THE ECONOMICS OF PLATFORMS IN A WALRASIAN FRAMEWORK

ANIL K. JAIN* AND ROBERT M. TOWNSEND†

Abstract. We present a tractable model of platform competition in a general equilibrium setting, allowing multiple platforms to emerge. We endogenize the size, number and type of each platform, allowing for different utility functions, different endowments for varying types of agents, and different capital costs. Contrary to the prior macro-financial literature on network architecture and the partial equilibrium industrial organization literature on two sided markets, our economy is efficient. Platforms internalize the network effects of adding more and different types of users by offering bundles which state both the number and composition of users. We use a Walrasian equilibrium concept and allow the price of joining a platform to depend not only on the characteristics of the platform, but the identified type of user. The sum of fees paid for a given active platform will cover its costs. With this extended commodity space and bundling, the first and second welfare theorems apply. We argue against distortions created through fees and the presumption that platforms with externalities have to be regulated.

Date: January 2016.
*Federal Reserve Board of Governors. Email: anil.k.jain@frb.gov and †Massachusetts Institute of Technology. Email: rtownsen@mit.edu. Anil Jain is grateful for financial support from the Consortium on Financial Systems and Poverty (CFSP). Robert Townsend is grateful for financial support from the Private Enterprise Development in Low-Income Countries (PEDL), the Eunice Kennedy Shriver National Institute of Child Health and Human Development (NICHD) under grant R01 HD027638, NSF, the John Templeton Foundation, and the Consortium on Financial Systems and Poverty at the University of Chicago through a grant from the Bill & Melinda Gates Foundation. We would like to thank seminar participants at the Boston Federal Reserve, the International Industrial Organization Conference and Princeton University for helpful comments. The findings and conclusions in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System, the views of any other person associated with the Federal Reserve System or the views of the funders. All mistakes are our own.

1. Introduction

We are interested in economic platforms which inherently depend on attracting multiple different types of users. For instance, the quality or usefulness of a credit card will depend on the merchants who accept the card and which consumers use the card. Each side cares about the other. A mobile phone network is attractive only if it allows a user to message her contacts on the same technological platform. A dark pool for the trading of financial instruments needs to attract both buyers and sellers in somewhat proportionate numbers if it is to allow trade and coexist with other public exchanges. Likewise, a clearinghouse is a mechanism to net trades and mitigates obligations that continue beyond an end-of-day settlement period. Even traditional financial intermediaries can be thought of in this way, in the sense that they stand between savers and borrowers and transform the risk and time structure of funds.

We ask, in these types of markets, with multiple competing platforms how does one define a Walrasian equilibrium. Typically does it exist or are there inherent problems? If an equilibrium exists, is it efficient in the allocation of costs or is there a case for the regulation of prices? Finally, what is the relationship between competitive equilibria and the distribution of welfare, specifically, does one side or the other have an inherent advantage?

To take one example of a platform competition that has attracted significant academic and regulatory attention, *the interchange fee*. The interchange fee is a charge for the acquiring bank (the bank that processes a credit card payment on behalf of the merchant) levied by the issuing bank (the bank that issues a consumer's credit card) to balance the credit card's costs between the merchant and the consumer's bank. Consumers' utility and merchants' profits from using/accepting a credit card depends on the number of users of both types, as well as their respective costs via the interchange fee. In this environment, since a user's utility or profit depends on the composition of the card's users, does this cause a network externality? Can the interchange fee correct this network externality? Does the market-determined interchange fee require regulation? Finally, how does the size of the interchange fee distribute costs between consumers and merchants?

These questions have been asked but only partially addressed. "The large volume of theoretical literature on interchange fees has arisen for the simplest of reasons: understanding their termination and effect is intellectually challenging"Evans et al. [2005]. Baxter [1983] first modeled multiple platforms (banks) that provided services for merchants to interact with consumers. Baxter argued the platform is unable to internalize the merchant's marginal utility gain from an extra consumer, which leads to an unpriced 'externality', and subsequently a market inefficiency. This concept of an unpriced externality is repeated in further work by Rochet and Tirole [2003a], Evans et al. [2005], Armstrong [2006], Hagiu [2006], Rochet and Tirole [2006], Rysman [2009], Weyl [2010], Weyl and White [2015].

In contrast to the previous literature, our paper demonstrates that this 'unpriced externality' is not a consequence of platform technologies per se, but is actually a theoretical consequence of a combination of market power and an insufficiently rich contracting space.

We extend the original literature in two key ways: (i) we use tools from General Equilibrium theory to model platform competition and (ii) we allow platforms to offer bundles that detail the composition of a platform's users. Through, the the use of this modified contract, we show that the prices for the platform membership overcomes the inherent externality, in a similar manner as suggested by Meade [1952] and Arrow [1969]. Further, the competitive equilibrium is Pareto optimal, and the usual First and Second Welfare theorems hold in our economy.

We use tools from standard General Equilibrium Theory in this modified environment, where there is an obvious externality: a user's willingness to pay for a product is dependent on the composition of the product's user base.

Our paper has three main results: first, building on Prescott and Townsend [2006] who analyzed firms as clubs in general equilibrium, we provide a framework which shows that platforms can internalize the above-described externality if the platforms do not exhibit ever increasing economies to scale. Additionally, we characterize the equilibrium among competing platforms: which types of platforms exist, the prices paid by user types, and the fees charged by intermediaries.

Second, we prove that both the first and second welfare theorems hold in this environment; a competitive equilibrium is Pareto optimal and any optimal allocations of resources can be achieved by lump sum taxes and transfers on underlying wealth.

Third, using the framework we characterize how the equilibrium prices for each type of user to join the platform (for example, buyers and sellers or consumers and merchants) and the composition of a platform's users change as we alter parameters of the underlying economic environment. For example, we consider decreasing the costs of creating platforms or altering the initial distribution of wealth among the agents. Indeed, we make a distinction between a fundamental type of user versus within-a-type users that differ only in wealth – for instance, we can examine how the equilibrium changes as we alter different consumers' wealth. The latter allows us to see how higher wealth for a certain type leads to more advantageous matches for that type and subsequently spilling over to others' and to their own utility.

Our framework offers a compelling model for certain forms of financial intermediation. For instance, there are over 40 different platforms for trading listed securities available to traders in 2008 (O'Hara and Ye [2011]). To ensure each platform is a price-taker, we assume that platforms do not have ever increasing returns to scale. This assumption has empirical support with both Altinkiliç and Hansen [2000] and O'Hara and Ye [2011], which document non-increasing returns to scale in equity underwriting and equity exchanges respectively.

Our framework builds heavily on club theory and in particular, the firms as club literature. Koopmans and Beckmann [1957] discuss the problem of assigning indivisible plants to a finite number of locations and its link to more general linear assignment/ programming problems. A system of rents sustains an optimal assignment in the sense that the profit from each plant-location pair can be split into an imputed rent to the plant and an imputed rent to the location. At these prices landowners and factory owners would not wish to change the mix of tenants or location. As Koopmans and Beckman point out, the key to this beyond linear programming is Gale et al. [1951]'s theorem which delivers Lagrange multipliers on constraints. Every location has a match and the firms and location are not over- or under- subscribed. A linear program ignores the intrinsic indivisibilities – the integer nature of the actual problem – yet nevertheless achieves the solution.

In the well-known labor assignment model of Sattinger [1993], workers are assigned to jobs and the contribution of a worker with a mix of skills depends not only on the particular type of job being performed but the assignment of others – that is, the work(er) environment. Hornstein and Prescott [1993] consider a Lucas [1978] managerial span-of control problem in which agents can choose to be workers or firms – an indivisibility – and also a second problem in which the number of hours a firm operates its plants and the numbers of workers assigned to each plant is endogenous. These environments appear to introduce a non-convexity in the production set. But with a large number of agents one can approximate the environment with a production set that has constant returns, that is when the non-convexity is small relative to the size of the economy. Essentially, the production set becomes a convex cone, as in McKenzie [1959, 1981]'s formulation of general equilibrium.

The economics underlying McKenzie's formulation – in contrast to Arrow and Debreu [1954] – makes endogenous the ownership of shares in firms, i.e. profits must be earned through entrepreneurial rents rather than through shares which are given a priori. The basic tool in Hornstein and Prescott is the use of lotteries as developed in Prescott and Townsend [1984] for private information environments in which incentive constraints introduce a non-convexity. The common element is that lotteries are a way at the aggregate level to assign fractions of agent types to contracts, clubs, occupations, and so on, even though individual assignments are discrete. Likewise, Hansen [1985] and Rogerson [1988] in macro determine the fraction of the population working overtime, a discrete choice. Pawasutipaisit [2010] assigns one male and one female type to common marriage. The firms as clubs methodology is well suited for our setting because it allows us to solve for which platforms emerge in equilibrium, the size of each platform, and who is part of each platform.

In this paper, we show that the first and second welfare theorems hold in our economy with platforms. A competitive equilibrium exists, and in this equilibrium, platforms are able to internalize the effects of interdependencies through the composition of users. Each basic user type faces a user price for each of a (infinite) number of potential platforms, which vary in the number of own-type participants and other-type participants. In equilibrium at given prices, the solution to these decentralized problems delivers the mix and number of participants in active platforms that each user anticipated when they choose platforms. That is, in equilibrium the club or platform is populated with user types exactly as anticipated. The solution is efficient because the market price for joining a platform, which a user takes as given, changes across platforms in a way which internalizes the marginal effect of altering the composition of the platform. Put differently, each agent of each type (having tiny, negligible influence), is buying a bundle which include the composition and number of total participants, that is, the commodity space is expanded to include the intrinsic externality feature of the platform. Essentially we solve the externality problem in the way suggested by Meade [1952] and Arrow [1969].

Having established the economy is efficient, we demonstrate how the size of platforms, prices, and individuals' utilities change as we alter parameters of the environment, such as the cost of building a platform, the disutility of having too many users (congestion), and the underlying wealth (endowments of the capital good) of user types. Higher costs and congestion naturally tend to reduce the relative number of equilibrium platforms. But there are distributional aspects as higher costs make the capital used to construct platforms more valuable, and this favors wealthy agents who are abundantly endowed with that capital. The poor are thus hurt in this comparison. A change in the wealth distribution toward a favored type not only increases the competitively determined utility of that type, it also changes the utility of others. In particular, it potentially increases the utility of those that the favored types wish to be matched with and also decreases the utility of other types with lower wealth who are in direct competition to populate platforms. We exploit in these latter comparative statics the fact that changing Pareto weights is equivalent to changing wealth, that is, we use a programming problem to maximize Pareto weighed sums of utilities and then change the weights, tracing out all Pareto optimal equilibria. A given optimum requires lump sum taxes and transfers or equivalently a change in the initial underlying distribution of wealth.

Finally, we demonstrate the generality of our framework for modeling platforms. First, we extend the model to allow for heterogeneous agent preferences, and second, we extend the model to allow agents to join multiple platforms (multi-homing).

The closest literature to our work is on two-sided markets. The two-sided markets literature consider platforms which sell to at least two different user groups, and whose utility is dependent on who else uses the platform. In general, the two–sided markets literature uses an industrial organization, partial equilibrium framework. The main finding in Rochet and Tirole [2003b, 2006], Armstrong [2006], Weyl [2010], Weyl and White [2015] is that two-sided markets lead to market failure. In particular, in the two-sided market literature a key concern is how the distribution of users' fees will cover the platform's fixed and marginal costs. There are many controversies: whether the allocation of fees alters the outcome (one definition of a two sided market is that the distribution of fees matters to the outcome Rochet and Tirole [2003b]); whether there are implicit subsidies; whether users are, or should be aware of what the price is covering (should payment charges be a separate part of the bill), and how to regulate the interchange fee that the issuing bank charges redeeming banks (and again how fees are passed to merchants – i.e. small merchants versus block entities such as Walmart).

Rysman [2009]'s comprehensive overview of the empirical and theoretical work on two-sided market states "the main result (in the two-sided market literature) is that pricing to one side of the market depends not only on the demand and costs that those consumers bring but also on how their participation affects participation on the other side". This highlights two of the main advantages of our general equilibrium framework with a Walrasian allocation mechanism: First, we show that net prices are appropriate – the indirect effect on the 'other side' is priced in – and outcomes are efficient. Second, we show how the equilibrium changes – the prices for joining a platform, the size of platforms, and the resulting agent utilities – as we alter the underlying wealth distribution or the cost of building a platform.

