010 employment shares and output per capita, we in fact target the linear trend from 1980 to 2010 rather than their exact values. However, since most trends are in fact linear, using the exact values barely change any of our results.



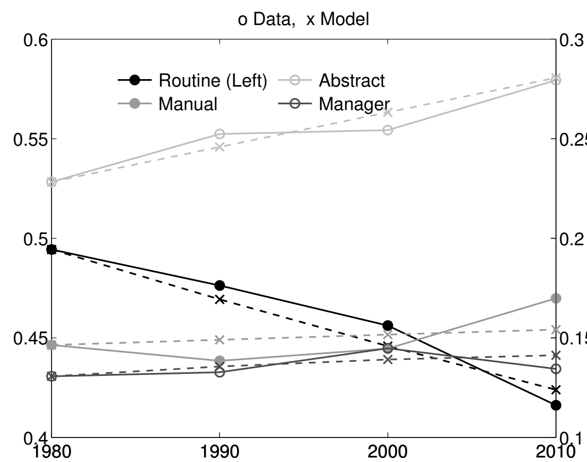
(a) Aggregate



(b) Services Share By Task



(c) Manufacturing



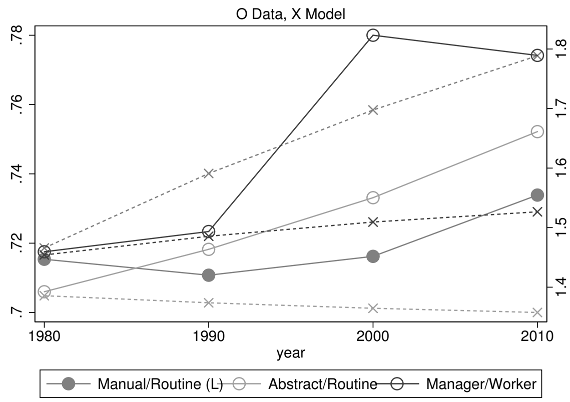
(d) Services

Fig. 13: Data vs. Model, Employment Shares by Task

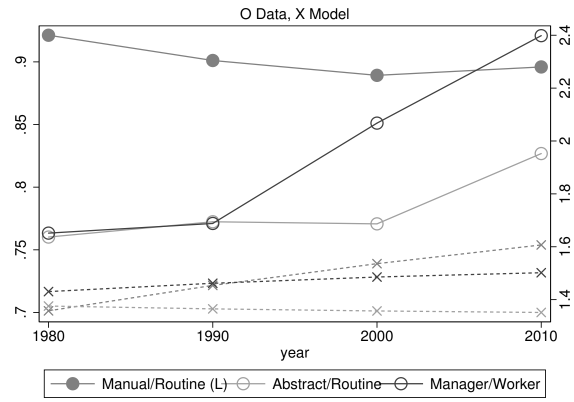
less exactly fit as seen in Figure 13(a). On the other hand, while we target the starting points for all the shares (services employment share, and within-sector employment shares by task), these 11 trends were calibrated using only the 2 elasticity parameters (σ, ω). Nonetheless, the calibrated model more or less exactly replicates structural change by occupation, as seen in Figure 13(b) and also within-sector polarization, as seen in Figures 13(c)-(d).

The fit is not as satisfactory for relative wages, plotted in figure 14. Here we plot the mean relative wages in aggregate and by sector. Here, the only targeted moments were the 1980 average wage ratios in Figure 14(a); all other moments were not. Manual and abstract wages are relative to routine jobs, and manager wages are relative to all workers.

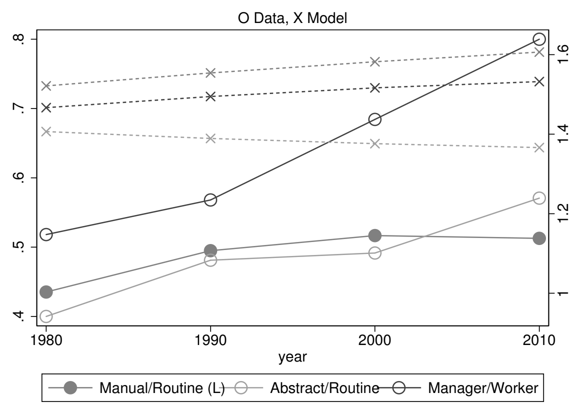
Recall that in the model, efficiency wages $(w_z, w_1, \ldots, w_9)$ are equal since we assume



(a) Aggregate



(b) Manufacturing



(c) Services

Fig. 14: Data vs. Model, Relative Wages by Task

that individuals are indifferent across sectors, and we constrained our attention to equilibria in which mean skill levels within occupations were equal across sectors. Hence, within relative wages are equal across sectors for any given occupation; the only reason they differ in Figures 14(b)-(c) is because we aggregate sub-occupation groups 3 broader categories; and for managers since they are compared against all workers. Compared to the data, all model-implied relative wages are too low in manufacturing, and too high in services. Hence to some degree, our model is missing something that causes relative wages to be lower in services.

Some explanation is in order. Due to the multiple layers of complementary between occupations and sectors ($[\omega, \sigma, \epsilon] < 1$), the calibrated growth rates m_j are smaller for those occupations that are growing, as shown in Table 4. Then for all occupations in the middle ($j = 1, \ldots, 9$), whether their relative wages grow or shrink depends on the magnitude of negative selection (that comes from having more or less low-skill workers from the left-side of the h-skill distribution) and positive selection (that comes from having more or less high-skill workers from the right-side of the h-skill distribution), since whether the employment share grows or shrinks, it will either absorb or lose employment from both sides of the distribution. This was proven in Proposition 1 and in Figure 10, can be seen from $\hat{h}_1$ increasing and $\hat{h}_2$ decreasing following a rise in the TFP among routine jobs.

For manual jobs, average skill is assumed to be constant at $\bar{h}$, and it is only those workers with the lowest h skill that work in this job. It turns out that routine jobs as a whole display enough negative selection so that the wages of manual workers relative to routine workers rise, although only slightly. This is in fact consistent with the data in aggregate and in the services sector, although in the manufacturing sector, manual wages slightly dropped.

We also proved in Proposition 1 that as long as $\omega < \sigma$, which it is (Table 3), managers' relative wages would rise relative to workers as long as their task-TFP grew slower than workers'. However, the quantitative magnitude of this rise is small. This is because as the employment share of managers grow, there is a negative selection along z-skills. In Figure 10, this can be seen from $\hat{z}$ decreasing to increase the mass of managers.

According to our model assumptions, all workers with the highest h-skill work in the highest-skill worker occupation ("professionals" in the data). Since their task-specific TFP grew relatively less, the average skill of workers in this occupation becomes lower, since employment growth leads to negative selection. Consequently, both because of lower TFP growth and negative selection, relative wages decline for abstract jobs, in contrast to the data.

