

Looking forward: theory-based
measures of chronic poverty and
vulnerability

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Abstract

Conventional poverty analysis is ill-equipped to answer questions concerning the future persistence of observed poverty. Are those observed to be poor at a particular point in time chronically poor, or are they simply in a transitory state? While a number of analysts have struggled with this question, this paper employs economic theory of asset accumulation and poverty traps to derive estimable chronic poverty measures. These measures in turn provide a conceptual foundation for understanding and measuring vulnerability. The analysis identifies two sorts of chronic poverty. The first type (intrinsic chronic poverty) is experienced by those who are intrinsically disadvantaged by lack of skill or unfavourable economic environment. The second (multiple equilibrium chronic poverty) is experienced by those who have the potential to be non-poor given their skills and circumstances, but who lack sufficient assets to craft a pathway out of poverty. The policies needed to address these two types of chronic poverty are distinct. Moreover, the analysis shows that the second group of chronically poor are especially vulnerable to shocks. Social protection policies are likely to be an especially effective means for addressing this multiple equilibrium chronic poverty. After illustrating these concepts with simulated data, the paper closes with an empirical application to South Africa.

Keywords: poverty measures, chronic poverty, poverty traps, South Africa

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Acronyms

HSL household subsistence line

FGT Foster-Greer-Thorbecke class of poverty measures

[The historian] becomes a crab. The historian looks backward;
eventually he also believes backward.

Freidrich Nietzsche, *Twilight of the Idols*

Introduction: From backward-looking to forward-looking poverty analysis

Conventional quantitative poverty analysis invariably looks backwards to the most recent living standards survey to enumerate (the past) extent and nature of poverty. Living standards surveys, with their 7-day, 30-day and 12-month recall periods look yet further backward. While there is no reason to follow Nietzsche and assert that backward-looking poverty analysis 'believes backwards', there are clearly (forward looking) questions that conventional poverty analysis is ill-equipped to answer. Perhaps the most important of these questions concerns the future persistence of observed poverty status: Are the observed poor chronically poor, or are they in a transitory state?

Others have struggled with this question. One approach (used by the Chronic Poverty Report, 2005) is empirical. With numerous repeated observations of the same households, the chronically poor can be identified as those who have been 'frequently' poor in the observed past. While this approach has much to recommend it, it is expensive and has an *ad hoc* element (how frequently must an individual be observed to be poor in order to be classified as chronically poor). More importantly, it is also backward looking.

The approach put forward in this paper is rather different. Using guidance from the microeconomic theory of poverty traps, this paper uses the past to identify structural patterns of change – asset dynamics – rather than past levels of poverty. The statistical identification of these patterns then permits the creation of forward-looking poverty measures that tell us where we expect the poor to be in the future, not where they have been in the past.¹ While these new measures do not eliminate the need for other approaches (indeed, when combined with standard approaches they provide a more complete poverty dialogistic for a particular economy), they do offer a promising approach for the conceptualisation and measurement of chronic poverty. They also carry important policy implications.

Building on the work of Buera (2005) and Barrett, Carter and Ikegami (2007), Section 1 of this paper develops a theoretically grounded approach to chronic poverty that emphasizes the role of individual heterogeneity and clarifies the role that vulnerability to economic shocks plays in producing chronic poverty. The key theoretical construct that emerges from this analysis is the Micawber frontier, defined as the level of assets below which an individual of a particular skill level is unable to successfully accumulate assets and move ahead economically over time.

Section 2 then shows how knowledge of the Micawber frontier can be used to generate two classes of chronic poverty measure. The first class generalises a suggestion put forward by Carter and Barrett (2006) and is based on the individual's distance from the Micawber frontier. The second uses information on the Micawber frontier to simulate future asset (and income) changes. When combined with the family of chronic poverty measures put forward by Calvo and Dercon (2006), these asset dynamics open yet another window into chronic poverty that is forward looking and based on a theoretically well-specified model. Numerical simulation of a stylised 100 household economy is used to illustrate both sets of measures. In addition, both sets of chronic poverty measures can be used to derive well-structured measures of vulnerability, where vulnerability is understood as the fraction of all chronic poverty that would be eliminated in a world without economic shocks (or with perfectly well-positioned social safety nets).