Weyl and White [2015] consider the general equilibrium implications of two-sided markets with imperfect competition. White and Weyl provide a new solution concept – Insulated Equilibrium – on how platforms may induce agents to coordinate over which platforms to join. Our paper focuses on modeling perfect competition with the full observability of an agent's type. In contrast to our paper, White and Weyl argue there remains a potential for market failure due to an unpriced consumption externality. The key difference in our papers' predictions, arise from our differing modeling choices. Our paper's economy is perfectly competitive, whereas White and Weyl assume an oligopolistic platform economy where each platform has market power and cannot extract the full consumer surplus. This potentially leads the platforms to charge socially inefficient prices. In our economy, the platforms are perfectly competitive and earn no rents – removing this source of the social inefficiency.[1]

The literature on middlemen is also related to our work. Middlemen facilitate trade between two different agents. Rubinstein and Wolinsky [1987]'s seminal paper outlines a model where middlemen increase the efficiency of the market through the reduction of search costs for buyers and sellers. Mortensen and Wright [2002] extend Rubinstein and Wolinsky [1987] and internalize a search externality using directed search into segregated submarkets that promise different expected waiting times. Further, Guerrieri et al. [2010] model an economy with both adverse selection and search frictions in matching agents to principals. Our paper concentrates on how intermediaries are platforms which facilitate trade between different parties. In contrast to the search and middlemen literature, we allow a competitive, constant returns to scale intermediary sector with free entry, a large (continuum) number of agents of each type, and *no* search frictions.

Indeed, we need to emphasize the limitations of what we are doing, specifically what we are not doing. We do not consider agents having any pricing power. We do not consider the problem of establishing new products/platforms in the sense of innovation and entry into an existing equilibrium outcome and the problem of changing client expectations. Relatedly, we do not discuss the historical development of platforms nor consider current regulatory restrictions, including well intended but potentially misguided regulations which may limit our ideal market design. Nor do we model monopolistic competition though we do allow our platforms to be configured with different compositions of customers, so there is clear product differentiation (just no market power). Finally, we do not allow ever increasing economies of scale in platform size.

1.1. **Applications.** Our paper is relevant to many different environments which involve the intermediation between different agents who have a choice of which platform to use and simultaneously have preferences about the size and composition of the intermediary's users. We explain some of our examples in more detail below.

Stock Exchanges, Dark Pools, Swap Execution Facilities and Underwriters

'Dark Pools' are collections of buyers and sellers which join a platform as a mechanism to anonymously trade bonds and stocks. There are many seemingly similar platforms (The Economist [2011] refers to at least 80 dark pools as of August 2011) which are steadily increasing the proportion of total equity trading (the percentage of consolidated U.S. equity trading on dark pools is estimated to have risen from 6.5% in 2008 to 12% in 2011, Tabb Group [2012]). Thus the market for exchanges is competitive and large,

[1]Our framework is sufficiently flexible to include a monopolist platform sector. In the model, if we substitute the perfectly competitive platform sector with a monopolist platform provider, the monopolist will maximize profits by severely restricting supply and producing a negligible mass of platforms.

that is, it is not concentrated; relatedly, O'Hara and Ye [2011] argue that after controlling for sample selection issues, new equity exchanges (not necessarily dark pools) have lower bid-ask spreads and faster execution times.[2]

'Swap Execution Facilities' (SEFs) are platforms for swap trading that provide price information and execute swaps among platform members. Similar to the dark pool market there many different SEFs – as of July 2015,[3] there were 25 different SEFs registered with the CFTC.

Altinkiliç and Hansen [2000] empirically document increasing marginal costs and diseconomies of scale for underwriting larger bond and equity issuances after controlling for firm characteristics. Altinkiliç and Hansen [2000] argue that underwriting larger capital issuances have larger placement costs because (i) adverse selection problems are more severe for larger placements and (ii) procuring more buyers for larger placements is harder.

Zhu [2014] distinguishes three types of dark pools: in the first, platforms match customer order without own account trading, in the second platforms are operated by broker dealers who operate as continuous non displayed limit order books; and in the third, dark platforms act as fast electronic market makers that instantaneously accept or reject incoming orders. In Zhu's model, the likelihood of being able to execute a trade depends on the availability of counterparties, in particular, the side with more orders – will fail to be executed. Our main point is obvious; for a platform to be successful it must attract both buyers and sellers for the potential transaction.

Clearinghouses

Clearinghouses (and depository trusts) net trades across platform participants, often at the end of each business day. A clearinghouse can increase the efficiency of trading between users by guaranteeing trades amongst its users.[4] This may lead to three main advantages: first, it can reduce a user's exposure through netting their total obligations across multilateral counterparties; second, it can reduce systematic risk by reducing the probability of defaults propagating across counterparties; and third it can increase the speed of transactions. For a clearinghouse to effectively settle and process trades, it must attract multiple buyers and sellers to its platform.

Credit Cards, Debit Cards, and other Payment Systems

Consumers and merchants use an array of different platforms to transfer cash or value: credit cards, debit cards, cash, checks, interbank transfers, mobile applications or online

[2]O'Hara and Ye [2011] estimate those stocks which are listed on multiple exchanges have trading costs which are 0.33-0.34 cents lower than those which are listed on only the major exchanges.
[3]Source U.S. Commodities Futures Trading Commission (CFTC) http://sirt.cftc.gov/SIRT/SIRT.aspx?Topic=SwapExecutionFaci
[4]In particular, the clearinghouses introduce two new contracts, whereby each party bilaterally trades with the clearinghouse rather than each other in a process called 'novation'.

transfers.[5] Further, each mechanism has a plethora of competitors, for instance, in credit cards, there are MasterCard, Visa, American Express and Discover and in the rapidly growing space of online payments, there are Square, Paypal, and Levelup.[6] Each of these payment platforms must attract merchants and consumers to use their platform.

Internet Service Providers and Net Neutrality

Internet service providers (ISPs) (such as Comcast, Verizon, AT&T) deliver internet content to consumers, from internet content producers (such as Netflix, Google and Amazon). Consumers desire ready and speedy access to internet content, and content producers want reliable access to consumers.

Current 'Net Neutrality' legislation provisions in the U.S. and Europe require ISPs to treat all internet traffic equally. This is equivalent to requiring ISPs to charge the same price (per byte) to every internet content producer. ISPs have consistently argued that the net neutrality reduces investment, and subsequently increases congestion on the network (Arstechica [2014]).

Our paper argues under perfect competition (which may not be true for ISPs), charging differing prices – that is *non-net neutrality* – may be Pareto optimal. Internet content producers have different preferences for the speed of sending data – Netflix may want to deliver internet content rapidly, whereas an academic transferring data to a co-author may prefer to transfer data at off-peak times. By utilizing differential prices to transfer data, the allocation of scarce capacity may be improved, while simultaneously incentivizing greater investment in capacity.

2. Model

There are two types of individuals, merchants (A) and consumers (B).[7] There is a continuum of measure one of each. There is variation within these types – namely, there are sub-types of merchants and consumers who differ in their endowment levels. We index each agent by (T, s) where T is the type (merchant or consumer) and s is the sub-type. There are I subtypes of merchants, A (and indexed by i) and J subtypes of consumers, B (and indexed by j). By introducing variation in an agent's wealth, we can analyze how

[5]The average US consumer uses a portfolio of payment mechanisms; Foster et al. [2013] found the average consumer used 5.2 different payment instruments (out of a possible 9).

[6]Hard numbers about the growth of mobile payments is difficult to procure, although there are a couple of statistics which hint at the possible growth of mobile payments; 25% of Starbucks transactions are paid for via a Starbucks' prepaid account (Tavilla [2012]) and Nielsen [2012] report that 9% of survey respondents paid for goods and services through their phone.

[7]For clarity, we restrict our model to two types, but our model is sufficiently general to accommodate multiple types.

changes in the economic environment both affect the composition of a platform and an agent's utility.[8]

There is a fraction $\alpha_{T,s}$ of each type T and subtype s, and there is a measure of each one of each type T, $\sum_s \alpha_{T,s} = 1 \; \forall T \in \{A, B\}$. Clearly the fraction of each subtype $\alpha_{T,s}$ are arbitrary real numbers on the unit interval – not integers. Each agent has an endowment of capital, denoted by $\kappa_{T,s} > 0$.

We model utility at a reduced form level, and assume agents procure utility from being matched with other agents. Although this is not realistic per se, we presume the process of being matched with other agents facilitates trade over the platform. We do not model that underlying environment – otherwise our setup is fairly general. For instance, agents may prefer larger platforms as that increases the number of potential trades. Further, we could generalize and introduce a term for any private benefit the platform provides over and above its matching service. In short, utilities are to be thought of as indirect.

We only allow non-negative integers of merchants and consumers to join a platform. The utility of a merchant (of any subtype i) matched with N_A merchants and N_B consumers is:

$$U_{A,i}(N_A, N_B) = U_A(N_A, N_B) = \begin{cases} 0 & \text{if } N_A \text{ or } N_B = 0 \\ \left[\left(\frac{N_B}{N_A}\right)^{\gamma_A} + N_B^{\epsilon_A}\right] & \text{else} \end{cases}$$

Note that the baseline utility of not being on any platform is zero.[9] This is the 'opt-out' option and is always available.

Symmetrically, for a consumer (of any subtype j) it is:

$$U_{B,j}(N_A, N_B) = U_B(N_A, N_B) = \begin{cases} 0 & \text{if } N_A \text{ or } N_B = 0 \\ \left[\left(\frac{N_A}{N_B}\right)^{\gamma_B} + N_A^{\epsilon_B}\right] & \text{else} \end{cases}$$

Where γ_A, γ_B, ϵ_A, $\epsilon_B \in (0, 1)$.

Individuals will compete between agents of their own type, whereas, they prefer more of the other type. For example, in the general merchant and consumer case, we are presuming that merchants dislike more merchants, since this will lead to greater competition and possibly reduce the good's price. Therefore, this is a reduced form specification for competition between agents of the same type which is highlighted by Armstrong [2006].[10]

[8] Section (6.1) extends the baseline model to allow subtypes to have different preferences.

[9] This is a natural assumption for the opt-out utility since the lower bound for N_A is one (since as soon as a merchant joins a platform, there must be at least one merchant on the platform) and if N_A is positive the limit of $U_A(N_A, N_B)$ as N_B goes to zero is zero ($\lim_{N_A \geq 1, N_B \to 0} \left(\frac{N_B}{N_A}\right)^{\gamma_A} + N_B^{\epsilon_A} = 0$).

[10] It could be argued that the presence of more merchants could be beneficial for a merchant, since it could lead to 'economies of agglomeration', however, if the reasoning of the beneficial effects by agglomerating

This utility function exhibits two important features which Ellison and Fudenberg [2003] highlight:

(1) **Market Impact Effects:** Each type prefers more of the other type and less of it's own.

$$U_A(N_A+1, N_B) - U_A(N_A, N_B) = \left[\left(\frac{1}{N_A+1}\right)^{\gamma_A} - \left(\frac{1}{N_A}\right)^{\gamma_A}\right] N_B^{\gamma_A} < 0$$

$$U_A(N_A, N_B+1) - U_A(N_A, N_B) = \frac{[(N_B+1)^{\gamma_A} - N_B^{\gamma_A}]}{N_A^{\gamma_A}} + [(N_B+1)^{\epsilon_A} - {N_B}^{\epsilon_A}] > 0$$

Individuals prefer to be on larger platforms as it offers greater possibilities to trade for a given ratio of participants.

(2) **Scale effects:** An individual prefers larger platforms for a given ratio - assume $\tau > 1$, therefore:

$$U_A(\tau N_A, \tau N_B) - U_A(N_A, N_B) = (\tau - 1)N_B^{\epsilon_A} > 0$$

Symmetrically, both effects also apply for the type B utility function.

In the model, agents buy personal contracts which stipulate the number of merchants and consumers on the platform.

We denote the contract by $d_T(N_A, N_B)$ where N_A and N_B are the number of merchants and consumers respectively in the given platform and T denotes the type of individual the contract is for, whether it is merchants (A) or consumers (B). Types are observed and Type T cannot buy a contract indexed by T'. Further one can think of an agent of a given type T and subtype s as allowed to join only one platform. Thus we can create a function $x_{T,s}[d_T(N_A, N_B)] \geq 0$ such that $\sum_s x_{T,s}[d_T(N_A, N_B)] = 1$ which is an indicator (or more generally a probability distribution) for the assignment of an agent (T, s) to contract $d_T(N_A, N_B)$.[11]

The set of contracts for type A is denoted as D_A and similarly the set of contracts for type B is denoted D_B.

The consumption set of type A, i agents can be written as:

$$X_{A,i} = \left\{ x_{A,i}[d_A(N_A, N_B)] \geq 0 \ \forall d_A \in D_A, \ \sum_{d_A \in D_A} x_{A,i}[d_A(N_A, N_B)] = 1, \ x_{A,i}[d_B(N_A, N_B)] = 0 \ \forall d_B \in D_B \right\}$$

is that it attracts more consumers, then this is still achieved by the utility function posited . The utility function models that merchants prefer less merchants *for a given number of consumers.*

[11] In subsection 6.2, we extend the model to allow multi-homing (agents can join multiple platforms) by omitting the requirement that an agent is matched to only one platform ($\sum_s x_{T,s}[d_T(N_A, N_B)] = 1$).

