Is there an efficiency case for land redistribution in Philippine brackishwater aquaculture? Analysis in a ray production frontier framework

Xavier Irz and James R. Stevenson

Abstract: We investigate the possible existence of an inverse relationship (IR) between farm size and productivity in Philippine brackishwater pond aquaculture. The study is motivated by the fact that fish ponds have so far been exempted from the Comprehensive Agrarian Reform Laws and suggestions in the literature of inefficient management of fish farms. The analysis of technical efficiency is based on the estimation of a multi-product ray production function estimated in a stochastic frontier framework. There is some evidence of an IR but of only limited strength. Hence, it is unlikely that agrarian reform is the key to unlocking the productivity potential of brackishwater aquaculture in the Philippines.

While global production of capture fisheries stagnated over the last decade, output from aquaculture expanded steadily. The FAO (2002) reports that global catch from capture fisheries barely returned in year 2000 to the level observed in the early 1990s at roughly 78 million tonnes. Meanwhile, production growth in aquaculture took place at an average annual rate of 7.1% in the 1980s and 5.1% in the 1990s, which makes aquaculture one of the fastest growing food-producing sub-sectors (Ahmed and Lorica, 2002). This spectacular development has sometimes been described as a ‘blue revolution’, with the underlying idea that aquaculture could potentially solve some aspects of the world’s chronic hunger and malnutrition problems (Coull, 1993). While there is no arguing with the increase in aquaculture production, it is however necessary to acknowledge that this development has generated a number of social, environmental and economic problems. Hence, questions have been raised about the ecological impact of aquaculture, in particular with regard to biodiversity (Jana and Webster, 2003; Tisdell, 2003) and mangrove destruction (Primavera, 2000); about the equity of its development (Primavera, 1997; Alauddin and Tisdell, 1998; Coull, 1993) and about its food security benefits (Naylor et al., 2000; Primavera, 1997).

The Philippines conform to these general trends. Yap (1999) reports that aquaculture output in the country has grown at the average annual rate of 5.4% in the 1990s and that its share of total fisheries production keeps increasing. Yet, its development has had a detrimental effect on mangroves, resulted in the salinisation of previously productive agricultural land, generated conflicts over the use of natural resources (Yap, 1999) and some have even argued that it has been responsible for the marginalisation of some coastal communities and an increase in the rate of unemployment (Primavera, 1997). Against this background, the aim of this article consists in addressing one equity aspect of aquaculture development in the

---

1 As there is increasing doubts regarding the validity of Chinese fisheries statistics, our statements refer to the world excluding China. See FAO (2002) for a discussion of this issue.
Philippines that relates to the distribution of fishpond holdings. We investigate whether there is any evidence of an inverse relationship (IR) between farm size and productivity in brackishwater aquaculture in order to evaluate the case, on efficiency ground, for reform of the existing tenurial system, land redistribution, or other policies aimed at improving the functioning of the land market.

The study is motivated first by a common perception that the vast areas of Philippine brackishwaters represent a valuable resource that is not exploited optimally and is not contributing fully to the development process of coastal areas. We believe that it will make a contribution to an important and ongoing policy debate that emerges from the fact that, while the Philippines adopted several land reform laws in the late 1980s, aquaculture ponds have so far been exempted. As a result, the distribution of holdings in brackishwater aquaculture remains very unequal as indicated by a Gini coefficient of 0.72 for the two regions that form the focus of our study and it is well-known that fish farms of more than a hundred hectares are not uncommon. Naturally, large fishpond owners and leaseholders believe that agrarian reform would, if anything, only worsen the severe problems of poverty and inequality in the communities where fish farming represents an important activity. Yap (1999) cites a telling extract from the newsletter of Negros Prawn Producers and Marketing Cooperative:

“The implementation of the (land reform) law is liable to cause widespread strife among the landowners…. There is no showing that land reform will enliven the plight of the poor. Without undermining their capabilities, it is also doubtful whether they (the farmers) can put up the necessary capital to maximize land use. Having been used to having a landlord on whom to call in times of need, this plunge to independence may have a crippling effect.”

This view stands in sharp contrast with the common belief in agriculture that small farmers tend to achieve higher productivity and efficiency levels than large farmers, i.e. that there usually is an IR, as demonstrated in Sen’s seminal paper (Sen, 1962). Besides, the experience of Thailand, where the extremely dynamic prawn industry is supported by relatively small

---

2 Although it is not always specified, our study relates only to brackishwater pond aquaculture.
3 Yap (1999) reports that there are 239,323 hectares of brackiswater fishponds in the Philippines. The electronic data that we obtained from the Bureau of Agricultural Statistics gives a total harvested area of 415,272 hectares in year 2000.
4 The most recent one is the Comprehensive Agrarian Reform Law (CARL) of 1988 that imposes land redistribution with a five hectare retention limit set on all agricultural land.
5 Source of data: Bureau of Agricultural Statistics’ inventory of fishponds from 1997.
6 A recent review of the IR literature is Fan and Chan-Kang (2003). It concludes to the lack of consensus on the validity of the IR hypothesis.
farmers (Yap, 1999), suggests that there is no particular impediment to the development of a competitive aquaculture sector based on smallholders. We therefore believe that testing the IR in Philippine brackishwater aquaculture will generate important policy insights; in particular, a strong IR would suggest that institutional changes leading to a more equal size distribution of holdings could increase both equity and efficiency.

Our analysis is based on the analysis of a sample of 127 farms in two of the three main regions for brackishwater aquaculture in the Philippines and investigates the level and determinants, including farm size, of their technical efficiency. From a methodological standpoint, we believe that the article makes three contributions to the agricultural economics literature. First, we represent the technology by a ray production function first proposed by Löthgren (1997), which, to the best of our knowledge, has not been previously attempted on farming data. This approach presents several advantages that we explain in the next two sections. Second, we explore the properties of the output ray function which arise from its duality with both the minimum cost and maximum revenue functions. This is important in interpreting our estimations results, and could be useful in the future to use the model to analyse issues of allocative efficiency. Finally, we propose two approaches to quantify the explanatory power of the inefficiency effect variables in the Battese and Coelli (1995) stochastic frontier model. This is extremely useful in the empirical section to measure, in a way that is entirely consistent with the underlying frontier model, the strength of the IR relationship.

The paper is organised as follows. The next two sections present the different approaches to the measurement of efficiency in polyculture systems, insisting on the advantages and properties of the ray production function. Section three presents the estimation strategy and proposes an approach to quantify the explanatory power of the inefficiency effect variables of the econometric model. The remaining sections discuss the data and empirical model, present the empirical results, and offer conclusions.

**Measuring the productivity/efficiency of polyculture farms**

The IR literature started with the simple observation that yields, defined as output per unit of surface area, differed according to the size of farms. In this context, output is measured either in quantity or value terms and the negative relationship seemingly implies that reallocating land from large to small farms would result in a net increase in production. However, output per hectare is only a partial productivity indicator which cannot satisfactorily measure overall farm productivity (Jha, Chitkara and Gupta, 2000). Hence, farm A can achieve a higher yield than farm B not because it is more efficient in transforming inputs into outputs but
simply because it uses more fertilisers or pesticides. Furthermore, any introductory economics textbook makes it clear that economic optimality usually differs from yield maximisation. To address this concern, albeit only partially, one can investigate the relationship between gross margin and farm size, but this again fails to account for the input of primary factors, such as labour and machinery services, in comparing farm performance. There is also a concern that gross margins do not only reflect the productive ability of farm managers but also the price environment in which they operate (Coelli, Rahman and Thirtle, 2002). Hence, a farm can generate a high gross margin per hectare due to its proximity to a particular market rather than its productive performance. There is therefore a strong case to investigate the IR relationship within the confines of production economics, which can accommodate the multi-dimensional aspect of farm production.