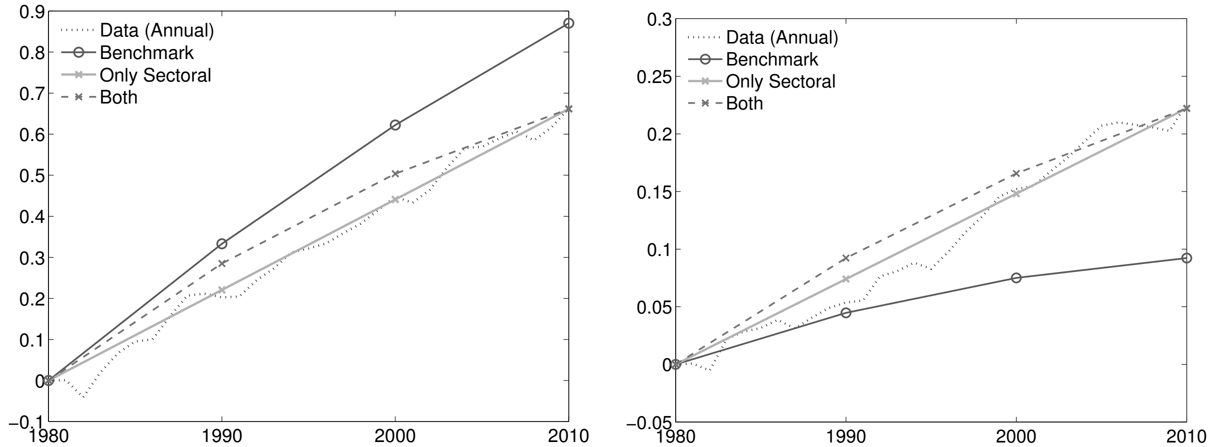
Overall, the model targeted only to aggregate moments delivers a good fit by task even within and across sectors in terms of employment shares, but not in terms of relative wages. In what follows, we focus only on employment shares and investigate how much each of these trends is explained by task-specific TFP's, and its implications for other outcomes such as sectoral TFP's.

8 Results

8.1 Counterfactuals

In this subsection, we analyze the role of task-specific TFP's on structural change, , we conduct two counterfactuals:

(1) First, we set all task-specific TFP growth to be equal, that is, we set $m_j = m$, and instead let both (A_m, A_s) change at rates (a_m, a_s). We jointly recalibrate



(a) Log TFP per Worker, Manufacturing (b) Log TFP per Worker, Services

Fig. 15: Benchmark vs. Counterfactuals, TFP

1980 levels are normalized to 0, so the slope of the lines are the growth rates.

(m, a_m, a_s) to match the empirical growth rate of TFP (i.e., the Solow residuals) in aggregate, and also in manufacturing and services, from 1980 to 2010. This yields the model's predictions in the absence of any exogenous, task-specific TFP growth.

(2) Second, we allow both exogenous task- and sector-specific TFP growth, and re-calibrate $(\{m_j\}_{j=z,0}^{9}, a_m, a_s)$ to match the change in employment shares *and* the empirical growth rates of TFP in aggregate, manufacturing and services, from 1980 to 2010. This gives the model the best chance to explain the data.

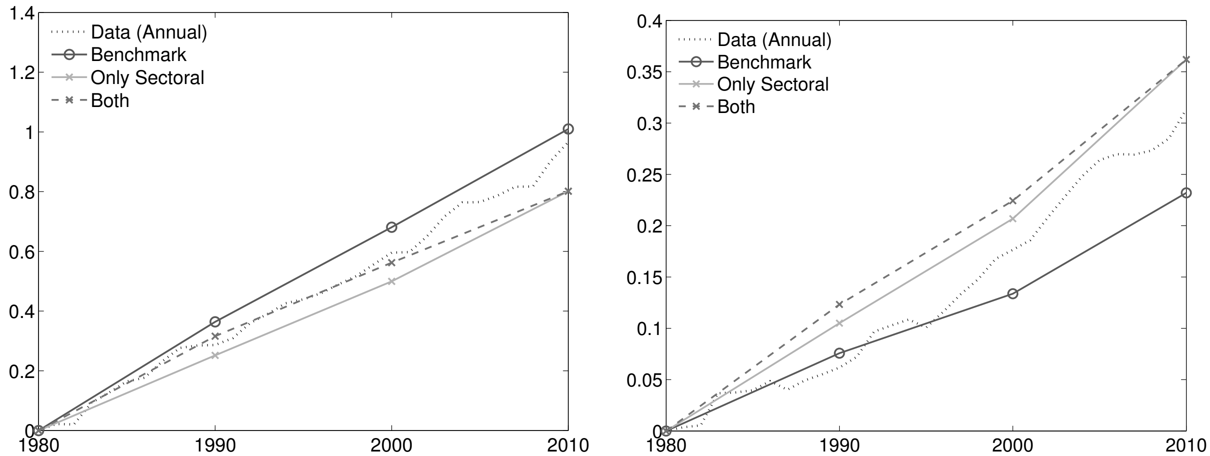
For both counterfactuals, we keep all other parameters at their benchmark values in Table 3-4, and only recalibrate the growth rates.

We focus on sectoral TFP's since in our model, structural change only results from the differential TFP growth across sectors—expressed in closed form in (19)—whether it is exogenous (caused by a_m and/or a_s) or endogenous (as in Section 4.2). The recalibrated parameters for the counterfactual scenarios are summarized in Appendix Table 6.

TFP and output growth In Figure 15, we compare the path of log sectoral TFP in the data, in our benchmark calibration, and two counterfactual scenarios. All scenarios match aggregate TFP and GDP growth from 1980-2010 in the calibration as shown in Appendix Figure 22, so we do not discuss them here.[19]

In our benchmark, we over shoot the growth rate of manufacturing TFP by about

[19] Denote aggregate TFP as Z_t. Since $Y_t = Z_t K_t^{\alpha}$ (labor is normalized to one), and we plug in the empirical values of K_t for all calibrations, matching aggregate TFP and GDP are the same things.



