

# A Class of Chronic Poverty Measures

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# Hypothetical Challenge

- The government would like to create an official chronic poverty indicator
- Desiderata
  - It must be understandable and easy to describe
  - It must conform to “common sense” notions of poverty
  - It must fit the purpose for which it is being developed
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable
- What would you advise?

# Not Exactly Hypothetical

- Mexico
- August 2002
  - Committee with Bob Michael, John Iceland
  - Precise desired attributes
- June 2003
  - Estimates for 2000, 2002 released by government
  - Public discourse on measurement
  - Later, on public action to combat poverty
- Reference
  - “Números que Mueven al Mundo: La Medición de la Pobreza en México,” Miguel Szekely, Coordinador (2006)

# My Aim

- Specific Task
  - “Explore whether ‘duration’ can be incorporated into the FGT measure”
- Short Answer
  - Yes
- Demystify

# Outline

- Review of Static Framework
- Presentation of Family of Measures
- Empirical Example
- Reflection

# Review: Static Poverty Measurement

Caveat: Single variable, e.g., consumption, income

Sen (1976) two steps

Identification step “who is poor?”

Typically use poverty line

Absolute, meaning unchanging over time

Cutoff is always somewhat arbitrary

Aggregation step “which overall indicator?”

Headcount ratio  $P_0 =$  percentage poor

Example: Incomes = (7,3,4,8) poverty line  $z = 5$

Who's poor?  $g^0 = (0,1,1,0)$

Headcount  $P_0 = \mu(g_0) = 2/4$

**Example: (7,3,3,8) No change!**

# Review: Static Poverty Measurement

Per capita poverty gap  $P_1$

Example: incomes = (7,3,4,8) poverty line  $z = 5$

Normalized gaps =  $g^1 = (0, 2/5, 1/5, 0)$

Poverty gap =  $P_1 = \mu(g^1) = 3/20$

Example: (7,3,3,8)  $P_1 = 4/20$  (sensitive to decrements)

However: (7,2,4,8)  $P_1 = 4/20$  (insensitive to inequality)

FGT  $P_2$

Example: incomes = (7,3,3,8) poverty line  $z = 5$

Squared Normalized gaps =  $g^2 = (0, 4/25, 4/25, 0)$

FGT =  $P_2 = \mu(g^2) = 8/100$

Example: (7,2,4,8)

Squared Normalized gaps =  $g^2 = (0, 9/25, 1/25, 0)$

$P_2 = 10/100$  (sensitive to inequality)

# Chronic Poverty Strategy

Twin cutoffs

poverty line

duration line

Based on  $P_\alpha$  family

Note:

No transfers across periods

No discounting; ignore direction of time and “bunching”

Crude 0-1 approach to observations over time

Alternative: see Foster and Santos (2006), Calvo and Dercan (2006)

Caveat: Single dimensional, cardinal variable





# Chronic Poverty: Normalized Gaps

$$\sigma_1 = \begin{matrix} & \text{Persons} \\ \begin{matrix} \text{Dates} \\ \text{Dates} \\ \text{Dates} \\ \text{Dates} \end{matrix} & \begin{bmatrix} \mathbf{0.4} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.6} & \mathbf{0} \end{bmatrix} \end{matrix}$$

Matrix of normalized gaps =  $(z - \text{income})/z$  if income below  $z$   
= 0 if equal or above

# Chronic Poverty: Poverty Spells

$$\sigma_e = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \text{Dates} \end{array}$$

Matrix of poverty spells = 1 if income is below z  
= 0 if equal or above

# Chronic Poverty: Duration

$$\sigma_e = \begin{matrix} & \text{Persons} & & & \\ & & & & \\ & & & & \\ \begin{matrix} \sigma_e \\ = \end{matrix} & \left[ \begin{array}{cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] & & \text{Dates} \end{matrix}$$

$\mathbf{d} = (0.25, 0.75, 0.75, 0)$   
= percent of time in poverty

# Chronic Poverty: Duration Line

$$\sigma_e = \begin{matrix} & \text{Persons} \\ \begin{matrix} \text{1} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{1} & \text{1} & \text{0} \\ \text{0} & \text{1} & \text{1} & \text{0} \\ \text{0} & \text{1} & \text{1} & \text{0} \end{matrix} & \text{Dates} \end{matrix}$$

$d = (0.25, \mathbf{0.75}, \mathbf{0.75}, 0)$   
= percent of time in poverty

Duration line

$\tau = .70$

Identify chronically poor

# Chronic Poverty: Duration Line

$$\sigma_e = \begin{matrix} & \text{Persons} \\ \begin{matrix} \text{1} & \text{0} & \text{0} & \text{0} \\ \text{0} & \text{1} & \text{1} & \text{0} \\ \text{0} & \text{1} & \text{1} & \text{0} \\ \text{0} & \text{1} & \text{1} & \text{0} \end{matrix} & \text{Dates} \end{matrix}$$

$$d = (0.25, \mathbf{0.75}, \mathbf{0.75}, 0)$$

= percent of time in poverty

Duration line       $\tau = .70$       Identify chronically poor

Now turn to Aggregation question.

