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Modelling the Mtera-Kidatu Reservoir System to improve Integrated Water Resources Management

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ABSTRACT:
Failure of the Mtera-Kidatu Reservoir System within the Rufiji River Basin in Tanzania in the early 1990s is investigated by considering four possible causes (i.e. sudden decrease in inflows, sudden increase in losses, sudden increase in hydropower generation or unnecessary spills; or a combination of these) and it was found out that unaccounted spillage seems to be the main cause. Consequently, the system's simulation model has been proposed in this paper that takes into consideration the flows that are generated in the intervening catchment (i.e. catchment between Mtera and Kidatu) with the operational policy that maximum power is produced at Kidatu most of the time because Kidatu has higher head for greater power generation than at Mtera. The paper shows that if this proposed model had been in place then the Mtera Reservoir should not have gone dry in the 1991-1994 period. The validity of the proposed model is tested with the TALSIM 2.0 Model and the regression analysis of the water levels at Mtera Reservoir produced by the models had an efficiency of 95%, indicating a very good correlation.

The proposed model operates the reservoir system in an integrated manner by considering the flows into the Mtera Reservoir as well as accounting for the flows generated by the intervening catchment.

Keywords: Reservoir Systems; Simulation Model; Operational Policy; Integrated Water Resources Management.

INTRODUCTION

Electricity generation and its distribution play a major role in a country's development, and in this technological era where electricity is classify as an engine of supporting economic growth, maximum attention has to be paid to its generation and efficient operation if the country is to enjoy the comfort and benefits it brings. This has to be done in an integrated manner.

This paper deals with the development of a simulation or an operational model for the Mtera-Kidatu Reservoir System by way of investigating the failure of the system that occurred in the early 1990s. The reservoir system comprises two reservoirs - the Mtera Reservoir and the Kidatu Reservoir, with the former being upstream of the latter. The investigation was intended to establish the cause of the failure of the reservoir system in 1994 when water level in the Mtera Reservoir went very close to its minimum level.

The estimation of inflows into the reservoirs is also dealt with in this paper since rivers entering into them are not gauged at the rim of these reservoirs. The gauges were destroyed after the impoundment of the river. Therefore, one cannot say with a high level of confidence how much water enters the reservoirs on a day-to-day basis without adequate mathematical model of the same. This information is absolutely necessary for optimal operation of the reservoir system as well as to help in the investigation of the failure of reservoir system. Land use change can also adversely affect the accuracy of the inflows' estimates as well.

In addition, estimation of loss of water from the Mtera Reservoir was carried out. This was done because it was anticipated that perhaps the amount of water lost through evaporation and also to a certain extent due to seepage and percolation is much higher than what was assumed at the design stage. The developed simulation/operational model was tested with the TALSIM 2.0.

THE STUDY AREA

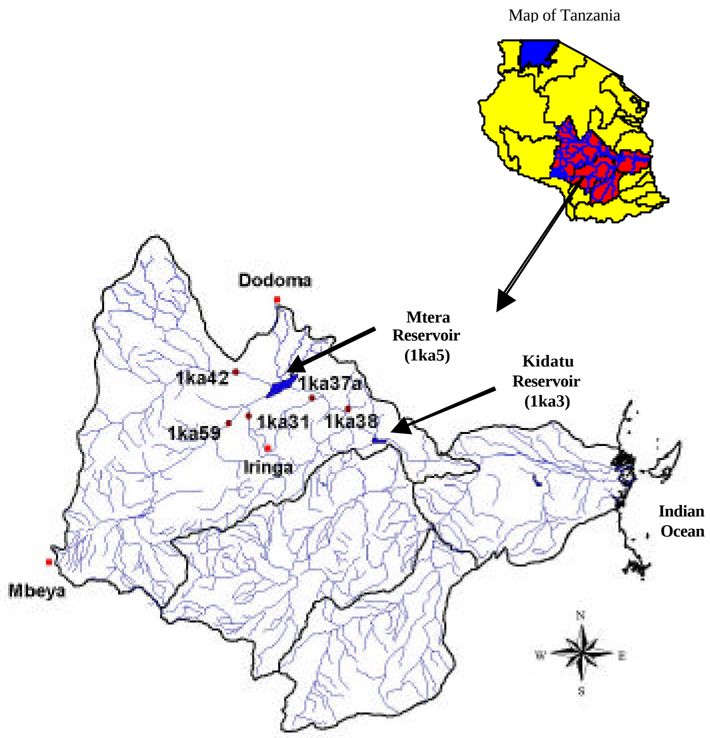

Figure 1 The location of the reservoir system within the Rufiji River Basin in Tanzania

The location of the reservoir system within the Rufiji River Basin in Tanzania is shown in Figure 1. Mtera Reservoir is the larger than Kidatu Reservoir with surface area of 620 km^2 at full capacity. It is 8.5 m deep ranging from 690.0 m above Mean Sea Level (MSL) to 698.5 m above MSL. Corresponding values for the Kidatu Reservoir are a surface area of 9.5 km^2 at full capacity and a depth of 17 m ranging from 433 m above MSL to 450 m above MSL. The storage capacity of Mtera Reservoir is 125 million cubic metres. It is roughly 25 times larger than the Kidatu Reservoir. The installed capacity at Mtera is 80 MW of power whereas at Kidatu it is 200 MW. There are 2 turbines at Mtera and 4 turbines at Kidatu. This information is summarised in Table 1.

