Multidimensional Poverty and its Discontents[1]

EUDN Conference, Paris

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November 2010

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Introduction

The multidimensionality of poverty is not in dispute.[2] Poverty can mean poor health, inadequate education, low income, precarious housing, difficult or insecure work, political disempowerment, food insecurity, and the scorn of the better off. The components of poverty change across people, time, and context, but multiple domains are involved.

An emerging question is how multidimensionality should be reflected in measures of poverty. The launch of a new 104 country multidimensional poverty index in 2010 attracted enormous attention and interest by the global media, as well as lively discussion among poverty scholars. [3] It is vital to

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[1] I am grateful to James Foster, Jeni Klugman, Maria Emma Santos & Gaston Yalonetzky for comments and suggestions.

[2] Grusky and Kanbur 2006, Jenkins and Micklewright 2007, Ravallion 1996, Ravallion 2010, Sen 1992, Sen 1993, Thorbecke 2008. For example, Cappellari and Jenkins write (in Jenkins and Micklewright 07), 'It is widely agreed nowadays that being poor does not simply mean not having enough money' (opening sentence, p 166).

[3] After the launch of the Multidimensional Poverty Index (MPI) in July 2010, the Oxfam Blog carried a posting by Martin Ravallion on the MPI as well as a number of responses and subsequent exchanges. The World Bank Blog

recognise and engage the issues that emerged in order to move the discussion forward constructively. This paper will examine whether and in what circumstances an aggregate measure of multidimensional poverty adds value to an assemblage of deprivation and income poverty indicators. For example, could a multidimensional poverty measure improve our ability to track overall poverty over time, to target the poor, or to inform policy interventions? If so, it is important to clarify what value-added such a measure might have to a dashboard poverty profile, what inaccuracies it might generate, and what would be the opportunity cost of investing in its construction. Given that income is the leading indicator of poverty, it is also vital to explore how a multidimensional poverty measure could complement analyses based on income poverty measures.

These issues have become vivid both due to an increasing body of studies on the interrelationships among indicators of disadvantage,[4] as well as to the increased possibility of creating multidimensional poverty measures.[5] More data on non-income dimensions of poverty are available than at any previous time in history. Alongside this, multidimensional measurement methodologies have advanced considerably in the past fifteen years. These advances together have created new possibilities to measure multidimensional poverty at the local, national and international level. Yet the fact that one can construct a multidimensional poverty measure does not mean that it will necessarily add value. As Sen writes, 'The passion for aggregation makes good sense in many contexts, but it can be futile or pointless in others... The [overall] view does have its uses, but it has no monopoly of usefulness. When we hear of variety, we need not invariably reach for our aggregator.'[6] This paper will focus on the question of when, how and why certain multidimensional poverty measures may add value, sketch the limits of the contribution, and introduce a set of standing questions.

The relevance of understanding interconnections among multiple deprivations was highlighted in the 2009 Report of the Commission on the Measurement of Economic Performance and Social Progress, which argues that "Some of the most important policy questions involved relate to how developments in one area (e.g. education) affect developments in others (e.g. health status, political

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continued the discussion with a number of interesting contributions, including James Foster's input. Martin Ravallion (2010) posted a more general *World Bank Policy Research* working paper that engages MPI as well as other measures, and a further exchange on VoxEU.org. Another significant exchange was with the Government of Morocco (the exchange is found on the World Bank Blog). OPHI also released robustness tests of the MPI to changes in weights, showing that MPI country rankings are quite robust.

[4] For example, in the European context Nolan and Marx argue that the multidimensionality of poverty requires additional measures:

> Both national and cross-country studies suggest that … low income alone is not enough to predict who is experiencing different types of deprivation: poor housing, neighborhood deprivation, poor health and access to health services, and low education are clearly related to low income but are distinct aspects of social exclusion (Nolan and Marx 2009 See also Balestrino 1996, Balestrino and Sciclone 2001, Brandolini and D'Alessio 1998, Chiappero-Martinetti 2000)

Whelan, Layte and Maître 2004 study multidimensional poverty and income poverty in a dynamic context across five waves of the ECHP, and find neither a direct nor a lagged relationship between them, hence call for further work on multidimensional poverty dynamics. Similarly, in a 16 country study Mitra *et al* (2010) find that disability is not significantly associated with consumption poverty in most countries, but is significantly associated with multidimensional poverty (using a range of functional forms and thresholds for the multidimensional poverty measures).

[5] Of course, there is also a parallel rise in multidimensional welfare assessments. See for example Becker, Philipson and Soares 2005, Brighouse and Robeyns 2010, Fleurbaey 2009, Fleurbaey and Gaulier 2009, Jones and Klenow 2010, Kreitler and Kreitler 2006, McGillivray 2007, Robeyns and Van der Veen 2007, Stiglitz, Sen and Fitoussi 2009 among others.

[6] Sen 1987b at page 33.

voice and social connections), and how developments in all fields are related to those in income." The report also highlights the particular relevance of joint distribution when studying disadvantage: "For example, the loss of quality of life due to being both poor and sick far exceeds the sum of the two separate effects, implying that governments may need to target their interventions more specifically at those who cumulate these disadvantages" (p 55). The conclusion affects both survey design as well as the measurement approach: "Developing measures of these cumulative effects requires information on the "joint distribution" of the most salient features of quality of life across everyone in a country through dedicated surveys."[7]

This paper argues that, provided that the methods are explicit and robustness tests are available, multidimensional poverty measures can add value for measuring poverty, for understanding poverty and for targeting poor people. A key strength of the methodologies on which I focus is that they reflect the joint distribution of disadvantages.

To explore these issues, the paper presents one class of poverty measures only, namely an extension to the FGT class of measures (James Foster, Greer, & Thorbecke, 1984) proposed by Alkire and Foster (2007; 2009), with some consideration given to related axiomatic measures (Bourguignon & Chakravarty, 2003; S. R. Chakravarty & Silber, 2008). It also introduces one recent implementation of one measure within this family: the Multidimensional Poverty Index, in order to illustrate some results. Measurement comprises a subset of the broad range of techniques that have been developed to assess multidimensional poverty, and include qualitative and participatory techniques, as well as dashboards and poverty profiles, dominance techniques, multivariate techniques, and multidimensional inequality indices. Among multidimensional poverty measures, this paper also covers a narrow terrain, and does not address relevant and interesting measures that use information theory,[8] fuzzy set theory,[9] latent variable techniques,[10] multiple correspondence analysis,[11] alternative counting approaches,[12] alternative axiomatic approaches,[13] or dominance.[14] While a number of the research questions are shared among approaches, in the limited space available we can only formulate the issues for one approach. However, it seems possibly useful to set out a clear account of this particular approach, so that its strengths and limitations can be grasped, and areas for further research advanced efficiently.

In the absence of such an account, multidimensional measures of poverty may be viewed as a somewhat sweet distraction. In an eloquent criticism of the parsimony for which economics is known A.O. Hirschman (1984) proposed complicating economic discourse by, among other things, introducing a more adequate treatment of love. Love, Hirschman argued, is poorly handled in economics, being neither a scarce resource nor an augmentable skill. Lofty as Hirschman's suggestion might have been it did not, in practice, take off. There could be many reasons that

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[7] The history of consideration of these issues can be traced to Atkinson and Bourguignon 1982

[8] Deutsch and Silber 2005, Maasoumi and Lugo 2008

[9] Balestrino 1998, Cerioli and Zani 1990, Cheli and Lemmi 1995, Chiappero-Martinetti 1994, Chiappero 2006, Deutsch and Silber 2005, Lelli 2001, Lemmi and Betti 2006, Qizilbash 2002

[10] Kakwani and Silber 2008b, Krishnakumar 2004, Krishnakumar and Ballon 2008, Schokkaert and Van Ootegem 1990

[11] Asselin 2009

[12] Atkinson 2003, Erikson 1993, Gordon, Nandy, Pantazis, Pemberton and Townsend 2003, Nolan and Whelan 1996, Subramanian 2007

[13] Bossert, D'Ambrosio and Peragine 2007, Bourguignon and Chakravarty 2002, Chakravarty 1998, Chakravarty and D'Ambrosio 2006, Chakravarty and Silber 2008, Deutsch and Silber 2005, Tsui 2002

[14] Duclos, Sahn and Younger 2006, Duclos, Sahn and Younger 2006,

parsimony endured in this respect; perhaps it was not sufficiently clear when and how such a complication would add value, or perhaps it has yet to find its time. While multidimensional poverty measurement might seem more familiar to economists than Hirschman's favored topic, it runs the risk of seeming to threaten legitimate parsimony if its potential contribution – and the limits of its contribution – are not sketched more precisely.

I. An Approach to Multidimensional Poverty

Poverty is multidimensional. So can we measure multidimensional poverty? That is a more vexing – yet important – question.

The motivation to take a multidimensional view of poverty arises primarily from the empirical mismatch between poverty measured in any single space such as income and additional important single and multidimensional measures of disadvantage. If it were the case that income (or any other unidimensional measure) were a sufficient proxy of other disadvantages for practical purposes (such as targeting or tracking change over time or guiding policy) then, in the interests of parsimony, one need not go further.[15] A set of indicators – or a multidimensional measure – may add value if no single dimensional index can be found that is adequate for practical purposes. For this reason, attention has been paid to both the levels of deprivation in different domains, and to the joint distribution of those deprivations.