¹ This approach of course still relies on the past, but uses it to identify patterns of asset dynamics. If those past patterns of change are not stable, providing a poor guide to future patterns, then the approach put forward here also becomes backward-looking.

Section 3 then takes some first steps toward implementing the ideas put forward in this paper, using estimated asset dynamics in South Africa over the 1993 to 1998 period to calculate chronic poverty measures based on distance from the asset threshold. While based on somewhat stringent assumptions, these forward-looking estimated chronic poverty measures provide a more in-depth look at the nature of poverty than do standard FGT-based measures. At the same time, data from a later time period (2004) illustrate weaknesses of the estimated chronic poverty measures and point the way toward more reliable estimation needed to capture the full richness of a theory-based approach to chronic poverty. Finally, the paper closes with reflections on the use of the proposed chronic poverty measure for policy.

The paper closes with some reflections on the implications of the analysis for the design of social protection policy which potentially will payoff with a double dividend of reduced chronic poverty.

1. A theory-based approach to chronic poverty

This section summarises recent theoretical work by Barrett, Carter and Ikegami (2007) (hereafter cited as BCI) on the economics of poverty traps. Building on the dynamic model of Buera (2005) that explicitly incorporates the intrinsic capacity or ability differences of individuals, BCI show that there are two types of chronic poverty:

- i) *Intrinsic chronic poverty* suffered by those of relatively low skill and possibilities who are inevitably trapped in a poor, low level equilibrium trap (given the structure of wages and opportunity in their economy); and,
- ii) *Multiple equilibrium chronic poverty* suffered by a middle-ability group that has the potential to be non-poor in their extant economy, but whose histories have placed them below the minimum asset threshold needed to initiate and sustain the accumulation needed to escape poverty.

In addition to these two groups, the BCI model identifies a third, high-ability group that may be consumption poor for an extended period of time, but who are expected to surmount a poor standard of living given a sufficiently long period of time in which to accumulate assets. We refer to this third group as the *intrinsically upwardly mobile*.

This section proceeds in two steps. First, it considers the implications of the BCI model in the absence of economic shocks. While unrealistic, this simplification underwrites basic insights into the economics of asset thresholds and chronic poverty. In addition, when paired with the analysis of shocks and risk pursued later in this section, the simplified model will suggest measures of vulnerability and its effect on chronic poverty.

1.1 Heterogeneous ability and poverty traps

Building on the model of Buera (2005), BCI assume that each individual is endowed with a level of innate ability (α) as well as an initial level of capital (k_0). Every period t , the individual has the choice between two alternative technologies for generating a livelihood, f :

$$f(\alpha, k) = \begin{cases} f_L(\alpha, k) = \alpha k^{1/2} & \text{under the low technology} \\ f_H(\alpha, k) = \alpha k^{1/2} - E & \text{under the high technology.} \end{cases}$$

Both technologies are skill sensitive (for any given technology, more able people can produce more than less able people). One technology (the 'high' technology) is subject to fixed costs, E , meaning that the technology is not worth using low amounts of capital. Figure 1 illustrates these technologies for an individual with a given skill level α . As can be seen in the figure, the individual (interested in maximising income or livelihood possibilities) will optimally shift to employing the high technology only after k reaches the critical level:

$$k^*(\alpha) = \{k \mid f_L(\alpha, k) = f_H(\alpha, k)\}.$$

Using this basic setup, BCI analyse when it is possible and desirable for the individual to save and accumulate assets in order to surpass $k^*(\alpha)$, employ the high technology and reach a higher standard of living. As summarised in the Appendix, the BCI model assumes that individuals divide their total income (f) between consumption (c_t) and investment (i_t) in order to maximize their inter-temporal stream of utility. Assets evolve according to the following rule:

$$k_{t+1} = i_t + (1 - \delta)k_t. \quad (1)$$

where $i_t = f(\alpha, k_t) - c_t$ is investment and δ is the rate at which capital depreciates. Critically, the model assumes that the individual cannot borrow against future earnings to build up capital and can only pursue autarchic accumulation strategies.