Parameter	Mtera	Kidatu
Live Storage (MCM)	3,200	125
Spillway Capacity (m^3/s)	4,000	6,000
Generating Capacity (MW)	80	200
Turbine Discharge Capacity (m^3/s)	96	140
Full Storage Level (m.a.s.l.)	+698.5	+450.0
Minimum Storage Level (m.a.s.l.)	+690.0	+433.0
Catchment Area (km^2)	67,884	80,040

Table 1: Summary of Mtera-Kidatu Reservoir System's Details

Three main rivers contribute to the Mtera Reservoir, namely; the Great Ruaha River, the Little Ruaha River and the Kisigo River. The Great Ruaha River at Msembe Ferry (1ka59) provides about 56% of the runoff into the reservoir. The Little Ruaha River at Mawande (1ka31), which joins the Great Ruaha River downstream of Msembe Ferry, provides an additional 18%, whilst the Kisigo (i.e. 1ka42, joining it further downstream) is about 26% (Danida/World Bank, 1995).

There are several tributaries between Mtera and Kidatu Reservoirs that also contribute to the inflows at Kidatu. The ones considered in this paper are flows from Mtera, Lukosi at Mtandika (1ka37a), Yovi (1ka38) and of course the contribution of rainfall within this intervening catchment (i.e. between Mtera and Kidatu).

TECHNIQUES OF INVESTIGATION

The methodology used to investigates the possible causes of the failure of the Mtera-Kidatu Reservoir System are as follows:

(1) Sudden decrease in inflows.
(2) Sudden increase in losses.
(3) Sudden increase in generated power.
(4) Unnecessary spill.

Any one or a combination of these reasons could or must be the cause of failure of the Mtera Reservoir to recover to full conditions in 1991 and in 1992. The failure of the reservoir in 1994 is, in fact, caused by the phenomenon that started in 1991 and persisted through 1992. If the reservoir had gone to the full capacity or nearly full conditions in 1991 and in 1992 then the reservoir would not have failed in 1994. Therefore, the focus of this investigation was on 1991 and on 1992 rather than on 1994 and 1997 when the reservoir levels actually were the lowest.

Inflows

Three main rivers - the Great Ruaha, the Little Ruaha and the Kisigo - contribute inflows into the Mtera Reservoir. These rivers are gauged some distance away from the rim of the reservoir. As a result, one cannot directly calculate inflows into Mtera Reservoir without modelling these inflows into the reservoir. Hence, these inflows must be estimated using mathematical/hydrological models of flow routing. Again, there is no gauging at the rim of the Kidatu Reservoir and similar approach is needed for inflow estimates there. The gauging stations were destroyed after the impoundment of the river for the hydropower generation.

A number of mathematical/hydrological models were tried and tested on daily basis. All of them were found satisfactory for the purpose of this investigation. Detailed results of these models are presented in Table 2. The models are Simple Linear Model (SLM), Linear Perturbation Model (LPM) and Linear Varying Gain Factor Model (LVGFM). These models were used to relate the flow at Mtera (1ka5) with the flows from 1ka31, 1ka42 and 1ka59, and to relate the flow at Kidatu (1ka3) with the flows from 1ka37a, 1ka38 and the average rainfall within the intervening catchment. The SLM is a multiple regression model where the dependent variable is the runoff at the outlet of the catchment/basin and the independent variables are rainfall and/or upstream flow values. The LPM is a modification of the SLM where seasonal variations in the variables are accounted for in the regression equation, and the LVGFM is a further extension of the regression equation, where non-linearity due to high intensity of rainfall is accounted for. The mathematical details of these models are presented in Appendix 1.

Since Mtera Reservoir was impounded in 1980, the model was calibrated for pre-impoundment period of 1957 to 1975. The data of 19 years was used for the calibration of the model and the remaining 4 years, 1976 to 1979, was used for the verification of the model. All the three models registered an efficiency (R^2) of above 90% during calibration, with LVGFM having the highest efficiency of 94.26% and LPM the least of 90.35%. The results are presented in Table 2. The same order of efficiency was observed during verification with LVGFM having an efficiency of 73.53% while LPM had an efficiency of 69.85%. SLM was used for further investigations of the system' failure because it had the best estimate of flow volume compared to LPM and LVGFM. It is also the simplest among the three models and the results are not vastly different from the others.

River flow data at Kidatu (1ka3) was consistently available from 1954 to 1975; prior to impoundment of the reservoir. Scanty discharge data after impoundment of the reservoir was also available for the period 1982 to 1985. Observed flow at three flow stations; 1ka5, 1ka37a and 1ka38 were combined with the average rainfall over the intervening catchment to estimate flow at the Kidatu gauging station. Similarly, the three models (SLM, LPM and LVGFM) were calibrated over a period of 12 years from 1958 to 1969. Model verification was done from 1970 to 1975 (6 years). Good model efficiencies were obtained, with all models registering an average R^2 of above 91% during calibration and 89% during the verification period. These results are also

Name of Catchment		Calibration Period	Model efficiency (R^2 in %) during calibration			Verification Period	Model efficiency (R^2 in %) during verification			Catchment Area (km^2)
Output	Inputs		SLM	LPM	LVGFM		SLM	LPM	LVGFM	
1ka5	-1ka31 -1ka42 -1ka59	1957-1975	93.87	90.35	94.26	1976-1979	72.21	69.85	73.53	67,884
1ka3	-1ka5 -1ka37a -1ka38 -Intervening catchment areal rainfall	1958-1969	91.83	92.02	91.98	1970-1975	89.18	89.48	89.68	80,040

presented in Table 2 and, again, SLM was selected over the others.

Table 2: Model efficiency results for SLM, LPM and LVGFM in estimating flows at Mtera (1ka5) and Kidatu (1ka3)

Losses

It had been mentioned that perhaps the amount of water lost through evaporation is much higher than what was assumed at the design stage, hence causing the failure of the reservoir system. Main losses that occur in a reservoir are due to evaporation followed by losses due to seepage or ground water percolation and direct pumping (but there is no evidence of direct pumping from the Mtera Reservoir).