(a) Log GDP per Worker, Manufacturing

(b) Log GDP per Worker, Services

Fig. 16: Benchmark vs. Counterfactuals, GDP per Worker

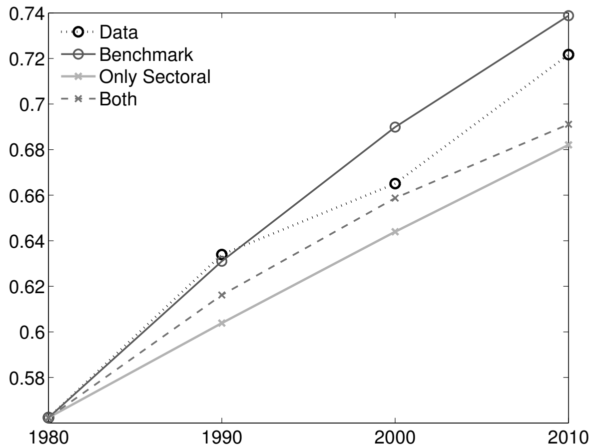
1980 levels are normalized to 0, so the slope of the lines are the growth rates.

half a percentage point, while undershooting the services TFP growth rate by about half a percentage point. However, note that when we look at the growth rates of sectoral output, in Figure 16, this gap almost disappears. This is because while the model assumes that the sectoral capital input shares are equal to labor input shares, as shown in (20) earlier, in the data they are not. In fact, counterfactuals (1) and (2) in Figure 16 show that when sectoral TFP growth is matched exactly, manufacturing output grows more slowly, and services output more quickly, compared to the data. This implies that capital input ratio between manufacturing and services grew slower than the labor input ratio, although the differences are small.[20]

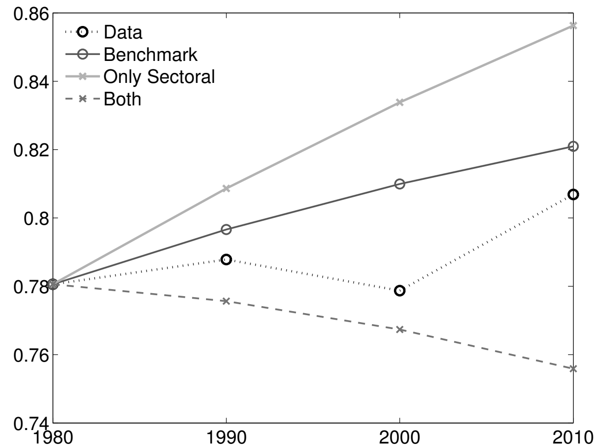
Structural Change and Polarization Since structural change is solely determined by output ratios (Section 4.2, equation (20)), the fact that sectoral output growth nearly tracks the data implies that the the benchmark model more or less fully explains structural change (in terms of employment), as shown in Figure 17(c). Both counterfactuals (1) and (2) undershoot the full extent of structural change, since sectoral output growth is too low and high in manufacturing and services, respectively. Moreover, when we look at structural change within occupation categories, the benchmark model outperforms both counterfactuals (1) and (2), especially among managers.

Lastly, we investigate whether exogenous growth in sectoral TFP's can explain polarization, as we also discussed in Section 4.3. In Figure 18, we see that sectoral forces alone can account for about 15-20 percent of horizontal and vertical polarization.

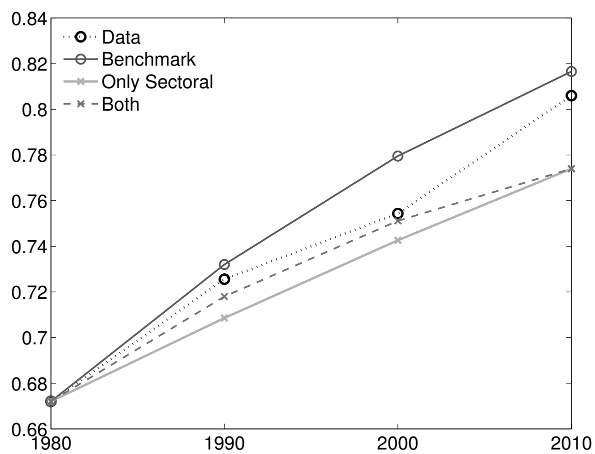
[20]If sectoral production functions remain Cobb-Douglas, this means that capital intensity is higher in manufacturing, as analyzed in Acemoglu and Guerrieri (2008).



(a) Services Employment Share, Routine Jobs



(b) Services Employment Share, Managers

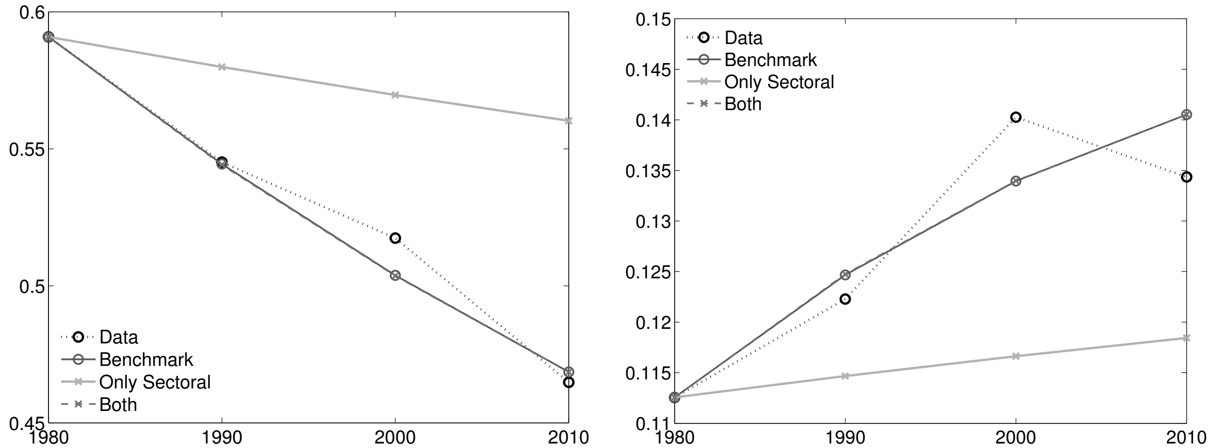


(c) Services Employment Share, All Jobs

Fig. 17: Benchmark vs. Counterfactuals, Structural Change

However, remember that this would not cause any changes in *within* sector employment shares by occupation.

In sum, task-specific TFP growth can more or less fully account for sectoral output growth, and consequently for the observed levle of structural change from 1980 to 2010. Due to the vertical and horizontal polarization induced by changes in task-specific TFP's, employment shifts to the sector that uses the routine task less and management more intensively. Conversely, sector-specific productivities can only account for 15-20 percent of polarization, and furthermore we have shown, both analytically and quantitatively, that it does not cause any polarization within sectors, that contrary to the data.