Aquaculture production in the study area involves the polyculture of prawns, fish (tilapia and/or milkfish) and crabs, which are produced simultaneously in the same ponds. This raises a number of interesting modelling issues that we now attempt to tackle. Obviously, all of the inputs, with the exception of the fry and fingerlings, are largely non-allocable, i.e. it is not possible to determine the amount of each input used in the production process of each individual output. Hence, land cannot be allocated to different productions, and neither can the feeds or the labour input used to exchange the pond water or maintain the mud dykes. This introduces a first linkage among the different outputs of the aquaculture farm. Second, it is necessary to recognize the possible jointness of production as it is likely that the different species interact with each other in the aquaculture pond. For instance, biologists and aquaculture experts often consider that the association prawn/tilapia in ponds tends to reduce the rate of prawn mortality because tilapias, through their filtering activity and consumption of organic matter lying at the bottom of the pond, improve the bacteriological quality of the pond water (Corre et al., 1999). If that is so, output of any single species depends not only on the inputs used in the production process of that species but also on the quantities of other species grown simultaneously in the pond. We therefore conclude that the production process relies on a truly multiple-output technology, and that it is not possible to specify different production functions for each output.

There exist several possible avenues to measure efficiency in a multiple-output context. Maybe the most common approach involves the estimation of dual cost, revenue or profit functions (Löthgren, 2000). However, this group of methods relies on relatively restrictive behavioural assumptions of economic optimization, such as that of profit maximisation, which might not be expected to hold in developing country aquaculture as farmers are likely to adopt complex livelihood strategies in the face of multiple market failures. For instance, in
addition to expected returns, it is likely that the riskiness of alternative enterprises is taken into account by farm operators when formulating production plans because perfect insurance markets simply do not exist. Furthermore, estimation of these dual functions requires data on prices of inputs and/or outputs that present sufficient variability to allow the use of regression techniques. However, such variability is unfortunately often not present in cross-sectional data because input and output markets are relatively well integrated within regions. We therefore believe that a primal approach is better suited to the analysis of efficiency and productivity for this particular study.

In a primal setting, a straightforward method of productivity/efficiency measurement consists in aggregating all outputs into a single index, which can in turn be used in the estimation of a production function/frontier. The method, suggested by Mundlak (1963) in the agricultural economics literature, has frequently been used in the study of efficiency in aquaculture (e.g., Karagianis, Katranidis & Tzouvelekas (2002) analyse seabass and sea bream production in Greece; Irz and McKenzie (2003) study polyculture systems in the Philippines; and Sharma and Leung (1998) as well as Sharma (1999) investigate the efficiency of carp polyculture systems in Nepal and Pakistan respectively). The single output index is usually obtained as the total weight of production, which, although not theoretically sound, seems acceptable for relatively similar products.

When the products are not close substitutes, however, it is necessary to use output prices to aggregate them. The simplest method consists in expressing output in value terms, but there is a concern that in that case the resulting index reflects not only output quantities but also the prices at which the farm products are sold. This general index number problem is partially circumvented by the use of superlative output indices, such as the Fischer index or Tornqvist-Theil index, first proposed by Caves, Christensen and Diewert (1982) and presented in details in Coelli, Rao and Battese (1998) (chapters 4 and 5). However, even superlative indices measure output in a satisfactorily manner only under a number of restrictive assumptions, most notably that output markets are perfectly competitive. Also, estimating an aggregate production function implicitly imposes restrictions on the form of the underlying multi-product technology. The very existence of an aggregate output index that can be built from output quantities and prices depends on the technology being separable in outputs and inputs (Orea, Alvarez and Morrison Paul, 2002). Hence, we believe that, given the complexity of the input-output relationship in aquaculture, it is desirable to use a framework of analysis that offers more flexibility in the representation of the multi-product technology.

That is how all of the studies mentioned above proceeded, with the exception of Irz and McKenzie (2003).
Fortunately, such a framework has recently become available with different ways of representing the technology. An intuitive idea consists in re-writing the transformation function so as to express one particular output as a function of the input vector and the quantities of all other outputs. There are two difficulties with this approach. First, the choice of output that is used as dependent variable in the regression analysis is arbitrary and this introduces an artificial asymmetry in the method. Second, and most problematic, is the fact that not only the technology parameters but also the efficiency scores depend on the particular output that is chosen as dependent variable. The efficiency scores are therefore output specific and there is no guarantee that the rankings obtained from alternative formulations of the model be consistent with each other, as is easily demonstrated in Figure 1 for the two-output case. The technology is represented by the production possibility frontier PP’. Farm A is clearly closer to the frontier than farm B when efficiency is measured according to a ‘fish’ orientation, but the reverse is true when a ‘prawn’ orientation to efficiency measurement is adopted. Furthermore, when a farm is not producing the output used as dependent variable in the regression, the interpretation of the efficiency scores becomes difficult.

For these reasons, the efficiency literature has moved away from the estimation of transformation functions. A first alternative that has become popular in recent years corresponds to the estimation of input or output distance functions (Coelli and Perelman, 2000; Morrison Paul, Johnston and Frengley, 2000; Brümmer, Glauben and Thijssen, 2002; Irz and Hadley, 2003). The output distance function introduced by Shephard (1970) is defined formally from the output set \( P(x) \) by:

\[
D_o(x,y) = \text{Min} \{ \theta > 0 : y/\theta \in P(x) \} \tag{1}
\]

It measures the fraction of maximum achievable output \( y/\theta \) that the firm produces, given a vector of inputs \( x \) and the technology, and assuming that any increase in production would involve a proportional increase in all individual outputs. For any input-output combination \((x,y)\) belonging to the technology set, the distance function takes a value no larger than unity, with a value of unity indicating technical efficiency. For instance, in Figure 1, farm C is clearly inefficient as its output vector does not lie on the border of the output set, and the resulting value of the distance function is equal to ratio \( OC/OC^d \). The output distance function gives directly the well-known Farrell (1957) output-based index of technical efficiency (Brummer, Glauben and Thijssen, 2002). The output distance function is always homogenous.

---

8 This is a problem for the empirical application presented here as there is great heterogeneity within our sample with regard to the subset of the four species actually produced on the farm, as discussed in detail in the data section.
of degree one in outputs and inherits properties from the parent technology as detailed in Färe and Primont (1995).  

The last set of techniques available to investigate the efficiency of multi-output firms relies on the estimation of a ray frontier production function, first proposed by Lothgren (1997). His basic insight consists in expressing the output vector in polar coordinates, which makes it possible to represent the technology by a function relating the Euclidian norm of the output vector to the inputs and output mix, represented by the output polar coordinates. Formally, the output vector $y$ of dimension $M$ is expressed as:

$$ y = \|y\| m(\theta(y)) $$  \hfill (2)

where $\|y\|$ denotes the Euclidian norm of vector $y$ ($\|y\| = \sqrt{\sum_{i=1}^{M} y_i^2}$), $\theta(y)$ is an $(M-1)$ vector of polar coordinate angles of the output vector $y$, and the $M$ functions $m_i : [0, \pi/2]^{M-1} \rightarrow [0,1]$ define the coordinates of the normalized output vector. This is illustrated in the two-output case in Figure 1. The output vector of farm C is expressed in terms of its norm, $OC/OC^r$, and a single angle $\theta$ measuring the relative proportions of fish and prawn outputs, i.e. the output mix. The two functions $m_f$ and $m_p$ of the polar-coordinate angle $\theta$ simply define the (regular) coordinates of the normalized output vector $OC^r$ obtained by radial projection of vector OC on the circle of radius 1. Formally, the $(M-1)$ polar coordinate angles are obtained by applying recursively the following formulae (Löthgren, 1997):

$$ \theta_i(y) = \cos^{-1}\left(\frac{y_i}{\|y\|\prod_{j=0}^{i-1} \sin \theta_j(y)}\right), i = 1, ..., M - 1 $$  \hfill (3)

where $\sin \theta_0 = \cos \theta_M = 1$. Hence, the first angle $\theta_1$ is equal to $\cos^{-1}(y_1/\|y\|)$; the second angle $\theta_2$ is equal to $\cos^{-1}(y_2/\|y\|\sin \theta_1)$ and so on. Note that all $(M-1)$ functions $\theta(y)$ are homogenous of degree zero in outputs, which simply reflects that they capture only the proportions of outputs in vector $y$. The coordinates of the normalized output vector are also easily recovered as:

$$ m_i(\theta(y)) = \cos \theta_i(y) \prod_{j=0}^{i-1} \sin \theta_j(y) $$  \hfill (4)

This set up allows us to represent any technology by a multi-output ray production function $f(x, \theta(y))$ as follows:

\footnote{In particular, as described in Lovell et al. (1994), the output distance function is non-decreasing, positively linearly homogeneous and convex in $y$, and decreasing in $x$.}
\[ f(x, \theta(y)) = \max \{ \rho > 0 : \rho \cdot m(\theta(y)) \in P(x) \} \]  
(5)

This function gives the maximum norm of the output vector that the firm can produce, given a vector of inputs \( x \) and the existing technology, and assuming that any increase in production would involve a proportional increase in all individual outputs. Hence, any technology feasible input-output combination \((x, y)\) is defined by the inequality \( f(x, \theta(y)) \geq \|y\| \). In terms of Figure 1, the value of the ray production function is simply equal for farm \( C \) to the ratio \( OC^d / OC^c \). Under the assumption of strong input disposability, the ray function is positively monotonic in inputs (Löthgren, 2000).