It is hardly likely that losses due to percolation increased suddenly at the end of 1990, i.e., after the reservoir has been in operation for nearly seven years. It is also very unlikely that losses due to evaporation increased suddenly at the end of 1990. But for the completeness of investigation, annual and expected seasonal losses due to evaporation, using the Morton model, are compared with the combined losses due to evaporation and percolation calculated by the water balance of the reservoir. It is assumed that the losses due to percolation and seepage are small compared to the losses due to evaporation. As a result, it is expected that the losses calculated by water balance should be comparable in magnitude to the evaporation losses estimated by the mathematical model (i.e. Morton model).

Hydropower generation

It is well known that there was no increase in the generation of hydropower in 1991 and 1992. In fact, the opposite is true. There was rather an acute shortage of power in the early 1990s in Tanzania. However, for the sake of completeness, a comparison is made between the turbine discharge from Mtera and Kidatu in 1991 and 1992 with releases made in previous years.

Prior to 1988, hydropower was only generated at Kidatu. In 1988, an additional turbine was installed at Mtera to increase the generating capacity of the reservoir system, but it is unlikely to be the cause of persistent low water levels in the Mtera Reservoir in the early 1990s. Power is mainly generated at Kidatu but water is stored at Mtera. Although, Kidatu is a much smaller reservoir, its function is, largely, to maintain sufficient head for power generation. Prior to installation of a turbine at Mtera water was spilled from Mtera to feed Kidatu. After the installation of the turbine all the water that was necessary to be released is not spilled but a part of it is passed through the turbine. The amount of water that passes through the turbine at Mtera would have been spilled anyway to feed the Kidatu Reservoir. Therefore, installation of a turbine at Mtera Reservoir is not likely to have any adverse effect on decrease in water levels at Mtera.

Unnecessary Spill

If the amount of water that was spilled from Mtera was more than what was necessary to feed the Kidatu Reservoir then it could have been the cause of low water levels in 1991 and 1992. Investigation of this possible cause was carried out in two different ways. The first course of investigation relied on searching for clues from the records of spill from the Mtera and the Kidatu Reservoirs, comparing the total outflow from the Mtera and from the Kidatu and by comparing the observed losses from the Mtera Reservoir with losses estimated from the reservoirs using mathematical models.

The second course of investigation relied on simulation of water levels in the Mtera Reservoir from the very start of the operation of the reservoir. The details of this investigation, which is the core of this paper, are presented under a separate heading called ‘Simulation of the Reservoir System’.

RESULTS OF INVESTIGATION

Open water potential evaporation from the reservoir system using data of daily duration from 1972 to 1994 was calculated using the Morton model. Calculated potential value was about 200 mm/month and the corresponding lake evaporation, calculated using the Morton’s model, was about 160 mm/month.

Comparison of expected monthly losses, based on model estimates (i.e. Morton model), and those based on the water balance of the Mtera Reservoir revealed that the latter were much higher for the months of January to May. For the remaining months, the water balance losses compared satisfactorily with the model estimated values.

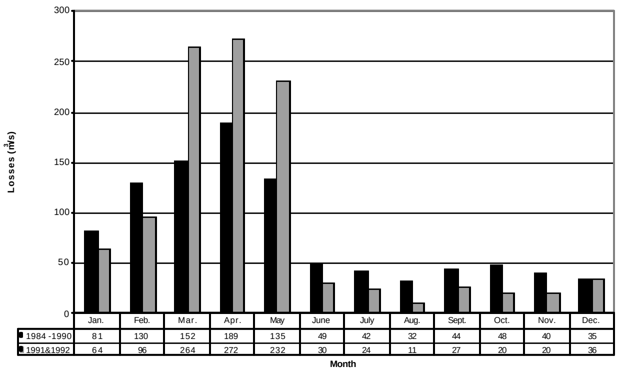

Figure 2

From Figure 2, i.e. mean monthly losses for the period 1984 to 1990 and also for 1991 and 1992, it can be seen that the expected monthly losses calculated during the draw down period of the reservoir (i.e. from June to November) are comparable in magnitude with the model estimates of evaporation from the Mtera Reservoir. But for the months of January to May (i.e. during the filling up period of the reservoir) the calculated losses are much higher than the model estimates. There is no obvious reason as to why this should be the case unless, of course, there is (1) an error in the estimated inflow data, or (2) an error in the recorded reservoir levels, or (3) an error in the recorded outflow.