(a) Routine Employment Share

(b) Manager Employment Share

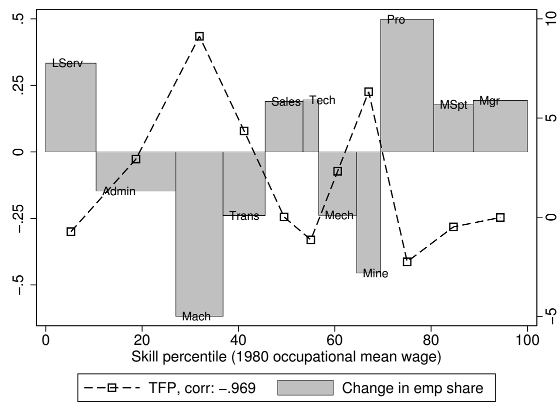
Fig. 18: Benchmark vs. Counterfactuals, Polarization

8.2 What are Task-Specific Productivities?

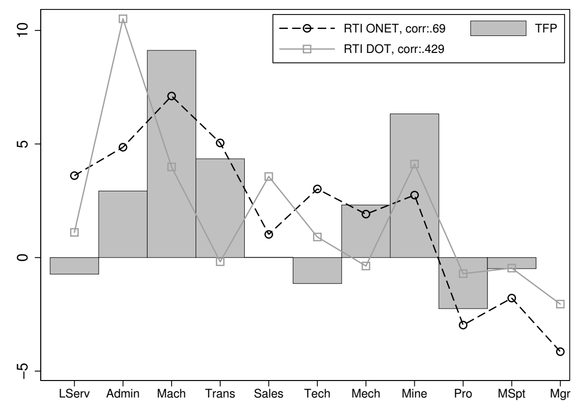
Despite having skill selection, horizontally and vertically differentiated jobs, and multiple sectors, Figure 19(a) shows that the bulk of the changes in occupational employment shares are still directly accounted for by task-specific TFP's, with a (negative) correlation of 0.97. This is also confirmed from a simple regression analysis shown in Appendix Table 7. This leads us to conclude that in order to understand changes in the employment structure, it is important to identify what these task-specific TFP's are.

How much of the variation in the TFP growth rates can be explained by routinization? As a first pass, we correlate the TFP growth rates with the RTI measure used in Autor and Dorn (2013), which itself is constructed from Autor et al. (2003) using the DOT, and the RTI measure from Acemoglu and Autor (2011), which was constructed from O*NET. While the TFP growth rates are positively correlated with both indices, and more strongly with the latter, it is visually clear that there is much left to be explained. More precisely, both the correlation and R^2's are still quite low, as can be seen in Appendix Table 7.

What about college? Skill-biased technological change (SBTC) has been a usual suspect for changes in the employment structure since since Katz and Murphy (1992), e.g. Krusell et al. (2000); Buera et al. (2015). In the SBTC literature, "skill" is usually a stand-in for whether or not an individual went to college, or obtained a college degree. However, as is evident from Figure 20(a), neither the fraction of college graduates within each occupation in 1980, nor the change in the fraction of graduates from 1980 to 2010, have much of a relationship with the task-specific TFP's calibrated from our

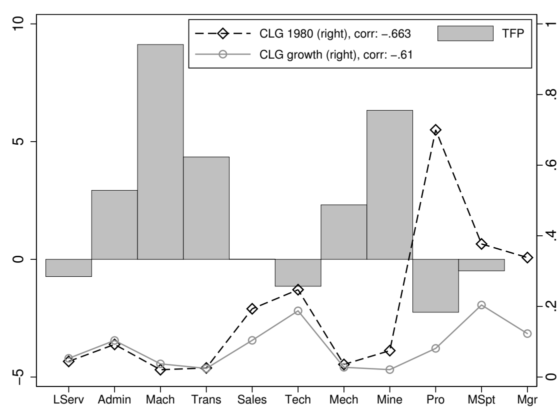


(a) Task-specific TFP growth and Employment Shares

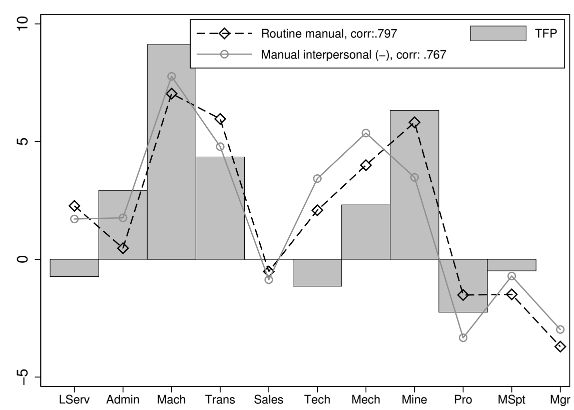


(b) Task-specific TFP growth and RTI

Fig. 19: Employment Shares, Task TFP growth and the Routinization Index



(a) Task-specific TFP Growth and College



(b) Task-specific TFP Growth and O*NET

Fig. 20: Task TFP growth, College Shares and O*NET-based indices

model. Since the TFP's more or less entirely explain the employment shifts observed in the data, this means college can not explain occupational employment shifts. Moreover, as is clearer in Appendix Table 7, the correlation between employment shifts and college measures are *negative*, that is, those occupations with more college graduates, or in which the college graduate share grew the fastest, in fact *shrank*. This is the opposite of most of propositions made in the SBTC literature.

What we do find, however, is that the TFP growth rates correlate strongly with disaggregated components of the RTI index in O*NET, in particular the routine-manual and interpersonal skills indices. Appendix Table 7 shows that the R^2 for both are also high. This means that those occupations with a higher share of routine-manual tasks have shrunk, while those with higher share of interpersonal tasks have grown.

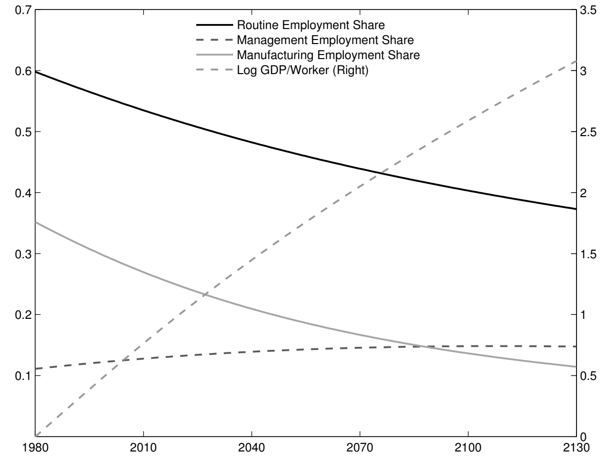


Fig. 21: Long-run Dynamics

While we conclude that productivity growth has been high in routine-manual tasks and low among interpersonal tasks, and that this can explain a significant part of shifts in occupational employment, polarization, and consequently structural change, it is evident from the regressions that this is not the end of the story. The unexplained part of task-specific TFP growth may also come from endogenous changes in the distribution of skill, and in an open economy setting from off-shoring, both of which we have abstracted from.[21]

8.3 Long-Run Growth Path

Lastly, we show the long-run dynamics of the model from Section 5, extended to the 10 horizontally-differentiated tasks we have analyzed thus far. Assuming a CRRA coefficient of $\theta = 2$, and that the economy starts in 1980, we target an asymptotic interest rate of 2%, implying an approximately equal discount rate ρ. The depreciation rate is set to $\delta = 0.065$, as computed from the NIPA accounts. As can be seen, both routine and manufacturing continue to decline, but the speed of the decline in manufacturing slows down as routine jobs continue to disappear. Likewise, managerial employment continues to rise, albeit at a slower pace. Finally, note that the first 150 years displays structural change and near balanced growth, consistently with the Kuznets and Kaldor facts.