The above condition states that type A, i agents can buy any non-negative amount of contract $d_A \in D_A$, but none of the type B contracts.

Symmetrically the consumption set of type B, j agents can be written as:

$$X_{B,j} = \{x_{B,j}[d_B(N_A, N_B)] \geq 0 \ \forall d_B \in D_B, \sum_{d_B \in D_B} x_{B,i}[d_B(N_A, N_B)] = 1, \ \ x_{B,j}[d_A(N_A, N_B)] = 0 \ \forall d_A \in D_A\}$$

Since individuals may only join a single platform, this introduces an indivisibility into an agent's consumption space. To overcome this problem we allow individuals to purchase mixtures, or probabilities of being assigned to a platform of a certain size including the opt-out option.[12] For example, consider an agent who buys two different contracts: the first contract assigns the agent to a platform consisting of four merchants and three consumers with probability one-third, and the second contract assigns the agent to a platform consisting of three merchants and one consumer with probability two-thirds. The deterministic case, where an agent buys a contract which matches them with a platform of size (N_A, N_B) with certainty, can be seen as a special case. We do not insist that there is mixing in a competitive equilibrium but it can happen as a special case. For instance, when agents are poor there can be mixing between a given platform and an opt-out contract.

As a technical assumption, we assume there is a maximal platform of size $(\overline{N_A}, \overline{N_B})$, and any platform up to this size can be created. Assuming there is a maximal platform size bounds the possible set of platforms and hence makes the commodity space finite. This is for simplicity, since we can choose $\overline{N_A}$ and $\overline{N_B}$ arbitrarily large such that this condition does not bind.

The commodity space is thus:

$$L = \mathbb{R}^{2(\overline{N_A} \times \overline{N_B} + 1) + 1}$$

There are contracts for every possible platform size, in turn indexed by the two types and further there is capital. Thus as we define the maximal platform size to be $(\overline{N_A}, \overline{N_B})$ and there is always the opt-out contract, there are $\overline{N_A} \times \overline{N_B} + 1$ contracts for each type. Since there are two types, we multiply this number by two for the number of contracts available. Finally there is a market for capital.

All contracts $d_T(N_A, N_B)$ are priced in units of the capital good and the type T price for contract $d_T(N_A, N_B)$ is denoted as $p[d_T(N_A, N_B)]$ for types A and B (where $T \in \{A, B\}$).

[12]A similar modeling approach is used in Prescott and Townsend [1984], Prescott and Townsend [2005], Pawasutipaisit [2010].

2.1. **Agent's Problem.** In summary agent T, s take prices $p[d_T(N_A, N_B)]$ $\forall d_T \in D_T$ as given and solves the maximization problem:

$$\max_{x_{T,s} \in X_{T,s}} \sum_{N_A, N_B} x_{T,s}[d_T(N_A, N_B)] U_T[d_T(N_A, N_B)] \tag{1}$$

$$\text{s.t.} \sum_{N_A, N_B} x_{T,s}[d_T(N_A, N_B)] p[d_T(N_A, N_B)] \le \kappa_{T,s} \tag{2}$$

$$\sum_{N_A N_B} x_{T,s}[d_T(N_A, N_B)] = 1 \tag{3}$$

where each type of individual has a endowment of $\kappa_{T,s}$ of capital and the price of capital is normalized to one, that is, capital is the numeraire.

Equation (1) is the agent's expected utility from the assignment problem. Equation (2) is the agent's budget constraint. Equation (3) is the agent's matching constraint, which requires the agent to join a platform with certainty.[13]

2.2. **Platforms.** We assume there are intermediaries or marketmakers who create platforms and sell contracts for each type to join the platform. As is evident, there is constant returns to scale for the intermediaries, so for simplicity we can envision just one marketmaker is needed in equilibrium. We denote $y_A[d_A(N_A, N_B)]$, as the *number of contracts* produced for type A of size (N_A, N_B) and $y_B[d_B(N_A, N_B)]$ as the *number of contracts* produced for type B of size (N_A, N_B). These are counting measures and there is nothing random. Also, these numbers are on a continuum and so do not have to take on integer values. Further, we denote the *number of platforms* of size (N_A, N_B) as $y(N_A, N_B)$. Thus $N_A \times y(N_A, N_B)$ is the number of type A's in total on the type of platform $y(N_A, N_B)$. Similarly, $N_B \times y(N_A, N_B)$ for type B.

The total number of agents of each type on a platform must be consistent with the total number of contracts offered for that type. Thus as indicated above, the intermediary must satisfy the following matching constraint:

$$\frac{y_A[d_A(N_A, N_B)]}{N_A} = \frac{y_B[d_B(N_A, N_B)]}{N_B} = y(N_A, N_B) \ \forall d_A \in D_A, \forall d_B \in D_B \tag{4}$$

This constraint states that the measure of contracts created of size (N_A, N_B) for type A must have the equivalent number of contracts created for type B normalized by the number of agents in each platform. The actual mathematics takes into account the continuum measure of each type of mass 1. For example, if we multiply each type by 100 we will

[13]Since an agent can join a platform which is only populated by that agent (autarky/singleton platform), this matching constraint essentially requires agents to join at most one platform in equilibrium.

have larger numbers for each type but the same proportions. For example, consider 0.1 platforms are created which match three merchants and two consumers. This would require $0.1 \times 3 = 0.3$ merchant contracts and $0.1 \times 2 = 0.2$ consumer contracts for a platform of size $(3, 2)$. We could also multiply this by 100 to have the numbers 10, 30 and 20 respectively.

A platform of size (N_A, N_B) requires the following amount of capital:

$$C(N_A, N_B) = \begin{cases} 0 & \text{if } N_A = 0 \text{ or } N_B = 0 \\ c_A N_A + c_B N_B + c N_A N_B + K & \text{else} \end{cases}$$

The capital requirement of a singleton/opt-out platform is normalized to zero as it costs nothing to produce and is always available. The amount of capital required for a platform has a positive marginal cost for an extra agent on each side of the platform (captured by c_A and c_B) and for the multiple of agents on both sides (captured by the interaction term c). Additionally, we model there can be some fixed cost, K, in creating a platform. For a more flexible specification we allow c_A and c_B to be different. We assume that c_A, c_B, $c \in (0, \infty)$ and $K \in [0, \infty)$. We require c_A, c_B and c to be strictly larger than zero, this ensures we can bound the size of the equilibrium platforms.

We denote the amount of capital input purchased by the intermediary as y_κ – this has to be sufficient to build the proposed platforms. So we can write the intermediary's capital constraint as:

$$\sum_{N_A, N_B} y(N_A, N_B)[C(N_A, N_B)] \leq y_\kappa \tag{5}$$

Hence, the intermediary's production set is:

$$Y = \Big\{(y, y_A, y_B, y_\kappa) \in \mathbb{R}^{2(\overline{N_A} \times \overline{N_B} + 1) + 1} | \text{ (4) and (5) are satisfied}\Big\}$$

It is a convex cone as in McKenzie [1959].

We explore the role of market power on platform supply and agent welfare by modeling two different environments: first, we model a price-taking intermediary and second, we model a price-setting intermediary who has market power and can set both quantity and price of each platform contract.

2.3. **Competition: price-taking intermediary.** The intermediary takes the Walrasian prices $p[d_T(N_A, N_B)]$ $\forall d_T \in D_T, T \in \{A, B\}$ and maximizes profits constructing platforms and selling type specific matchings (again we normalize the price of capital to be one):

$$\pi = \max_{y_A, y_B, y_\kappa \in Y} \sum_{N_A, N_B} \{p[d_A(N_A, N_B)] \times y_A[d_A(N_A, N_B)] + p[d_B(N_A, N_B)] \times y_B[d_B(N_A, N_B)]\} - y_\kappa \tag{6}$$

Equation (6) states the intermediary maximizes how many platforms of a given size (N_A, N_B) to produce given the prices for each position in the platform. The intermediary's profits are equal to the number of contracts it constructs multiplied by their respective price minus the cost of the capital input.

The intermediary's first order condition for creating a platform $y(N_A, N_B)$ is:

$$C(N_A, N_B) \geq p[d_A(N_A, N_B)] * N_A + p[d_B(N_A, N_B)] * N_B \tag{7}$$

where equation (7) holds with equality if there are positive number of platforms of that size (N_A, N_B). If equation (7) is a strict inequality then no such platform exists in equilibrium. Notice this natural condition requires that the payments/memberships received by the platform must cover all the platform's costs.

2.3.1. *Competition: Market Clearing.* For market clearing we require the following conditions to hold

$$\sum_s \alpha_{T,s} x_{T,s}[d_T(N_A, N_B)] = y_T(d_T(N_A, N_B) \; \forall N_A, N_B, \; T \in \{A, B\} \tag{8}$$

$$\sum_{T,s} \alpha_{T,s} \kappa_{T,s} = y_\kappa \tag{9}$$

Equation (8) ensures that the (decentralized) amount of demand for each contract for each type equals the (decentralized) supply of that contract. Equation (9) states the total endowment of capital (the supply) must equal the amount of capital used by the intermediary.

2.3.2. *Competitive Equilibrium.* Let us define x as the vector of contracts bought $x_{T,s}[d_T(N_A, N_B)]$ for all subtypes (T, s), then a competitive equilibrium in this economy is $(p, x, \{y, y_A, y_B, y_\kappa\}) \in L \times X \times Y$ such that for given prices $p[d_T(N_A, N_B)]$:

(1) The allocation $\{x_{T,s}[d_T(N_A, N_B)]\}$ solves the agent's maximization problem [i.e. $x_{T,s}[d_T(N_A, N_B)]$ solves equation (1) subject to equations (2 and 3)].
(2) The allocation $\{y, y_A, y_B, y_\kappa\}$ solves the platform's maximization problem [i.e. $\{y, y_A, y_B, y_\kappa\}$ solves equation (6) subject to $\{y, y_A, y_B, y_\kappa\} \in Y$].
(3) The market clearing conditions hold [equations (8) and (9) hold].

In equilibrium, the pricing mechanism will determine the size and number of each platform and subsequently the relative proportions of merchants and consumers on each platform.

2.4. **Monopoly: price-setting intermediary.** In contrast to section (2.3), we model the intermediary as a price-setting monopolist, who sets prices $p[d_T(N_A,N_B)]$ $\forall d_T \in D_T, T \in \{A,B\}$ and quantities $y_T(d_T(N_A,N_B)$ $\forall d_T \in D_T, T \in \{A,B\}$ to maximize profits subject to quantity demanded being greater than or equal to quantity supplied:

$$\pi = \max_{p,y_A,y_B,y_\kappa \in L\times Y} \sum_{N_A,N_B} \{p[d_A(N_A,N_B)] \times y_A[d_A(N_A,N_B)] + p[d_B(N_A,N_B)] \times y_B[d_B(N_A,N_B)]\} - y_\kappa \tag{10}$$

$$\text{s.t. } \sum_s \alpha_{T,s} x_{T,s}[d_T(N_A,N_B)] \geq \; y_T(d_T(N_A,N_B) \; \forall N_A,N_B, \; T \in \{A,B\} \tag{11}$$

Equation (10) states the intermediary problem maximizes how many platforms of a given size to produce and the price to each side of the market ($p[d_A(N_A,N_B)]$ and $p[d_B(N_A,N_B)]$) for each position in the platform subject to quantity supplied being less than or equal to total demand for each contract.

2.4.1. *Monopolistic Equilibrium.* Then a monopolistic equilibrium in this economy is $(p, x, \{y, y_A, y_B, y_\kappa\}) \in L \times X \times Y$ such that:

(1) The allocation $\{x_{T,s}[d_T(N_A,N_B)]\}$ solves the agent's maximization problem [i.e. $x_{T,s}[d_T(N_A,N_B)]$ solves equation (1) subject to equations (2 and 3)].
(2) The allocation $\{y, y_A, y_B, y_\kappa\}$ and prices p solves the platform's maximization problem [i.e. $(p\{y, y_A, y_B, y_\kappa\})$ solves equation (10) subject to $(p\{y, y_A, y_B, y_\kappa\}) \in L \times Y$ and equation (11)].

3. Social Planner's Problem

First, we set up the social planner's problem and determine the set of all Pareto optimal contracts. We show (i) a competitive equilibrium is Pareto optimal, (ii) *any* Pareto optimal allocation can be achieved with lump-sum transfers and taxes among agents and (iii) there exists a competitive equilibrium. So these results have two important implications (i) the decentralized problem is Pareto optimal and (ii) when solving for the competitive equilibrium, we can use the simpler social planner's problem to compute the allocation. Subsequently, we can use the Lagrangian multipliers to impute the competitive equilibrium prices and wealth associated with that allocation.