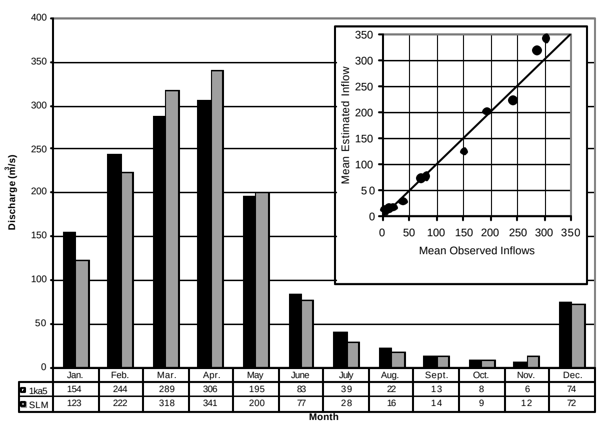

Figure 3

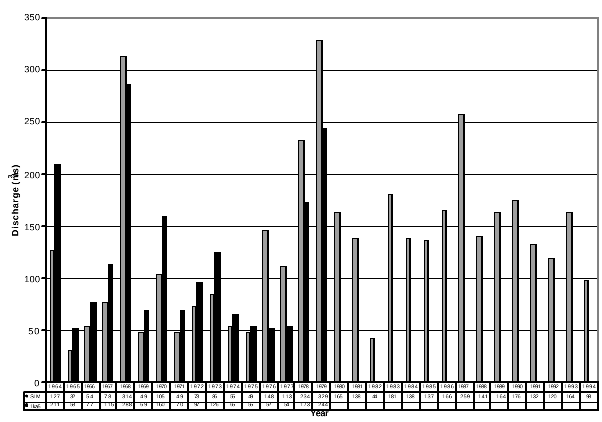

Figure 4

Figure 3 presents a comparison of the observed monthly flow data at 1ka5 and the flow estimated by the SLM for 16 years prior to the start of construction of Mtera Reservoir in 1980, and the same figure showing the relationship between them. The estimated annual flow data prior to and after the construction of the reservoir is presented in Figure 4 in a time series format. From the estimated annual flows from 1983 to 1993 (see Table 3), one can easily see that the inflows in 1991 and in 1992 were not exceptionally low. They were in the same order of magnitude as in previous years. It is interesting to note that the average estimated inflow (177 m^3/s) calculated between 1983 and 1990 is not much greater than the average inflow of 134 m^3/s between 1991 and 1992. The average observed flow at 1ka5 prior to the construction of the reservoir was equal to 118 m^3/s. This must have been the value for which the Mtera Reservoir was designed. During the operation period, from 1983 to 1994, the estimated inflow in the reservoir has always been higher than 118 m^3/s.

Model	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**Average**
ADD	181	138	137	166	259	141	164	176	132	120	164	162
SLM	213	174	187	169	238	148	240	248	175	146	184	193
NNM	149	130	148	127	209	130	193	189	120	111	160	151
Average	181	147	157	154	235	140	199	204	142	126	169	

ADD** is for Sum of the inputs without assigning weights and **NNM** is for Neural Network Model that makes use of **ADD** and **SLM

Table 3: Estimated Annual Flow into Mtera Reservoir from 1983 to 1993 (expressed in m^3/s)

Based on this analysis, it was ruled out that decrease in the inflows to the reservoir was the reason as to why the reservoir did not fill up in 1991 and 1992. To support this conclusion, observed rainfall data in the region had to be analysed for decreasing trend. The average annual rainfall prior to 1991 was 855 mm/year compared to 1,022 mm in 1991 and 922 mm in 1992. There is no obvious increasing or decreasing trend in the amount of annual rainfall within the catchment.

There was no sudden increase or any significant increase in the machine discharges at Kidatu Reservoir in 1991 and 1992. The values for these two years are comparable with the previous years. Therefore, increased activity of hydropower development at Kidatu Reservoir was ruled out as the cause for the failure of the system.

For the recorded spills from the Kidatu Reservoir in 1991 and in 1992, the spill volume is 621 Mm^3 and 353 Mm^3, respectively. This is a substantial amount of water and comprises about 30% of the volume of the Mtera Reservoir. This spill was allowed to happen when the Mtera Reservoir was not full and was struggling to raise its water level to its historical average values. It is difficult to accept that any enlightened authority would release water from a struggling reservoir only to be spilled.

Comparing the total water released (machine discharge plus spill) from the Kidatu Reservoir as well as from the Mtera Reservoir, one would have expected the two quantities to be equal to each other when compared over a period of time. The available records show that that is not the case. Average outflow from Kidatu (101 Mm^3) is higher than that of Mtera (65 Mm^3). This reconfirms the previous conclusion that the water released from Mtera is much more than what had been recorded.

Analysis of spills revealed that most of the spills occurred during the refilling phase of the reservoir (i.e. during the months January to June). Since this is generally the period during which the discrepancy between the observed and the model estimated losses was noted it could be concluded that the actual amount of water that was released form the Mtera Reservoir as spill must have been much higher than what was recorded.

Analysis of the estimated inflows between January and June showed that in 1989 and in 1991 the amount of inflow into the reservoir was more or less the same, 294.3 m^3/s and 247.1 m^3/s, respectively, for 1989 and 1991. The value in 1989 was slightly higher than that of 1991. The total recorded discharge was also similar. It was 89.3 m^3/s in 1989 and 67.3 m^3/s in 1991. Yet in 1989 the water level rose by 3.5 m between January and June whereas in 1991 it rose by only 1 m. Clearly, water was lost through spill in 1991 that was not recorded.

Similarly, the estimated inflow for 1992 between January and June was equal to 219.3 m^3/s. In 1990, the reservoir level rose by about 2 m but in 1992 the rise was about 1 m. It is interesting to note that while Mtera Reservoir was struggling to get refilled in 1991 and 1992 the Kidatu Reservoir recorded large amounts of spill – enough to bring the Mtera to its full condition. The average water released from the Mtera Reservoir (61 m^3/s per year) is about two-thirds of the water released from the Kidatu Reservoir (93 m^3/s per year). All these values indicate that a large amount of unrecorded water might have been spilled from the Mtera Reservoir for no reason.

SIMULATION OF THE RESERVOIR SYSTEM (OR SIMULATION MODEL)

The operation policy adopted during the simulation exercise was based on the following guidelines:

(1) Power generation was maintained at historical levels for both Mtera and Kidatu Reservoirs. That is, maintaining the machine discharges at the recorded levels for both reservoirs ensured that this aim was achieved.
(2) Kidatu Reservoir level was maintained at allowed maximum capacity at all times.
(3) Spill from both reservoirs was minimised at all times.
(4) No allowance was made to leave safety storage space for any unexpected flood in the Mtera Reservoir.