In order to understand how the ray production function can be used to measure efficiency, it suffices to recognize that the ray production and output distance functions are closely related to each other. It follows from equation (5) that, for any observed output vector \( y \), the radial frontier output vector is simply defined by \( f(x, \theta(y)).m(\theta(y)) \) so that the distance function is recovered as:

\[ D_x(x, y) = \frac{\|y\|}{f(x, \theta(y))} \]  
(6)

This is indeed observed in our graphical example, as ratio \( OC / OC^d \) is obviously equal to \( OC / OC^c \) divided by \( OC^d / OC^c \). This relationship is most important because we know that virtually all the properties of a multi-output technology can be recovered from the distance function. For instance, Brummer, Glauben and Thijssen (2002) and Irz and Hadley (2003) use it to characterize technological change and productivity growth, while Kim (2000) derives measures of output substitutability from it. Equation (6) therefore implies that the same can be done from the ray production function. For our purpose, it is sufficient to recognize that output elasticities are easily derived from the ray production function as:

\[ \varepsilon_{y,x_j} = \frac{\partial \ln \|y\|}{\partial \ln x_j} = \frac{x_j}{f(x, \theta(y))} \frac{\partial f(x; \theta(y))}{\partial x_j} \]  
(7)

This expression gives the percentage change in all outputs resulting from a one percent change in input \( j \) and is expected to take a positive value (Fousekis, 2000). Alternatively, Appendix 1 demonstrates that because the ray production function entertains some duality with both the maximum revenue and minimum cost functions, this elasticity can be interpreted as the revenue elasticity or the scale-adjusted cost share of input \( j \). The scale elasticity follows immediately (Löthgren, 2000):

\[ \varepsilon_{\text{scale}} = \frac{\nabla_j f(x; \theta(y)).x}{f(x, \theta(y))} \]  
(8)
This elasticity should be compared to unity to establish whether the firm operates under decreasing, constant or increasing returns to scale. Finally, the derivatives with respect to the coordinate angles reflect the change in output norm when the output mix is changed along the production frontier. They therefore relate to the degree of substitutability of the different outputs, but in a rather indirect way as demonstrated in Appendix 1, where the expression for the marginal rate of transformation between any two outputs $i$ and $j$ is derived as:

$$\frac{P_{ij}}{P_{ji}} = \frac{y_j \left( \sum_{m=1}^{M-1} \frac{\partial \ln f}{\partial \ln \theta_m} \frac{\partial \ln \theta_m}{\partial \ln y_i} - 1 \right)}{y_i \left( \sum_{m=1}^{M-1} \frac{\partial \ln f}{\partial \ln \theta_m} \frac{\partial \ln \theta_m}{\partial \ln y_j} - 1 \right)}$$

We note, however, that these derivatives are simultaneously equal to zero if and only if the PPF is, at the point of approximation, a perfect sphere in output space. Furthermore, this expression could easily be used to derive to Morishima-like elasticities of output substitution.

We conclude from this analysis that, from a theoretical point of view, the ray production function and the output distance function are superior, as a basis for efficiency measurement of aquaculture farms, to the alternatives presented in the literature (dual functions, aggregate production functions and transformation functions).

**Implementation issues**

While from a theoretical point of view the output distance function and ray production function appear equally satisfactory for our purpose of measuring efficiency in aquaculture, the same does not hold from an empirical/econometric point of view. A first issue relates to the fact that the distance function is linear homogeneous in outputs. Imposing this property globally requires the use of a logarithmic functional form that cannot accommodate zero values on either inputs or outputs. In fact, all of the published papers on distance functions of which we are aware use a transcendental logarithmic functional form, in order to impose homogeneity while conferring sufficient flexibility to the parametric function. A common practice consists then in replacing zero values by ‘small numbers’ (see Morrison-Paul, Johnston and Frengley (2000) and Fousekis (2002) for two recent examples) but this seems highly unsatisfactory as the logarithmic function goes asymptotically to minus infinity at zero. In fact, Battese (1997) explores this problem in the context of a Cobb-Douglas production function to conclude that replacement of zero values by small numbers can seriously bias the

---

10 Note that distance functions can be used in a non-parametric setting in order to measure technical efficiency, as is the case for instance in Data Envelopment Analysis. Because these techniques do not account for noise, and production shocks seem important in aquaculture as indicated for instance by the levels and variability of mortality rates of prawns, we do not think that they are suitable for our purpose. We therefore focus our discussion on parametric techniques.
parameter estimates. Given that most farms in our sample do not produce all four outputs, this problem represents a major obstacle to the estimation of an output distance function on our data.

The second issue arises from the fact that the value of the distance function is unobservable so that an expression of the form $D=f(x,y)$ is not estimable directly by standard regression techniques. Following Lovell et al. (1994), this problem is usually circumvented by modifying the regression equation based on the homogeneity properties of the distance function. However, it is feared that this clever transformation of the estimable equation might lead to possible endogeneity of the regressors (Grosskopf et al., 1997; Löthgren, 2000).

By contrast, no homogeneity restriction needs be imposed on the ray production function, which can therefore be represented by non-logarithmic functional forms and hence accommodate zero values. Furthermore, it is also believed that the endogeneity problem highlighted above for the distance function does not apply to the ray production function (Löthgren, 2000). Hence, we choose to pursue our investigation of efficiency of aquaculture farms in the Philippines based on the estimation of a ray production function.

**Estimation Strategy**

The estimation of firm-level efficiency scores from a ray production function follows the stochastic frontier methodology initially proposed by Aigner, Lovell and Schmidt (1977). Accordingly, a scalar-valued composed error term is introduced in the empirical ray production function$^{11}$:

$$\|y\| = f(x, \theta(y); \beta) + v - u$$

where $\beta$ is a vector of parameters to be estimated; $v$ is a symmetric random variable that is independently and identically distributed across individuals; and $u$ is a non-negative random variable. This specification recognizes the fact that production is first affected by random shocks and measurement errors, which are captured by the disturbance term $v$. However, the productive performance of farms is also determined by the quality of managerial decisions and it is likely that some farmers make mistakes, i.e., that they are technically inefficient. This is formally captured by the random variable $u$ that describes the deviation of the norm of the observed output vector $y$ from the maximum achievable norm $f(x, \theta(y); \beta)e^v$, which is conditional on the exogenous shock $v$.

$^{11}$ Notice that the error term is introduced in an additive rather than multiplicative way because, as explained earlier, we do not want to use a logarithmic functional form due to the ‘zero value’ problem.
Given a parameterisation of the ray production function and distributional assumptions on the random terms, equation (10) can be estimated by the maximum likelihood methods that have now become commonplace in the stochastic frontier literature. All models consider that the random error term $v$ follows a normal distribution $N(0, \sigma_v^2)$ but differ with respect to the distribution of inefficiencies $u$. A first generation of models considers that this term is identically and independently distributed, following a half-normal distribution (Aigner, Lovell and Schmidt, 1977), truncated normal (Stevenson, 1980) or gamma (Greene, 1990) distribution. However, by assuming that inefficiencies are identically distributed, all of these models implicitly assume that there is no relationship between efficiency and farm-specific characteristics, as was first noted by Kumbhakar, Gush and McGuckin (1991). Consequently, they are obviously ill-suited to the analysis of the inverse relationship.