# Headcount Ratio

$$\sigma_e = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \text{Dates} \end{array}$$

$$d = (0.25, \mathbf{0.75}, \mathbf{0.75}, 0)$$

= percent of time in poverty

Duration line  $\tau = .70$

Headcount ratio  **$H = 1/2$**

# Problem

Suppose time in poverty increases for person 3

$$\sigma_e = \begin{matrix} & \text{Persons} \\ \begin{matrix} \left| \begin{matrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{matrix} \right| \\ \text{=} \\ \left[ \begin{matrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{matrix} \right] \end{matrix} \quad \text{Dates}$$

$$d = (0.25, \mathbf{0.75}, \mathbf{0.75}, 0)$$

= percent of time in poverty

Duration line  $\tau = .70$

Headcount ratio  $H = 1/2$



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Suppose time in poverty increases for person 3

$$\sigma^0 = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \text{Dates} \end{array}$$

$$d = (0.25, \mathbf{0.75}, \mathbf{1}, 0)$$

= percent of time in poverty

Duration line  $\tau = .70$

Headcount ratio  $H = 1/2$

# Time Monotonicity

Suppose time in poverty increases for person 3

$$\sigma^0 = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \text{Dates} \end{array}$$

$$d = (0.25, \mathbf{0.75}, \mathbf{1}, 0)$$

= percent of time in poverty

Duration line  $\tau = .70$

Headcount ratio **H = 1/2** No change! Violates Time Mon

# Fixing the Problem

Return to original data

$$\sigma_e = \begin{matrix} & \begin{matrix} \text{Persons} \\ \text{Dates} \end{matrix} \\ \begin{matrix} \text{Persons} \\ \text{Dates} \end{matrix} & \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix} \end{matrix}$$

$$\begin{aligned} d &= (0.25, \mathbf{0.75}, \mathbf{0.75}, 0) \\ &= \text{percent of time in poverty} \end{aligned}$$

# Fixing the Problem

Now remove all poverty spells of persons not chronically poor

$$\sigma_e = \begin{matrix} & \begin{matrix} \text{Persons} \\ \text{Dates} \end{matrix} \\ \begin{matrix} \text{Persons} \\ \text{Dates} \end{matrix} & \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix} \end{matrix}$$

$$d = (0.25, \mathbf{0.75}, \mathbf{0.75}, 0)$$

= percent of time in poverty

# Focus on Chronically Poor

Now remove all poverty spells of persons not chronically poor

$$g^0(\tau) = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \end{array} \quad \begin{array}{c} \text{Dates} \end{array}$$

$$\mathbf{d}(\tau) = (0, \mathbf{0.75}, \mathbf{0.75}, 0)$$

= percent of time in poverty **for chronically poor**

# Duration Adjusted Headcount Ratio

Need to augment the information of H

$$g^0(\tau) = \begin{matrix} & \text{Persons} & & & \\ & \begin{matrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{matrix} & & & \\ & & & & \text{Dates} \end{matrix}$$

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= percent of time in poverty **for chronically poor**

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$$\mathbf{d}(\tau) = (0, \mathbf{0.75}, \mathbf{0.75}, 0)$$

= percent of time in poverty **for chronically poor**

**D = average duration among chronically  
poor = 0.75**

# Duration Adjusted Headcount Ratio

Duration adjustment headcount =  $P_0 = HD$

$$g^0(\tau) = \begin{matrix} & \text{Persons} \\ \begin{matrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{matrix} & \text{Dates} \end{matrix}$$

$$\mathbf{d}(\tau) = (0, \mathbf{0.75}, \mathbf{0.75}, 0)$$

= percent of time in poverty **for chronically poor**

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poor = 0.75



# Duration Adjusted Headcount Ratio

Duration adjustment headcount =  $P_0 = HD = \mu(g^0(\tau))$

$$g^0(\tau) = \begin{matrix} & \text{Persons} \\ \begin{matrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{matrix} & \text{Dates} \end{matrix}$$