Using multiple indicators does not, however, require a multidimensional poverty *index*; each indicator could be considered individually. I will argue that a multidimensional poverty index adds value insofar as it is robust and rigorously implemented, and can convey additional information not captured in unidimensional measures, particularly the joint distribution of disadvantage, and the composition of poverty among the multiply deprived. To argue this is not to suggest that single-dimensional measures be abandoned; only that they be supplemented. The remainder of this section recalls basic features of this approach to poverty measurement: that deprivations are identified using dimension-specific cutoffs, that aggregation occurs first across dimensions then across people, and that poverty measures are created the reflect the vector of deprivations for poor people.

Bourguignon and Chakravarty use the term 'multidimensional poverty' to designate a situation in which deprivations with respect to multiple dimensions are used to identify whether a person is multidimensionally poor and to describe the extent of their poverty. They argue for the use of dimension-specific cutoffs: "a multidimensional approach to poverty defines poverty as a shortfall from a threshold on each dimension of an individual's well being. In other words, *the issue of multidimensionality of poverty arises because individuals, social observers or policy makers want to define a poverty limit on* each *individual attribute: income, health, education, etc…* All the arguments presented in this paper are based on this idea." Other approaches such as the 'counting' approaches widely implemented in Europe also use this approach (Atkinson 2003), and that is also used in Alkire and Foster 2007.

Sen's seminal paper "Poverty: An Ordinal Approach to Measurement" opens with the following sentence:

[15] This issue is discussed in Foster and Sen's Appendix 7 of Sen 1997, which discusses various forms of income poverty measures as well as indicators of other functionings.

> In the measurement of poverty two distinct problems must be faced, viz., (i) identifying the poor among the total population, and (ii) constructing an index of poverty using the available information on the poor. [16]

Based on that paper, most poverty measures have largely acknowledged the need for two components: an identification approach, and aggregation mechanism. What follows builds upon Sen's approach to poverty measurement.

Whereas in income poverty measures, a person is identified as poor if their income falls beneath a poverty line, identification in multidimensional space is more complex. If a deprivation vector is constructed for each person, then multidimensionally poor persons can be identified on the basis of their vector, but there are various ways of proceeding. For example, a person is identified as multidimensionally poor by the union approach if the person is deprived in *any* dimension (Atkinson 2003; see also Duclos Sahn and Younger 2006). A person is identified as multidimensionally poor by the intersection approach if and only if they are deprived in *all* dimensions. James Foster and I have adopted an intermediary approach, in which a person can be identified as multidimensionally poor if they are poor in some (weighted) sum or 'count' of dimensions that can include union and intersection as well as intermediary cutoffs (Alkire and Foster 2007). The final step is to aggregate the information regarding persons who have been identified as multidimensionally poor into a summary index which satisfies a set of desirable properties.

We describe our general measurement approach thus:

> A methodology $\mathcal{M}$ for measuring multidimensional poverty is made up of an identification method and an aggregate measure (Sen 1976). Following Bourguignon and Chakravarty (2003) we represent the former using an *identification function* ρ: $R_{+}^{d} \times R_{++}^{d} \to \{0,1\}$, which maps from person *i*'s achievement vector $y_i \in R_{+}^{d}$ and cutoff vector z in R_{++}^{d} to an indicator variable in such a way that $\rho(y_i; z)=1$ if person *i* is poor and $\rho(y_i; z)=0$ if person *i* is not poor.[17] Applying ρ to each individual achievement vector in y yields the set $Z \subseteq \{1,\ldots,n\}$ of persons who are poor in y given z. The aggregation step then takes ρ as given and associates with the matrix y and the cutoff vector z an overall level $M(y; z)$ of multidimensional poverty. The resulting functional relationship $M: Y \times R_{++}^{d} \to$ R is called an *index*, or *measure*, of multidimensional poverty.[18] A methodology is then given by $\mathcal{M} = (\rho, M)$. (Alkire & Foster 2009)

The identification and aggregation steps will now be illustrated in the following sections, then their value-added scrutinized, beginning with our most basic and applicable index M_0 drawn from the class M_a introduced in Alkire and Foster (2007).

One Multidimensional Poverty Measure: the M_0

This section briefly introduces the Alkire-Foster class of M_α measures that build on the FGT index.

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[16] Sen 1976. In that paper, Sen's focus is on (ii), given that Atkinson, Rowntree, Weisbrod and Townsend among others had focused on identification.

[17] Note that this representation assumes that the underlying identification method is individualistic (in that *i*'s poverty status depends on y_i) and symmetric (in that it uses the same criterion for all persons). It would be interesting to explore a more general identification function which abstracts from these assumptions.

[18] A 'poverty focus axiom' ensures coherence between identification function and poverty measure; see section 6 below.

Let us consider poverty in d dimensions across a population of n individuals. Let $y = \left[y_{ij} \right]$ denote the $n \times d$ matrix of achievements for i persons across j dimensions. The typical entry in the achievement $y_{ij} \geq 0$ represents individual i's achievement in dimension j. Each row vector $y_i = (y_{i1}, y_{i2}, \ldots., y_{id})$ gives individual i's achievements in the each dimension, whereas each column vector $y._j = (y_{1j}, y_{2j}, \ldots., y_{nj})$ gives the distribution of achievements in dimension j across individuals. To weight the dimensions, define a weighting vector w whose j^{th} element w_j represents the weight that is applied to dimension j. We set $\sum_{j=1}^{d} w_j = d$, that is, the dimensional weights sum to the total number of dimensions.

The M_0 measurement methodology can be summarized in three steps. First we construct a deprivation matrix g^0. Let $z_j > 0$ be the deprivation cut-off in dimension j, and z be the vector of deprivation cutoffs. Define a matrix of deprivations $g^0 = [g_{ij}^0]$, whose typical element is defined by $g_{ij}^0 = w_j$ when $y_{ij} < z_j$, and $g_{ij}^0 = 0$ when $y_{ij} \geq z_j$. From the g^0 matrix construct a column vector c of *deprivation intensity*, whose i^{th} entry $c_i = \sum_{j=1}^{d} g_{ij}^0$ represents the sum of the entries in a given row, and represents the weighted deprivations suffered by person i.

The second step is to identify who is multidimensionally poor. Select a poverty cutoff k, such that $0 < k \leq d$ and apply it across this column vector c. A person is identified as poor if her weighted deprivation count $c \geq k$. This can be called a *dual cutoff* identification method, because it uses the *deprivation* cutoffs z_j to determine whether a person is deprived or not in each dimension, and the *poverty* cutoff k to determine who is to be considered multidimensionally poor.

To compute the M_0 value that is used for the MPI, construct a second matrix $g^0(k)$, obtained from g^0 by replacing its i^{th} row g_i^0 with a vector of zeros whenever $c \geq k$. This matrix contains the weighted deprivations of exactly those persons who have been identified as poor and *excludes* deprivations of the non-poor. M_0 is the mean of the matrix $g^0(k)$. That is $M_0 = \mu(g^0(k))$, where μ denotes the arithmetic mean operator.

M_0 can also be expressed as the product of two the (multidimensional) headcount ratio (H) and the average deprivation share among the poor (A). H is simply the proportion of people that are poor, or q/n where q is the number of poor people. A is the average of fraction of deprivation intensity among the poor: $A = \sum_{i=1}^{n} c_i(k) \Big/ dq$. A represents the *intensity* of multidimensional poverty.

M_0 satisfies dimensional monotonicity: if a poor person becomes deprived in an additional dimension, the M_0 will increase. M_0 is also decomposable by population subgroups. Additionally, after identification, M_0 can be broken down by dimension. The intuition of M_0 – that it is the product of headcount and intensity – together with these three properties in particular, enable M_0 to be taken apart in various ways to generate multiple insights.[19] If data are cardinal and satisfy

[19] M_0 also satisfies other properties: replication invariance, symmetry, poverty focus, deprivation focus, weak monotonicity, non-triviality, normalisation, and weak re-arrangement. These axioms are joint restrictions on the identification and aggregation methodologies.

additional assumptions, we identify other measures within the M_α family that can be computed to reflect the depth and severity of multidimensional poverty. These replace the binary g^0 matrix with a matrix of normalized gaps, or with normalized gaps raised to a positive power α; apply the identification function, censor deprivations of the non-poor, and take the mean of the respective matrix to generate higher order measures.