The solution to this inter-temporal choice problem defines an investment rule, $i^*(k_t \mid \alpha)$. Using this rule, we can define expected capital stock of a household in year $t+1$ as follows:

$$k_{t+1}^e(k_t \mid \alpha) = i^*(k_t \mid \alpha) + (1 - \delta)k_t$$

If individuals had access to only one technology, they would optimally accumulate capital up to the steady state values shown in Figure 1, $k_L^*(\alpha)$ for the low technology and $k_H^*(\alpha)$ for the high technology.² To make it easy to discuss the model, we will assume that the poverty line just equals $f(k_L^*(\alpha))$ for a high-skill individual. That is, individuals can only become non-poor if they adopt the higher livelihood strategy.³

The key question addressed by the BCI model is whether individuals whose initial capital stock is below $k^*(\alpha)$ gravitate toward the high or the low technology? Consider an individual who begins life with the asset position at the level marked by the dot in Figure 1. Will this individual optimally move to the right over time, accumulating assets and ending up at $k_H^*(\alpha)$ and a non-poor standard of living? Alternatively, will the individual deaccumulate, move to the left and settle into a poor standard of living with capital stock $k_L^*(\alpha)$?⁴ More formally, is there an initial asset threshold, which we will denote $k^*(\alpha)$, below which individuals stay at the low equilibrium (remaining chronically poor), and above which she or he will move to the high equilibrium (eventually becoming non-poor)?

² Note that these steady state values are increasing in the level of skill, α . The steady state values are also influenced by the individual's discount rate, meaning his or her willingness to sacrifice current consumption in order to save and gain higher future consumption.

³ If this assumption is not true, individuals may get trapped at the low equilibrium, but they would not necessarily be poor.

⁴ Note that beyond the low level equilibrium, $k_L^*(\alpha)$ the immediate marginal returns to additional capital are lower and less than the value of the consumption that the individual must sacrifice in order to accumulate additional capital. In the rationality of the model, an individual will only make that sacrifice if future returns (when he finally accumulates at least $k^*(\alpha)$) are large enough and close enough (and sufficiently certain, when there is uncertainty in the model).