[21] In an open economy setting, cheaper foreign labor would be observationally equivalent to higher productivity.

9 Conclusion

We presented a new model which encompasses job polarization, structural change, and a modified span of control technology. We showed analytically and the quantitatively that the model can be a useful tool for analyzing macroeconomic dynamics.

Appendices

A Census Employment/Wages/Occupations

Occupation Group	`occ1990dd`
Managers	4– 19
Management Support	22– 37
Professionals	43–199
Technicians	203–235
Sales	243–283
Administrative Support	303–389
Low Skill Services	405–472
Mechanics and Construction Workers	503–599
Miners and Precision Workers	614–699
Machine Operators	703–799
Transportation Workers	803–899

Table 5: Census Occupation Groups

322 non-farm occupations according `occ1990dd` (Dorn, 2009), itself harmonized from `occ1990` (Meyer and Osborne, 2005), are grouped into 11 occupation groups in order of their `occ1990dd` code. Except for management support, technicians and sales, all occupation groups correspond to their 1-digit census occupation group. Groups are presented in their (contiguous), ascending order of their codes, excluding agricultural occupations 473–498 which are dropped. In the main text, occupation groups are presented in ascending order of skill (mean hourly wage).

We use the 5% census samples from IPUMS USA. We drop military, unpaid family workers, and individuals who were in correctional or mental facilities. We also drop workers who work either in an agricultural occupation or industry.

For each individual, (annual) employment is defined as the product of weeks worked times usual weekly hours, weighted by census sampling weights. Missing usual weekly hours are imputed by hours worked last week when possible. Missing observations are imputed from workers in the same year-occupation-education cell with 328 occupations×6 hierarchical education categories: less than high school, some high school, high school, some college, college, and more than college.

Hourly wages are computed as annual labor income divided by annual employment at the individual level. Hence while employment shares include the self-employed, hourly wages do not include self-employment income.[22] We correct for top-coded

[22] While we have only considered labor income in the paper, we have conducted robustness checks by including business income as well. Hourly business income is defined similarly as hourly wages. We also separately corrected for top-coding (the top-codes for labor and business income differ) and bottom-coded in a similar fashion.

incomes by multiplying them by 1.5, and hourly wages are set to not exceed this value divided by 50 weeks × 35 hours (full-time, full-year work). Low incomes are bottom-coded to first percentile of each year's wage distribution.

For the line graphs in Figures 2–4, we ranked occupations by their hourly wages defined as above, and smoothed across skill percentiles using a bandwidth of 0.75 for employment and 0.4 for wages; these are the same values used in Autor and Dorn (2013). For the bar graphs in Figures 2–4 and 19–20, we grouped the 322 occupations vaguely up to their 1-digit census occupation codes, resulting with the following 11 categories summarized in Table 5 and used for our quantitative analysis. In the figures and in Tables 1–6, these groups are then ranked by the mean wage of the entire group. In particular in Figures 2–4 and 19(a), the horizontal length of a bar is set to equal the corresponding group's 1980 employment share, which does not necessarily coincide with the 3-digit occupations used to generate the smooth graphs by percentile.

B Proofs

B.1 Proof of Theorem 1

We first show that for fixed $[q_h(j), q_z] \in (0,1)$, the within-sector equilibrium is unique. For an arbitrary guess of $\hat{z}(j)$, Assumptions **??**-3 imply existence of a solution to the differential equation (**??**) by Picard-Lindelöf's existence theorem. Similarly, a solution to (**??**) exists by Brouwer's fixed point theorem once we apply a minimum value for $\hat{z} \geq \underline{z} > 0$ such that the denominator does not converge to zero.

To show that the solution is unique, we first prove the following Lemma:

Lemma 4 *Suppose $[q_{ih}(j), q_{iz}]$ are fixed and that $[\hat{h}(j), \hat{z}]$ and $[\hat{h}^1(j), \hat{z}^1]$ are both an equilibrium for one sector. For any connected subset $\mathcal{J}^1 \subseteq \mathcal{J}$, $\hat{h}$ and $\hat{h}^1$ can never coincide more than once on $\mathcal{J}^1$.*

Proof: We proceed by contradiction as in Lemmas 3-6 in Costinot and Vogel (2010). Suppose **(i)** $\hat{h}(j_a) = \hat{h}^1(j_a)$ and $\hat{h}(j_b) = \hat{h}^1(j_b)$ such that both $(j_a, j_b) \in \mathcal{J}^1$. Without loss of generality, we assume that $j_a < j_b$ are two adjacent crossing points. Then, since $[\hat{h}, \hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that

1. **(ii)** $\hat{h}^{1\prime}(j_a) \geq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \leq \hat{h}'(j_b)$; and **(iii)** $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j_a, j_b)$; or
2. **(ii)** $\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_a)$ and $\hat{h}^{1\prime}(j_b) \geq \hat{h}'(j_b)$; and **(iii)** $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (j_a, j_b)$.

Consider case **1**. Condition **(ii)** implies

$$\hat{h}^{1\prime}(j_b)/\hat{h}^{1\prime}(j_a) \leq \hat{h}'(j_b)/\hat{h}'(j_a)$$

and using (**??**)-(**??**) and (**??**), and applying $\hat{h}^1(j) = \hat{h}(j)$ for $j \in \{j_a, j_b\}$ we obtain

$$
\begin{aligned}
0 &< [\alpha + \sigma(1-\alpha)] \cdot \left[\int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}^1(j'), j')}{\partial j'} dj' - \int_{j_a}^{j_b} \frac{\partial \log b(\hat{h}(j'), j')}{\partial j'} dj' \right] \\
&\leq \log \left[F(\hat{z}^1(j_b)|\hat{h}(j_b)) / F(\hat{z}(j_b)|\hat{h}(j_b)) \right] - \log \left[F(\hat{z}^1(j_a)|\hat{h}(j_a)) / F(\hat{z}(j_a)|\hat{h}(j_a)) \right]
\end{aligned} \tag{41}
$$

where the first inequality follows since (**??**), the log-supermodularity of b, implies

$$
\partial \log b(h^1, j)/\partial j > \partial \log b(h, j)/\partial j \qquad \forall h^1 > h, \tag{42}
$$

and applying **(iii)**. Next, since (**??**) and Assumption **??.??** implies that $\hat{z}'(j) = \tilde{z}'(h)\hat{h}'(j) > 0$, Assumption **??.??** implies that the strict inequality in (41) holds only if

$$
\hat{z}^1(j_b)/\hat{z}(j_b) > \hat{z}^1(j_a)/\hat{z}(j_a) \quad \Leftrightarrow \quad \log\left[\tilde{z}^1(h_b)/\tilde{z}^1(h_a)\right] > \log\left[\tilde{z}(h_b)/\tilde{z}(h_a)\right]
$$