Fortunately, several models have been developed to simultaneously measure inefficiencies and identify their farm-level determinants. We adopt the formulation of Battese and Coelli (1995) who relax the assumption of identically distributed inefficiency terms by considering that $u_i$ is obtained by truncation at zero of a normal variable $N(\mu_i; \sigma_u^2)$ where:

$$\mu_i = z_i \delta$$

The term $z_i$ denotes a vector of potential determinants of inefficiencies, including farm size, while $\delta$ is a vector of parameters to be estimated. Note that because the inefficiency effects enter the model in a highly non-linear way, there is no identification problem when using the same variable in the specification of the ray production function and as an inefficiency effect. The likelihood function is derived algebraically as in Battese in Coelli (1993) and it can then be maximised numerically to produce estimates of both the ray production function and the vector of parameters $\delta$. Further, while the individual inefficiency levels are not directly observable, the method allows for calculation of their predictors by applying the procedure first proposed by Jondrow et al. (1982). As the expressions for these predictors are presented in Battese and Coelli (1993) only for the multiplicative model, while ours is additive, they are worth reporting here. First, the conditional expectation of the inefficiency term $u$ given a total residual $e = v - u$ is derived from the expression of the conditional density function of $u$ given $e$ derived in full in Battese and Coelli (1993):

---

13 The individual subscript $i$ was ignored up to this point for notational clarity.
14 An example of a stochastic production frontier where land appears both as an input and as an inefficiency effect is Ngwenya, Battese and Fleming (1997), cited on page 212 of Coelli, Rao and Battese (1998). The issue of identification is also discussed in Battese and Coelli (1995) where a time trend is used to capture both technological change and inefficiency change over time.
\[ E(u|e) = \mu_e + \sigma_{\mu_e} \frac{\phi(\mu_e / \sigma_e)}{\Phi(\mu_e / \sigma_e)} \]  

(12)

where:

\[ \mu_e = \frac{\sigma_{\mu_e}^2 \delta - \sigma_{\mu_e}^2 \epsilon}{\sigma_{\mu_e}^2 + \sigma_{\epsilon}^2} \]  

(13a & b)

\[ \sigma_e^2 = \frac{\sigma_{\mu_e}^2 \sigma_{\epsilon}^2}{\sigma_{\mu_e}^2 + \sigma_{\epsilon}^2} \]

These expressions express mathematically that the random variable \( u \), conditional on \( e \), is simply obtained by truncation at zero of the normal variable \( N(\mu, \sigma^2) \). The Farell output-oriented efficiency score follows immediately:

\[ \hat{TE} = \frac{\hat{y} - E(u|e)}{\hat{y}} \]  

(14)

where \( \hat{y} \) denotes the fitted output norm \( f(x, \theta(y); \hat{\beta}) \) and \( \hat{e} \) is the estimated residual.

Next we turn to the issue of quantifying the explanatory power of the inefficiency effects introduced in vector \( z \), which is motivated by our primary aim of exploring the robustness of any potential IR in Philippine aquaculture. This problem has been largely ignored in the literature, as the only attempt at tackling it of which we are aware is Pascoe and Coglan (2002) who, on a fisheries model, develop a procedure to isolate the variation in inefficiencies attributable to the inefficient effect variables. The procedure simply involves regressing the estimated technical efficiency scores against the variables introduced as inefficiency effects by ordinary least squares. This approach is ad hoc and seems unsatisfactory because it fails to recognize the highly non-linear way in which the inefficiency effects enter our model. From equation (11), it is evident that the mean of the normal variable truncated at zero to model inefficiencies is a linear function of the \( z \) variables but this implies that the relationship between predicted efficiencies (14) and the \( z \) variables takes a complex non-linear form. We therefore prefer to investigate this question differently.

A first approach compares the full specification of the model to a restricted one where this variable is dropped from vector \( z \) in equation (11). The comparison is based on the decomposition of the total variance term \( e \) into its random shock and inefficiency components \( u \) and \( v \). Coelli (1995) establishes that the relative contribution of inefficiency to the variance of the error is given by:

\[ \gamma^* = \gamma / [(\gamma + (1-\gamma))\pi/(\pi-2)] \]  

(15)
where parameter $\gamma = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$. This quantity captures the variation in production not accounted for by physical factors that is attributed to inefficiencies rather than random shocks. Hence, the difference between this quantity for the full model and the restricted model gives us directly a measure, in percentage terms, of the explanatory power of the inefficiency effect $z_i$.

We would also like to be able to measure the strength of the relationship between any $z_i$ variable and technical efficiency by calculating a standard elasticity but once again, the literature seems to have ignored this issue. From equations (12) and (13), one can derive the responsiveness of the conditional predictor of $u$ to a change in any inefficiency effect variable:

$$\frac{\partial E(u|\gamma)}{\partial \ln z_i} = \frac{z_i \gamma_i (1 - \gamma)}{\sigma^2} \left[ \Phi(\mu^*/\sigma^*) \Phi(\mu^*/\sigma^*) - \Phi(\mu^*/\sigma^*) \right]$$  

Using this expression in equation (14) defining the efficiency score, one obtains:

$$\frac{\partial TE}{\partial \ln z_i} = -\frac{z_i \gamma_i (1 - \gamma)}{\gamma(\Phi(\mu^*/\sigma^*))^2} \left[ \Phi(\mu^*/\sigma^*) \Phi(\mu^*/\sigma^*) - \Phi(\mu^*/\sigma^*) \right]$$  

This elasticity gives the percentage change in efficiency resulting from a unit percentage change in the inefficiency effect variable $z_i$. Note that it depends not only on the parameter estimates but also on the data so that it can be estimated at any sample point or at the sample mean. The empirical section of the paper uses this expression to derive what we call the technical efficiency elasticity of farm size. Alternatively, Kumbhakar and Lovell (2002) propose to use the mode of the distribution of $u$ given $e$ as predictor of the inefficiency variable, which gives for our model:

$$M(u|e) = \begin{cases} \mu^* & \text{if } \mu^* \geq 0 \\ 0 & \text{otherwise} \end{cases}$$ (18)

The resulting elasticity of technical efficiency with respect to any inefficiency effect variable follows immediately:

$$\frac{\partial TE}{\partial \ln z_i} = \begin{cases} -\frac{z_i \gamma_i (1 - \gamma)}{\gamma} \text{if } \mu^* \geq 0 \\ 0 & \text{otherwise} \end{cases}$$ (19)

**Data and estimable model**

The two main regions of the Philippines for brackishwater pond aquaculture were selected for this particular study. Region three covers the central part of the northern island of Luzon and has brackishwater fish ponds in the four provinces of Pampanga, Bulacan, Bataan and Zambales. Region six is located in the Western Visayas, central Philippines, and includes the
provinces of Iloilo, Capiz, Negros Occidental and Aklan. The sample was stratified by farm size and by province, based on census data from 1997 provided by the Bureau of Agricultural Statistics. Production and socio-economic data were then collected by interviews with farm operators and caretakers (salaried supervisors). A total of more than 150 farms were initially surveyed but several observations were dropped because of inconsistencies and/or missing values, so that our analysis is based on a sample of 127 individuals.

Table 1 presents the summary statistics of the production variables. Starting with the land input, it is apparent from our data that the farms in the study area are relatively large, with an average surface area of more than eleven hectares\(^\text{15}\). Further, land is also unequally distributed, ranging from one tenth of a hectare for the smallest farm to 130 hectares for the largest, and the Gini coefficient of land concentration for our sample is equal to 0.67. This heterogeneity in farm size gives some relevance to the investigation of the IR pursued in this paper. Table 1 also informs us about the type of aquaculture practiced by the farms in the sample. First, in monetary terms, the main intermediate input corresponds to the seeds (“fry” for prawns, “juveniles” for crabs and “fingerlings” for milkfish and tilapia), followed by the feeds and, finally, fertilisers. This simple fact reflects the extensive nature of brackishwater aquaculture in the Philippines, as even in semi-intensive production systems, the feeds account for the major share of cash costs. Also, the substantial cost of fertilisers reveals that farm operators attempt to bolster the natural productivity of aquaculture ponds, while the production process in intensive aquaculture relies solely on the provision of feeds from an external source for the growth of the cultivated species. Finally, the summary statistics also suggest that labour represents an important cost of production, as the wage rate for farm labour is approximately 150 PhP/day in region 3 and 100 PhP/day in region 6, so that the total wage bills exceeds, on average, the cost of any individual intermediate input.