The algorithm of the operation policy is presented in Appendix 2 and it is, basically, based on a water accounting principles.

The system's operation was simulated on a ten-daily basis, although the developed model does operate in any user-defined time interval. The idea was to compare the estimated reservoir water level with the observed levels for the Mtera Reservoir at each ten-day interval. Figure 5 shows a plot of the results of the simulation for the period starting on 1st January 1983.

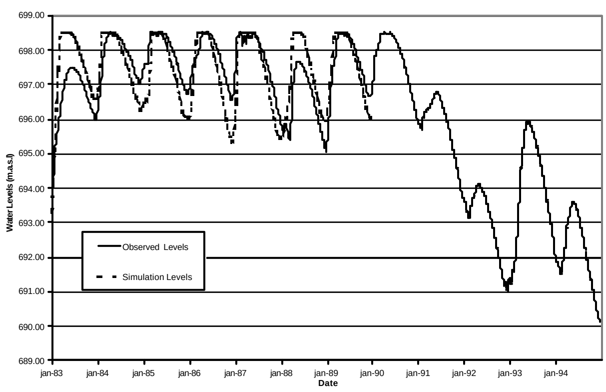

Figure 5

The results show that there was a good agreement between the estimated and recorded water levels for the years 1983 up to 1990 but the estimated water levels were much higher than the recorded water levels for the period 1991 and 1992. This simulation exercise confirms the conclusion drawn from the analysis that the recorded outflows from the Mtera Reservoir are incorrect and if the system was operated efficiently then it should have filled up in 1991 and every other year thereafter. In simple terms, the reservoir system should not have failed in 1994.

It was also observed during the simulation exercise that there should have been more spills for both Mtera and Kidatu Reservoirs compared to the historical spills. The mean monthly and yearly spills at the Mtera Reservoir are shown in Figures 6 and 7. Clearly, the simulation exercise expected more spills than historically recorded spills and with that the reservoir system should not have failed.

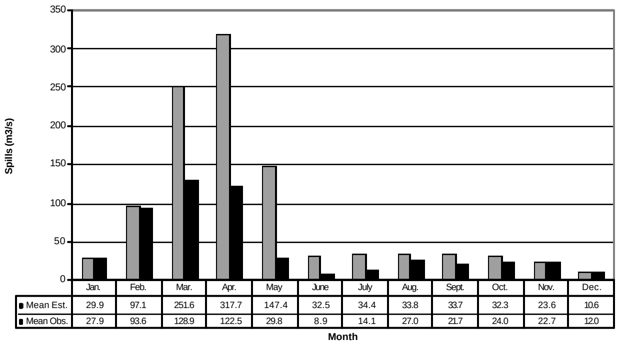

Figure 6: Plot of Mean Monthly Simulated Spills versus Mean Monthly Historical Spills from the Mtera Reservoir

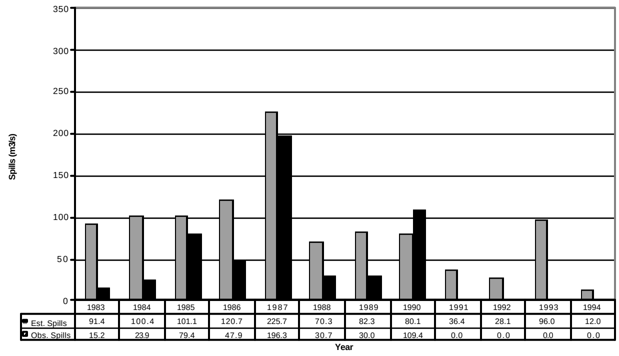

Figure 6 **Figure 7**

TESTING OF THE SIMULATION MODEL

The performance of the developed simulation model that can be used in an operational mode was tested using TALSIM 2.0, a reservoir simulation model developed by the Institute for Hydrualics and Water Resourses Engineering, Section for Hydrology and Water Management of the Technical University of Darmstadt, Germany.

The TALSIM 2.0 was calibrated by using 1983 – 1990 data by way of only observed data (i.e. inflows, rainfall, releases, spill, etc.). By comparing the resulting computed water levels with the observed water levels, the unknown factors of the model (e.g. characteristics of the spillway) can be calibrated so that the simulation of the reservoir system is as accurate as possible. The calibration is done by means of trial and error.

To determine the likely causes of the dropping water levels (1991 – 1993), that is once the model has been calibrated using the time period from 1983 to 1990, a continuation of the simulation from 1991 to 1993 determines which factors are the likely causes of the dramatic drop in water levels that occurred during that period.

From 1983 to 1990, the model was able to represent the annual rise and fall of the water levels quite well, although the simulated water levels drop to about 1 m below the observed value in 1987. However, the simulation results again coincide very closely with the observed values in 1989 and 1990. In 1991 and 1992, although inflow into the reservoir is relatively low, the model predicts that reservoir levels should remain at a relatively high level, while the observed water levels drop significantly. The failure of the reservoir system, i.e. the dropping of water levels in Mtera Reservoir, started from 1991 onwards. In the simulation, however, the water levels remain at a high level even after 1991.

The only option that seems to justify the difference of the observed and the simulated levels is that the recorded discharges do not represent what was actually released from the reservoir from 1991 to 1993. In other words, large amounts of water were discharged from Mtera Reservoir during 1991, 1992 – and maybe 1993 – without being (fully) recorded.

It is also a possibility that the discharge from the intervening catchment is not being computed correctly in the normal operation of the system. The discharge from the intervening catchment is computed using TALSIM's internal rainfall-runoff model. The intervening catchment, which is about 8,500 km^2, contributes between 15% and 40% of the inflow into Kidatu Reservoir each year. Thus, representing a sizable amount that could possibly and greatly influence the water levels in Kidatu Reservoir.