$\mathbf{d}(\tau) = (0, \mathbf{0.75}, \mathbf{0.75}, 0)$   
= percent of time in poverty **for chronically poor**

D = average duration among chronically  
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# Duration Adjusted Headcount Ratio

$$\begin{aligned} \text{Duration adjustment headcount} = P_0 = \text{HD} &= \mu(g^0(\tau)) \\ &= \mathbf{6/16 = 0.375} \end{aligned}$$

$$g^0(\tau) = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \end{array} \begin{array}{c} \text{Dates} \end{array}$$

$$\begin{aligned} \mathbf{d}(\tau) &= (0, \mathbf{0.75}, \mathbf{0.75}, 0) \\ &= \text{percent of time in poverty for chronically poor} \end{aligned}$$

D = average duration among chronically poor = 0.75

# Time Monotonicity

$$\begin{aligned} \text{Duration adjustment headcount} = P_0 = \text{HD} &= \mu(g^0(\tau)) \\ &= 6/16 = 0.375 \end{aligned}$$

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$$\begin{aligned} \mathbf{d}(\tau) &= (0, \mathbf{0.75}, \mathbf{0.75}, 0) \\ &= \text{percent of time in poverty for } \mathbf{\text{chronically poor}} \end{aligned}$$

Obviously if person 3 is in poverty longer,  $P_0$  will rise

# Time Monotonicity

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Obviously if person 3 is in poverty longer,  $P_0$  will rise  
**Satisfies time monotonicity**

# Problem

$$\begin{aligned} \text{Duration adjustment headcount} = P_0 = \text{HD} &= \mu(g^0(\tau)) \\ &= 6/16 = 0.375 \end{aligned}$$

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Q/ However, what happens when an income below  $z$  falls even further (for a chronically poor person)?

# Income Monotonicity

$$\begin{aligned} \text{Duration adjustment headcount} = P_0 = \text{HD} &= \mu(g^0(\tau)) \\ &= 6/16 = 0.375 \end{aligned}$$

$$g^0(\tau) = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \text{Dates} \end{array}$$

Q/ However, what happens when an income below  $z$  falls even further (for a chronically poor person)?

A/  $P_0$  is unchanged! **Violates income monotonicity.**

# Duration Adjusted Poverty Gap

Need to augment the information of  $P_0$

$$g^0(\tau) = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{array} \right] \end{array} \begin{array}{c} \text{Dates} \end{array}$$

# Duration Adjusted Poverty Gap

Return to normalized gaps

$$\sigma^1(\tau) = \begin{matrix} & \text{Persons} & & & \\ & & & & \\ \begin{matrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.6} & \mathbf{0} \end{matrix} & & \text{Dates} & & \end{matrix}$$



# Duration Adjusted Poverty Gap

$$G^1(\tau) = \begin{matrix} & \text{Persons} \\ \begin{matrix} \text{Dates} \\ \left[ \begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.6} & \mathbf{0} \end{array} \right] \end{matrix} \end{matrix}$$

Average gap across all poverty spells of the chronically poor:  $G(\tau) = 2.4/6 = \mathbf{0.4}$

# Duration Adjusted Poverty Gap

$$\text{Duration Adjusted Poverty Gap} = P_1 = \mathbf{P}_0 \mathbf{G} = \mathbf{HDG}$$

$$g^1(\tau) = \begin{matrix} & \text{Persons} \\ \begin{matrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.6} & \mathbf{0} \end{matrix} & \text{Dates} \end{matrix}$$

Average gap across all poverty spells of the chronically poor:  $G(\tau) = 2.4/6 = \mathbf{0.4}$



# Duration Adjusted Poverty Gap

$$\text{Duration Adjusted Poverty Gap} = P_1 = P_0 G = \text{HDG} = \mu(g^1(\tau)) \\ = \mathbf{0.15}$$

$$g^1(\tau) = \begin{matrix} & \text{Persons} \\ \begin{matrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.2} & \mathbf{0.6} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.4} & \mathbf{0.6} & \mathbf{0} \end{matrix} & \text{Dates} \end{matrix}$$

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Obviously, if an income under  $z$  falls even further (for a chronically poor person), then  $P_1$  will rise.

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Obviously, if an income under  $z$  falls even further (for a chronically poor person), then  $P_1$  will rise.

**Satisfies income monotonicity**

# Problem

$$\text{Duration Adjusted Poverty Gap} = P_1 = P_0 G = \text{HDG} = \mu(g^1(\tau)) \\ = \mathbf{0.15}$$

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Problem: A decrement in income in poorest spell has same impact as a decrement in income in least poor spell.