With respect to outputs, milkfish is the dominant production in volume. This is not surprising as the polyculture production system described here represents a recent evolution of the traditional milkfish monoculture system (Chong et al., 1984). The average milkfish yield of less than 500kg per hectare confirms the extensive nature of the production process. The volumes produced of the other species appear relatively small compared to that of milkfish but the relative importance of the species is different in value terms. Given that prawns fetch a price nearly ten times as high as that of milkfish per weight unit, they actually represent the

\(^{15}\) This can be compared to an average size of prawn farms in Thailand of only 2.16 hectares (Yap, 1999).
dominant production in terms of revenue share. This price differential is explained in part by the fact that milkfish and tilapia are consumed domestically, while an important proportion of the prawns are exported to the high-income markets of Japan and the United States. However, notice that crabs, which are also exclusively sold on domestic markets, also receive high prices and are therefore important productions in economic terms.

Finally, Table 1 also suggests that the farms in the sample choose different associations of species, as average output for each species differs whether it is computed on the whole sample of farms or on the sub-sample with a strictly positive output for that particular species. Figure 2 represents the distribution of the number of species grown on the farm and brings two valuable insights. First, a large majority of farms do indeed practice the polyculture of at least two species, hence justifying our earlier discussion on multi-product technologies. And second, the association of all four species is only adopted by a relatively small fraction of the sample farms, implying that there is a large number of zero output values in the sample.

We choose a quadratic functional form as a first step in estimating the output ray function defined in equation (7):

$$\|y\| = \alpha_0 + \sum_{j=1}^{M-1} \alpha_j w_j + \sum_{j<k}^{M-1} \sum_{k=1} \beta_{jk} w_j w_k + D - u + v$$

(20)

where the vector $w$ includes each of the $(M-1)$ polar coordinate angles $\theta(y)$ and the $K$ inputs, and $D$ is a regional dummy taking a value of unity for the farms located in region 6. The quadratic production function is a flexible functional form in the sense that it can serve as a local second-order approximation to any unknown production function. This specification therefore gives flexibility to the model which can accommodate zero values on both inputs and outputs.

The empirical specification includes the three following inputs: land and labour, defined as in Table 1 as the total surface area of the aquaculture farm and the number of man days of labour used on the farm; and intermediate inputs, expressed in value terms, and hence representing an aggregate of the feed, fry/fingerling and fertiliser inputs. On the output side, all four productions were used to define the three polar coordinate angles for tilapia, crabs and prawns. The last step in specifying the model consists of choosing the inefficiency effects that enter equation (11). The literature suggests at that level that one should choose variables

---

16 The average prices per kilogram for our sample are 45PhP for milkfish, 31PhP for tilapia, 412PhP for prawns and 210 PhP for crabs.

17 We introduce the regional dummy because the preliminary OLS regressions discussed below suggest that there might be technological differences between the two regions.
susceptible of influencing the adoption of particular management practices or the determinants of their adoption (Irz and McKenzie, 2003). Given the focus of this paper on the IR relationship, farm size is included as it is our aim to establish whether small and large farms adopt different management practices that lead to differences in efficiency. It is also possible that management differs across regions, and we therefore include the regional dummy as well as inefficiency effect. Other variables, such as training and experience of the operator, probably have an influence on efficiency but our data unfortunately does not allow for their inclusion.

**Empirical Results**

*Partial productivity indicators*

We start our analysis of the IR relationship by investigating the relationship between farm size and land productivity. Although imperfect as outlined earlier, partial productivity indicators have played an important role in the development of the IR literature and are therefore worth reporting. Table 2 presents the results of three OLS regressions relating a measure of land productivity to farm size and, in order to account for possible regional effects, a regional dummy taking a value of unity for the farms located in region 6. The first regression uses the crudest possible measure of land productivity, i.e. harvest weight per hectare, and the results seemingly indicate a significant and positive relationship between farm size and productivity.

However, it makes little sense to add weights of species that fetch widely different prices and the second regression tackles this problem by measuring land productivity in terms of revenue per hectare. The regression, which represents the equivalent of those presented in the influential paper of Berry and Cline (1979), has a surprisingly large explanatory power, as indicated by a R-square value of 0.42. Further, it reveals a significant and negative relationship between farm size and revenue per hectare. The coefficient of the farm size variable is easily interpreted as an elasticity and indicates that a 10% increase in farm size results in a 2.2% decrease in revenue per hectare. Finally, the coefficient of the regional dummy is also negative and significant, indicating that farms tend to be substantially less productive in region 6 than in region 3.

The last regression accounts for differences in use of intermediate inputs when comparing farms as it measures land productivity by gross margin per hectare\(^\text{18}\). It does confirm to some

---

\(^{18}\) Note that for this regression, the dependent variable is the level of the gross margin and not its logarithm. This is so because some farms have negative gross margins which prohibits the use of a log-log functional form for this regression.
extent the results of the previous regression but in a much weaker way. In particular, the negative relationship land productivity-farm size persists, with an elasticity of -0.18, but with only a modest level of statistical significance; the explanatory power of the regression declines to a mere 13%; and it transpires once again that land productivity in region 3 is significantly higher than in region 6.

The difference in results between the two first regressions imply that, on a per hectare basis, larger farms tend to produce more in weight but less in value terms than smaller ones. Hence, it is likely that larger farms tend to choose output combinations with greater emphasis on lower value species (tilapia, milkfish). The difference in results between the two last regressions indicates that the higher revenue per hectare achieved by smaller farms, is, to a large extent, explained by a more intensive use of intermediate inputs. All in all, we conclude from this simple analysis that there is clear evidence of a negative relationship between intensity of land use and farm size but only weak evidence of an inverse relationship between land productivity and farm size.

**Specification tests and the structure of the technology**

The general specification of the stochastic frontier described above was tested against a number of simpler alternatives in order to gain some insights into the structure of the technology and inefficiencies. A second objective is to define a more parsimonious specification as the full model requires estimation of a relatively large number of parameters given the sample size. The results of likelihood ratio tests are presented in Table 3.

First, we test the composed error specification against the hypothesis of absence of inefficiencies by comparing the log-likelihood of our model against that obtained by standard OLS regression. The likelihood ratio statistic of 39.2 exceeds by far its critical value and we therefore conclude to the presence of substantial inefficiencies across our sample farms. From a methodological angle, this result implies that the modelling of the technological relationship between inputs and outputs as a stochastic ray production function rather than a deterministic one is strongly supported by our data.

---

19 The total number of parameters in specification (20) is equal to 34, for a sample size of 127.

20 The test statistic is \( LR = -2 \times (\ln(L(H_0)) - \ln(L(H_1))) \), where \( L(H_0) \) and \( L(H_1) \) denote the values of the likelihood function under the null and alternative hypotheses (Battese and Coelli, 1998).

21 Note that the null hypothesis includes the restriction \( \sigma_u = 0 \). As this parameter is necessarily positive, the test statistic follows a mixed chi-square distribution, the critical values of which are found in Kodde and Palm (1986).
The second test investigates the explanatory power of the two variables introduced as inefficiency effects in our specification. It is also strongly rejected, implying that the regional dummy and farm size variables have, jointly, a statistically significant influence on efficiency. The third test considers the null hypothesis that the regional effects, introduced into the model through the regional dummy in the ray production function and in the inefficiency effect component of the model (11), are inexistent. The hypothesis is accepted as dropping these two variables from the model results in a decrease in the likelihood function of only 0.8, which is marginal. This absence of regional effects in the ray production function stands in sharp contrast to the results obtained earlier based on partial productivity indicators. There is no inconsistency here, however, because the ray production function can accommodate possible differences in output mix across regions, while partial productivity indicators fail to do so\textsuperscript{22}. Next, the explanatory power of farm size on inefficiencies is tested and the null hypothesis of no farm-size effect is strongly rejected. We therefore conclude from these four tests that the regional dummy variable can be dropped from the specification of the model, while farm size as an inefficient effect should be retained.

The last two tests investigate the structure of the technology. Most interesting is the question of whether inputs and outputs are separable, which is tested by comparing our model to a restricted version where the parameters of all cross-terms between inputs and polar coordinates angles in (20) are set equal to zero. The null hypothesis is rejected at any sensible level of significance, which implies that it would not be possible to aggregate consistently the four outputs into a single index. This is why the ray production frontier is used rather than a frontier production function, which requires output aggregation prior to estimation. Finally, the last test considers the null hypothesis that the parameters associated with all the cross-terms among inputs and among polar coordinate angles are equal to zero. It is strongly rejected.