Figure 1 Assets and livelihood options

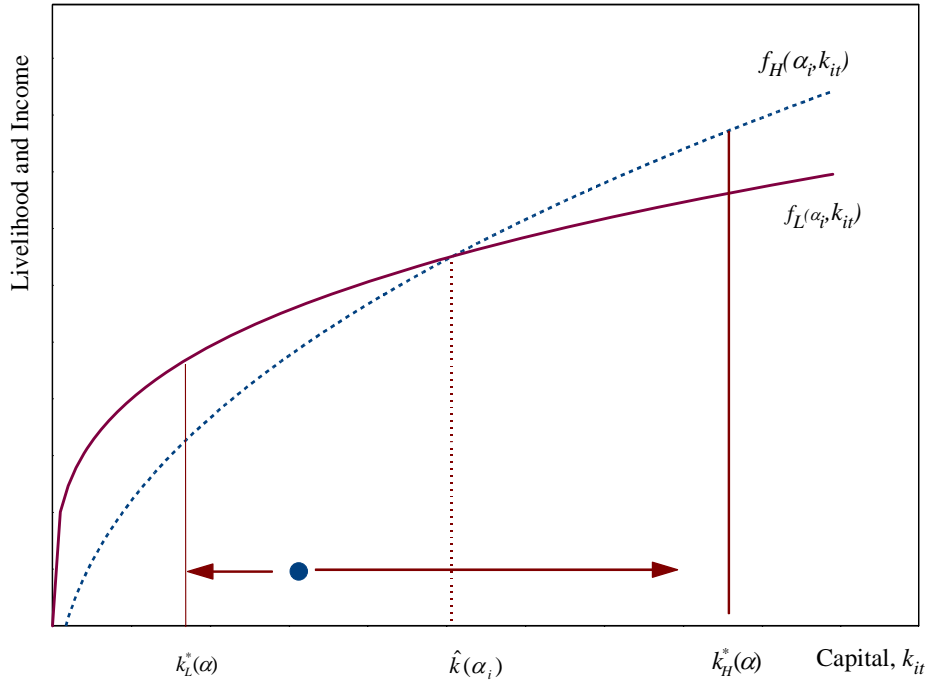
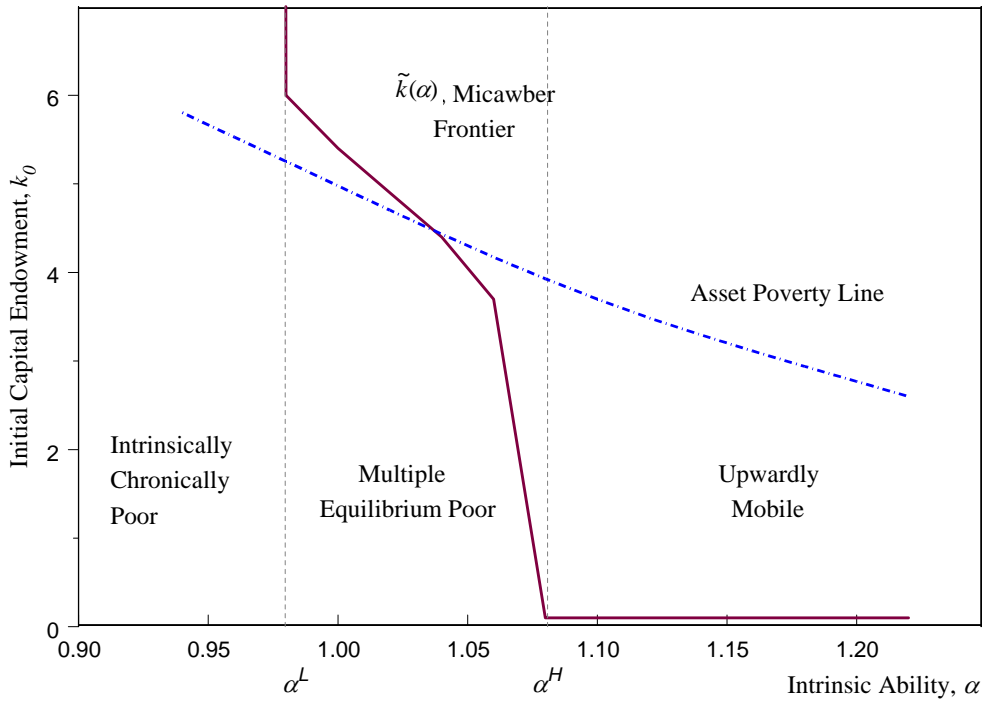


Figure 2 The Micawber frontier (non-stochastic case)