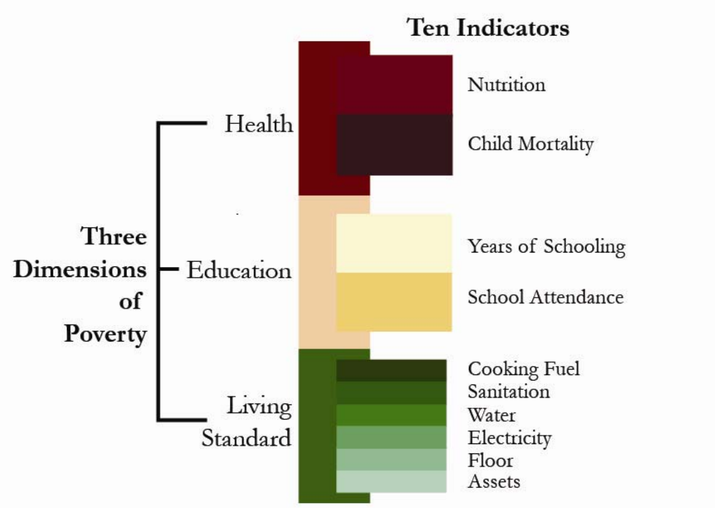
II. One particular application of the M_0 measure: the MPI

In 2010, the UNDP Human Development Report Office and the Oxford Poverty & Human Development Initiative (OPHI) released an acute Multidimensional Poverty Index (MPI) for 104 developing countries. This section describes the MPI's structure, and subsequently some key results, drawing on our working paper (Sabina Alkire & Santos, 2010).

Before beginning, a word on terms: in the methodological discussion each entry in the matrix constituted a 'dimension'. In describing the MPI, the terminology changes, and each entry in the matrix is termed an 'indicator'; the term dimension is used in the MPI to reflect conceptual categories ('health') that do not appear in the g^0 matrix directly.

The MPI generates multidimensional poverty measures by analyzing existing publicly available data sources. In particular, the MPI is based on Demographic and Health Surveys (DHS) for 48 countries, on Multiple Indicator Cluster Surveys (MICS) for 35 countries, and on the World Health Survey (WHS) for 19 countries. Distinct surveys are used for Mexico and urban Argentina. All surveys used are between 2000 and 2008. All questions for each country were drawn from the same household survey for that country, which enables the MPI to reflect the joint distribution of deprivations a person experiences at the same time.

The MPI is constructed using ten indicators covering three dimensions. The three dimensions are health, education, and standard of living. The indicators are: Health: nutrition (anthropometric measures) and child mortality; Education: years of schooling and school attendance; Living Standard: electricity, water, sanitation, cooking fuel, flooring, and asset ownership. Each dimension is equally weighted at one-third. Each indicator within a dimension is also equally weighted. Thus the health and education indicators are weighted at one-sixth each, and the standard of living at one-eighteenth.[20]



The MPI implements the first measure in the dual-cutoff approach of Alkire and Foster (2007) introduced above: M_0. This methodology was chosen because it can be used for ordinal and even categorical data; it builds upon the FGT income poverty measures, is straightforward and intuitive in construction, and satisfies a number of desirable axioms. In terms of policy relevance, the resulting

[20] In fact, as there are 10 indicators, the weights must sum to d=10, so in fact each dimension weighs 3.33 and k = 3; however for simplicity of presentation we discuss the fractions.

measure can be decomposed by population group and broken down by factor to show the composition of poverty, hence can describe how the extent and composition of multidimensional poverty varies across states or ethnic communities, or across time.

Deprivation cutoffs: The MPI first identifies who is deprived in each of the 10 indicators. The deprivations are dichotomous: each person is identified as deprived or non-deprived in each indicator with respect to a cutoff or threshold selected for each indicator. A person is deprived in nutrition of any member of their household for whom we have nutritional data is malnourished. A person is deprived in child mortality if any child in their household has died.[21] A person is deprived in years of schooling if no member of their household has completed five years of schooling. A person is deprived in school attendance if any school-aged child is out of school, where the national age of schooling is used, and school attendance considered for the first eight years of school. Please note that the health and education indicator definitions differ considerably from standard headcount definitions; we will return to this point later.

Among the standard of living indicators, person is deprived in electricity if they do not have a connection; in water if they do not have clean water by the MDG definitions or if they must walk more than 30 minutes to obtain clean water. They are deprived in sanitation if they do not have adequate sanitation by MDG standards or if their sanitation is shared. A person is deprived in cooking fuel if they cook with wood, charcoal, or dung; and in flooring if their floor is dirt, sand or dung. Finally, they are deprived in asset ownership if their household does not own a car or truck, and does not own more than one of the following: telephone, television, radio, refrigerator, bicycle, or motorcycle.

The indicators, cutoffs and weights are summarized in the figure below, which also mentions the links between eight of the MPI indicators and the MDG indicators.

Figure 1: Dimensions, indicators, cutoffs and weights of the MPI

Dimension	Indicator	Deprived if…	Related to…	Relative Weight
Education	Years of Schooling	No household member has completed five years of schooling	MDG2	16.7%
	Child Enrolment	Any school-aged child is not attending school in years 1 to 8	MDG2	16.7%
Health	Mortality	Any child has died in the family	MDG4	16.7%
	Nutrition	Any adult or child for whom there is nutritional information is malnourished*	MDG1	16.7%
Standard of Living	Electricity	The household has no electricity		5.6%
	Sanitation	The household´s sanitation facility is not improved (according to the MDG guidelines), or it is improved but shared with other households	MDG7	5.6%
	Water	The household does not have access to clean drinking water (according to the MDG guidelines) or clean water is more than 30 minutes walking from home.	MDG7 MDG7	5.6%
	Floor	The household has dirt, sand or dung floor		5.6%
	Cooking Fuel	The household cooks with dung, wood or charcoal.	MDG7	5.6%
	Assets	The household does not own more than one of: radio, TV, telephone, bike, motorbike or refrigerator, and does not own a car or truck.		5.6%

Note: **MDG1** is *Eradicate Extreme Poverty and Hunger*, **MDG2** is *Achieve Universal Primary Education*, **MDG4** is *Reduce Child Mortality*, **MDG7** is *Ensure Environmental Sustainability*.
* Adults are considered malnourished if their BMI is below 18.5. Children are considered malnourished if their z-score of weight-for-age is below minus two standard deviations from the median of the reference population.

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[21] Angus Deaton has suggested that future MPI versions consider adding a third health indicator to reflect the tragic burden of HIV-AIDS. For example, one might use life expectancy at the age of five. If life expectancy at the age of 5 years was less than some deprivation cutoff, then all individuals in a country would be deprived in that indicator.

Poverty Cutoff: Once it has been identified who is deprived in each indicator, the next step is to determine who is multidimensionally poor. This depends upon the weighted sum of their deprivations. Having a single deprivation does not establish that a person is multidimensionally poor. A deprivation might not necessarily indicate poverty. A person might cook with wood, but have a separate kitchen and ventilation system, so that does not indicate poverty. An uneducated person may nonetheless be a self-made millionaire blossoming with good health. Furthermore, deprivations may also be caused by inaccuracies in the data themselves; or by inappropriate indicators for that context. In some climates a natural floor may not indicate deprivation, for example.

Thus we set a second cutoff, called the 'poverty cutoff' k. In the case of MPI, $k = 3$ and we have 10 indicators, so every person is identified as multidimensionally poor if and only if they are deprived in at least 30% of the weighted indicators. That is, a person is poor if she is deprived in any two health or education indicators, in all six standard of living indicators, or in three standard of living and one health or education indicator.

Whenever the poverty cutoff k requires deprivation in more than indicator there will be people that, despite experiencing some deprivation will not be considered multidimensionally poor, simply because their total weighted deprivations is less than the k poverty cutoff. In the Alkire and Foster methodology, those deprivations are censored since they correspond to people that are not multidimensionally poor. Their values are replaced in the $g^0(k)$ matrix (which differs from the g^0 matrix precisely in the censoring of these deprivations). *All subsequent analyses are not based on the original raw data* (that would appear in a poverty profile or dashboard for example and that it is contained in the g^0 matrix) but rather *only the deprivations of multidimensionally poor people.* This censoring of any deprivations of non-poor people is a novel step, so is easily overlooked. It influences all subsequent analysis, at times considerably.

As described previously, the MPI, as the more general M_0 measure, is the mean of the censored matrix of weighted deprivations. It can equivalently be calculated as the product of the headcount or *incidence* of poverty – the percentage of people who are multidimensionally poor – and the *intensity* or average proportion of weighted deprivations a poor person experiences. For example, if a person is deprived in nutrition, years of schooling, and three standard of living indicators, then her intensity is 50% (1/6 + 1/6 + 3/18). If – on average – every person in a country is deprived in 50% of the weighted indicators, and 40% of the population is poor in that country, then the MPI for that country is 0.20.

Data Constraints

The indicators which could be compared across the DHS, MICS, and WHS datasets were limited in several ways; indeed data limitations proved to be a binding constraint for the MPI. First, many indicators were only present in some surveys, not for all countries. Even across the indicators that were available for most countries there were data constraints that had to be solved. In the first place, members of households with non-applicable populations for certain indicators were considered non-deprived in that indicator. For example, people in households with no children in school-age were considered non-deprived in school attendance. Households with no women in reproductive age, were not asked the mortality questions in the surveys, and so were not considered deprived in that indicator.[22] In the case of countries which lack one or more indicator, the remaining indicators within each dimension were re-weighted such that the dimension is still weighted at one-third and the indicators are weighted equally within it. Furthermore, some subgroups are inadequately represented in the survey data (elderly, imprisoned, homeless, etc). Further details are given in Alkire and Santos (2010).