where we have written $h_x \equiv \hat{h}(j_x)$ for $x \in \{a, b\}$. Plugging in for $\tilde{z}(\cdot)$ using (**??**) we obtain

$$
\Leftrightarrow \quad \int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' > \int_{h_a}^{h_b} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'
$$

and since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(iii)** implies that $\hat{j}^1(h) < \hat{j}(h)$ for all $h \in (h_a, h_b)$. But (**??**), the log-supermodularity of b, implies

$$
\partial \log b(h, j^1)/\partial h < \partial \log b(h, j)/\partial h, \qquad \forall j^1 < j. \tag{43}
$$

a contradiction. Case **2** is symmetric. □

Lemma 4 implies, in particular, that any within-sector equilibria must have identical $\hat{h}(j)$, since $\hat{h}(0) = 0$ and $\hat{h}(J) = h_M$ in all equilibria. Moreover, the lemma also implies that $\hat{h}(j)$ is determined independently of $\hat{z}$, which is uniquely determined by $\hat{h}(j)$ given (**??**). Hence, the within-sector equilibrium is unique, and furthermore, the solution to q_z in (**??**) is also unique given $q_h(0)$.

Existence of the between sector equilibrium is straightforward, since the LHS of (**??**) increases smoothly from 0 to ∞ as $q_h(0)$ varies from 0 to 1, while the RHS is always positive and strictly bounded regardless of the value of $q_h(0)$. To show uniqueness then, it suffices to show that the RHS cannot cross LHS more than once. We will consider the log derivatives of the RHS of (**??**) term by term.

Let Δ_x denote the log-derivative of x w.r.t. $q_h(0)$. Since Assumption **??.??** implies that

$$
\Delta_{B_j(j)} = \int_0^j \frac{\partial^2 \log b(\hat{h}(j'), j')}{\partial h \partial j'} \cdot \frac{d\hat{h}(j')}{dj'} \cdot dj' < \varepsilon \tag{44}
$$

for all $\varepsilon > 0$, we obtain from (**??**) that

$$\Delta_{\pi_{ih}} = (1-\alpha)(\sigma-1)\cdot\Delta_{B_j(j)} \approx 0$$

so $\Delta_{\Pi_{ih}} \approx 0$. Likewise, Assumption **??.??** also implies that

$$\Delta_{B_h(h)} = \int_0^h \frac{\partial^2 \log b(h', \hat{j}(h'))}{\partial h' \partial j} \cdot \frac{d\hat{j}(h')}{dh'} \cdot dh' < \varepsilon \tag{45}$$

for all $\varepsilon > 0$. This implies that $\hat{h}(j)$ is not affected by the choice of $q_h(0)$, and it is independent of the determination of $\hat{z}$ by Lemma 4. Intuitively, Assumption **??.??** makes the model behave as if there were no log-supermodularity. Then since we assume a constant returns technology, all worker allocations approach constant multiples of H_0 and does not depend on its particular value. Since $\Delta_{\Pi_{ih}} \approx 0$, $\Delta_{\Pi_{K_i}}$ only depends on $\Delta_{\hat{z}}$ since from (15) and (17),

$$\Delta_{\pi_{iz}} = (1-\alpha)(\omega-1)\Delta_{\hat{z}} \quad \Rightarrow \quad \Delta_{\Pi_{K_i}}\Pi_{K_i} = \pi_{iz}\cdot(1-\alpha)(\omega-1)\Delta_{\hat{z}},$$

Similarly, $\Delta_{\Pi_{L_i}}$ only depends on $\Delta_{\hat{z}}$ as well, since from (**??**) and (45) we obtain

$$\Delta_{\tilde{z}(h)} = \Delta_{\hat{z}} + \Delta_{B_h(h)} \approx \Delta_{\hat{z}}. \tag{46}$$

so using Leibniz' rule,

$$\begin{aligned}
\Delta_Z \cdot Z &= -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh, \\
\Delta_{L_z} \cdot L_z &= -\Delta_{\hat{z}} \cdot \int \left[\tilde{z}(h) \cdot f(\tilde{z}(h)|h)\right] g(h)dh, \\
\Rightarrow \quad \Delta_{\bar{z}} = \Delta_Z - \Delta_{L_z} &= \Delta_{\hat{z}} \cdot \underbrace{\int \{\tilde{z}(h)\left[1/L_z - \tilde{z}(h)/Z\right] \cdot f(\tilde{z}(h)|h)\} g(h)dh}_{\equiv \Lambda \in (0,1)}
\end{aligned} \tag{47}$$

where the inequality follows from selection and Assumption **??.??**, so using this and (45), from (18) we obtain

$$\Delta_{\Pi_{L_i}}\Pi_{L_i} = (\hat{z}/\bar{z})\pi_{iz}\cdot\left[\alpha + \omega(1-\alpha) - \Lambda\right]\Delta_{\hat{z}}.$$

Now rearranging (**??**), plugging in (47), and using (**??**) at $j = 0$ we obtain

$$\begin{aligned}
&\left\{\alpha + \omega(1-\alpha) + \hat{z}f(\hat{z}|0)/F(\hat{z}|0) + \int \left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right] g(h)dh\right\}\Delta_{\hat{z}} \\
&= \Delta_{q_z} - 1 \equiv \Gamma(X),
\end{aligned}$$

since $H_s(0) = q_h(0)H(0)$, $\Delta_{\hat{h}'(0)} = 0$ as it does not vary with $q_h(0)$, and $\Gamma(X)$ is defined from (**??**):

$$\Gamma(X) = q_h(0)(X-1)/\left[q_h(0) + (1-q_h(0))X\right],$$

$$\text{where}\quad X \equiv \frac{\nu_s(0)}{\nu_m(0)} \cdot \frac{\eta_m(1-\eta_s)}{(1-\eta_m)\eta_s} \cdot \left(\frac{V_{sh}}{V_{mh}}\right)^{\frac{\sigma-\omega}{1-\sigma}}.$$

So it follows that the log-slope of the RHS in (**??**) is

$$-\Bigg\{(1-\epsilon)(1-\alpha)[\alpha+\omega(1-\alpha)] \cdot \left[\frac{\pi_{sz}}{\Pi_{K_s}} - \frac{\pi_{mz}}{\Pi_{K_m}}\right]$$
$$+ (\hat{z}/\bar{z})[\alpha+\epsilon(1-\alpha)][\alpha+\omega(1-\alpha)-\Lambda] \cdot \left[\frac{\pi_{sz}}{\Pi_{L_s}} - \frac{\pi_{mz}}{\Pi_{L_m}}\right]\Bigg\}$$
$$\times \frac{\Gamma(X)}{\alpha+\omega(1-\alpha)+\hat{z}f(\hat{z}|0)/F(\hat{z}|0)+\int\left[\tilde{z}(h)^2 \cdot f(\tilde{z}(h)|h)\right]g(h)dh}.$$

The log-slope of the LHS in (**??**) is $1/\left[1-q_h(0)\right]$, which increases from 1 to ∞ as $q_h(0)$ increases from 0 to 1, and is larger than $\Gamma(X)$ for all $X>0$. Hence it suffices to show that the absolute value of all terms multiplying $\Gamma(X)$ are less than 1, which is true in particular due to Assumption **??**.**??**.