The social planner's welfare maximizing problem with Pareto weights $\lambda_{A,i}$ and $\lambda_{B,j}$ for types (A,i) and (B,j) respectively is:

$$\max_{x\geq 0, y\geq 0} \sum_i \lambda_{A,i} \left\{ \sum_{d_A(N_A,N_B)} \alpha_{A,i} x_{A,i}[d_A(N_A,N_B)] U_A(N_A,N_B) \right\} + \sum_j \lambda_{B,j} \left\{ \sum_{d_B(N_A,N_B)} \alpha_{B,j} x_{B,j}[d_B(N_A,N_B)] U_B(N_A,N_B) \right\}$$

$$\text{s.t.} \sum_{b(N_A,N_B)} x_{T,s}[d_T(N_A,N_B)] = 1 \ \forall T, s \tag{12}$$

$$\sum_s \alpha_{T,s} x_{T,s}[d_T(N_A,N_B)] = y(N_A,N_B) \times N_T \ \forall d_T \in D_T, \forall T \in \{A,B\} \tag{13}$$

$$\sum_{N_A,N_B} y(N_A,N_B)[C(N_A,N_B)] \leq \sum_{T,s} \alpha_{T,s} \kappa_{T,s} \tag{14}$$

Equation (12) ensures that each individual is assigned to a platform, equation (13) ensures that the total purchase of contracts equals the number of contracts produced, and equation (14) ensures the total number of contracts produced is resource feasible.

3.1. **Dual.** The Pareto Problem can also be written in terms of its dual equivalent:

$$\min_p \sum_{T,s} \alpha_{T,s}(p_{T,s} + p_\kappa \kappa_{T,s})$$

$$\text{s.t. } p_{T,s} + p_T[d_T(N_A,N_B)] \geq \lambda_{T,s} U_T(N_A,N_B) \ \forall i, \forall T, \forall (N_A,N_B) \tag{15}$$

$$C(N_A,N_B) - \{p_A[d_A(N_A,N_B)] \times N_A + p_B[d_B(N_A,N_B)] \times N_B\} \geq 0 \ \forall (N_A,N_B)$$

In this formulation $p_{T,s}$ is the imputed valuation for type T,s. So $p_{T,s}$ will be higher, the scarcer the type. And p_κ is the imputed valuation for capital. The dual problem minimizes the aggregate cost of the economy (in terms of prices of each type and total capital) such that each type of agent receives a given level of Pareto weighted utility. The primal problem maximizes the Pareto weighted expected utility of each type subject to the matching and resource constraints.

The Pareto problem is well defined in both the primal and dual form therefore, by the 'strong duality property'[14] there must exist an optimal solution (p^*, x^*, y^*) such that:

[14]See Bradley et al. [1977] pages 142-143 for more details.

$$\sum_{T,s} \lambda_{T,s} \left\{ \sum_{d_T(N_A,N_B)} \alpha_{T,s} x^*_{T,s}[d_T(N_A,N_B)] U_T(N_A,N_B) \right\} = \sum_{T,s} \alpha_{T,s}(p^*_{T,s} + p^*_\kappa \kappa_{T,s})$$

In the proofs of going between the Pareto allocation and competitive equilibrium we will assume that individuals are non-satiated – this is not a crucial assumption since we can always expand the commodity space such that this holds.

The following theorems show that for all Pareto weights, there is a competitive equilibrium which replicates the social planner's problem.

Theorem 1. *If all agents are non satiated, a competitive equilibrium* (p^*, x^*, y^*) *is a Pareto optimal allocation* (x^*, y^*)*. [First Welfare Theorem]*

Proof follows from Prescott and Townsend [2005].

Theorem 2. *Any Pareto optimal allocation* (x^*, y^*) *can be achieved through a competitive equilibrium with transfers between agents subject to there being a cheaper point for all agents and agents are non-satiated.*

The proof relies on using the Pareto weights $\lambda_{T,s}$ and the dual variables from the planner problem, to claim the Pareto optimal allocation (x^*, y^*) can be supported as a competitive equilibrium with transfers between agents. The proof follows from Prescott and Townsend [2005].

Theorem 3. *There exists a competitive equilibrium.*

To provide a more general proof of existence of a competitive equilibrium, where the distribution of wealth across individuals is taken as given, but there is no restriction on the mass of agents requires the use of a fixed point theorem. Negishi [1960] alters the Pareto weights in the economy such that the budget constraints binds for all agents at the fixed point. The proof follows from Prescott and Townsend [2005].

4. Results

4.1. **How does market power affect the allocation of resources and rent?** In section (3) we showed that the competitive equilibrium is a Pareto optimal allocation. In this subsection, we will analyze the monopolistic equilibrium. The main difference between the competitive and the monopolistic is the number and type of platforms created. In particular, the price-setting intermediary in the monopolistic equilibrium will restrict the supply of platforms to maximize his rent.

Theorem 4. *The price-setting intermediary in the monopolistic equilibrium will capture all the rent in the economy and will produce less slots than the price-taking intermediary in the competitive equilibrium.*

The proof relies on demonstrating that in the monopolistic equilibrium, where the intermediary has price-setting power, the intermediary will only produce a negligible amount of platforms to maximize profits. Whereas, in the competitive equilibrium, the equilibrium supply of platforms will ensure that the entire resources in the economy will be used to build platforms. The proof is provided in section (8.1) in the Appendix.

Overall, market structure changes both the allocation of rents and the allocation of resources within the economy. In particular, competition ensures that the intermediary makes no profits, and that surplus is accrued by the agents. Further, competition ensures that the all resources in the economy are used to produce platforms.

4.2. **Prices for joining a platform in a competitive equilibrium.** To understand in greater detail how prices are determined in the competitive equilibrium we can analyze the agent's maximization problem in more detail. The agents' maximization problem can be written as the following Lagrangian maximization problem. We can use this to show which contracts the agent of type T buys.

$$\begin{aligned} L &= \sum x_{T,s}[d_T(N_A,N_B)]U_T[d_T(N_A,N_B)] - \mu^W_{T,s}\left(\sum x_{T,s}[d_T(N_A,N_B)]p[d_T(N_A,N_B)] - \kappa_{T,s}\right) - \\ &\quad \mu^P_{T,s}\left(\sum_{N_A N_B} x_{T,s}[d_T(N_A,N_B)] - 1\right) \end{aligned}$$

The first order condition for Type T and contract $x_{T,s}[d_T(N_A,N_B)]$ is:

$$U_T(N_A,N_B) - \mu^P_{T,s} - \mu^W_{T,s} * p[d_T(N_A,N_B)] \le 0 \tag{16}$$

Where $\mu^P_{T,s}$ is the Lagrangian multiplier associated with the individual being assigned to some platform and $\mu^W_{T,s}$ is the Lagrangian multiplier associated with the agent's budget constraint. Furthermore, for any platform the agent buys with positive probability ($x_{T,s}[d_T(N_A,N_B)] > 0$), then the equation will hold with equality. If the left hand side of equation (16) is strictly less than zero, this implies that agent will not purchase that contract.

Let us consider what equation (16) implies. Consider an agent of Type T who purchases positive probabilities of two different contracts $d_T(N_A,N_B)$ and $d_T(N'_A,N'_B)$. Let us define the variable $\Delta U \equiv U_T(N_A, N_B) - U_T(N'_A, N'_B)$ and $\Delta p \equiv p[d_T(N_A, N_B)] - p[d_T(N'_A, N'_B)]$, then we can state:

$$\Delta U = \mu^W_{T,s} * \Delta p$$

Therefore, if an individual buys two contracts with positive probability, the difference in utility she will derive will be a constant multiplied by the change in price. Intuitively, if an individual is indifferent between two contracts, then the change in utility must be proportional to the change in price.

In general, an agent is unwilling to pay proportionally more for a contract which confers proportionally more utility (that is, $\frac{\Delta U}{U} \neq \frac{\Delta p'}{p}$) – this is only true when the individual's matching constraint is not binding (i.e. $\mu^P_{T,s} = 0$). Intuitively, when an individual's matching constraint binds, this individual would prefer to join more platforms but is constrained by the ability to only join one platform. In turn, this will ensure the percentage increase in the individual's willingness to pay to join the platform which confers the greater utility will be more than the percentage change in utility; both platforms require the same assignment of type component, but one platform confers greater utility.

5. Competitive Equilibrium Examples

In a general equilibrium framework we can analyze both how the composition of platforms and how the resulting utilities change as we alter some of the parameters. First, as a useful benchmark we examine an equilibrium where we have symmetric parameters for both sides of the market, that is the same costs, preferences and wealth. Second, we provide an example which varies the wealth *within* and *across* types. We show that even with symmetric preferences a subtype with lower wealth may be better off than an alternative subtypes.

Third, we are interested how the equilibrium – and subsequently agents' utilities – change as we redistribute wealth within our economy. Fourth, we examine how the equilibrium utilities change as we alter the Pareto weights. We show that even if an agent's relative Pareto weight falls, their equilibrium utility can actually rise – it depends on the General Equilibrium effects.

Fifth, given that the cost of producing platforms changes over time (for instance due to technological improvement) we demonstrate how the equilibrium utilities change as we alter the fixed cost of producing platforms. We show that increasing fixed costs leads to heterogeneous effects and potentially increases inequality.

5.1. **Example 1: Symmetric wealth, preferences and cost parameters in a competitive equilibrium.** Our initial example has two subtypes for each type and is symmetric – there is equal fractions of each type ($\alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2}$), each subtype has

TABLE 1. Equilibrium platforms and user utility for Example 1.

Equilibrium Platforms		
Platform Size (N_A, N_B)	Number of Platforms created $y(N_A, N_B)$	Cost of Production $C(N_A, N_B)$
$(2, 2)$	0.5	8

Equilibrium user utility and platform choice.					
Type T, s	Wealth $\kappa_{T,s}$	Platform joined (N_A, N_B)	Price of joining $p(d_T[N_A, N_B])$	Pr(joining) $x_{T,s}(d_T[N_A, N_B])$	Utility on Platform $U_T(N_A, N_B)$
$A, 1$	2	$(2, 2)$	2	1	2.41
$A, 2$	2	$(2, 2)$	2	1	2.41
$B, 1$	2	$(2, 2)$	2	1	2.41
$B, 2$	2	$(2, 2)$	2	1	2.41

the same Pareto weight ($\lambda_{A1} = \lambda_{A2} = \lambda_{B1} = \lambda_{B2}$), the cost function is the same for both types ($c_A = c_B$) and the utility functions' parameters are the same ($\gamma_A = \gamma_B$ and $\epsilon_A = \epsilon_B$) .[15] In this initial example, although there are nominally two subtypes, they are in fact identical therefore there is no variation by subtype.

In this equilibrium only one type of platform is created. All users pay a price of two units of capital to join a platform which matches them with two users of the other type, and one more user of their own type, so the total for each type is two.

5.2. **Example 2: Different wealth but same preferences and cost parameters in a competitive equilibrium.** Our second example varies wealth both *within* and *across* types. To improve intuition, let us consider a payment platform which connects merchants to consumers. There are two subtypes of merchants – Small $(A, 1)$ and Big $(A, 2)$; and two subtypes of consumers – Rural $(B, 1)$ and Urban $(B, 2)$. Each consumer would prefer to be on platform with more merchants (more places to pay) and less consumers (less congestion). Similarly, merchants want as many consumers to use the same platform but would like less rival merchants.

There is equal fractions of each type ($\alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2}$), the cost function is the same for both types ($c_A = c_B$) and the utility functions' parameters are the same ($\gamma_A = \gamma_B$ and $\epsilon_A = \epsilon_B$), however the agents vary in wealth.[16]

[15]The parameter values are:
$\alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2} = \frac{1}{2}$; $c_A = c_B = c = 1,\ K = 0$; $\gamma_A = \gamma_B = \epsilon_A = \epsilon_B = \frac{1}{2}$
$\lambda_{A1} = \lambda_{A2} = \lambda_{B1} = \lambda_{B2} = \frac{1}{4}$

[16]The parameter values are:
$\alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2} = \frac{1}{2}$; $c_A = c_B = c = 1,\ K = 0$; $\gamma_A = \gamma_B = \epsilon_A = \epsilon_B = \frac{1}{2}$

TABLE 2. Equilibrium platforms and user utility for Example 2.