Overall, 63 of the 104 countries have all ten indicators and 93 countries have nine or ten indicators; eight countries lack two indicators and three countries lack three indicators. In all cases, the indicators weights are re-adjusted such that each dimension is weighted one-third (in no cases are all indicators for a dimension missing). In terms of years, three countries' data come from 2008, twelve from 2007, twenty-five from 2006, and twenty-four from 2005, so 64 countries' data date from 2005 to 2008. In addition, six countries' data are from 2004; twenty three countries' data are from 2003 (including all WHS data); two each from 2002 and 2001, and seven from 2000. For all countries, the data were used directly; no interpolations were constructed that would attempt to predict changes in the intervening years. The 2010 MPI figures thus form a baseline for each country, drawing on the most recent publicly available dataset containing the MPI indicators. As data are collected, the MPIs will be re-calculated using the newer data. This will allow an explicit tracking of changes for each country across time.

Interpretation: These data limitations affect cross-country comparability. The MPI values cannot be used to compare the 104 countries' acute poverty in a definitive way, as they are drawn from different years, vary in the definition of certain variables, and some countries lack indicators. The study does claim to

a. provide a more comprehensive and accurate baseline of acute multidimensional poverty that reflects joint deprivations than is possible using a dashboard of the same indicators,
b. provide an estimate of acute multidimensional poverty in each of the 104 countries using available information about three core dimensions of human development, and
c. demonstrate the AF methodology for measuring multidimensional poverty - which can be adapted to national or regional settings having different objectives or more and better data

Illustrative Results of the MPI[23]

About 1.7 billion people in the 104 countries covered – 32% of the entire population – are poor according to the MPI.[24] As our aim is to complement income poverty measures with a direct measure of deprivation, we compare the MPI headcount to the income poverty headcounts in those countries that have data for both measures (92 of our 104 countries), and find that it lies between the $1.25 and $2/day poverty. Across these countries, 25% of the population are estimated to live on US $1.25 a day or less and 48% live on less than US $2 a day. Moving down to the national level, we find a clear overall relationship between income and multidimensional poverty, but considerable differences for particular countries. The MPI captures deprivations directly – in health and educational outcomes and key services such as water, sanitation and electricity – hence we would

[22] A different issue is the case of missing information. In general we have used all the available information as much as possible. For example, if at least one woman in reproductive age answered the child mortality questionnaire and reports a child death, the household members are considered deprived in that indicator. Only when the household had eligible population for the questionnaire and no one answered it, it is dropped from the calculations. For further details, please see Alkire and Santos 2010.

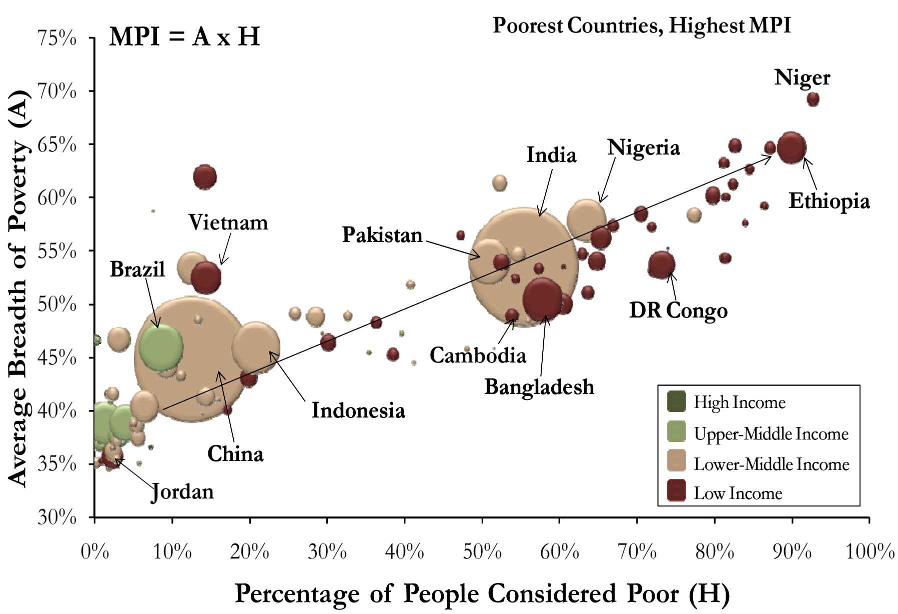
[23] This section very briefly mentions some results from Alkire and Santos 2010; for a fuller discussion the reader is referred to that paper.

[24] In all of these figures we use 2007 population data.

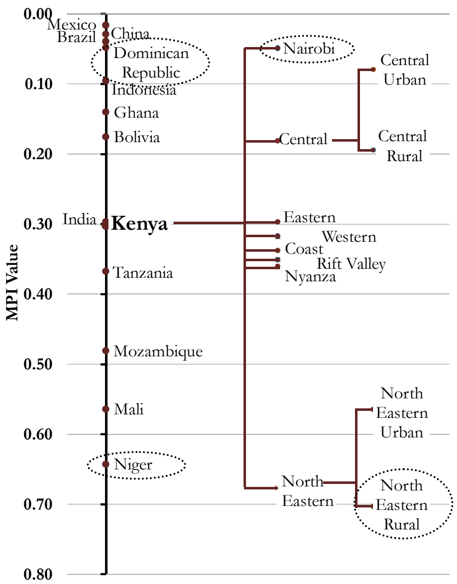
expect some mis-match, based on the literature. To some extent the MPI and income values will also differ due to different survey years as well as measurement error and data inaccuracies. Further analysis is required to understand the differences more fully.

In terms of regional distribution of acute multidimensional poverty, we find that 51% of the world's poor as measured by the MPI live in South Asia (844 million people) and 28% in Africa (458 million). In Sub-Saharan Africa 64.5% of people are MPI poor; in South Asia it is 55%. The intensity of poverty – the average number of deprivations experienced by each household – is also greatest in Sub-Saharan Africa and South Asia.

India's MPI is 0.296 and 55.4% of people are MPI poor. As India's population exceeds that of the 37 African countries we decomposed India's MPI, to explore the variation of the MPI across its large states. We found that the MPI headcount ranges from 16% in Kerala to 81% in Bihar. Given that the range of MPI ranges from zero to 0.64 across the 104 developing countries for which it was estimated, we also identified countries whose MPI $\geq$ 0.32, the midpoint value of the MPI range. We found that 27 countries have MPIs equal or higher than 0.32. These are Nepal and 26 African countries. Interestingly, eight Indian states have an MPI that is also equal or higher than 0.32. These are Bihar, Chhattisgarh, Jharkhand, Madhya Pradesh, Orissa, Rajasthan, Uttar Pradesh, and West Bengal. These eight states are home to 421million MPI poor people – a greater number than the 26 African nations where 410 million MPI poor people live. Note that these comparisons could not be accomplished if we did not have information on the intensity of poverty as well as the headcounts. Having this information, this exercise suggests that comparisons between South Asia and Africa cannot simplistically be used to state that poverty is more acute in Africa based on national averages, because there is great within-country variation and because population size matters. Multidimensional poverty in both continents is troubling both in regards to the number of people who are multiply deprived and the intensity of their poverty.



MPI Intensity increases with Headcount



The MPI is the product of poverty incidence (H) and its intensity (A). Thus, we compare these partial indices in the 104 countries and find a disconcerting relationship: countries with higher incidence of multidimensional poverty tend to have higher average intensity. The figure below plots average intensity (A) *vs.* headcount (H). The size of each bubble represents each country's population, and the color, its GDP per capita. Niger has both the highest incidence *and* the highest intensity of all countries; and the trend line has a positive slope, indicating an overall positive relationship between the two MPI components. There are some exceptions: Vietnam is to the left of the trend line for example, has relatively low headcount for its intensity.

There is an encouraging disconnect between GDP per capita and the MPI. Overall low income countries have higher MPI rates and upper middle income countries (light green) have lower MPI rates, as one would expect. Yet the percentage of people in low income countries (dark red) who are MPI poor ranges from 2% in Uzbekistan to 93% in Niger. The variation *among* low income countries shows that it is possible for low income countries to have low levels of acute multidimensional poverty.

Decompositions reveal considerable disparity in MPI among population subgroups. For example, Nairobi, in Kenya, has the same MPI value as the Dominican Republic, which ranks in the middle of the countries analysed, whereas the poorest rural areas of northeastern Kenya have a lower MPI value than Niger.

We can also break down the MPI by indicators. This is a post-identification decomposition, hence results still exclude the deprivations experienced by those not identified as poor. This decomposition reveals the structure of poverty among the poor. For example, among the Kikuyu ethnic group in Kenya, deprivation in child mortality and malnutrition (both health indicators) contribute most to the poverty. Deprivations in electricity, sanitation and cooking fuel, contribute most to the poverty of the Embu, another ethnic group. Decomposition of poverty in India and Bolivia also reveals interesting differences among ethnic, caste, and religious groups.