Altogether, we conclude from this series of tests that there are substantial inefficiencies among the sample farms, which are partially explained by farm size. However, regional effects are not present, and the regional dummy is therefore dropped from the model’s specification. Further simplification of the specification is not possible as the tests indicate that the technology is truly multi-product and the relationship among inputs and outputs is a complex one.

\textsuperscript{22} In terms of figure 1, the efficiency of farm C is measured radially, which means that this farm is implicitly compared to farms with a similar output mix. By contrast, gross margin or revenue per hectare measures fail to account for possible differences in output combinations when comparing farms.
The results of the maximum likelihood estimation for our preferred specification are presented in Table 4. We note that many of the coefficients present relatively low levels of statistical significance but this should be expected as there is a high level of collinearity among the covariates. The individual parameters of the technology are not directly interpretable and we therefore compute in Table 5 the elasticities of the production ray function at the sample mean, together with their standard errors. Most straightforward to interpret are the input elasticities described in equation (7). First, there is a significant and positive relationship between land input and production, as a one percent increase in farm size results in a 0.58% increase in all outputs. Hence, land stands out as a key production factor which can be explained by the extensive nature of the technology. Second, the elasticity with respect to intermediate inputs is also highly significant, with a one percent increase in that aggregate resulting in a 0.36% increase in production. Finally, the elasticity with respect to labour is very small, negative and not statistically significant, which means that the model fails to capture a positive relationship between labour input and production. There are several possible explanations for this negative result. One relates to the difficulty of measuring labour input properly, in particular as far as farm operators are concerned. We had to make sometimes crude assumptions in building the labour variable, which might explain in part this statistically insignificant elasticity. Second, the labour variable presents a high degree of collinearity with the other inputs, which can be explained by the fact that most farm operators seem to adhere to the rule of thumb ‘one caretaker for ten hectares’. Finally, we note that the finding of a negative and/or insignificant labour elasticity, although paradoxical, represents an empirical regularity (Whiteman, 1999). The scale elasticity (8) is obtained by summation of all three input elasticities to give a value of 0.92, with a standard error of 0.11. Hence, the technology exhibits slightly decreasing returns to scale at the sample mean but the hypothesis of constant returns to scale cannot be rejected.

On the output side, the elasticities of the ray function with respect to the polar coordinate angles are more difficult to interpret. We note, however, that the last elasticity is large and strongly significant, which implies that the PPF, at the sample mean, differs significantly from a perfect sphere. We conclude that the representation of the technology that we obtain appears reasonably consistent with theoretical expectations and provides an ex-post

---

23 This is not unusual when using flexible functional forms. For instance, in the full translog specification of his model, Löthgren (2000) reports only five significant coefficients (5%) from a total of 21 in the specification of the technology.

24 For instance, we had to assume that the operator was either working full time or half-time on the farm, which probably does not reflect the heterogeneity of situations regarding the labour contribution of the operator.
justification for the ray production function approach that was chosen over the standard estimation of an aggregate production function.

Inefficiencies and the inverse relationship

The likelihood ratio tests established that inefficiencies in our sample were statistically significant, and that is confirmed by the fact that parameter $\gamma$ in Table 4 has a large t-ratio. The mean efficiency score for the sample is equal to 0.37, which is very low and implies that the sample farms could potentially increase production 2.7 times without any increase in inputs or change in technology. This finding confirms that of Irz and McKenzie (2003) and suggests that there is considerable room for managerial improvement of the farms in the study area. It represents an empirical validation of Yap’s contention that many brackishwater ponds are ‘underdeveloped and under-productive’ (Yap, 1999). It can also be explained by the fact that extensive production systems have not been the focus of much research and extension activity in the Philippines, which stands in sharp contrast with the situation of the semi-intensive systems of tilapia production in fresh water that have benefited from large R&D investment. The interviews carried out with farmers confirmed that formal extension services are simply not regarded as an important source of technical information by the operators of extensive farms. Finally, it is also necessary to recognize that the extensive production systems considered here are intrinsically complex and offer numerous opportunities for farmers to make mistakes. This is so because these systems are open, due to the frequent exchange of the pond’s water, which limits the farmer’s control of the production process. Furthermore, the production process depends on the natural productivity of the pond, which itself relates to the populations of various plankton and filamentous algae species that are difficult to manage and sensitive to temperature, salinity, soil conditions and the chemical and nutrient composition of the culture water (Arfi and Guiral, 1994). The situation is very different in intensive production where the growth of the target species depends primarily on the feeds brought from outside of the farm and the pond has little biological function beyond the provision of oxygen to the fish/crustaceans (Kautsky et al., 2000).

Figure 3 presents the frequency distribution of efficiency scores and indicates a high level of heterogeneity within the sample. The distribution is very flat, as reflected by a standard error of 0.26, and is spread over the whole possible range, from a minimum of zero to a maximum.

---

25 In the additive model presented here, the predicted efficiency scores can take negative values, which is theoretically impossible. We therefore replaced negative values by zeros when that occurred (in only a few cases) prior to calculating this average.

26 We are thankful to Pierre Morrisens for this idea.

27 An analogy with agriculture might be useful here. Extensive aquaculture, like organic farming, seems to be management intensive while intensive aquaculture, like conventional farming, tends to rely on the application of standard technological packages that leave little initiative to the farmer.
of 0.97. We now turn to the direct analysis of the IR relationship by investigating whether these large variations in technical efficiency scores are related to farm size. The likelihood ratio tests already demonstrated the existence of a significant relationship between farm size and efficiency, which can also be seen in Table 4 by the large t-ratio of parameter δ. Furthermore, note that this parameter takes a strictly positive sign, indicating that larger farms in our sample are less efficient than smaller ones. Hence, we conclude to the existence of a statistically significant IR in Philippine brackishwater aquaculture. We would like, however, to go further in identifying the strength of this relationship, which cannot be established directly from the parameter estimates and we now implement for that purpose the two approaches discussed in the methodological section.

Our first contention was that a natural way of investigating the explanatory power of farm size as an inefficiency effect consists of comparing the full model to a restricted version where farm size is dropped as an inefficiency effect. We find that for the full specification, inefficiencies account for 86% of the total variance term, implying that the bulk of the variation in production not accounted for by physical factors is attributed to inefficiencies rather than random shocks (i.e., what Pascoe and Coglan (2002) referred to as luck). For the restricted model, where farm size is dropped from the z vector, inefficiencies account for only 73% of the total variance term. It is therefore logical to conclude that variations in production not accounted for by inputs are attributable to random shocks for 14%; farm size for 13%; and unexplained inefficiencies for 73%. This implies that the IR, although statistically significant, appears to be of only limited quantitative importance.

Next, we compute the efficiency elasticity of farm size corresponding to equation (17) and obtain a value of -0.137 at the sample mean. This indicates that a 10% increase in farm size decreases the level of farm-level efficiency by a modest 1.4% for the average farm and confirms the previous result of an IR of only limited strength. When farm-level efficiency is predicted by the mode of the distribution of u given e, as in equation (18), the efficiency elasticity at the sample mean takes the same value at the three-digit level. The results are therefore robust to the choice of predictor used to infer farm-level efficiency scores.

From a methodological point of view, it is also interesting to compare our results to those obtained by application of the procedure suggested by Pascoe and Coglan (2002) to quantify the explanatory power of the inefficiency effect variables. When regressing by OLS the efficiency scores against the logarithm of farm size, we obtain results that are simply inconsistent with the first-stage maximum-likelihood estimation. The estimated efficiency elasticity of farm size at the sample mean is 0.24, with a t-ratio of 3.52, and the R-square for
this regression is only 9%. Clearly, the sign of the elasticity is inconsistent with that of parameter $\delta_a$ in Table 4. Furthermore, these results suggest that farm size explains only 7.7% ($=0.09*0.86$) of the variation in outputs not accounted for by physical inputs, while we find a value almost twice as large. Hence, we conclude that this procedure, which is not consistent with the underlying model of efficiency measurement, can lead to erroneous conclusions regarding both the direction and the strength of the relationship between inefficiency effect variables and efficiency scores. We therefore believe that our methodological contribution is important in deriving the policy implications of the popular Battese and Coelli (1995) model.