Platform Size (N_A, N_B)	Number of Platforms created $y(N_A, N_B)$	Cost of Production $C(N_A, N_B)$
$(3, 2)$	0.25	11
$(1, 2)$	0.25	5

	Type T, s	Wealth $\kappa_{T,s}$	Platform joined (N_A, N_B)	Price of joining $p(d_T[N_A, N_B])$	Pr(joining) $x_{T,s}(d_T[N_A, N_B])$	Utility on Platform $U_T(N_A, N_B)$	Expected Utility
A	Merch, Small $(A, 1)$	1.37	$(3, 2)$	1.37	1	2.23	2.23
	Merch, Big $(A, 2)$	1.64	$(3, 2)$	1.37	0.5	2.23	2.53
			$(1, 2)$	1.91	0.5	2.8	
B	Cons, Rural $(B, 1)$	1.54	$(1, 2)$	1.54	1	1.7	1.7
	Cons, Urban $(B, 2)$	3.45	$(3, 2)$	3.45	1	2.96	2.96

In this equilibrium, there are two different types of platforms created. One platform is larger than the other and is populated with more merchants; the richer urban consumers join this platform. Urban consumers have higher resultant utility since this platform is both bigger and has a more favorable ratio of merchants to consumers. Whereas, the poorer rural consumers join the smaller platform, this platform is both smaller and is populated with a less favorable ratio of merchants to consumers causing lower utility for rural consumers (and lower prices for consumers to join that platform).

The urban and rural consumers, and the small merchants all buy degenerate lotteries of contracts, where they are assigned to a particular platform with probability one. However, the big merchants buy probabilities in two different platforms, therefore 50% of them are allocated to platforms of size $(3, 2)$ and 50% are allocated to platforms of size $(1, 2)$. The respective prices for these two different contracts are 1.37 and 1.91.

The cost of the platform is primarily borne by the type which receives the most utility from the platform (the consumers); therefore, in the platforms of size $(3, 2)$ consumers pay 63% of the platform's cost, even though, they are only 40% of the platform's population.

Further, rural consumers are *wealthier* than small merchants, yet they are *worse off.* Urban consumers are much richer than the other participants hence they are willing to sponsor larger platforms. This allows merchants to contribute less towards joining a platform. Another way to look at this is that the merchants are in scarcer supply (since consumers are so much wealthier – average consumer wealth is 2.5 and average merchant wealth is only 1.5), therefore the price schedule they face is lower than the consumers' schedule.[17]

[17]Recall that the cost parameters $c_A = c_B$ therefore, the asymmetric prices and allocations are solely driven by agents' different capital endowments.

5.3. **How does the competitive equilibrium change as we redistribute endowments?** If we redistribute wealth in our economy, this will change the relative demand for merchants and consumers and subsequently change the relative prices to join a given platform. To examine the general equilibrium effects of redistributing wealth, we construct two placebo interventions which reallocate wealth within our economy whilst holding the total resources constant.

Figure (1) shows the effects of redistributing wealth within our economy while holding total resources constant (with the same cost and preferences as in the previous examples). The left panel shows the effects on utility from redistributing wealth *across* types,[18] in particular between $(B,1)$ and $(A,2)$ – i.e. $\kappa_{B,1} + \kappa_{A,2} \approx 2.4$. The right panel shows the effects on utility from redistributing wealth *within* type,[19] in particular between $(B,1)$ and $(B,2)$.

Recall our payment platform example which connects merchants to consumers. There are two subtypes of merchants – Small $(A,1)$ and Big $(A,2)$; and two subtypes of consumers – Rural $(B,1)$ and Urban $(B,2)$. As we redistribute wealth from big merchants $(\kappa_{A,2})$ to rural consumers $(\kappa_{B,1})$, the utility of urban consumers $(B,2)$ falls (holding both urban consumer's wealth $(\kappa_{B,2})$ and and small merchant's wealth $(\kappa_{A,1})$ constant).

As we increase rural consumer's wealth $(\kappa_{B,1})$, the demand to join platforms with merchants rises. Subsequently the price for consumers to join platforms for a given number of merchants will also rise. Therefore, since urban consumer's wealth $(\kappa_{B,2})$ is a constant and they now face higher prices, their utility must fall. Symmetrically, a similar result holds when we increase the wealth of small merchants on big merchants' utility.

As one would expect the utility of merchants and consumers are increasing in their respective wealth.[20]

Further, we can consider how the equilibrium changes as we adjust the endowments within a type (consumers), and hold the endowments of the other types (merchants) fixed. There is no effect on merchants' utilities since any reduction in purchasing power by one of the consumer subtypes is compensated by an equal change in the other slightly richer consumer subtype. This example is shown in the right panel of figure (1).[21]

[18]We solve the model using the Pareto problem and then impute the wealth and prices which replicate the same allocation. We simulate 2880 equilibria for different Pareto weights, and then collect only the equilibria in which $0.51 < \kappa_{A,1} < 0.59$ and $1.01 < \kappa_{B,2} < 1.19$. We then 'join up' all the points to plot a smooth curve.

[19]We solve the model using the Pareto problem and then impute the wealth and prices which replicate the same allocation. We simulate 2880 equilibria for different Pareto weights, and then collected only the equilibria in which $0.51 < \kappa_{A,1} < 0.59$ and $1.01 < \kappa_{A,2} < 1.1$. We are approximately holding the endowment of $(A,1)$ and $(A,2)$ constant.

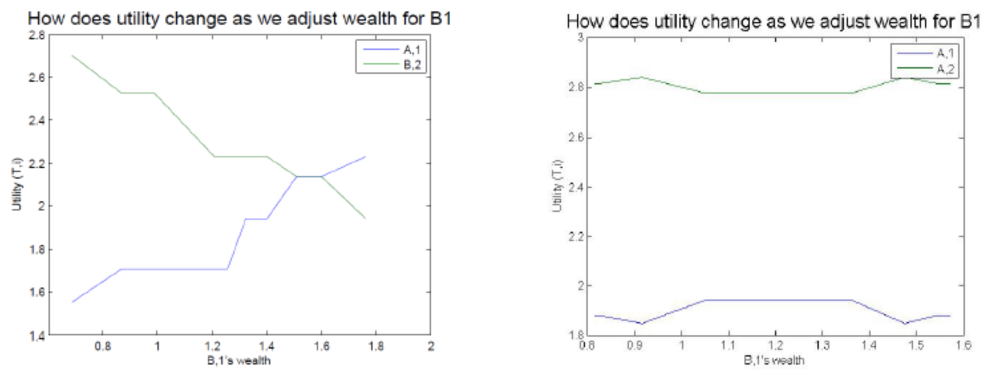
[20] Figure (6) in the appendix shows the effect of the wealth changes on all agents' utilities.

[21]There is a tiny change in the utility of $(A,2)$ this is due to the discrete nature of the possible platform combinations and the changes in the platforms type (B) can purchase.

FIGURE 1. Redistributing wealth within and across type

Redistributing wealth *across* type: | Redistributing wealth *within* type:

The effect on utility from transferring wealth from $(A, 2)$ to $(B, 1)$ | The effect on utility from transferring wealth from $(B, 2)$ to $(B, 1)$



Note we do not change $(A, 1)$ or $(B, 2)$'s wealth in these examples. The changes in $(A, 1)$ and $(B, 2)$'s utility is due to the general equilibrium effects of redistributing wealth from $(A, 2)$ to $(B, 1)$.

Note we do not change $(A, 1)$ or $(A, 2)$'s wealth in these examples. The changes in $(A, 1)$ and $(A, 2)$'s utility is due to the general equilibrium effects of redistributing wealth from $(B, 2)$ to $(B, 1)$.

5.4. **How does the competitive equilibrium change as we alter the Pareto weights?** We can also consider how the equilibrium changes as we adjust the Pareto weights[22] on only one subtype $(B, 2)$.

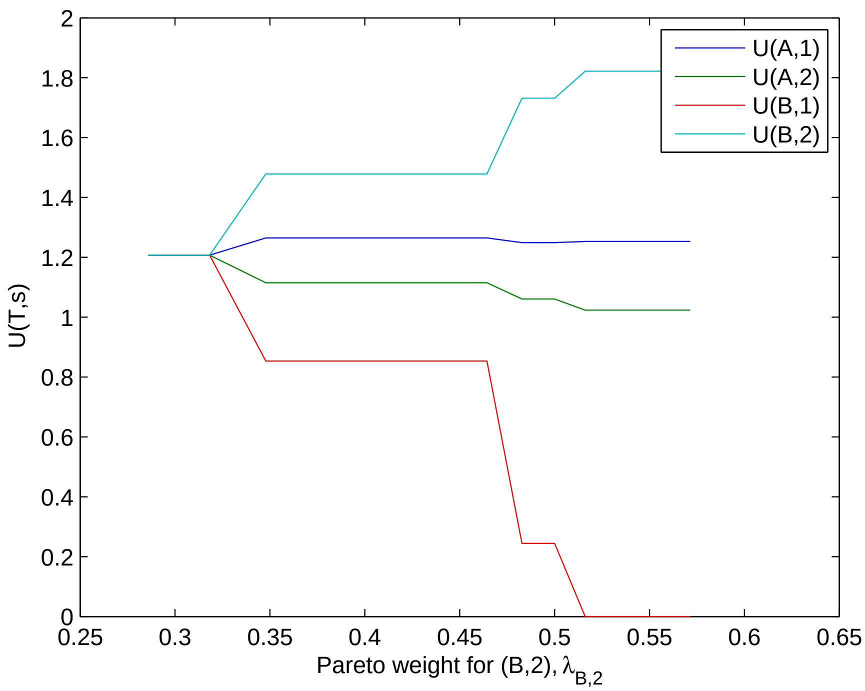
Figure (2) demonstrates (for given parameters) how the resulting utilities change as the Pareto weight for type $(B, 2)$ increases. First it is clear and intuitive the utility of $(B, 2)$ monotonically weakly increases with their respective Pareto weight. This is a general result, as the Pareto weight on $(B, 2)$ rises, it must be true that the utility for $(B, 2)$ weakly increases.

As figure (2) shows, type $(B, 1)$ is clearly disadvantaged. This is a general result and follows from the utility of subtype $(B, 2)$ rising.

Theorem 5. *If we increase the Pareto weight on subtype $(B, 2)$ by Δ and reduce the Pareto weight on subtype $(B, 1)$ by Δ, the utility of type $(B, 1)$ monotonically increases in Δ and the utility of subtype $(B, 1)$ monotonically falls in Δ, where $0 < \Delta < \lambda_{B,1}$.*

[22]The parameter values are:
$\alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2} = \frac{1}{2}$; $c_A = c_B = c = 1, K = 0$;$\gamma_A = \gamma_B = \epsilon_A = \epsilon_B = \frac{1}{2}$
$\lambda_{A1} = \frac{1.01-x}{3}, \lambda_{A2} = \frac{0.99-x}{3}, \lambda_{B1} = \frac{1-x}{3}, \lambda_{B2} = x$; We introduce a tiny wedge between $(A, 1)$ and $(A, 2)$ to highlight the effects on a favored subtype.

FIGURE 2. How does the utility for each subtype change as we alter the Pareto weight for Urban Consumers (B,2)



This follows from the result that a subtype's utility is monotonically increasing in their own Pareto weight and that the different subtypes compete amongst themselves to be matched with other type.

Recall our merchant and consumer example from before. If we increase the Pareto weight on urban consumers ($\lambda_{B,2}$), the allocation will match them in both larger platforms and with more merchants. This has two effects: first, there is less resources left for the rural consumers, and second, there are less merchants left unmatched.

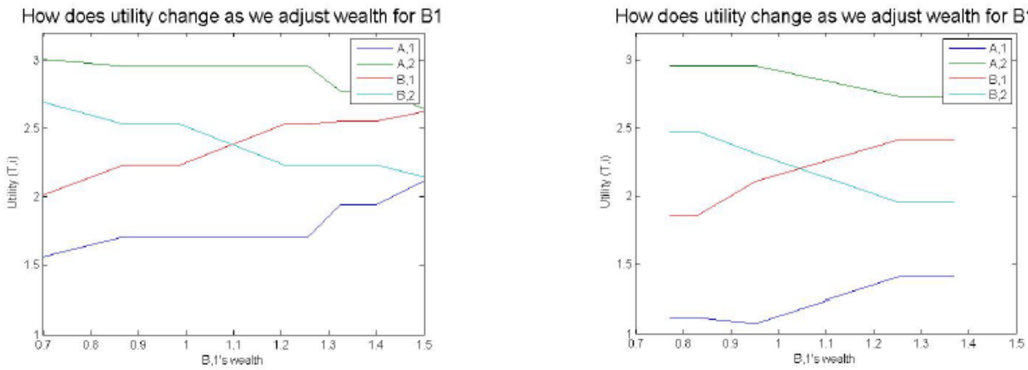
The story is more complicated for the merchants. An increase in the urban consumers Pareto weight can lead to lower or *higher* utility for merchants. One of the merchants subtypes will always be made worse off $(A, 1)$ by the rise in $(\lambda_{B,2})$ since some platforms are comprised of relatively more merchants – favoring consumers on those platforms.