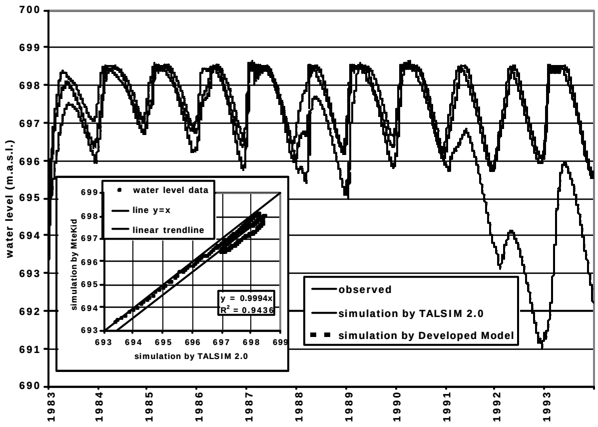

Figure 8

The water levels of the two simulations coincide quite closely. Figure 8 shows a regression analysis of the water levels produced by these two simulations with a R^2-value of almost 95%, indicating a very good correlation, which is an indication of the correctness of the developed simulation model for the system.

CONCLUSION

The possible causes for the failure of the Mtera-Kidatu Reservoir System had been investigated in this paper and had been concluded that inefficient management of the system was the cause. The possible causes considered were sudden decrease in inflows, sudden increase in losses, sudden increase in hydropower generation, and unnecessary spillage. There is enough evidence that a large amount of unrecorded water that might have been spilled from the Mtera Reservoir. Simulation of reservoir operation based on maintaining historical machine discharges at Mtera and Kidatu Reservoirs indicated that much higher reservoir water levels should have been attained in 1991 and 1992 than what was attained and that would have averted the subsequent failure of the reservoir system in 1994.

Clearly, there is a serious problem with the reservoir operation policy of the Mtera-Kidatu Reservoir System. It is likely a possibility that flow contribution of the intervening catchment of Mtera and Kidatu (about 12,000 km^2) is not considered in the operation of the system. It is, therefore, proposed in this paper that the operation of the system should be done in an integrated manner that account for the flows generated by the intervening catchment.

Since estimation of the loss of water is central to the management and operation of reservoirs, this component must not be over look in the operational policy of the system. The suitability and validity of the proposed simulation model was tested with TALSIM 2.0 model and a correlation of 95% was achieved.

APPENDIX 1 – Mathematical Details of the Inflow Models Used

1. Simple Linear Model (SLM)

Let x(t) be the average rainfall (or precipitation) on a catchment and y(t) be the resultant discharge from the catchment, then SLM stipulates that

$$y(t) = \int_0^\infty h(t)x(t-t)dt \qquad (1.1)$$

with the gain factor, $G = \int_0^\infty h(t)dt \cong \sum y(t) / \sum x(t) \leq 1.0$

If the rainfall is effective rainfall, then G = 1.0.

In discrete non-parametric form,

$$y_i = x_i h_1 + x_{i-1} h_2 + \ldots + x_{i-m+1} h_m + e_i$$

$$y_i = \sum_{j=1}^{m} x_{i-j+1} h'_j + e_i, \qquad i = 1,2,\ldots,n \qquad (1.2)$$

where

x_i = total rainfall at i[th] time.
y_i = total discharge at i[th] time.
m = memory length of the system.
h'_j = j[th] discrete pulse response ordinate.
n = number of observations.
e_i = error term at the i[th] time.

If x_i and y_i are in the same units of measurements, then $G = \sum_{j=1}^{m} h'_j$, the gain factor.

Hence, $$y_i = G\sum_{j=1}^{m} x_{i-j+1} h_j + e_i, \qquad i = 1,2,\ldots,n \qquad (1.3)$$

where h_i = j[th] standardized pulse response ordinate and $\sum_{j=1}^{m} h_j = 1.0$.

Equations (1.2) and (1.3) can be solved by Ordinary Least Squares (OLS).

By OLS method,

$$\hat{H} = \left(X^T X\right)^{-1} X^T Y \qquad (1.4)$$

with the objective function for the equation being

$$F_{OLS} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2, \qquad i = 1,2,\ldots,n \qquad (1.5)$$

If we should add a weighting factor, w_i, then

$$F_{WLS} = \sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2, \qquad i = 1,2,\ldots,n \qquad (1.6)$$

and hence $\hat{H} = \left[X^T W X\right]^{-1}\left[X^T W Y\right]$ (1.7)

Estimated discharge is $\hat{y}_i = \sum_{j=1}^{m} x_{i-j+1}\hat{h}_j$.

Solving the system of equations with ordinary least squares without any constraints, the pulse response function could have the following:

(1) Negative ordinates at the beginning and/or at the end of the response function.
(2) Non-existence of high damping condition.
(3) Non-stability of pulse response function.

Solution is, therefore, to impose constraints on the shapes of the response function.