**Discussion and conclusion**

This paper uses a stochastic ray production function in order to investigate a potential inverse relationship in Philippines brackishwater aquaculture, based on a cross-section of 127 farms. The novelty of our approach is threefold: at a theoretical level, we derive the dual properties of the ray production function, which are useful in interpreting the parameter estimates; at an econometric level, we offer two different approaches to quantity the explanatory power of the inefficiency effects, including farm size, that are introduced in the model; and at an empirical level, ours is the first attempt to model a farming technology by a ray production function.

The estimated multi-product technology is not separable in inputs and outputs, implying that our approach is superior to the estimation of a stochastic production function, which requires the aggregation of outputs into a single index. Returns to scale are slightly decreasing at the sample mean but the CRS hypothesis cannot be rejected. The distribution of efficiency score is spread out over the whole possible range with an average value of 0.37, which is extremely low. Large potential productivity gains are therefore achievable in the study area, without any change in the technology, output mix or input combination. Is land redistribution or an improvement in the functioning of the land market a key to achieving these efficiency gains? Our analysis reveals that it is probably not the case. We find that there clearly exists a significant inverse relationship between farm size and productivity, but that the strength of this relationship is limited. Farm size explains only 13% of the variability in outputs not accounted for by physical inputs, against 73% for unidentified factors, and 14% for random shocks. The elasticities that we derive indicate that when farm size doubles, efficiency decreases only by a modest 14%. It is therefore likely that application of the land reform laws to brackishwater fish ponds, which so far have secured exemptions via intense political lobbying by the pond owners and lease holders, does not constitute a panacea to unlock the productive potential of these areas. There might be legitimate reasons, on equity grounds, to call for the removal of these exemptions, but the efficiency case for this policy carries only limited weight. We know that the cost of implementing land redistribution programs is always
high, and that is likely to be particularly so in the Philippines where issues of corruption, weak law enforcement and slow-moving bureaucracy in coastal areas are well documented (Primavera, 2000).

Although this is an important result for policy formulation, it is unfortunately a negative one as we are left with the conclusion that variations in efficiency relate to unexplained factors. The best we can do at this level is therefore to speculate on the underlying reasons leading to the poor average technical performance of the farms. Here, we believe that the lack of R&D investment in brackishwater aquaculture is a key constraint to the production and productivity growth of the sector. Even aquaculture specialists recognize the difficulty to manage these systems, and it therefore seems that there is a need to generate knowledge before even considering investment in extension services. It remains to be shown that such investments are economically desirable, but our results suggest that the potential gains from improved farm management are very large.
Figure 1: Alternative representations of a multi-output technology.

Figure 2: Distribution of Number of Species Grown on the Farm
Figure 3: Frequency distribution of efficiency scores

Table 1: Summary statistics*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Mean producers**</th>
<th>Std Dev.</th>
<th>Std Dev. Producers**</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OUTPUTS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Milkfish (Kg)</td>
<td>4,356</td>
<td>5,075</td>
<td>1,098</td>
<td>13,339</td>
<td>0</td>
<td>80,000</td>
</tr>
<tr>
<td>Tilapia (Kg)</td>
<td>674</td>
<td>2,950</td>
<td>230</td>
<td>4,886</td>
<td>0</td>
<td>25,600</td>
</tr>
<tr>
<td>Prawns (Kg)</td>
<td>691</td>
<td>1,111</td>
<td>202</td>
<td>2,832</td>
<td>0</td>
<td>22,240</td>
</tr>
<tr>
<td>Crabs (Kg)</td>
<td>311</td>
<td>878</td>
<td>79</td>
<td>1,330</td>
<td>0</td>
<td>8,000</td>
</tr>
<tr>
<td><strong>INPUTS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Land (ha)</td>
<td>11.5</td>
<td>-</td>
<td>1.9</td>
<td>-</td>
<td>0.1</td>
<td>130.0</td>
</tr>
<tr>
<td>Labour (man days)</td>
<td>1,160</td>
<td>-</td>
<td>220</td>
<td>-</td>
<td>187</td>
<td>26,312</td>
</tr>
<tr>
<td>Feeds (Pesos)</td>
<td>95,259</td>
<td>-</td>
<td>39,617</td>
<td>-</td>
<td>0</td>
<td>4,420,893</td>
</tr>
<tr>
<td>Fert (Pesos)</td>
<td>33,578</td>
<td>-</td>
<td>5,867</td>
<td>-</td>
<td>0</td>
<td>403,260</td>
</tr>
<tr>
<td>Fry/fingerlings (Pesos)</td>
<td>183,770</td>
<td>-</td>
<td>46,518</td>
<td>-</td>
<td>0</td>
<td>4,140,000</td>
</tr>
</tbody>
</table>

* All variables are expressed on a per year basis.
** Mean and standard deviation computed over the sub-sample of farms producing a positive quantity of the product.
Table 2: Inverse relationship based on partial productivity measures

<table>
<thead>
<tr>
<th>Regressors:</th>
<th>Dependent Variable</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Log(harvest weight</td>
<td>Log(harvest value per</td>
<td>Gross Margin per</td>
</tr>
<tr>
<td></td>
<td>per hectare)</td>
<td>hectare)</td>
<td>hectare)</td>
</tr>
<tr>
<td>Constant</td>
<td>6.52</td>
<td>11.60</td>
<td>90,940</td>
</tr>
<tr>
<td></td>
<td>(37.32)</td>
<td>(71.78)</td>
<td>(6.78)</td>
</tr>
<tr>
<td>Log(Farm size)</td>
<td>0.87</td>
<td>-0.22</td>
<td>-8,953</td>
</tr>
<tr>
<td></td>
<td>(11.91)</td>
<td>(-3.30)</td>
<td>(-1.60)</td>
</tr>
<tr>
<td>Regional Dummy</td>
<td>-0.85</td>
<td>-1.69</td>
<td>-63,528</td>
</tr>
<tr>
<td></td>
<td>(-4.04)</td>
<td>(-8.69)</td>
<td>(-3.93)</td>
</tr>
</tbody>
</table>

R2 0.55 0.42 0.13

Table 3: Specification tests

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Log-likelihood</th>
<th>LR statistic</th>
<th>Critical Value (5%)</th>
<th>Critical Value (1%)</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 No inefficiencies</td>
<td>-113.0</td>
<td>39.2</td>
<td>8.8</td>
<td>12.5</td>
<td>Reject</td>
</tr>
<tr>
<td>2 No inefficiency effects</td>
<td>-104.3</td>
<td>21.9</td>
<td>6.0</td>
<td>9.2</td>
<td>Reject</td>
</tr>
<tr>
<td>3 No regional effects</td>
<td>-94.1</td>
<td>1.6</td>
<td>6.0</td>
<td>9.2</td>
<td>Accept</td>
</tr>
<tr>
<td>4 No farm size effect</td>
<td>-100.0</td>
<td>13.4</td>
<td>3.8</td>
<td>6.6</td>
<td>Reject</td>
</tr>
<tr>
<td>5 Input-output separability</td>
<td>-195.2</td>
<td>203.8</td>
<td>16.9</td>
<td>21.7</td>
<td>Reject</td>
</tr>
<tr>
<td>6 No cross-terms</td>
<td>-136.2</td>
<td>85.7</td>
<td>12.6</td>
<td>16.8</td>
<td>Reject</td>
</tr>
</tbody>
</table>
Table 4: estimation results of ray production frontier