As seen in figure (2) it is possible for one of the merchant subtypes $(A, 2)$ to be made better off – even as their relative Pareto weight *falls*. This follows from the most favored consumer subtype (urban) are matched to proportionally more merchants as $\lambda_{B,2}$ increases, this means the proportion of merchants to consumers remaining declines. Hence, those merchants who are not matched with urban consumers may be matched at favorable ratios of consumers to merchants – increasing their utility.

FIGURE 3. How does utility change as we alter the platform's cost function

Utility for each subtype as the wealth for (B,1) varies and $K = 0$

Utility for each subtype as the wealth for (B,1) varies and $K = 1$



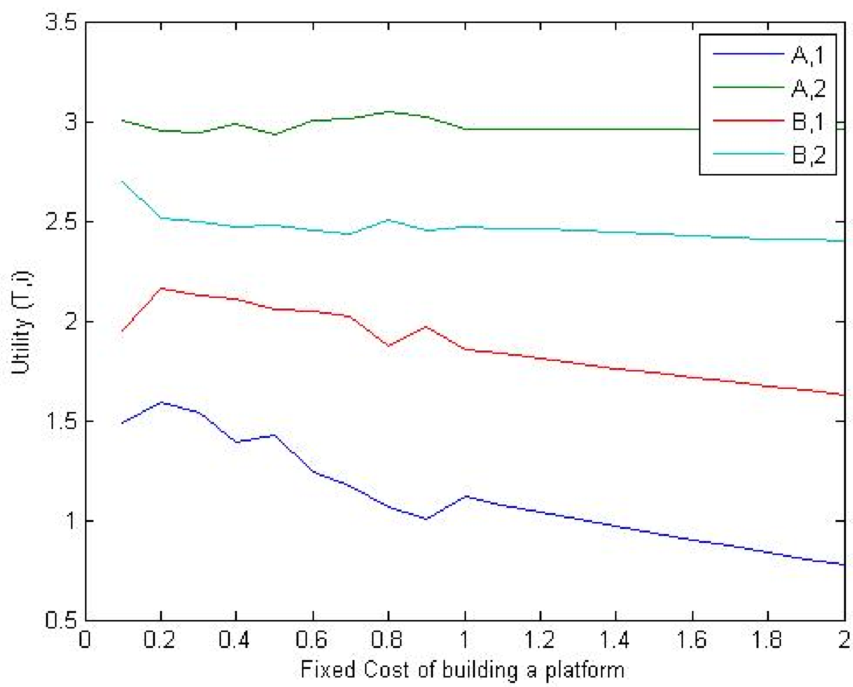
5.5. **How does the competitive equilibrium change as we alter the costs of building platforms?** A further important consideration is how the equilibrium changes as we adjust costs, for instance, technological innovations may decrease the costs of creating a platform. Figure (3) shows how the equilibrium changes for two different fixed costs for building a platform; the left panel shows the equilibrium utilities if the fixed cost is zero and the right panel shows the equilibrium utilities if the fixed cost is one unit of capital. As one would expect for a given distribution of wealth, utility is lower. However, the distribution itself changes. For larger fixed costs of producing a platform, the distribution becomes more dispersed, and inequality between different subtypes becomes more pronounced.

To gain intuition for this result, recall our interpretation that agents are endowed with two assets: labor and capital. As we increase the costs of producing platforms of a given size, this will lead to the relative value of capital to labor becoming larger. Therefore, those agents who are endowed with more capital are less hurt by the rise in costs, leading to greater inequality.

For example, consider the equilibrium with $\kappa_{B,1} = 1.09$, with the introduction of the fixed cost, the utility of subtype $(A, 1)$ falls by over 25% whereas for the richest subtype $(A, 2)$ the fall in utility is about 3%. A similar qualitative result holds for subtypes $(B, 1)$ and $(B, 2)$.

A further way to demonstrate how changes in a fixed cost have distributional impacts is to vary the fixed cost of building a platform whilst holding each agent's wealth constant. Figure (4) shows the equilibrium utility for each subtype when we vary the fixed cost of building a platform for given parameters.[23] As one can see the richest subtype $(A, 2)$ is barely affected by the rise in platform costs. Yet, the poorest subtype utility, $(A, 1)$'s utility, falls by about 50% as we increase the fixed cost of building a platform from 0.2 units of capital to 2 units of capital. Further supporting the evidence that changes in fixed costs of building platforms have heterogeneous effects and in particular the most adverse effects occur on the poorest agents.

FIGURE 4. How does the utility for each subtype change as we alter the fixed cost of building a platform



Do these distributional impacts suggest a rationale to regulate prices? No – our environment is Pareto optimal, so the optimal government intervention would be to introduce lump-sum taxation on the rich and transfers to the poor. This would increase the poorest's utility, achieving a more equitable division of utility while maintaining a Pareto optimal allocation. Other interventions would be distorting.

[23]The economy's parameters are $\alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2} = \frac{1}{2}$; $c_A = c_B = c = 1$;$\gamma_A = \gamma_B = \epsilon_A = \epsilon_B = \frac{1}{2}$; $\kappa_{A1} = 0.5, \kappa_{A2} = 1.5, \kappa_{B1} = 0.8, \kappa_{B2} = 1.1$.
For computational simplicity, we allow the equilibrium wealth levels to be close to the desired wealth levels ($\kappa_{A1} = 0.5, \kappa_{A2} = 1.5, \kappa_{B1} = 0.8, \kappa_{B2} = 1.1$). We only plot the equilibrium utilities for those equilibria such that the maximum difference between the desired wealth endowment and the plotted capital endowment is less than 0.1 units of capital for each subtype.

6. Extensions to the model

6.1. **How does user heterogeneity in preferences (within type) affect the competitive equilibrium?** We have concentrated on all types having the same preferences, but potentially varying in their wealth endowments. In this subsection, we consider how varying preferences *within* type affect the competitive equilibrium.

In our reformulated economy we introduce three new parameters $(\beta_1^{T,s}, \beta_2^{T,s}, \beta_3^{T,s})$, which potentially vary across type (T) *and subtype (s)*. The merchant (A,i)'s utility function is now:

$$U_{A,i}(N_A, N_B) = \begin{cases} 0 & \text{if } N_A \text{ or } N_B = 0 \\ \left[\beta_1^{A,i}\left(\frac{N_B}{N_A}\right)^{\gamma_A} + \beta_2^{A,i} N_B^{\epsilon_{A,}} + \beta_3^{A,i}\right] & \text{else} \end{cases}$$

In particular note that: $\beta_1^{A,i}$ alters the merchant (A,i)'s utility with respect to the ratio of consumers and merchants on the platform. $\beta_2^{A,i}$ alters the merchant (A,i)'s utility with respect to the size of the platform (holding the ratio of consumers and merchants constant). Finally, $\beta_3^{A,i}$ is the merchant (A,i)'s intrinsic value from joining a platform. Therefore, the introduction of the parameters $(\beta_1^{T,s}, \beta_2^{T,s}, \beta_3^{T,s})$ facilitate the comparison of how users who vary in their preferences alter the resulting equilibrium.[24]

Symmetrically, consumer (B,j)'s utility function is:

$$U_{B,j}(N_A, N_B) = \begin{cases} 0 & \text{if } N_A \text{ or } N_B = 0 \\ \left[\beta_1^{B,j}\left(\frac{N_A}{N_B}\right)^{\gamma_A} + \beta_2^{B,j} N_A^{\epsilon_{B,}} + \beta_3^{B,j}\right] & \text{else} \end{cases}$$

Recalling our prior example describing a payment platform, it is natural to consider that rural and urban consumers will vary in preferences as well as wealth. For instance, a rural consumer may be both poor and prefer to be on *any* platform (high $\beta_3^{B,j}$). Whereas, the urban consumer may prefer to have a choice of merchants (higher $\beta_1^{B,j}$).

Contrary to the Armstrong [2006], Weyl and White [2015], which show that heterogeneity in user preferences leads to market failure, our economy's competitive equilibrium remains Pareto efficient. The main difference in our papers' results from our differing modeling choices. In Armstrong [2006] and Weyl and White [2015]'s models, each oligopolistic platform potentially serves users with varying preferences and can only partially extract consumer surplus, leading to potentially socially inefficient prices. Whereas our model has free entry for platforms (as opposed to exogenously fixing the number of platforms), which (i) allows the possibility of complete platform differentiation and (ii) prevents pricing

[24]Note if $\beta_1^{T,s} = 1, \beta_2^{T,s} = 1$ and $\beta_3^{T,s} = 0$ for all types and subtypes, we have the same utility function as previous sections.

distortions due to market power. Subsequently, users may separate according to their preferences; for instance, if there are subtypes who strongly prefer larger platforms, they can join other agents who strongly prefer larger platforms.

The introduction of consumer heterogeneity in preferences leads to interesting comparative statics. To understand in greater detail how the differences in preferences affect the competitive equilibrium, we apply the new utility function to the experiment from section (5.5).

In figure (5) we plot how the equilibrium utilities (for the new utility function) for each subtype vary as we alter the fixed cost of building a platform for some given parameters.[25] To introduce differences in user preferences, we assume that (i) urban consumers $(B, 2)$ strongly prefer to be on a platform with a large number of merchants ($\beta_1^{B,2} = 3$) relative to rural consumers ($\beta_1^{B,1} = 1$) and (ii) the urban consumers are relatively indifferent about the size of the platform ($\beta_2^{B,2} = 0.01$), whereas the rural consumers prefer larger platforms ($\beta_2^{B,1} = 1$) – so the rural consumer's utility function is identical from previous sections. Similarly, the merchants' utility functions are unchanged from previous sections of the paper.

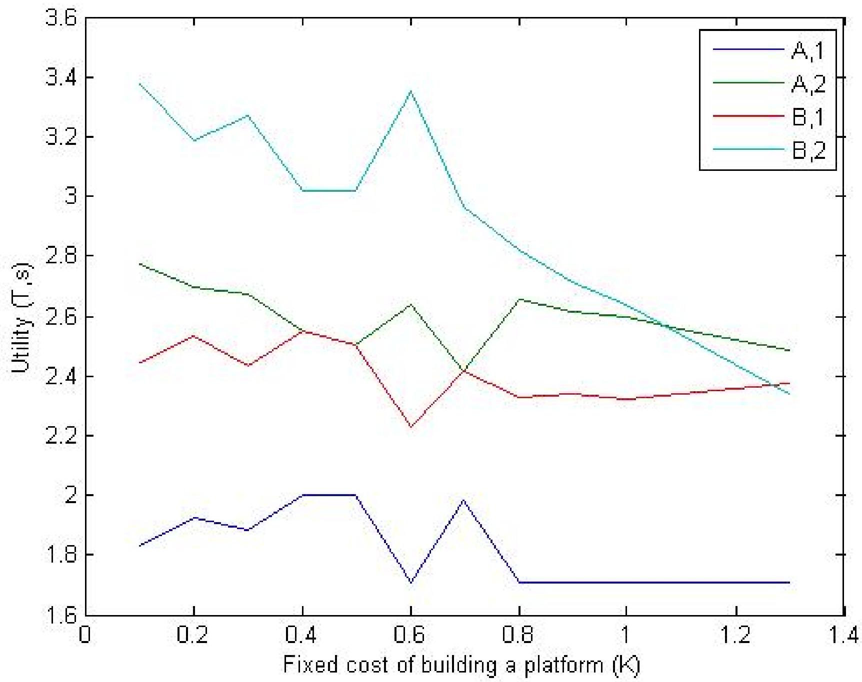
In figure (5) as we increase the cost of building platforms, urban consumers (B,2) – who prefer smaller platforms with a higher fraction of merchants – are affected the most. Whereas the rural consumers – who prefer larger platforms – are barely affected at all.

The increase in the fixed cost of building a platform causes the relative price of smaller platforms to become larger, which leads the competitive equilibrium to comprise of larger platforms. Therefore, even though the cost of building platforms is larger (and subsequently the production possibility frontier of the economy is shrinking), the equilibrium utility of rural consumers and the merchants are relatively unchanged. The big loser in this experiment are the urban consumers. In the new equilibrium, platforms which are larger and are populated by a more equal fraction of consumers and merchants are relatively cheaper to produce – hurting urban consumers, who prefer platforms which are largely populated by merchants.