Analyses are also underway to explore trends in MPI over time for a number of countries. For example, in Bangladesh, 68% of people were MPI poor in 2004; by 2007 multidimensional poverty had fallen to 58%. Although progress was made in a number of indicators, an improvement in school attendance was the most striking aspect of poverty reduction in Bangladesh. In contrast, Ethiopia reduced poverty by improving nutrition and water, whereas Ghana improved several indicators at once.[25]

Robustness to weights

Amartya Sen among others sees the need to set weights in multidimensional measures as a strength not an embarrassment: "There is indeed great merit… in having public discussions on the kind of

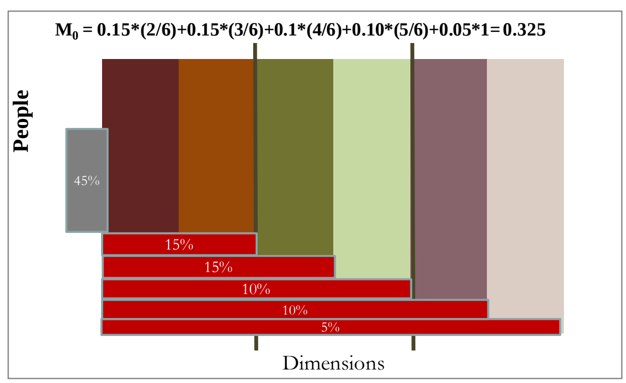
[25] Apablaza, Ocampo and Yalonetzky 2010

weights that may be used" (1997). After all, any national budget implicitly sets weights on many dimensions of welfare, often with little debate. The weights on the MPI are explicit: equal weights on each dimension, and each on indicators within a dimension. Yet given the legitimate diversity of human values, Sen also argues that it may not be necessary to agree on a precise set of weights: ideally, measures would be developed that are robust to a range of weights.

Is the MPI robust to a range of weights?[26] As an initial exploration, we estimate the MPI using three additional weighting structures: (i) giving 50% weight to health and 25% weight each to education and standard of living, (ii) giving 50% weight to education and 25% weight each to health and standard of living, and finally (iii) giving 50% weight to standard of living and 25% weight each to health and education. Then we verify if the country rankings are stable using four approaches. First, we calculate the correlation coefficients between the MPI rankings and each of the three new methods. We find that the minimum of the three Pearson's correlation coefficient is 0.989, the minimum of the three Spearman's coefficient is 0.981 and the minimum of the three Kendall's Tau-b is 0.903.[27] Next, we estimate the concordance between all four rankings using three methods: Kendall and Dickinson-Gibbon (KDG), the multi-rank version of Spearman's coefficient (by Kendall, KS) and the multiple-rank concordance index of Joe (J), and perform a well-known test of rank independence by Friedman. The concordance is high (0.975 an above) and the null hypothesis of rank independence across the four rankings is rejected with 99% confidence by the Friedman test. In terms of large changes in ranking, among the 60 countries whose MPI scores range from 0.05 to 0.64, only five countries exhibit rank changes of 10 or more places.[28] We also explore pairwise country comparisons and find that 88% of rankings are robust for all weighting structures. On the basis of this we conclude that the MPI country rankings are quite robust to weights.

Intensity of Deprivation

A distinctive feature of the M_0 measures is the partial index we call intensity. This is constructed, recall, by taking the average proportion of dimensions in which poor people are deprived. For example, in the figure on the right, there are six dimensions and $k = 2$. We can see that 5% of people are deprived in 6/6 dimensions (100%), 10% each in 4/6 and 5/6 dimensions, and 15% each in 2/6 and 3/6 dimensions, so 55% of people are deprived in 59% of the dimensions on average, where 59% is the weighted sum of the above proportions. The area of the red bars is equivalent to M_0, as would be a single rectangle, 55% x 59%.



Applying the intensity to the headcount creates a measure that can be broken down by dimension: the headcount cannot.

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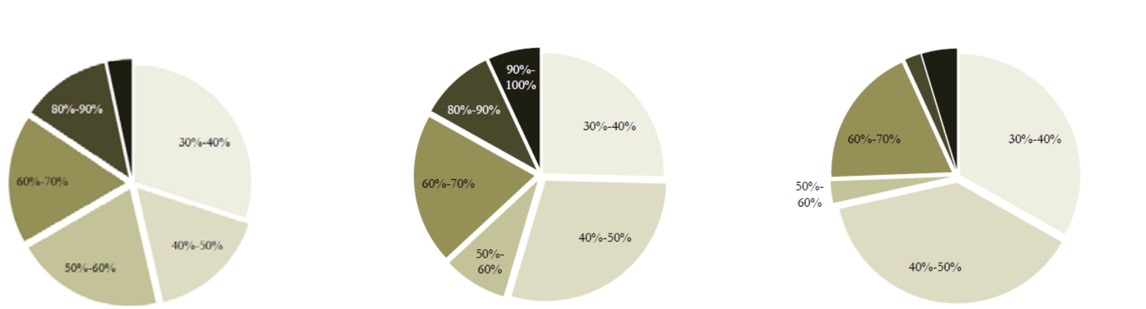
[26] The results in this paragraph draw on Alkire, Santos, Seth and Yalonetzky 2010.

[27] Pearson's correlation coefficient measures the linear relationship between a pair of rankings. The Spearman's rank correlation coefficient is based on the changes in country ranks between a pair of rankings. The Kendall's Tau-b coefficient is calculated by comparing each pair of countries in a pair of rankings.

[28] Among the 44 countries having MPI scores 0 to 0.05, 14 countries change rank 10 or more places, suggesting that the MPI is not finely tuned for very fine comparisons.

Also because k can vary, one can choose to focus on only a proportion of the population at a time. For example, if we increased k to 4, we can see that 25% of people would be identified as multidimensionally poor, if to 5, 15%, and when k= 6, 5% of people would be identified as poor. In each successive increment of the value of k, the people considered poor are deprived in more dimensions simultaneously. Thus identification itself, in this approach, interacts with the coupled deprivations.

Because of its construction, analyses of M_0 for any *given* value of $k<d$ can still explore the composition of *intensity* among poor people – the average proportion of dimensions in which multidimensionally poor people are deprived in a population. In the MPI, a person must be deprived in 30% of the weighted indicators in order to be identified as poor hence 30% $\leq A \leq$ 100%. We could break up the intensity in different ways. In each country briefing for the MPI, we present a pie diagram depicting the percentage of MPI poor people who are deprived in category of intensity. That is, the darkest slice shows the percentage of MPI poor people who are deprived in 90-100% of the dimensions (e.g. 90$\leq c_i/d \leq$100. The next lightest represent the percentage of people deprived in 80-89%, and so on down to 30-39%.[29] As is visually evident the configuration varies between countries. Consider for example India, Cameroon, and Kenya, which are adjacent in the MPI rankings. The average intensity of MPI poverty ranges from 50 to 54.7%. However the *composition* of average intensity varies: in Kenya – which has the highest MPI, the percentage of people who are deprived in 30-50% of dimensions is just under 70%, and about 25% are deprived in 60% or more dimensions. Cameroon has the highest intensity overall, and the highest percentage of people deprived in 60 to 90% of dimensions – about a third. However India, which is quite similar in intensity, has a lower percentage of poor people who are deprived in 30 to 50% of dimensions than Cameroon, and a higher percentage deprived in 50-60% and 80-90% of dimensions.



The intensity of deprivations among the MPI poor

India	Cameroon	Kenya
MPI: 0.296	MPI: 0.299	MPI: 0.302
Intensity: 53.5%	Intensity: 54.7%	Intensity: 50%

This ability to break apart a multidimensional poverty index by strata of intensity can be useful for targeting, as mentioned above. For example, if the government were able to provide services for 18% of the population, and if the multidimensional poverty measure were appropriately constructed for that country, then one could merely increase the value of the k cutoff until it identified about

[29] As the MPI weights are .167 and .055 on different dimensions, there are additional values of k at which the intensity actually changes; we showed decile bands for clarity only.

18% of people as poor.[30] These would be the 18% of people having the highest intensity of poverty. It can also be useful for policy interventions: in Cameroon, more people are deprived in 80 to 100% of indicators; thus here, targeting the poorest poor and providing an integrated spectrum of services might reduce poverty most quickly. Recall that if one brings a high intensity person out of poverty, the reduction in poverty is greater than if one brings a just-poor person out of poverty,[31] because in the former case the average intensity, as well as the headcount, would decrease. It could also be possible to combine this analysis with a subgroup decomposition to show commonalities in the structure of deprivation: for example, if 80% of the persons who are deprived in 40-50% are deprived in the *same* two indicators, this suggests different policies than if there was a great dispersion of deprivation combinations. Finally, it is possible and could be useful to undertake these analyses not only for a population as a whole but also for various subgroups – states, ethnic groups, or other subgroups for which the data are representative and the measure is valid.