As analysed by BCI, the answer to this question depends on the skill level of the individual. In particular, there will be three classes of individuals, each exhibiting distinct dynamically-optimal behaviour. Figure 2, which is created through the numerical analysis of the BCI model, illustrates these three classes (see the Appendix below for the full specification of the dynamic programming model). Along the horizontal axis are skill levels, ranging from least to most able. The vertical axis measures the stock of productive assets. The dashed curve is the asset poverty line, defined as the level of assets needed to generate an expected standard of living equal to the poverty line. The solid curve shows the asset level at which an individual is just indifferent between staying with the low technology versus building up stocks of assets such that a transition to the high technology eventually becomes feasible. Denote this frontier as $k^*(\alpha)$. An individual with ability level α will attempt to accumulate and move out of poverty if she enjoys an capital stock $k_0 > k^*(\alpha)$. Otherwise, he will only pursue the low technology, accumulating the modest levels of capital that it requires. Following Carter and Barrett (2006), we label $k^*(\alpha)$ as the Micawber frontier as it divides those who have the wealth needed to accumulate from those who do not.⁵

As illustrated in Figure 2, the numerical analysis identifies three distinct regions in the space of ability and initial asset holdings. High skill individuals are those with $\alpha > \alpha^H$ who will always move toward the high equilibrium, even if they find themselves with a zero stock of assets (as $k^*(\alpha) = 0$ for these individuals). When they reach $k^*(\alpha)$ they will optimally switch to the higher technology. Irrespective of their starting position, these individuals steadily converge to the steady state asset value for the high technology. These individuals are the *intrinsicly upwardly mobile*, perhaps consumption poor over some extended period as they save and accumulate assets, but eventually expected to become non-poor.

In contrast, those with an ability level below the critical level $\alpha < \alpha^L$ will never move toward the high technology if they find themselves with any finite stock of assets. These are *intrinsicly chronically poor* individuals who lack the ability or circumstance to achieve a non-poor standard of living in their existing economic context (CPRC 2004 gives examples of individuals who suffer such fundamental disabilities).⁶

Finally, and most interestingly, the intermediate-skill group with $\alpha^L < \alpha < \alpha^H$ have positive, but finite, values $k^*(\alpha)$. If sufficiently well-endowed with assets ($k_0 > k^*(\alpha)$), these individuals – the *multiple equilibrium poor* – will accumulate additional assets over time, adopt the high technology and eventually reach a non-poor standard of living. If they begin with assets below $k^*(\alpha)$, these individuals will no longer find the high equilibrium attainable and will settle into a low standard of living. Like the intrinsicly chronically poor, this subset of the multiple equilibrium poor will be chronically poor.⁷ The total number of chronically poor in any society will thus depend on the distribution of households across the ability-wealth space shown in Figure 2. The chronic poverty measures developed below rely on this insight.

1.2 Shocks, risk and poverty traps

While establishing the possibility of distinct types of poverty, the analysis in the prior section has ignored the reality of the economic shocks that threaten the wellbeing of less well-off

⁵ As discussed by Carter and Barrett, the phrase Micawber threshold was first used by Michael Lipton, and was then subsequently adopted by Zimmerman and Carter (2003) who give it a meaning similar to that used here.

⁶ Addressing the poverty of such individuals will require transfers and perhaps efforts like the Progressa programme to assure that the next generation acquires adequate human capital.

⁷ Unlike the disabled, this class can be helped to help themselves with safety nets and cargo nets.

people almost everywhere. In the presence of asset thresholds and poverty traps, economic shocks take on particular significance as Carter et al. (2007) explore in an empirical analysis of Ethiopia and Honduras.

There are at least three types of shocks that could generate risks that could have a major impact on the accumulation decisions of poor people. The first type is income shocks where households receive more or less than the expected amount of income from their assets at any point in time. Second, the marginal utility of income is also subject to shocks. Households, for example, may suffer a severe illness, creating new needs for cash that effectively drive up the marginal utility of consumption. Third, assets themselves are subject to shocks. Livestock may die, businesses may burn down, or productive equipment may unexpectedly break or be stolen. All three types of shocks have the capacity to derail households from the accumulation paths discussed in the prior section. We will here focus only on asset shocks.⁸

In the presence of asset shocks, next period assets depend not only on prior stocks plus investments, but also depend on realised shocks. To represent this possibility, BCI rewrite the rule that determines the evolution of capital stock (1) as follows:

$$k_{t+1} = \theta_t [i_t + (1 - \delta)k_t], \quad (2)$$

where θ_t is a random variable realised every period t . Note that if $\theta = 1$, there is no shock, whereas $\theta < 1$ indicates a negative shock that destroys some fraction of assets. While in principal shocks could be positive ($\theta > 1$), such events seem unlikely and we will restrict the analysis here to the case where only negative shocks are possible.