Some of suggested solutions are:

(1) Pulse response function constrained by ridge regression $y_i = \sum_{j=1}^{m} x_{i-j+1} h_j + e_i$ and applying penalty on the linear system.
(2) Pulse response function constrained to by Gamma function form $h(t) = \frac{1}{k\Gamma(n)} e^{-(t/k)} \left(t/k\right)^{n-1}$
where n = amplitude and nk = lag parameter.
resulting in a normal shape of the pulse response function.
Application: Determine the parameters n and nk to get the impulse response function using search optimisation technique with the objective function as the final variance, $F = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$.
(3) Volumetric constrained linear model $y(t) = \int_0^\infty h(t)x(t-t)dt$; $\int_0^\infty h(t)dt = G$
In discrete form,

$$G = \sum_{j=1}^{m} h_j = \sum_{i=1}^{n} y_i \Big/ \sum_{i=1}^{n} x$$

2. Linear Perturbation Model (LPM)

To account for the effect of seasonality, the following transformations were made of the SLM variables:

$$\left.\begin{aligned} R_i &= x_i - \bar{x}_S \\ Q_i &= y_i - \bar{y}_S \end{aligned}\right\} S = 1,2,...,365 \qquad i = 1,2,...,n \tag{2.1}$$

where

S = seasonal mean values.
n = number of daily values.
$\bar{x}_S = \frac{1}{L}\sum_{r=1}^{L} x_{s,r}$; $\bar{y}_S = \frac{1}{L}\sum_{r=1}^{L} y_{s,r}$
L = number of years.
R_i = rainfall departure from the seasonal expectation.
Q_i = discharge departure from the seasonal expectation.

The LPM then develops as follows:

$$Q_i = G\sum_{j=1}^{m} R_{i-j+1} h_j + e_i, \qquad i = 1,2,...,n \tag{2.2}$$

Likewise, the OLS method can be used to estimate the $\hat{h}_j$.

If in equation (2.2), it is assumed that the gain factor, G, is not a constant but varies linearly with the antecedent moisture condition, P_i, then

$$Q_i = b * P_i \sum_{j=1}^{m} R_{i-j+1} h_j + e_i , \qquad i = 1,2,..., n \tag{2.3}$$

where β is the slope of the assumed linear relationship between the gain factor, G_i, and the antecedent moisture condition, P_i.

An estimate of P_i can be given by discharge estimated from the use of the simple linear model (i.e. $\hat{y}_{SLM,i}$).

Thus,
$$Q_i = \sum_{j=1}^{m} R_{i-j+1} * \hat{y}_{SLM,i} * b * h_j + e_i \tag{2.4}$$

or simply
$$Q_i = \sum_{j=1}^{m} R^*_{i-j+1} h^*_j + e_i \tag{2.5}$$

Equation (2.5) is similar in form to equation (1.2) and hence may be estimated by OLS. Once $\hat{Q}_j$ is estimated from equation (2.5), it can be transformed back into discharge estimate.

i.e.
$$\hat{y}_i = \hat{Q}_i + \bar{y}_S \tag{2.6}$$

Equations (2.5) and (2.6) are known as Linearly Varying Gain Factor Perturbation Model (LVGFPM).

3. Linearly Varying Gain Factor Model (LVGFM)

Here, G is not considered as a constant but rather $G_i = a+bz_i$, where a and b are constants, and z_i = index of the soil moisture state.

Thus,
$$y_i = a\sum_{j=1}^{m} x_{i-j+1} B_j + bz_i \sum_{j=1}^{m} x_{i-j+1} B_j + e_i , \qquad i = 1, 2, \ldots, n$$

$$= \sum_{j=1}^{m} x_{i-j+1} B'_j + \sum_{j=1}^{m} \left(z_i x_{i-j+1}\right) B''_j + e_i \tag{3.1}$$

where B_j is a set of discrete weighting function ordinates such that the sum of the ordinates constituting the set is unity.

Note: This is non-linear model as well as multiple linear regression model.

A crude index of the current soil moisture state, z_i, could be obtained from the antecedent precipitation index (API) or $z_i = \frac{G}{\bar{y}} \sum_{j=1}^{m} x_{i-j+1} h_j$ obtained from the outputs of the SLM, where $\hat{G}$ = estimates of the gain factor, $\hat{h}_j$ = estimates of the pulse response ordinates of SLM and $\bar{y}$ = the mean discharge in the calibration period.

OLS can be used for estimating the ordinates B_j.

Alternative approach;
With the linearly varying gain factor,

$$y(t) = G(z(t)) \int_0^{\infty} w(t) x(t-t) dt \tag{3.2}$$

where G(z(t)) = varying gain factor with time.

W(ι) = standardized impulse response with unit area i.e. $\int_0^{\infty} w(t) dt = 1.0$.

(1) Using SLM as an auxilliary model to generate the catchment wettness index

$$y_a(t) = Ga\int_0^t \hat{h}(t\ )x(t - t\ )dt \text{ , Ga = constant}$$

$$\Rightarrow z(t) = \frac{y_a(t)}{\bar{y}} = \left(\frac{Ga}{\bar{y}}\right)\int_0^t \hat{h}(t\ )x(t - t\ )dt$$

where $\bar{y}$ = mean flow in calibration.

(2) Assume that the gain factor is the function of non-dimensional series z(t) i.e. $G(t) = f\left[z(t), P_1, P_2, ...\right]$

- the constraint here is that the gain factor at any time should be less than unity.
- The gain factor should increase monotically with the increase in the catchment wettness as indicated by z(t), i.e., G(t) = a + b*z(t), where a and b are constants.

APPENDIX 2 – Algorithm for the Reservoir System's Simulation

The continuity equation of the Mtera Reservoir is given by

$$\boldsymbol{ZM_{(t+1)} = ZM_{(t)} + IM_{(t)} - QM_{(t)} - OMS\,1_{(t)} - OMS\,2_{(t)} - EM_{(t)}} \qquad (1)$$

Subject to:

$\boldsymbol{ZM_{min}}$ £ $\boldsymbol{ZM_{(t)}}$ £ $\boldsymbol{ZM_{max}}$

where

$\mathbf{ZM_{(t)}}$	is the level at the beginning of time step (t).
$\mathbf{ZM_{(t+1)}}$	is the level at the beginning of the next time step (t+1).
$\mathbf{IM_{(t)}}$	is the total inflow during the time step (t).
$\mathbf{QM_{(t)}}$	is the total machine discharge during the time step (t).
$\mathbf{OMS1_{(t)}}$	is the total necessary spill during the time step (t) to feed the requirements of the Kidatu Reservoir.
$\mathbf{OMS2_{(t)}}$	is the total supplemental spill during the time step (t). This refers to the amount of water that must be spilled to avoid overflowing of the reservoir.
$\mathbf{EM_{(t)}}$	is the total loss (evaporation plus seepage and percolation) during the time step (t).
$\mathbf{ZM_{min}}$	is the minimum allowed storage for Mtera Reservoir.
$\mathbf{ZM_{max}}$	is the maximum allowed storage for Mtera Reservoir.