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Problem: A decrement in income in poorest spell has same impact as a decrement in income in least poor spell.

$P_1$  is not distribution sensitive.



# Duration Adjusted FGT

Consider the matrix of squared normalized shortfalls

$$\sigma^2(\tau) = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.16} & \mathbf{0.04} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.04} & \mathbf{0.36} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.16} & \mathbf{0.36} & \mathbf{0} \end{array} \right] \text{Dates} \end{array}$$

# Duration Adjusted FGT

Consider the matrix of squared normalized shortfalls

$$g^2(\tau) = \begin{matrix} & \text{Persons} & & & \\ & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.16} & \mathbf{0.04} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.04} & \mathbf{0.36} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.16} & \mathbf{0.36} & \mathbf{0} \end{bmatrix} & & \text{Dates} \end{matrix}$$

Average squared gap across all poverty spells of the chronically poor:  $S(\tau)$

# Duration Adjusted FGT

$$\text{Duration Adjusted FGT} = P_2 = P_0 S = \text{HDS} = \mu(g^2(\tau))$$

$$g^2(\tau) = \begin{array}{c} \text{Persons} \\ \left[ \begin{array}{cccc} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.16} & \mathbf{0.04} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.04} & \mathbf{0.36} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.16} & \mathbf{0.36} & \mathbf{0} \end{array} \right] \text{Dates} \end{array}$$

Average squared gap across all poverty spells of the chronically poor:  $S(\tau)$

Emphasizes lowest incomes

# Duration Adjusted $P_\alpha$

Duration Adjusted  $P_\alpha = \mu(\mathbf{g}^\alpha(\tau))$  for  $\alpha \geq 0$

$$\mathbf{g}^\alpha(\tau) = \begin{matrix} & \text{Persons} & & \\ \begin{matrix} \mathbf{0}^\alpha & \mathbf{0}^\alpha & \mathbf{0}^\alpha & \mathbf{0}^\alpha \\ \mathbf{0}^\alpha & \mathbf{0.4}^\alpha & \mathbf{0.2}^\alpha & \mathbf{0}^\alpha \\ \mathbf{0}^\alpha & \mathbf{0.2}^\alpha & \mathbf{0.6}^\alpha & \mathbf{0}^\alpha \\ \mathbf{0}^\alpha & \mathbf{0.4}^\alpha & \mathbf{0.6}^\alpha & \mathbf{0}^\alpha \end{matrix} & \text{Dates} \end{matrix}$$

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Satisfies numerous properties including decomposability

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Note:  $\mu(\mathbf{g}^\alpha(0))$  is average  $P_\alpha$  poverty across dates

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Hence  $T_\alpha = \mu(\mathbf{g}^\alpha(0)) - \mu(\mathbf{g}^\alpha(\tau))$  is corresponding index of transient poverty

# Example: Argentina

- **Data Source:** Instituto Nacional de Estadísticas y Censos (INDEC), Encuesta Permanente de Hogares (EPH).
- **Period:** Waves of October 2001, May 2002, October 2002 and May 2003. It is a panel of 4 observed periods.
- **Unit of Analysis:** Households.
- **Number of observations:** 2409.
- **Type of income used:** Equivalent Family Income per month



# Example: Argentina

- Head Count Ratio

% of households poor in 1 or more periods	% of households poor in 2 or more periods	% of households poor in 3 or more periods	% of households poor in the 4 periods
47%	38%	30%	20%

# Example: Argentina

- $P_0$

Of those who are poor 1 or more periods	Of those who are poor 2 or more periods	Of those who are poor 3 or more periods	Of those who are poor 4 periods
0.34	0.31	0.27	0.20

- $P_1$

Poverty Gap of...			
Those who are poor one or more periods	Those who are poor two or more periods	Those who are poor three or more periods	Those who are poor four periods
0.15	0.14	0.13	0.10

- $P_2$

Squared Poverty Gap of...			
Those who are poor one or more periods	Those who are poor two or more periods	Those who are poor three or more periods	Those who are poor four periods
0.085	0.082	0.077	0.064

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- Note: As we go from H to  $P_2$  the percent of total poverty that is chronic rises:

H: 64%    $P_0$ : 79%    $P_1$ : 87%    $P_2$ : 91%

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Second example in paper utilizing decomposability

# Review Challenge

- Desiderata
  - It must be understandable and easy to describe
  - It must conform to “common sense” notions of poverty
  - It must fit the purpose for which it is being developed
  - It must be technically solid
  - It must be operationally viable
  - It must be easily replicable

Thanks for your attention