Hence once one has created the censored matrix $g^0(k)$, the mean of which is the MPI, it is possible to generate a range of related descriptions from it: comparisons, subgroup decompositions, analysis by indicator, and by intensity. The data source for these various tools is the post-identification censored matrix, not the aggregate data. This $g^0(k)$ matrix is a function of the achievement matrix, the deprivation cutoffs vector, the weighting vector, and the poverty cutoff. In any identification other than the union approach (in which k takes the value of the lowest-weighted indicator), $g^0(k)$ will differ from the deprivation matrix.

'Operationality'
The MPI is a very elementary international baseline multidimensional poverty measure that is operational. It is deeply constrained by data, but it does implement a recognizable methodology, perform robustness tests, and invite improvements.

In the 2006 Grusky and Kanbur volume, Bourguignon argues that, "the key challenge in the field of poverty analysis is clear. It consists of building a set of instruments, starting with a satisfactory definition of poverty, that would meet part or all of the critiques of the [income poverty] paradigm described above, *while retaining at least part of its 'operationality'*. Current economic analysis of poverty clearly falls short of this objective… The poverty income paradigm is presently often used in situations calling for alternative definitions of poverty, essentially because instruments to handle these definitions are not available. The challenge is to create those instruments, rather than trying to make the initial paradigm artificially fit a different conceptual basis" (p 78-79). Essentially, the most basic claim of the MPI is that it is an operational instrument, whose strengths and limitations have been made quite clear, and which can be developed and strengthened in the future. The remainder of this paper seeks to identify further issues and catalyze research by which to advance.

III. The way forward: research questions and debates

The above summarizes the MPI, and the methodology that underlies it. The MPI is one possible implementation of M_0; which itself is one measure within the Alkire Foster (AF) family of M_α measures. The M_α family is itself one possible approach to multidimensional poverty measurement as here defined. The MPI has come under scrutiny and criticism; it could be useful to categorize the

[30] Alkire and Foster 2009. The degree of precision in adjusting the headcount by increasing k depends upon the weighting structure and the number of variables as well as the distribution of deprivations. Note that the poverty cutoff k is, in the example given, itself a policy tool (see Alkire and Foster 2009, section 8).
[31] For example, by identifying exactly the deprivation(s) lacked by those deprived in 30-40% in India and Kenya.

'discontent' in terms of their generality. Many of the issues raised with respect to the MPI would be shared by any multidimensional poverty measure that aggregates first across dimension and reflects the joint deprivations that poor people experience – for example, Bourguignon and Chakravarty 2003, Bossert *et al.* 2007, or a counting headcount. The latter include *methodological* issues, such as aggregation, weights and cutoffs; *data* issues, such as whether it is possible to obtain sufficiently accurate data on relevant dimensions from one survey; *political* issues, such as updating and manipulation; and *economic* issues, in particular the link between multidimensional poverty measures and welfare economics. Some issues pertain to the AF methodology directly – such as its neutrality with respect to compensation among dimensions, and the focus axiom. Others pertain to the implementation of AF methodology in the Multidimensional Poverty Index (MPI).

This final section discusses each of these sets of issues. They are issues in two senses: first, they may be 'critiques' that have been offered; and second, they may be questions for future research and innovation. I articulate most issues with respect to the MPI and the M_0 methodology; however many questions would be answered differently by different measurement approaches. The debate thus far has also passed over some issues within these categories that may be of equal or greater importance to those already articulated, so I take the liberty of proposing these as well.

III A. Multidimensional Poverty and Joint Distribution: General Issues

As was signaled above, poverty measures that reflect the joint distribution of deprivations for one person or household proceed by aggregating [weighted] information on deprivations across all dimensions for each person, identifying multidimensionally poor persons on that basis, and subsequently aggregating across poor people to construct a poverty measure. This section identifies some areas for further work, both on general-purpose methodologies and on how these methodologies are implemented in practice.

Dashboard vs Index

A clear preliminary question is why to measure multidimensional poverty at all? It is already possible instead to consider a vector of deprivations. A vector of deprivations can be constructed from different data sources; data from the same survey are not required. Also, weights and functional forms are not required, so a dashboard seems less controversial. And many of the analyses appear to be very similar.

The importance of a dashboard of indicators is not in dispute. The approach taken by the MPI, by AF, and by the related measures is that it would be *additionally* useful to construct a measure of multidimensional poverty. Following Sen (1976), the construction of a poverty measure entails *identification* of who is multidimensionally poor, and *aggregation* of data on poor people into an aggregate index. The value-added of a multidimensional poverty measure is that can reflect the level and joint distributions of poor people. Furthermore, an aggregate measure provides an overall assessment of poverty, overall comparisons between communities and, importantly, comparisons as to how poverty has evolved over time. Certainly in the MPI, which, following AF, censored the deprivations of non-poor people, the resulting measure, its decomposition and descriptive analysis, differ, at times greatly, from a 'dashboard' indicator – for example in identifying *which* of the families deprived in sanitation are also multidimensionally poor.

Weights

Every multidimensional poverty measure places some weight on every dimension. The form of the weight may be different, and it may also enter differently. Hence the issue of weights was an initial focus of discussion regarding the MPI.[32] However if one moves beyond a dashboard to a summary measure it is hard to avoid weights altogether. Shortly after the release of the MPI, Martin Ravallion (2010) drew attention, among other things, to questions regarding the robustness of the MPI to a plausible range of weights as well as to the space in which weights were articulated. Recall that the MPI sets weights between the incidence of deprivations, with health, education and standard of living being equally weighted, and the weights entering in a linear form. Our empirical analysis of the robustness of the MPI to a range of plausible weights has been sketched above. In addition, work is required on the following four topics:

Standards and Kinds of Robustness to Weights: First additional work on robustness to a range of weights is required. The 2010 MPI values are to be thought of as a baseline, and are not directly comparable for the data reasons already outlined. This limits the power of studies regarding the robustness of country MPI rankings. Furthermore, it is often not clear, even in the tests already implemented, what levels of robustness a multidimensional poverty measure *should* satisfy. Also, the AF measures are designed to inform poverty analysis not just rankings by country or subgroup, and the level of multidimensional poverty, as well as its break-down by indicator, is affected by the weighting structure, so additional approaches to robustness are required which explore the robustness of different relevant descriptive analyses to changes in weights.

Source of Weights: A second key question is how to generate weights, and what the conceptual as well as empirical considerations are in the choice of method. Approaches to setting weights that have been implemented include participatory consultations, survey data (questions on time trade-offs, gambling, socially perceived necessities, and subjective well-being), statistical techniques, expert opinion, axiomatic approaches,[33] and, most commonly, normative weights applied by the author.[34] Quantitative studies have implemented, compared and scrutinized a range of approaches to setting weights, for example in health economics and social policy; such studies are required for multidimensional poverty measures also.

Space of Weights: Third, even if the *normative* weights on indicators were brilliantly clear, there are questions about which *space* those normative weights pertain to. In the MPI, the weighting vector *w* applies to the incidence of deprivation. However one might transform the vector such that the normative weights apply in a different space. For example, if 10% of people are deprived in clean water and 40% are deprived in sanitation, then there might be arguments for an 80-20 or 20-80 weighting structure. Alternatively, the weights could be fixed in the space of social expenditure such that the policy maker would face an *equal incentive* to invest in reducing either deprivation. The appropriate space will depend on the purpose of the measure as well as the accuracy of potential transformations. It is necessary to set out the alternatives, their implications for not only the overall MPI but also the decompositions and further analyses, as well as their potential uses and limitations.

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[32] It will be interesting and fruitful to relate this discussion to the distinct but related discussion of 'indexing problem'. See Arneson 1990, Arrow 1973, Fleurbaey 2007, Rawls 1971, Sen 1991a and references therein.

[33] In the case of Mexico, the legal guidelines governing the development of their national multidimensional poverty measure include principles – for example, that economic and social progress had to balance each other, and that the achievement of a certain level in each social dimension should be seen as a human right. James Foster and I then developed an axiomatic approach showing that these principles, together with assumptions regarding the accuracy of data, were sufficient to set the weights across dimensions and to uniquely identify people as multidimensionally poor. Alkire, S and Foster, JE. 2009. "Memo to Coneval." posted as 'Research in Progress' on OPHI's website.

[34] See the papers in the 2008 OPHI workshop on weighting, which covered each of these methodologies, for example Saltelli, Dibben, and Decanq and Lugo forthcoming.

Preferences: A fourth key question is discussed in the last section below, and is the need for weights to connect to individual or social preferences and hence to welfare economics.

Clearly it is vital to develop and implement robustness methods and, where feasible, dominance tests; to develop explicit methodologies generating and updating weights; to articulate the space in which weights are to be applied, and to link weights to preferences and social preferences.

Deprivation Cutoffs

Just as weights can be generated by a participatory process, set by experts, by a convention or standard, or for the purposes of a specific measurement exercise, so too the deprivation cutoffs can be set by a number of mechanisms. Robustness tests exist,[35] yet the deprivation cutoffs are likely to affect the final measure in many instances so further reflections on the methodology for setting the cutoffs, as well as of testing them, are needed.