Intuitively, what the planner cares about is the marginal products of Z and H in total. So when the distribution of z has a fat tail, the response of $\hat{z}$ to the choice of $q_h(0)$ is minimal as it changes Z smoothly along its entire support.

B.2 Proof of Proposition 1

Part 1. By Lemma 4, we know that no crossing can occur on $(0,\underline{j})$ or $(\bar{j},J)$, since $\hat{h}$ and $\hat{h}^1$ already coincide at the boundaries 0 and J. Similarly, we also know from Theorem 1 that it can never be the case that there is no crossing ($\hat{h}^1(j) > \hat{h}(j)$ or $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in \mathcal{J} \setminus \{0,J\}$). Hence, there must be a single crossing in $\mathcal{J}^1$ since Lemma 4 also rules out multiple crossings.

At this point, the only possibility for j^* not to exist is if instead, there exists a single crossing j^{**} such that **(i)** $\hat{h}^1(j) < \hat{h}(j)$ for all $j \in (0,j^{**})$ and **(ii)** $\hat{h}^1(j) > \hat{h}(j)$ for all $j \in (j^{**},J)$. If so, since $[\hat{h},\hat{h}^1]$ are Lipschitz continuous and strictly monotone in j, it must be the case that $\hat{h}^{1\prime}(0) < \hat{h}'(0)$, $\hat{h}^{1\prime}(j^{**}) > \hat{h}'(j^{**})$ and $\hat{h}^{1\prime}(J) < \hat{h}'(J)$. This implies

$$\hat{h}^{1\prime}(j^{**})/\hat{h}^{1\prime}(0) \geq \hat{h}'(j^{**})/\hat{h}'(0), \qquad \hat{h}^{1\prime}(J)/\hat{h}^{1\prime}(j^{**}) \leq \hat{h}'(J)/\hat{h}'(j^{**}). \tag{48}$$

Let us focus on the first inequality. Using (**??**) and (**??**) we obtain

$$0 > [\alpha+\sigma(1-\alpha)] \cdot \left[\int_0^{j^{**}} \frac{\partial \log b(\hat{h}^1(j),j)}{\partial j}dj - \int_0^{j^{**}} \frac{\partial \log b(\hat{h}(j),j)}{\partial j}dj\right] \tag{49}$$

$$\geq (1-\sigma)m + \log\left[F(\hat{z}^1(j^{**})|\hat{h}(j^{**}))/F(\hat{z}(j^{**})|\hat{h}(j^{**}))\right] - \log\left[F(\hat{z}^1(0)|\hat{h}(0))/F(\hat{z}(0)|\hat{h}(0))\right]. \tag{50}$$

where the first inequality follows from (42), and applying **(i)**. Since $m > 0$, if $\sigma \in (0,1)$, Assumptions **??.??** and **??.??** imply that the strict inequality in (49) holds only if

$$\int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}^1(h'))}{\partial h'} dh' < \int_0^{h^{**}} \frac{\partial \log b(h', \hat{j}(h'))}{\partial h'} dh'$$

where we have written $h^{**} \equiv \hat{h}(j^{**})$. And since $\hat{j}(h)$ is the inverse of $\hat{h}(j)$, **(i)** implies that $\hat{j}^1(h) > \hat{j}(h)$ for all $h \in (0, h^{**})$. But this violates (43), the log-supermodularity of b. The case for the second inequality in (48) is symmetric.

Part 2. Let Δ_x denote the log-derivative of x w.r.t. m. From (**??**),

$$\begin{aligned} \Delta_{\Pi_{ih}} \cdot \Pi_{ih} &= (\sigma - 1) \int_{\underline{j}}^{\overline{j}} \pi_{ih}(j) dj + \int \left\{ \pi_{ih}(j) \cdot (1-\alpha)(\sigma - 1) \cdot \Delta_{B_j(j)} \right\} dj \\ &\approx (\sigma - 1) \int_{\underline{j}}^{\overline{j}} \pi_{ih}(j) dj \end{aligned} \tag{51}$$

where the approximation follows from Assumption **??.??** and (44). Hence $\Delta_{\Pi_{ih}} < 0$ if $\sigma < 1$. Rearranging (**??**) and using (**??**) at $j = 0$ we obtain

$$0 > \frac{\sigma - \omega}{1 - \sigma} \cdot \Delta_{\Pi_{ih}} - \Delta_{\hat{h}'(0)} = \left[\alpha + \omega(1-\alpha) + \hat{z} f(\hat{z}|0) / F(\hat{z}|0) \right] \Delta_{\hat{z}} - \Delta_Z \tag{52}$$

where the inequality holds if $\omega < \sigma < 1$, and since we know from part 1 that $\Delta_{\hat{h}'(0)} \geq 0$. Now suppose $\Delta_{\hat{z}} \geq 0$. Then for (52) to hold it must be the case that $\Delta_Z > 0$, but from (47), $\Delta_Z \leq 0$ if $\Delta_{\hat{z}} \geq 0$, a contradiction. Hence, $\hat{z}^1 < \hat{z}$, and $\tilde{z}^1(h) < \tilde{z}(h)$ for all h by (46).

C Additional Tables and Figures

Ranked by mean wage (except management)	(1) -	(2) m_j	BM m_j	Data
Low Skill Services	**1.973**	**-2.726**	**-0.731**	
Middle Skill				
Administrative Support	1.973	1.252	2.930	
Machine Operators	1.973	10.018	9.122	
Transportation	1.973	3.326	4.348	
Sales	1.973	-1.895	0.012	
Technicians	1.973	-2.484	-1.144	
Mechanics & Construction	1.973	1.742	2.315	
Miners & Precision Workers	1.973	6.367	6.328	
High Skill				
Professionals	1.973	-3.973	-2.248	
Management Support	1.973	-1.973	-0.489	
Management	**1.973**	**-1.438**	**-0.017**	
Aggregate TFP growth	**1.030**	**1.030**	**1.030**	**1.030**
a_m (Manu TFP growth)	0.252	0.252	2.943	2.229
a_s (Serv TFP growth)	-1.205	2.021	0.308	0.743

Table 6: Recalibrated TFP Growth Rates for Counterfactuals

Column (1) stands for the counterfactual in which we set $m_j = m$ and calibrate (a_m, a_s) to match sectoral TFP's, and (2) for when we let $(\{m_j\}_{j=z,0}^{9}, a_m, a_s)$ all vary simultaneously. "BM" stands for the benchmark calibration. For all scenarios, aggregate GDP growth (and consequently TFP growth) is matched exactly, shown in the first row of the bottom panel. For the "BM" and "Data" columns, the a_m and a_s rows show the empirical growth rates of the manufacturing and services sectors' TFP's, respectively.