The water Balance equation for the Kidatu Reservoir is similar to that of the Mtera Reservoir except the term that relates to spill.

$$\boldsymbol{ZK_{(t+1)} = ZK_{(t)} + IK_{(t)} - QK_{(t)} - OKS_{(t)} - EK_{(t)}} \qquad (2)$$

Subject to:

$\boldsymbol{ZK_{min}}$ £ $\boldsymbol{ZK_{(t)}}$ £ $\boldsymbol{ZK_{max}}$

where the terms have same meaning as that for the previous equation presented for the Mtera Reservoir. The word 'K' is used in the Kidatu equation instead of the word 'M' for the Mtera equation.

In the continuity equation for Mtera, there are two terms for spill. One refers to the usual understanding of the term spill and the other refers to spill created to feed the Kidatu Reservoir. However, in the case of Kidatu, the spill term refers to the amount released to avoid over flowing of the dam.

The simulation of the reservoir operation was performed using recorded machine discharge data from the Tanzania Electricity Supply Company (TANESCO), estimated daily inflows to Mtera, estimated daily runoff from the intervening catchment to Kidatu, and estimated daily evaporation losses for both Mtera and Kidatu Reservoirs.

A FORTRAN computer program (currently in a DOS version) was developed to implement this reservoir operation simulation based on the following procedure:

For Mtera Reservoir;

Step (1): Initial water level, $ZM_{(t)}$, was set equal to the the recorded water level on the first day of the simulation on 01/01/1983.

Step (2): The necessary spill from Mtera Reservoir ($OMS1_{(t)}$) was set on the first day as equal to zero and the supplemental spill ($OMS2_{(t)}$) was also set to zero.

Step (3): Inflow into the reservoir was estimated using the ADD model and the machine discharge was set at the recorded value.

Step (4): Daily loss was set equal to daily Penman open water potential evaporation.

Step (5): The storage-elevation relationship of the Mtera Reservoir was used to convert the water level into storage volume. The water balance equation was used in volumetric units and the resultant storage was converted into water level using the same storage-elevation relationship.

Step (6): The computed storage ($ZM_{(t+1)}$) was checked to see if it was greater than the allowed maximum storage (Zm_{max}).

If $ZM_{(t+1)}$ was greater than ZM_{max} then the necessary spill (i.e. $OMS1_{(t)} = ZM_{(t+1)} - ZM_{max}$) was allowed to happen. Computation was allowed to go back to step 5 and $ZM_{(t+1)}$ was recalculated using the new $OMS1_{(t)}$.

For Kidatu Reservoir;

Step (7): The initial water level for Kidatu, $ZK_{(t)}$, was set at the recorded water level on the first day of the simulation.

Step (8): Inflow to Kidatu was estimated as follows:

$$IK_{(t)} = OM_{(t)} + \alpha(AIC_{(t)})$$

where

$OM_{(t)}$ is the total release from Mtera during time step (t)
(i.e. $QM_{(t)} + OMS1_{(t)} + OMS2_{(t)}$),
$AIC_{(t)}$ is the total average rainfall in the intervening catchment during time step (t), and
α is a calibrated parameter equal to runoff coefficient of the intervening catchment.

Step (9): Total losses were set to be equal to estimated daily Penman potential evaporation. The machine discharge was set at the recorded daily machine discharge from the Kidatu Reservoir. The initial spill, $OKS_{(t)}$, was set at zero.

Step (10): The storage-elevation relationship of the Kidatu Reservoir was used to convert the water level into storage volume to compute the water balance in volumetric units. The resulting storage was converted into water level using the same storage-elevation relationship.

Step (11): The computed storage ($ZK_{(t+1)}$) was checked to see if it was greater or less than allowed maximum storage (ZK_{max}) at Kidatu.

(a) If $ZK_{(t+1)}$ was greater than ZK_{max}, then spill was calculated for Kidatu to be equal to $OKS_{(t)} = ZK_{(t+1)} - ZK_{max}$.

Computation was carried back to step (10) and water level for Kidatu $ZK_{(t+1)}$ was recalculated using the revised spill $OKS_{(t)}$.

(b) If $ZK_{(t+1)}$ was found to be less than ZK_{max} then supplemental spill from Mtera was set $OMS2(t) = ZK_{max} - ZK_{(t+1)}$.

Calculations were brought back to step (5) and water levels for Mtera, $ZM_{(t+1)}$, were re-calculated using the new value of $OMS2_{(t)}$.

The supplemental spill from Mtera, OMS2, is for topping up the storage at Kidatu to maintain the Kidatu Reservoir water level at full capacity at all times.

Step (12): Time step was changed and steps (1) to (11) were repeated until the end of the simulation period.

ABBREVIATIONS

LPM – Linear Perturbation Model
LVGFM – Linear Varying Gain Factor Model
SLM – Simple Linear Model
TALSIM – 'Simulation von Talsperren (Systemen)'
TANESCO - Tanzania Electricity Supply Company

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Captions of Tables and Figures