6.2. **Multihoming.** Agents may wish to join multiple platforms. For instance, some consumers may prefer to use multiple forms of payment, some companies may prefer to

[25]The economy's parameters are $\alpha_{A1} = \alpha_{A2} = \alpha_{B1} = \alpha_{B2} = \frac{1}{2}$; $c_A = c_B = 1$;$\gamma_A = \gamma_B = \epsilon_A = \epsilon_B = \frac{1}{2}$; $\kappa_{A1} = 0.7, \kappa_{A2} = 1.3, \kappa_{B1} = 1, \kappa_{B2} = 1$.
Further we make urban consumers $(B, 2)$ strongly prefer platforms which comprise of a favorable ratio of consumers to merchants ($\beta_1^{B,2} = 3$), be mostly indifferent about the size of the platform ($\beta_2^{B,2} = 0.01$) and have little to no benefit from being on a platform ($\beta_3^{B,2} = 0.01$). For all other types, we maintain the previous utility function ($\beta_1^{A,1} = \beta_1^{A,2} = \beta_1^{B,1} = 1$), ($\beta_2^{A,1} = \beta_2^{A,2} = \beta_2^{B,1} = 1$) and ($\beta_3^{A,1} = \beta_3^{A,2} = \beta_3^{B,1} = 0$).
For computational simplicity, we allow the equilibrium wealth levels to be close to the desired wealth levels ($\kappa_{A1} = 0.7, \kappa_{A2} = 1.3, \kappa_{B1} = 1, \kappa_{B2} = 1$).

FIGURE 5. How does the utility for each subtype change as we alter the fixed cost of building a platform



list their stock on multiple exchanges, some traders may prefer to trade over many dark pools.

Our framework is sufficiently flexible to allow endogenous multihoming (agents can choose to join multiple platforms). In previous sections, we restricted individuals to only join one platform via our matching constraint – $\sum_s x_{T,s}[d_T(N_A, N_B)] = 1$.

We can relax the matching constraint and yet retain the linear programming nature of the problem. Therefore, we can model various different forms of multihoming by altering the matching constraint. For instance, we could require agents join two platforms (the matching constraint would be $\sum_s x_{T,s}[d_T(N_A, N_B)] = 2$), a maximum of two platforms (the matching constraint would be $\sum_s x_{T,s}[d_T(N_A, N_B)] \leq 2$), or as many platforms as the agent as the agent can afford (no matching constraint).

Relaxing the matching constraint will tend to create smaller, more numerous platforms in equilibrium. For instance, consider some very rich subtype, with singlehoming (an agent is only allowed to join a maximum of one platform), the rich subtype would only be able to sponsor larger or more unequal platforms. With the possibility of joining more than one platform, the rich subtype could sponsor multiple, smaller platforms which would lead to a higher utility (since utility is concave in the number of users of each type) and would generally be cheaper to produce (to be precise, the cost function for producing platforms exhibits decreasing returns to scale if there are no fixed costs of platform production, that is, if $K = 0$).

A potential shortcoming of our model is that utility is additive in the number of platforms an agent joins. Therefore, if an agent joins two identical platforms, the agent's utility would be double the utility from joining only one platform.

7. Conclusion

There are many economic platforms which must cater to multiple, differentiated users, who in turn, care about who else the platform serves. For instance credit cards, clearinghouses and dark pools, to name but a few (Rochet and Tirole [2003b], Ellison and Fudenberg [2003], Rochet and Tirole [2006], Caillaud and Jullien [2003], Armstrong [2006], Rysman [2009], Weyl [2010] and Weyl and White [2015]). Over-the-counter markets can also be conceptualized in this way – who is trading with whom, what is the network architecture, and what is the overall degree of direct and indirect connectedness (Allen and Gale [2000], Leitner [2005], Allen et al. [2012], Acemoglu et al. [2015], Cohen-Cole et al. [2014], Elliott et al. [2014]). Modeling each of these arrangements is inherently difficult and there is much more to be done. Here we try to capture each of the applications in a stylized way by building a common conceptual framework for analysis.

Our paper has three four contributions.

Our first contribution is methodological. As in the prior work on firms as clubs by Prescott and Townsend [2006], which builds on Koopmans and Beckmann [1957], Sattinger [1993], Hornstein and Prescott [1993], Prescott and Townsend [1984] Hansen [1985] and Rogerson [1988], we model an economy with competing platforms in a general equilibrium framework, with platforms as clubs. Our framework is relatively general; we can analyze an economy with many (i.e. more than two) types of users, who may have heterogeneous preferences; an economy with heterogeneous costs for servicing different users; or an economy with inherent differences within a type's wealth.

Second, our economy incorporates that an individual's utility may be contingent on the actions of others – in short an externality. But we show how to internalize interdependencies, so they do not lead to an inefficient equilibrium overall. In particular the potential externality is 'priced' – in a manner suggested by Arrow [1969]. The competitive equilibrium is efficient.

Third, we demonstrate how changes in one agent's wealth (or Pareto weight) has interesting general equilibrium effects both within- and across-types. The matching in the economy is endogenous and the math of assignment has to work out in the general equilibrium. For instance, consider a payment platform for consumers and merchants, where there are two subtypes of consumers – rural and urban consumers. An increase in the rural consumer's wealth will lead to *decreases* in the *urban* consumer's welfare and ambiguous effects on the merchant's welfare. This follows from our assumption that agents do not like to be on a platform with more of their own type, and therefore as we increase the rural consumer's wealth, they will prefer platforms with more merchants (and less consumers). This is also bad for some merchants with low wealth, as they are now on platforms with fewer consumers and relatively more merchants. Further, the rise in the rural consumer's wealth will lead the rural consumers to pay a greater fraction of the costs of being on a platform – in turn, this has distributional general equilibrium effects on the other types' welfare.

Fourth, we show how technological progress may reduce inequality. A reduction in the fixed cost of building a platform reduces the relative value of capital (that is wealth) and subsequently allows both bigger and more platforms to be created which in turn creates more demand from the various subtypes. The biggest utility gain is for the lowest wealth subtypes, who can now join some platforms rather than reside in autarky/non participation.

We should make clear at the same time the limitations of our framework. First, our model is purely static, and we exclude any coordination failures (Caillaud and Jullien [2003], Ellison and Fudenberg [2003], Ellison et al. [2004], Ambrus and Argenziano [2009], Lee [2013], Weyl and White [2015]) and any possibility of innovation in platform design as an intrinsic part of the model.

Second, no platforms or agents have any pricing power in our model, which as Weyl [2010] and Weyl and White [2015] show may interact with the agent's preferences over other agent's actions to exacerbate or minimize market failures. That literature is concerned with the allocation of fees. In our Walrasian set up there is no rationale for the regulation of prices on a platform – if a social planner wishes to implement a more equitable allocation, a social planner should redistribute wealth and not regulate prices.

Third, the only source of platform differentiation arises from the size and composition of a platform's users. Relatedly, we also require that the characteristics of agents to be clearly identified and rules enforced (i.e. no adverse selection or false advertising). Some might find it implausible that the neighborhood composition can be so tightly controlled.

Fourth, we do not allow ever increasing economies of scale in platform size. The existence of economies of scale remains an empirical matter, depending on the particular platform and market one has in mind. But for some there is no presumption of ever increasing returns. Duffie and Zhu [2011] argues this with central counterparty clearing house (CCP) platforms but O'Hara and Ye [2011] for equity market platforms and Altinkiliç and Hansen [2000] for capital issuance find contrary evidence.

Our work differs from the existing two-sided market literature and the macro financial literature in two key ways. First, our method is different. We concentrate on modeling platforms in a Walrasian equilibrium with extended commodity space with complete contracts and exclusivity. In contrast, the two-sided market literature concentrates on modeling platforms in a partial equilibrium environment. The macro financial literature typically imposes incomplete contracts or a particular institutional arrangement or game. Second, the two sided market literature focuses on how market power and imperfect competition affect platform economics, while our framework considers perfect competition between platforms. The macro financial literature argues explicitly or implicitly for regulation, to ensure stability, where we argue for the appropriate design of markets ex ante and letting rights to trade be priced in equilibrium (see also Kilenthong and Townsend [2014]).

We do not view our paper as the final word. In some sense we are trying to arbitrage across distinct literatures, bringing some general equilibrium insights to applied problems in industrial organization and market design/ regulation. Ultimately, modeling and understanding platform economies with more nuanced but important details is crucial. We hope this paper ignites a discussion on how to model and analyze multiple, competing platforms.

References

D. Acemoglu, A. Ozdaglar, and A. Tahbaz-Salehi. Systemic risk and stability in financial networks. *American Economic Review*, 105(2):564–608, 2015.

F. Allen and D. Gale. Financial contagion. *Journal of Political Economy*, 108(1):1–33, 2000.

F. Allen, A. Babus, and E. Carletti. Asset commonality, debt maturity and systemic risk. *Journal of Financial Economics*, 104(3):519–534, 2012.

O. Altinkiliç and R. Hansen. Are there economies of scale in underwriting fees? evidence of rising external financing costs. *Review of Financial Studies*, 13(1):191–218, 2000.

A. Ambrus and R. Argenziano. Asymmetric networks in two-sided markets. *American Economic Journal: Microeconomics*, 1(1):17–52, 2009.

M. Armstrong. Competition in two-sided markets. *The RAND Journal of Economics*, 37 (3):668–691, 2006.

K. J. Arrow. The organization of economic activity: issues pertinent to the choice of market versus nonmarket allocation. *The Analysis and Evaluation of Public Expenditure: the PPB system*, 1:59–73, 1969.

K. J. Arrow and G. Debreu. Existence of an equilibrium for a competitive economy. *Econometrica: Journal of the Econometric Society*, pages 265–290, 1954.

Arstechica. Level 3 claims six isps dropping packets every day over money disputes. 2014. URL `http://arstechnica.com/information-technology/2014/05/level-3-claims-six-isps-dropping-packets-every-day-over-money-disputes/`.

W. F. Baxter. Bank interchange of transactional paper: Legal and economic perspectives. *The Journal of Law & Economics*, 26(3):541–588, 1983.

S. Bradley, A. Hax, and T. Magnanti. Applied mathematical programming. 1977.

B. Caillaud and B. Jullien. Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics*, pages 309–328, 2003.

E. Cohen-Cole, A. Kirilenko, and E. Patacchini. Trading networks and liquidity provision. *Journal of Financial Economics*, 2014.

D. Duffie and H. Zhu. Does a central clearing counterparty reduce counterparty risk? *Review of Asset Pricing Studies*, 1(1):74–95, 2011.

M. Elliott, B. Golub, and M. O. Jackson. Financial networks and contagion. *American Economic Review*, 104(10):3115–53, 2014.

G. Ellison and D. Fudenberg. Knife-edge or plateau: When do market models tip? *The Quarterly Journal of Economics*, 118(4):1249–1278, 2003.

G. Ellison, D. Fudenberg, and M. Möbius. Competing auctions. *Journal of the European Economic Association*, 2(1):30–66, 2004.

D. S. Evans, R. Schmalensee, et al. The economics of interchange fees and their regulation: an overview. (May):73–120, 2005.

K. Foster, S. Schuh, and H. Zhang. The 2010 survey of consumer payment choice. *FRB of Boston Public Policy Discussion Paper*, (13-2), 2013.

D. Gale, H. W. Kuhn, and A. W. Tucker. Linear programming and the theory of games. *Activity Analysis of Production and Allocation*, 13:317–335, 1951.

V. Guerrieri, R. Shimer, and R. Wright. Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862, 2010.

A. Hagiu. Pricing and commitment by two-sided platforms. *The RAND Journal of Economics*, 37(3):720–737, 2006.

G. D. Hansen. Indivisible labor and the business cycle. *Journal of Monetary Economics*, 16(3):309–327, 1985.

A. Hornstein and E. C. Prescott. The firm and the plant in general equilibrium theory. *General equilibrium, growth, and trade*, 2:393–410, 1993.

W. T. Kilenthong and R. M. Townsend. A market based solution to price externalities: A generalized framework. *National Bureau of Economic Research Working Paper No. 20275*, 2014.

T. C. Koopmans and M. Beckmann. Assignment problems and the location of economic activities. *Econometrica: Journal of the Econometric Society*, pages 53–76, 1957.

R. S. Lee. Vertical integration and exclusivity in platform and two-sided markets. *The American Economic Review*, 103(7):2960–3000, 2013.

Y. Leitner. Financial networks: Contagion, commitment, and private sector bailouts. *The Journal of Finance*, 60(6):2925–2953, 2005.

R. E. Lucas. On the size distribution of business firms. *The Bell Journal of Economics*, pages 508–523, 1978.

L. W. McKenzie. On the existence of general equilibrium for a competitive market. *Econometrica: Journal of the Econometric Society*, pages 54–71, 1959.

L. W. McKenzie. The classical theorem on existence of competitive equilibrium. *Econometrica: Journal of the Econometric Society*, pages 819–841, 1981.