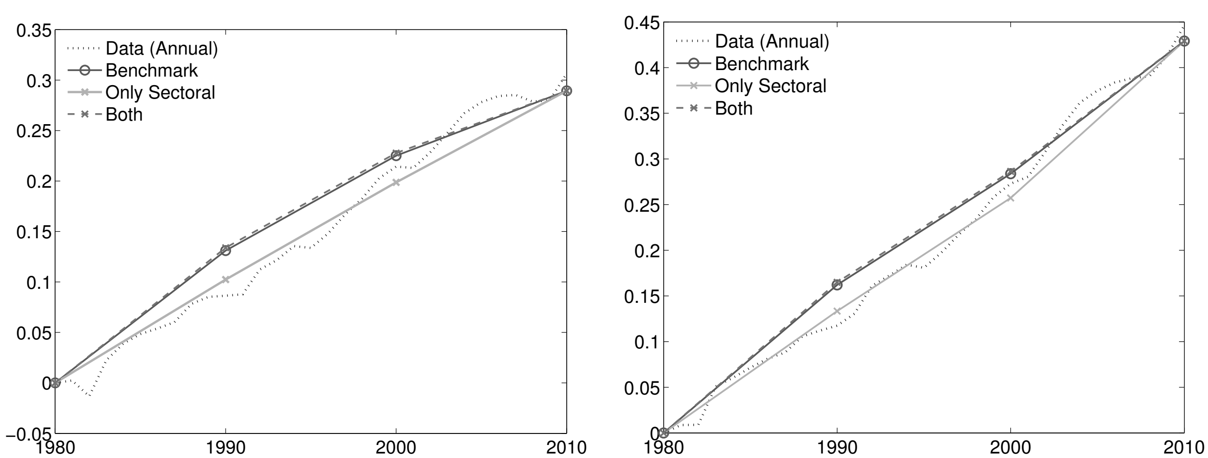
	ΔL_j
TFP	- 9.584 ***
R^2	0.939

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
RTI (DOT)	0.429						
	(0.268)						
Routine manual		0.797**		0.618			
		(0.206)		(0.527)			
Manual interpersonal			-0.767**	-0.192			
			(0.192)	(0.549)			
College share 1980					-11.142*		-7.994**
					(3.599)		(2.269)
ΔCollege share 1980-2010						-33.673*	-20.295*
						(17.410)	(13.547)
Constant	1.061	0.377	3.281**	1.065	4.035*	4.818*	5.204*
	(0.941)	(0.738)	(0.970)	(2.339)	(1.401)	(1.674)	(1.759)
R^2	0.184	0.635	0.588	0.640	0.439	0.372	0.539

Standard errors in parentheses, $^{\dagger}p < 0.10$, $^{*}p < 0.05$, $^{**}p < 0.01$

Table 7: Task-Specific TFP Growth, Employment, and Empirical Measures

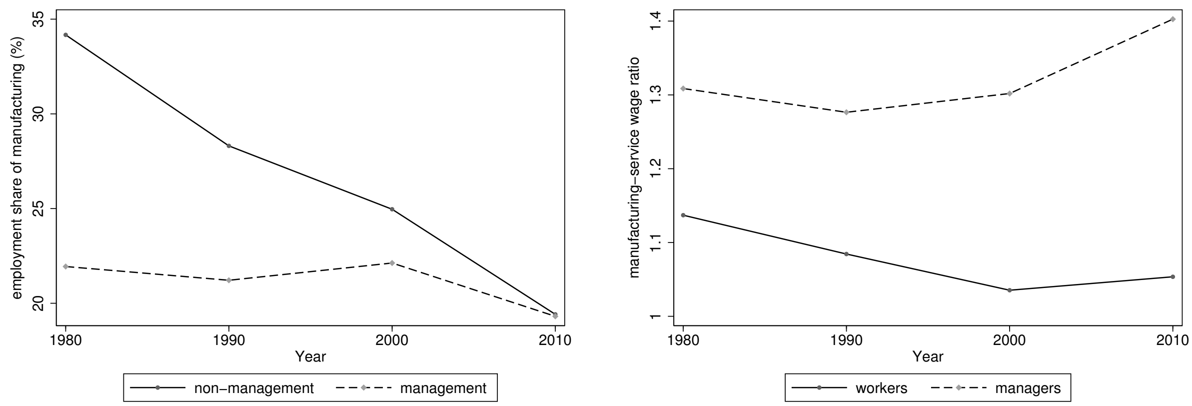
The first panel shows the results from regressing employment share changes on the calibrated task-specific TFP growth rates, m_j. The second panel shows the results from regression the growth rates on various empirical measures.



(a) Log TFP Growth

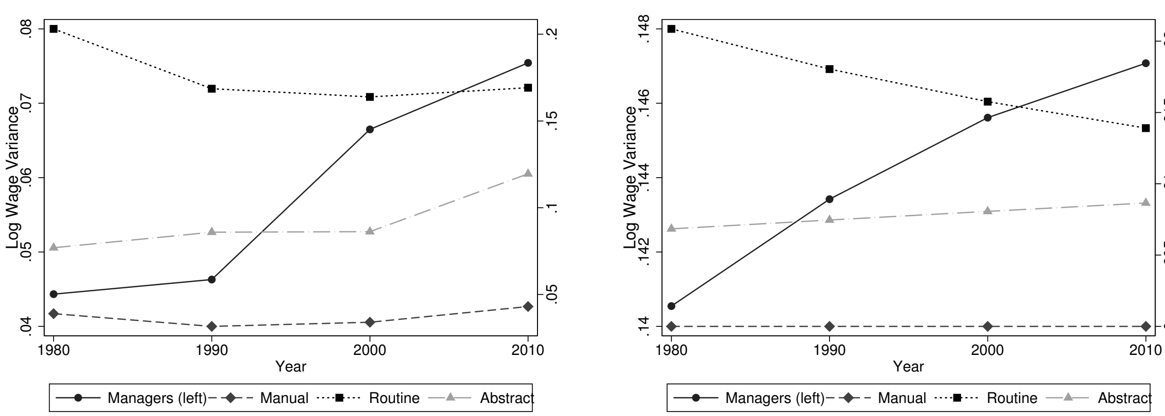
(b) Log GDP per Worker Growth

Fig. 22: Aggregate Output and TFP Growth



(a) Manufacturing Employment Share (b) Manufacturing-Services Average Wage Ratio

Fig. 23: Manufacturing vs. Services by Occuaption



(a) Data (b) Model

Fig. 24: Within Task Wage Inequality

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