Decomposability

A number of relevant multidimensional poverty measures including the M_α family satisfy the principle of *decomposability,* about which there is a longstanding debate in the unidimensional literature.[36] Chakravarty and Mukherjee (1998) argue that multidimensional poverty measures should have both subgroup and factor decomposability (M_α measures satisfy subgroup decomposability. Post-identification, deprivations of poor people can be decomposed by factor). However in the unidimensional literature Sen argues that decomposability can be useful, but full subgroup decomposability may not be required as usually it is only relevant to decompose by some – not all – population groups. Multidimensional poverty measures inherit this decomposability debate from unidimensional measures, and the possibility of generating relevant measures that are not fully decomposable will need to be re-visited in the new context.

Income

Considerable work is required to understand how multidimensional poverty measures relate to income poverty measures for the same population in the same time period as well as across time. Some questions include: what should be considered in deciding whether to include income in a multidimensional poverty measure, vs have a separate measure of income or consumption poverty? How do non-income measures of material deprivation (assets, housing) relate to income poverty; should they be combined or kept separate? How does one prevent 'double-counting' if a poverty measure includes both income and deprivations with respect to market goods? Where some services can be costed and entered into household income or entered directly in an achievement vector (e.g. health insurance), which is more useful and in what circumstances? Clearly the relevant non-income variables will change in rural and urban settings, raising the question of whether to set distinctive rural and urban multidimensional poverty measures as is customarily done for income measures.

Combining Individual and Household Data:

One of the features of multidimensional poverty measures is that they combine individual and household level data. It would be useful to consider further how to combine individual and

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[35] Alkire and Santos merely implemented the MPI with different deprivation cutoffs and used correlation among country rankings. Yalonetzky (2010) has proposed a stringent dominance test, but this has large sample size requirements and requires a measure to be robust to changes in the weights and poverty cutoff, as well as the deprivation cutoffs.

[36] There is an useful summary of the debate around decomposability in Foster and Sen 1997.

household level data for two circumstances: 1) when the unit of analysis is the person, and 2) when it is the household.[37]

There are three kinds of combinations that need to be examined in each circumstance: a) how to combine information that is available for each household member (as years of schooling) and how to address 'missing values' in some responses or attribute scores to ineligible respondents; b) how to attribute household level data to individuals (taking into account literature and empirical studies on equivalence scales for income, and on intra-household inequalities in distribution), and c) when it is justified to use a variable from a single respondent or from a subset of household members to represent all household members. These may require detailed expertise on each indicator area.

The combination methods must also consider biases due to differently sized households. In the 2010 MPI, larger households have greater probability of being deprived in the health and school attendance indicators, and less probability of being deprived in 'years of schooling' and at least the 'asset' indicator among the standard of living indicators. The overall effect is not yet clear.

Studies are needed to enumerate alternative methods of combining the data, what errors may be introduced by different methodologies, and how to check the robustness of results to choices made. Empirical studies also are needed to explore the magnitude of differences introduced by different methodologies, and to generate examples of careful and rigorously verified methods of combining individual and household data. Alongside quantitative work, qualitative and ethnographic studies can be used to explore the assumptions underlying different alternatives, and consider which equivalence scales and intra-household aggregation methods are most accurate in a given context.[38]

Endogeneity

The fundamental challenge for multidimensional poverty measures, which requires a great deal of further work, is the issue of endogeneity. It is clearly necessary to pursue analyses of multidimensional measures using instrumental variables and structural equation models, and to analyse component variables individually. It will also be useful to identify and test additional quantitative methodologies used in environmental economics, in health economics, and in other disciplines.

Data

The data restrictions on the MPI or any other global measure which requires internationally comparable indicators are considerable, as was detailed earlier. The data constraints at a country level are less binding, but it can be useful to itemize common constraints. Many of these are well recognised. For example, many household surveys omit institutionalized populations such as the imprisoned, the homeless, and the hospitalized; further, certain surveys exclude key groups such as the elderly or a gender group. The sampling frame, periodicity, and quality of household surveys are also regularly criticized. Multidimensional measures raise a distinctive set of questions in addition to these for two reasons.

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[37] Of course in other feasible applications of AF measures, the unit of analysis might be an institution (school, community health clinic), a community (datazone), a business or cooperative, or even a state or country (for governance indicators). However in the case of multidimensional poverty the unit is likely to be a person or household.

[38] For example, identifying a household as non-deprived if any member has 5 years of schooling, as the MPI, presumes that education is shared across household members; in some cultural contexts or in some kinds of households, that assumption may not be accurate.

Data on each variable must be available for the same person. If a multidimensional poverty measure follows Sen's approach, and identifies *who* is multidimensionally poor first, then information on joint deprivations is required. This same approach was advocated more recently by the Sarkozy Commission. The need for data on different dimensions for the same person or family is a fundamental issue that cannot be ignored.

In developing countries all questions may need to come from the same survey (or else be generated for the same household through matching or mapping[39]). Increasingly, multi-topic household surveys have specialized to explore multidimensional health *or* the quality of education *or* empowerment *or* water management, etc. Such surveys treat one or a few topics in some depth precisely because no single indicator has been identified as a sufficient single proxy for that dimension. A legitimate concern is whether it is possible to construct brief modules on each dimension such that the data generated are sufficiently accurate.[40] This requires the input of professionals from different disciplines and areas of expertise. It also requires participatory and qualitative work to explore the accuracy of indicators and measures after implementation.

Data must be accurate at the individual level. Second, because the data are aggregated first across each person or household, each variable must reflect deprivation at the level of the person or household itself – not merely when averaged. This affects survey design. For example, a question on "morbidity within the past two weeks" may provide useful data on average, but is unlikely to be a good indicator of the respondent's general health status. Research on each dimension-area is required to propose questions that reflect different dimensions of poverty accurately for the relevant unit of analysis and time period.

Politics

The MPI, like income poverty measures, faces a number of political challenges. These are well-known and common, it seems, to any poverty measure – unidimensional or multidimensional. Hence I will merely name two in passing: updating and flexibility.

Updating: As Angus Deaton has emphasized, a key feature of any poverty measure is not merely its construction, but its updating. In the case of the MPI and related measure, it could be possible to update the data keeping the methodology fixed. But periodically it may also be appropriate to change the indicators, cutoffs, and weights as populations vary or data improve. The politics as well as processes of doing so need to be considered from the outset, as the measures are designed.

Flexibility: The M_α measures allow others to choose the indicators, dimensions, cutoffs, and weights for the measure that generate the best measure for a given set of purposes. This flexibility raises concerns about who will decide these and whose interests will the final measure reflect. There are also concerns that the measure will be manipulated or implemented carelessly and as a result will be inaccurate. The sensation of overwhelming flexibility arises in large part because there are as yet far fewer 'conventions' than presently exist in income poverty measurement; with time conventions and standards will be developed. Income poverty measures, deprivation headcount measures, and their underlying data, can also be manipulated; no measure is strategy proof. An advantage of the M_α

[39] When all relevant indicators are not present on the same survey, where feasible, surveys could be matched to provide data for the same individual or household from different surveys. Poverty mapping techniques might also be used, when they provide sufficient accuracy at the individual/household level, and are appropriate for policy formulation.
[40] For example, Browning 2003.

measures is that their construction is easy to communicate, and uses explicit weights and cutoffs, so an informed public and technical advisors could identify serious shortcomings.

Welfare Economics

The welfare economics of previous generations focused on consumption. With the capability approach Sen has provided a very rich foundation for subsequent work.[41] As Atkinson (2003) emphasised, it is particularly important to establish and emphasise very clear connections between new multidimensional measures and welfare economics in order that the emerging literature can develop beyond measurement to guide public policy in a coherent and transparent fashion.[42] Further work is required on many areas, including topics such as individual and social preferences, individual utility, multidimensional welfare economics, and paternalism.

Data Qualities

If all variables are cardinal, it is possible to engage a number of measurement approaches. For example, all cardinal achievements could be aggregated and considered to be a well-being indicator. A cutoff could be taken across this measure, and poverty measures generated. The challenge is that such an aggregate must satisfy challenging assumptions, as the dimensions (with weights) must be made commensurate for all possible achievement levels for all possible dimensions.[43] These assumptions may or may not be valid.

An implication of this measurement approach is that it allows full compensation for one deprivation in any dimension by an achievement in another dimension. This may not be sufficiently coherent for a multidimensional poverty measure, because it may not reflect the extent of a person's real deprivation. Hence as mentioned at the outset of this paper, the set of multidimensional poverty measures on which this paper focuses applies cutoffs in each dimension (Bourguignon & Chakravarty, 2003; Stiglitz et al., 2009). Many of these measures still requires cardinally significant variables, but variables that reflect functionings are often only ordinally significant. This is relevant, because a measure based on ordinal variables must allow any monotonic transformation of the variable and its cutoff. For most measures above – including M_1 and M_2 – such a transformation will change the value of the poverty measure.

The MPI implemented a M_0 measure largely because the data available were binary or categorical or ordinal (the headcount does as well but that is a partial index in the multidimensional context). The MPI does not implement M_1 and M_2 as these require cardinal, ratio scale variables (that have a natural zero and linearity. Recall that unlike most other measures, the M_0 can be used with ordinal data. This is because the M_0 first sets a deprivation cutoff for each dimension, then identifies the poor using a weighted sum of deprivations, then aggregates weighted deprivations of the poor.