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>α_0</td>
<td>0.569</td>
<td>0.68</td>
</tr>
<tr>
<td>α_a</td>
<td>0.018</td>
<td>0.01</td>
</tr>
<tr>
<td>α_l</td>
<td>0.965</td>
<td>0.83</td>
</tr>
<tr>
<td>α_i</td>
<td>-0.890</td>
<td>-1.40</td>
</tr>
<tr>
<td>α_{θt}</td>
<td>-0.685</td>
<td>-0.64</td>
</tr>
<tr>
<td>α_{θc}</td>
<td>0.417</td>
<td>0.36</td>
</tr>
<tr>
<td>α_{θp}</td>
<td>-0.858</td>
<td>-1.87</td>
</tr>
<tr>
<td>β_{aa}</td>
<td>0.047</td>
<td>1.27</td>
</tr>
<tr>
<td>β_{ll}</td>
<td>0.001</td>
<td>0.08</td>
</tr>
<tr>
<td>β_{ii}</td>
<td>-0.092</td>
<td>-13.76</td>
</tr>
<tr>
<td>β_{θtθt}</td>
<td>0.140</td>
<td>0.20</td>
</tr>
<tr>
<td>β_{θcθc}</td>
<td>-0.320</td>
<td>-0.48</td>
</tr>
<tr>
<td>β_{θpθp}</td>
<td>0.448</td>
<td>2.00</td>
</tr>
<tr>
<td>β_{al}</td>
<td>0.034</td>
<td>0.98</td>
</tr>
<tr>
<td>β_{ai}</td>
<td>0.403</td>
<td>11.72</td>
</tr>
<tr>
<td>β_{θtθt}</td>
<td>-0.218</td>
<td>-5.16</td>
</tr>
<tr>
<td>β_{θtθc}</td>
<td>0.103</td>
<td>0.11</td>
</tr>
<tr>
<td>β_{θtθp}</td>
<td>-0.920</td>
<td>-0.86</td>
</tr>
<tr>
<td>β_{θcθp}</td>
<td>0.031</td>
<td>-0.09</td>
</tr>
<tr>
<td>β_{θcθc}</td>
<td>0.843</td>
<td>1.53</td>
</tr>
<tr>
<td>β_{θpθp}</td>
<td>0.580</td>
<td>0.69</td>
</tr>
<tr>
<td>β_{θcθp}</td>
<td>0.368</td>
<td>1.14</td>
</tr>
<tr>
<td>β_{θtθc}</td>
<td>0.120</td>
<td>0.22</td>
</tr>
<tr>
<td>β_{θtθp}</td>
<td>0.084</td>
<td>0.27</td>
</tr>
<tr>
<td>β_{θcθp}</td>
<td>0.066</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Inefficiency Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ_0</td>
<td>-2.356</td>
<td>-2.26</td>
</tr>
<tr>
<td>δ_a</td>
<td>2.292</td>
<td>4.43</td>
</tr>
</tbody>
</table>

Variance Parameters

\[ \sigma^2 = \sigma_u^2 + \sigma_v^2 \]

\[ \gamma = \sigma_u^2 / \sigma^2 \]

Log-likelihood -94.147

Subscript notations: a=land input, l=labour inputs, i=intermediate inputs, (θt, θc, θp)=three polar coordinate angles corresponding to tilapia, crabs and prawns respectively; α0 and δ0 are the constant parameters.
Table 5: Elasticities of estimated ray production function (at sample mean)

<table>
<thead>
<tr>
<th>Elasticity w.r.t.</th>
<th>Estimate</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land</td>
<td>0.58</td>
<td>4.51</td>
</tr>
<tr>
<td>Labour</td>
<td>-0.03</td>
<td>-0.43</td>
</tr>
<tr>
<td>Intermediate Inputs</td>
<td>0.36</td>
<td>7.90</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>0.03</td>
<td>0.12</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>0.08</td>
<td>0.34</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>0.61</td>
<td>7.37</td>
</tr>
</tbody>
</table>

References:


Appendix 1: Dual properties of the ray production function

The revenue maximisation problem can be written in terms of the ray function as:

\[ R(p, x) = \max_y (py) : \; f(x, \theta(y)) \geq \|y\| \]  
(A.1)

This constrained optimisation problem is solved by introducing the following Lagrangian:

\[ L(y, \lambda) = py + \lambda [f(x, \theta(y)) - \|y\|] \]  
(A.2)

The first order conditions are:

\[ \frac{\partial L}{\partial y_j} = p_j + \lambda \left[ \sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \frac{\partial \theta_m}{\partial y_j} - \frac{y_j}{f} \right] = 0 \]  
(A.3)

\[ \frac{\partial L}{\partial \lambda} = f(x, \theta(y)) - \|y\| = 0 \]  
(A.4)

which are more conveniently written as:

\[ p_j = -\lambda \left[ \sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \frac{\partial \theta_m}{\partial y_j} - \frac{y_j}{f} \right] \]  
(A.5)

\[ f(x, \theta(y)) = \|y\| \]  
(A.6)

Multiplying (A.5) by \( y_j \) and summing over all outputs, one obtains:

\[ \sum_{j=1}^{M} p_j y_j = -\lambda \sum_{j=1}^{M} \left[ \sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \frac{\partial \theta_m}{\partial y_j} - \frac{y_j^2}{f} \right] \]  
(A.7)

\[ \sum_{j=1}^{M} p_j y_j = -\lambda \sum_{j=1}^{M} \sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \frac{\partial \theta_m}{\partial y_j} y_j - \sum_{j=1}^{M} y_j^2 f^{-1} \]  
(A.8)

The first term of this sum can be re-written as \( \sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \sum_{j=1}^{M} \frac{\partial \theta_m}{\partial y_j} y_j \), but since the function \( \theta_m \) is homogenous of degree zero in \( y \), \( \sum_{j=1}^{M} \frac{\partial \theta_m}{\partial y_j} y_j = 0 \). Further, using (A.4), equation (A.8) reduces to:

\[ R(p, x) = \sum_{j=1}^{M} p_j y_j = \lambda f \Rightarrow \lambda = R(p, x) / f \]  
(A.9)

This expression means that the Lagrange multiplier is simply the unit value of the norm.

Applying the envelop theorem to the original problem (A.2) therefore gives us:

\[ \frac{\partial R}{\partial x_k} = \frac{\partial f}{\partial x_k} = \frac{R}{f} \frac{\partial f}{\partial x_k} \Rightarrow \frac{\partial \ln R}{\partial \ln x_k} = \frac{\partial \ln f}{\partial \ln x_k} \]  
(A.10)
Hence, the elasticities of the revenue function and output ray function with respect to any input $k$ are equal and are expected to be positive. We also use (A.9) to rewrite (A.3):

$$ p_j = -\frac{R}{f} \left[ \left( \sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \frac{\partial \theta_m}{\partial y_j} \right) - \frac{y_j}{f} \right] \quad (A.11) $$

It follows that the marginal rate of transformation between two outputs is:

$$ \frac{p_j}{p_i} = \frac{\sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \frac{\partial \theta_m}{\partial y_j}}{\sum_{m=1}^{M-1} \frac{\partial f}{\partial \theta_m} \frac{\partial \theta_m}{\partial y_i}} \frac{y_j}{f} \quad (A.12) $$

It can also be written in terms of log derivatives as:

$$ \frac{p_j}{p_i} = \frac{y_j \left( \sum_{m=1}^{M-1} \frac{\partial \ln f}{\partial \theta_m} \frac{\partial \ln \theta_m}{\partial y_j} \right) - 1}{y_i \left( \sum_{m=1}^{M-1} \frac{\partial \ln f}{\partial \theta_m} \frac{\partial \ln \theta_m}{\partial y_i} \right) - 1} \quad (A.13) $$

Suppose all the derivatives of the ray function with respect to the angles are equal to 0. This implies that the marginal of transformation becomes:

$$ \frac{p_j}{p_i} = \frac{y_j}{y_i} \quad (A.14) $$

The previous expression can only be valid if the PPF is perfectly approximated in the plane $(y_i, y_j)$ by a circle. Hence, the restriction that all derivatives of the ray production function with respect to the $(M-1)$ angles are equal to zero means that the PPF is a perfect sphere of dimension $M$.

The ray function also shares some dual properties with the minimum cost function. We proceed as before to rewrite the Lagrangian of the cost minimisation problem:

$$ L(x, \lambda) = -wx + \lambda [f(x, \theta(y)) - \|y\|] \quad (A.15) $$

The FOCs are:

$$ \frac{\partial L}{\partial x_k} = -w_k + \lambda \frac{\partial f}{\partial x_k} = 0 \quad (A.16) $$

$$ \frac{\partial L}{\partial \lambda} = f(x, \theta(y)) - \|y\| = 0 \quad (A.17) $$

Multiplying (A.15) by $x_k$ and summing over all inputs gives:

$$ \lambda = \frac{C}{f \cdot \mathcal{E}_{\text{scale}}} \quad (A.18) $$

It follows from (A.16) that:
\[
\frac{w_k x_k}{C} = S_k = \frac{\partial \ln f}{\partial \ln x_k} \frac{1}{\epsilon_{scale}}
\]  

(A.18)

The elasticity of the ray function with respect to any input \( x_k \) is therefore interpreted as the scale-adjusted (optimal) cost share of that input.