J. E. Meade. External economies and diseconomies in a competitive situation. *The Economic Journal*, pages 54–67, 1952.

D. T. Mortensen and R. Wright. Competitive pricing and efficiency in search equilibrium*. *International Economic Review*, 43(1):1–20, 2002.

T. Negishi. Welfare economics and existence of an equilibrium for a competitive economy. *Metroeconomica*, 12(2-3):92–97, 1960.

Nielsen. How u.s. smartphone and tablet owners use their devices for shopping. pages http://www.nielsen.com/us/en/newswire/2012/how–us–smartphone–and–tablet–owners–use–their–devices–for–shopping.html, 2012.

M. O'Hara and M. Ye. Is market fragmentation harming market quality? *Journal of Financial Economics*, 100(3):459–474, 2011.

A. Pawasutipaisit. Family formation in walrasian markets. *University of Chicago PhD Thesis*, 2010.

E. C. Prescott and R. M. Townsend. Pareto optima and competitive equilibria with adverse selection and moral hazard. *Econometrica: Journal of the Econometric Society*, pages 21–45, 1984.

E. S. Prescott and R. M. Townsend. Firms as clubs in walrasian markets with private information: Technical appendix. *Federal Reserve Bank Richmond*, 05-11, 2005.

E. S. Prescott and R. M. Townsend. Firms as clubs in walrasian markets with private information. *Journal of Political Economy*, 114(4):644–671, 2006.

J.-C. Rochet and J. Tirole. Platform competition in two-sided markets. *Journal of the European Economic Association*, 1(4):990–1029, 2003a.

J.-C. Rochet and J. Tirole. Platform competition in two-sided markets. *Journal of the European Economic Association*, 1(4):990–1029, 2003b.

J.-C. Rochet and J. Tirole. Two-sided markets: a progress report. *The RAND Journal of Economics*, 37(3):645–667, 2006.

R. Rogerson. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics*, 21(1):3–16, 1988.

A. Rubinstein and A. Wolinsky. Middlemen. *The Quarterly Journal of Economics*, 102 (3):581–593, 1987.

M. Rysman. The economics of two-sided markets. *The Journal of Economic Perspectives*, pages 125–143, 2009.

M. Sattinger. Assignment models of the distribution of earnings. *Journal of Economic Literature*, pages 831–880, 1993.

Tabb Group. Liquidity matrix. *Technical Report*, 2012.

E. Tavilla. Opportunities and challenges to broad acceptance of mobile payments in the united states. *Federal Reserve Bank of Boston*, 2012.

The Economist. Shining a light on dark pools. pages http://www.economist.com/blogs/schumpeter/2011/08/exchange–share–trading, 2011.

E. G. Weyl. A price theory of multi-sided platforms. *The American Economic Review*, pages 1642–1672, 2010.

E. G. Weyl and A. White. Insulated platform competition. *mimeo*, 2015.

H. Zhu. Do dark pools harm price discovery? *Review of financial studies*, 27(3):747–789, 2014.

8. Appendix

8.1. **Proof of Theorem (4).** *The price-setting intermediary in the monopolistic equilibrium will capture all the rent in the economy and will produce less slots than the price-taking intermediary in the competitive equilibrium.*

Proof. To begin we show that in the monopolistic equilibrium, the intermediary will produce a negligible amount of platforms. Then we show that the competitive equilibrium will produce platforms that use the entire endowment in the economy.

For simplicity, let us assume there are only two types of agents A and B with no subtypes. Assume a monopolistic intermediary produces X (where X is less than one) platforms of size[26] $(1,1)$ and sells each contract to type T at a price of κ_T/X, where κ_T is the agent T's wealth. The agents can either participate (that is, buy contracts) or not buy. If the agent does not buy any contracts, their resultant utility is zero.

Let us assume each agent buys X contracts of the platform of size $(1,1)$. Then type T's utility will be $XU_T(1,1)$, that is, the utility of being on a platform of size $(1,1)$ multiplied by the probability of being on that platform, X.

Could the agent buy any other contract? No, since the monopolist only produces one type of platform. Could the agent buy less of the contract? Yes, but utility is increasing in the purchase of this contract, X, therefore not optimal. Could the agent buy more of the contract? No, because the agent is constrained by their wealth endowment, κ_T.

The intermediary's profit is equal to: $\kappa_A+\kappa_B-X(c_A+c_B+c)$. Therefore, the intermediary's profit is decreasing in X. Therefore, the intermediary will produce the smallest positive number of platforms, X, as possible to maximize profits. Therefore, in the monopolistic equilibrium only a negligible number of platforms will be produced.

In the competitive equilibrium, from theorem (1) – the First Welfare Theorem – we know that the competitive equilibrium is a Pareto Optimal allocation, second, given the intermediary's constant returns to scale technology, we know the intermediary makes zero profits. Combining these two results, we know in the competitive equilibrium there will be a positive number of platforms and that the total cost of producing these platforms will be $\kappa_A + \kappa_B$ – the total amount of resources in the economy. □

[26]We restrict attention to the platform of size (1,1) for expositional ease, although the intermediary could construct platforms of any given size. Additionally, even though in equilibrium the platform will produce only a negligible amount of this platform, the platform of size $(1,1)$ would be the cheapest platform to produce.

8.2. **Computation.** Attempting to compute the Pareto problem can be difficult due to the large commodity space and the number of constraints, therefore, we transform the above Pareto problem by removing the club constraints therefore, allowing us to use simplex algorithms which are quicker and more capable to handle the large commodity and constraint space.

For ease of explanation let us assume there is only two subtypes of merchants and consumers, i.e $i \in \{1,2\}$ and $j \in \{1,2\}$.

First we eliminate the club constraints recall equation (13), this constraint can be rewritten in matrices for each contract $d_T(N_A, N_B)$ as

$$\begin{bmatrix} \alpha_{A,1} & \alpha_{A,2} & 0 & 0 & -N_A \\ 0 & 0 & \alpha_{B,1} & \alpha_{B,2} & -N_B \end{bmatrix} \begin{bmatrix} x_{A,1}[d_A(N_A,N_B)] \\ x_{A,2}[d_A(N_A,N_B)] \\ x_{B,1}[d_B(N_A,N_B)] \\ x_{B,2}[d_B(N_A,N_B)] \\ y(N_A,N_B) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{17}$$

Since, $x_{T,s}[d_T(N_A, N_B)]$ and $y(N_A, N_B)$ must be non-negative, with equation (17) let us define a polyhedral cone, with a single extreme point at the origin. Therefore, using the Resolution Theorem of Polyhedrons, the systems of equations can be represented as the set of all non-negative linear combinations of its extreme rays. Scaling each $y(N_A, N_B) = 1$, the extreme rays of this cone are:

$$\begin{gathered} \left(\frac{N_A}{\alpha_{A,1}}, 0, \frac{N_B}{\alpha_{B,1}}, 0, 1\right) \\ \left(\frac{N_A}{\alpha_{A,1}}, 0, 0, \frac{N_B}{\alpha_{B,2}}, 1\right) \\ \left(0, \frac{N_A}{\alpha_{A,2}}, \frac{N_B}{\alpha_{B,1}}, 0, 1\right) \\ \left(0, \frac{N_A}{\alpha_{A,1}}, 0, \frac{N_B}{\alpha_{B,2}}, 1\right) \end{gathered}$$

Let $y^{(i,j)}(N_A, N_B)$, the quantity of each ray, where i is the subtype A agent, j is the subtype B agent. Therefore, we can define the set of $\{x_{T,s}[d_T(N_A, N_B)]$,$y(N_A, N_B)\}$ that satisfies (17) as:

$$\{x_{T,s}[d_T(N_A,N_B)],y(N_A,N_B)\} = [y^{(1,1)}(N_A,N_B)]\left(\frac{N_A}{\alpha_{A,1}},0,\frac{N_B}{\alpha_{B,1}},0,1\right)+ \\ \ldots + [y^{(2,2)}(N_A,N_B)]\left(0,\frac{N_A}{\alpha_{A,2}},0,\frac{N_B}{\alpha_{B,2}},1\right)$$

Where $y^{(i,j)}(N_A,N_B) \geq 0$, $i = 1,2$ and $j = 1,2$. Intuitively, each ray is a different composition of types of agents to fulfill the contract, for example $y^{(1,1)}(N_A,N_B)$ corresponds to the measure of platforms which are fulfilled by agents $(A,1)$ and $(B,1)$. There are four extreme rays hence a linear combination of these four rays is able to replicate any combination of types of agents. In general, if there are I types of A and J types of B then there will be $I \times J$ extreme rays for each contract.

Furthermore, we have the following relations:

$$\begin{aligned} x_{A,i}(N_A,N_B) &= \sum_{i'} \frac{y^{(i,j)}}{\alpha_{A,i}} N_A \\ x_{B,j}(N_A,N_B) &= \sum_{i} \frac{y^{(i,j)}}{\alpha_{B,j}} N_B \\ y(N_A,N_B) &= \sum_{i,i'} y^{(i,j)}(N_A,N_B) \end{aligned}$$

Hence, we are now ready to redefine the Pareto problem in terms of our new definitions which satisfy the matching constraints.

$$\max_{y^{(i,j)}(N_A,N_B)\geq 0} \sum_i \lambda_{A,i}\left[\sum_j \sum_{(N_A,N_B)} y^{(i,j)}(N_A,N_B) \times N_A \times U_A(N_A,N_B)\right] + \\ + \sum_j \lambda_{B,j}\left[\sum_i \sum_{(N_A,N_B)} y^{(i,j)}(N_A,N_B) \times N_B \times U_B(N_A,N_B)\right]$$

Such that each agent is assigned to a platform with probability one (the counterpart to equation (12)).

$$\begin{aligned} \sum_j \sum_{(N_A,N_B)} \frac{y^{(i,j)}(N_A,N_B)}{\alpha_{A,i}} N_A = 1 \ \forall i, \\ \sum_i \sum_{(N_A,N_B)} \frac{y^{(i,j)}(N_A,N_B)}{\alpha_{B,j}} N_B = 1 \ \forall j \end{aligned}$$

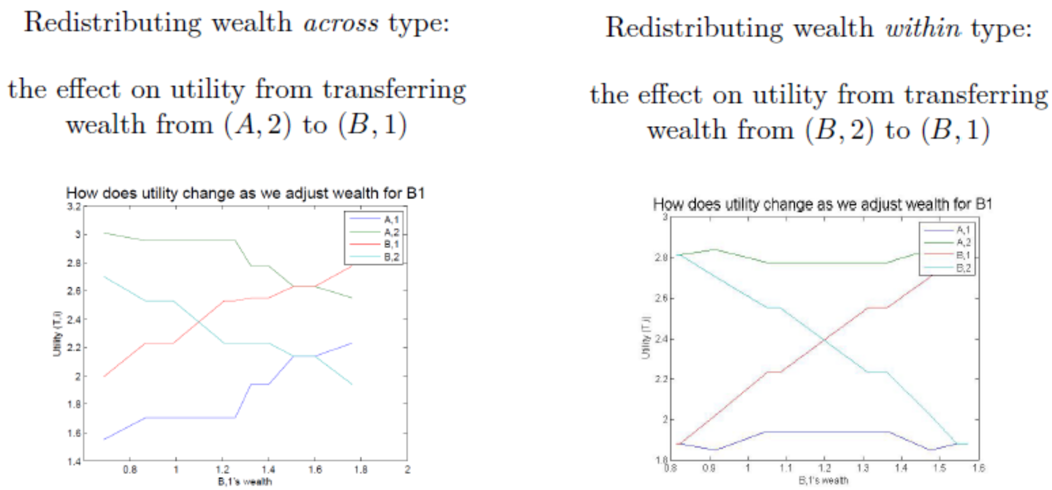
Such that the resource constraint is satisfied (the counterpart to equation(14)):

$$\sum_{(N_A,N_B)} \left[\sum_{i,j} y^{(i,j)}(N_A, N_B) \times C(N_A, N_B) \right] \leq \sum_{T,s} \alpha_{T,s} \kappa_{T,s}$$

Therefore, the advantage of writing the Pareto problem in the above formulation reduces the constraint set, in this example, there are only five constraints, however, the number of variables is very large.

We can use a linear programming solver to compute the reformulated Pareto program.

FIGURE 6. Redistributing wealth within and across type – the effects on all agents' utilities



These figures are identical to figure (1), except we also include the agents whose wealth changes. In the left panel, the x-axis redistributes wealth from subtype $(A, 2)$ to subtype $(B, 1)$. As one would expect this increases $(B, 1)$'s utility and simultaneously reduces $(A, 1)$'s utility.
In the right panel, the x-axis redistributes wealth from subtype $(B, 2)$ to subtype $(B, 1)$. As one would expect this increases $(B, 1)$'s utility and simultaneously reduces $(B, 2)$'s utility.

8.3. **Additional Graphs.**