Hence the AF M_0 methodology focuses on deprivations. It is at least plausible to compare and weight deprivations, and less challenging than trying to create comparability at every possible cutoff. The key point of interest is that if the cutoff and variables are changed by a monotonic

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41 Some key examples are Sen 1979, Sen 1982, Sen 1987a, Sen 1991b, Sen 1996a, Sen 1996b, Sen 1997, Sen 2002a, Sen 2002b

42 See also Thorbecke 2008. One limited yet illuminating example from the FGT index in income poverty, was the use of poverty orderings to provide a unifying framework linking poverty, inequality and well-being. Foster and Shorrocks 1988

43 See Alkire and Foster 2007, Alkire and Foster 2010

transformation, the level of M_0 or poverty remains unchanged, and the same people are identified as poor. The M_0 can also be used with categorical or dichotomous variables if a deprivation cutoff can be fixed.

Even when using ordinal or categorical data, some information is lost. Other techniques could be explored: for example, if all variables permitted, one could implement two sets of poverty cutoff vectors: one for extreme poverty and one for poverty. However this would entail assumptions regarding the comparability of cutoffs across variables which are unlikely to be substantiated. If the data include income alongside ordinal variables, then it would be natural to explore other ways of using the information on the poverty gap.

III B. The AF methodology

The AF methodology has certain desirable properties as sketched above, and certainly M_0 is quite applicable due to its ability to use ordinal variables. But there are also issues or questions that arise within the context of that methodology, two of which are raised below.

Complementarity and Substitutability

All M_α measures allow perfect substitutability among weighted deprivations. Others allow other kinds of relationships to hold between deprivations. For example Bourguignon and Chakravarty (2003) introduce complementarity and substitutability among dimensions into FGT multidimensional measures. While these measures have the appeal of generality, they do require all dimensions to be either complements or substitutes, and all dimensions to have the same elasticity of complementarity or substitutability with one another.[44] In contrast, in the M_α measures James Foster and I take a more 'neutral' approach.[45] The main motivation for this is *descriptive clarity*. The MPI directly reflects actual deprivations that each household experience at a point in time, and does not make assumptions regarding compensation between dimensions that might occur in the future. Also, if relevant, subindices might be constructed to reflect clearly analysed complementarities or substitutability among a subgroup of indicators, and entered into the g^α matrix as a single variable. Clearly multidimensional poverty measures should catalyse further study of the interconnections between dimensions. It may be that analysis of measures employing the 'neutral' AF approach will shed light on relevant interconnections, even if these relationships are not incorporated all at once into this poverty measure.

There would also be good justifications for implementing measures that do adjust for complementarity and substitutability between dimensions if it was possible to provide a clear basis for doing so. Such measures would be required, for example, in order to predict future poverty based on current achievements, or to approximate achievements in latent variables.[46]

Properties

The M_α measures satisfy a number of principles; however some of these principles have, and will, generate further debate. In the case of multidimensional poverty the axioms are joint restrictions on identification and measurement methodologies. One principle in particular has been discussed: the *focus axioms*. The M_α measures satisfy two focus axioms, which we call poverty and deprivation focus

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[44] These have been questioned by Dercon and by Thorbecke, both in Kakwani and Silber 2008a

[45] These are outlined in the concluding section of Alkire and Foster 2009.

[46] Calvo 2010, Calvo and Dercon 2007

axioms (Alkire and Foster 2009).[47] These cause discontinuity around the poverty line – a feature shared by income poverty headcount measures. They also preclude compensation, for example by very high achievements in non-deprived dimensions for which cardinal data are available. The justification for this is not dissimilar to the discussion of complementarity and substitutability: a very high level of education (e.g. PhD) does not eliminate the fact that a person is malnourished and deprived in assets, water and sanitation at present; while we might presume that an educated person would not have these deprivations, if indeed they do it may be worthwhile exploring why instead of assuming them to be voluntary. One might consider other examples where this feature is less satisfactory, and where identification might reflect achievements in non-deprived dimensions. These are likely to be related to the issue of complementarity and substitutability discussed above.

Areas for Further Research

The AF methodology itself can be complemented by further developments in the form of a basic toolkit of related methodologies that will facilitate its application and its analysis, and hence allow a fuller exploration of its empirical value-added in different contexts. These include statistics and poverty dynamics.

Statistics:

In order to generate multidimensional poverty measures which communicate clearly the strength or fragility of any claims, it is necessary to generate a basic statistical toolkit that in many ways parallels the toolkit developed for income poverty measures. Such a toolkit would include robustness tests and dominance tests for the deprivation cutoffs, poverty cutoffs and weights.[48] It would also include significance tests, for example, for differences in multidimensional poverty between two regions or sub-groups, and tests of statistical inference that are valid for the types of surveys used. Also required is a methodology for computing and reporting standard errors. A first generation set of statistical tests are under development for the M_α measures, both theoretically and empirically, which extend their unidimensional counterparts.[49]

Multidimensional Poverty Dynamics:

The M_α measures are decomposable by population subgroup and (post-identification) by factor, so they can show changes over time both overall and in each (censored) dimension. Time series comparisons can show the extent to which the incidence and intensity of poverty was reduced between two time periods, and decompositions can reveal the indicators in which deprivations increased or decreased. Panel data comparisons allow further exploration of multidimensional

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[47] If by M(x;z) we denote an aggregate measure of poverty; if deprivation simply indicates shortfall from a deprivation cutoff z_j whereas persons are poor if and only if identified as such through the identification function, then the axioms are as follows "POVERTY FOCUS: If x is obtained from y by a simple increment among the non-poor, then $M(x; z)=M(y; z)$. DEPRIVATION FOCUS: If x is obtained from y by a simple increment among the nondeprived, then $M(x; z)=M(y;z)$. (Alkire and Foster 2009).

[48] Dominance conditions for the AF measures are more complex due to the identification step, but a stringent and full set of dominance conditions that ensure the robustness of comparisons to widely varying weights, deprivation cutoffs, and poverty cutoff, has been derived in Yalonetzky 2010a. Statistical tests for these conditions are available inYalonetzky 2010a, Yalonetzky 2010c, for discrete variables; and in Anderson 2008, for continuous variables. The familiar challenge is finding samples that are sufficient to implement the test in the case of discrete variables and several dimensions.

[49] For tests of the AF measures based on a bootstrapping approach see Bennett and Singh forthcoming. For tests based on analytically derived standard errors see Yalonetzky 2010b

poverty dynamics. Methodologies are under construction to enable such time series and panel data comparisons (Apablaza et al., 2010). Detailed context-specific and historical studies will then be required to identify the policy ingredients and policy sequences that reduce multidimensional poverty and sustain reductions across time. It also may be interesting to construct measures of chronic multidimensional poverty related to the AF methodology as well as to Foster (2007), that aggregate first across time periods for each dimension, then across dimensions, and finally across persons.

Concluding remarks

This paper has introduced one approach to multidimensional poverty measurement, one particular methodology (AF), one implementation of it (MPI), and a series of research topics that are either being investigated or are issues for future research. The key strengths of the M_0 methodology are that it is a poverty measure, fulfilling the steps of identification and aggregation that Amartya Sen set out for poverty measures; that it is intuitive and easy to interpret, that it satisfies a set of desirability properties such as subgroup consistency, that it makes explicit the weights set upon dimensions, that it identifies joint deprivations and has multiple ways of presenting joint deprivation through the measurement of intensity. Finally, the AF methodology is flexible: the dimensions, cutoffs, and weights can all be chosen to reflect the purpose of the measure and its context.

We used the results from the recent 104-country Multidimensional Poverty Index (MPI) which implemented the M_0 methodology to illustrate some analyses that the measure can generate. We noted that MPI analyses differ from analyses using the original data and indicators because the basic matrix used by all MPI-related figures is 'censored' to focus *only* on the disadvantages of people who are jointly deprived in 30% (in this case) of dimensions.

The last section briefly introduced an incomplete yet substantive set of research topics, progress in which would take this work to the next stage. These include a set of issues related to many multidimensional measurement approaches – such as work on weights, cutoffs, income, combining individual and household data, policy analysis, linkages to preferences and welfare economics, treatment of ordinal and categorical data, and so on. There are also issues specific to the AF methodology – such as incorporating complementarity and substitutability, and relaxing the focus axiom at identification.

The fundamental question is whether undertaking the further field-building research and collecting missing data is likely to significantly advance various agents' abilities to reduce the incidence, intensity, depth and duration of human poverty. I have argued that investing further in multidimensional poverty measures has the potential to generate significant advances in understanding and to create useful policy tools. To develop this potential, it is vital to establish and convey good practices for the implementation of multidimensional poverty measures, such that measures are implemented with rigour and transparency in the upcoming phase. The paper also identified additional statistical and analytical methodologies that are required. If the methodologies of multidimensional poverty measurement adequately navigate these challenges, they may be seen not a as threat to economics' legitimate parsimony, but as an extension of its core strengths.

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