As summarised in the Appendix, the individual inter-temporal choice problem can be modified with (2) and the assumption that the individual knows the distribution of θ and chooses consumption and investment every period in order to maximize the discounted stream of expected utility. Denote the investment rule in the presence of asset shocks as $i_s^*(k_t | \alpha, \Omega)$, where Ω represents the set of information on the probability distribution that generates random shocks.

The impact of shocks on investment and the long-term evolution of poverty can be broken down into two pieces, the *ex post* effect of realised shocks and the *ex ante* effect of risk. The *ex post* effect of shocks comes about simply because negative events may destroy assets, knocking people off their expected path of accumulation. For intrinsically upwardly mobile individuals, such shocks may delay their arrival at the upper level equilibrium, or occasionally knock them down from it, necessitating a period of additional savings and asset reaccumulation.

For multiple equilibrium households, the *ex post* consequences of shocks can be rather more severe. Consider the case of a household that is initially only slightly above the Micawber frontier. A shock that knocks it below that frontier will knock the household into the ranks of the chronically poor, as the household will (optimally) alter its strategy and give up trying to reach the high equilibrium. Figure 3 illustrates such a case derived from the numerical simulation of the BCI model. The horizontal axis shows time, and the vertical measures accumulated capital stock. The two illustrated time paths show two different histories for a household that begins with initial assets above the Micawber frontier. Under the solid line trajectory, the household avoids severe shocks (at least early on) and manages a long-term escape from poverty. The dashed line trajectory shows that the household receives a more severe shock in year 5 and falls below the Micawber threshold. From that point on, the

⁸ The analysis of income and marginal utility shocks is more difficult, raising interesting issues of asset smoothing, as Zimmerman and Carter (2002) theoretically discuss, and Hoddinott (2006) empirically analyses.

household sinks into a long-term poverty trap. Under the more fortunate history, the household recovers and continues to move toward the high equilibrium steady state.

While these *ex post* effects of shocks are important, the anticipation that they might take place would be expected to generate a ‘sense of insecurity, of potential harm people must feel wary of – something bad can happen and “spell ruin”’, as Calvo and Dercon (2005) put it. Analysis of the BCI model shows that this sense of impending ruin will indeed discourage forward-looking households from making the sacrifices necessary to reach the high equilibrium. Numerical analysis of the model shows that the Micawber frontier shifts to the northeast once asset risk is introduced into the model. As shown in Figure 4, the solid line is the Micawber frontier in the absence of risk (as in Figure 2 above), while the dashed curve is the Micawber frontier in the face of risk. The boundaries marking the critical skill levels at which households move between the different accumulation regimes also shift right (to α^L and α^H), meaning fewer intrinsically upwardly mobile households and more intrinsically chronically poor households.

The most dramatic effects of risk are seen by considering a household whose skill and capital endowments place it between the two frontiers. Consider a household whose skill and initial asset endowments place at the solid circle illustrated in Figure 4. Absent of the risk of shocks, such a household would strive for the upper equilibrium and eventually escape poverty. In the presence of risk, such a household would abandon this accumulation strategy as futile and settle into a low level, chronically poor standard of living. In the face of asset risk, the extraordinary sacrifice of consumption required to try to reach the high equilibrium is no longer worth doing, and the household will optimally pursue the low level, poverty trap equilibrium. Again, shocks have their largest effects on mid-skill households.

Figure 3 The irreversible consequences of shocks

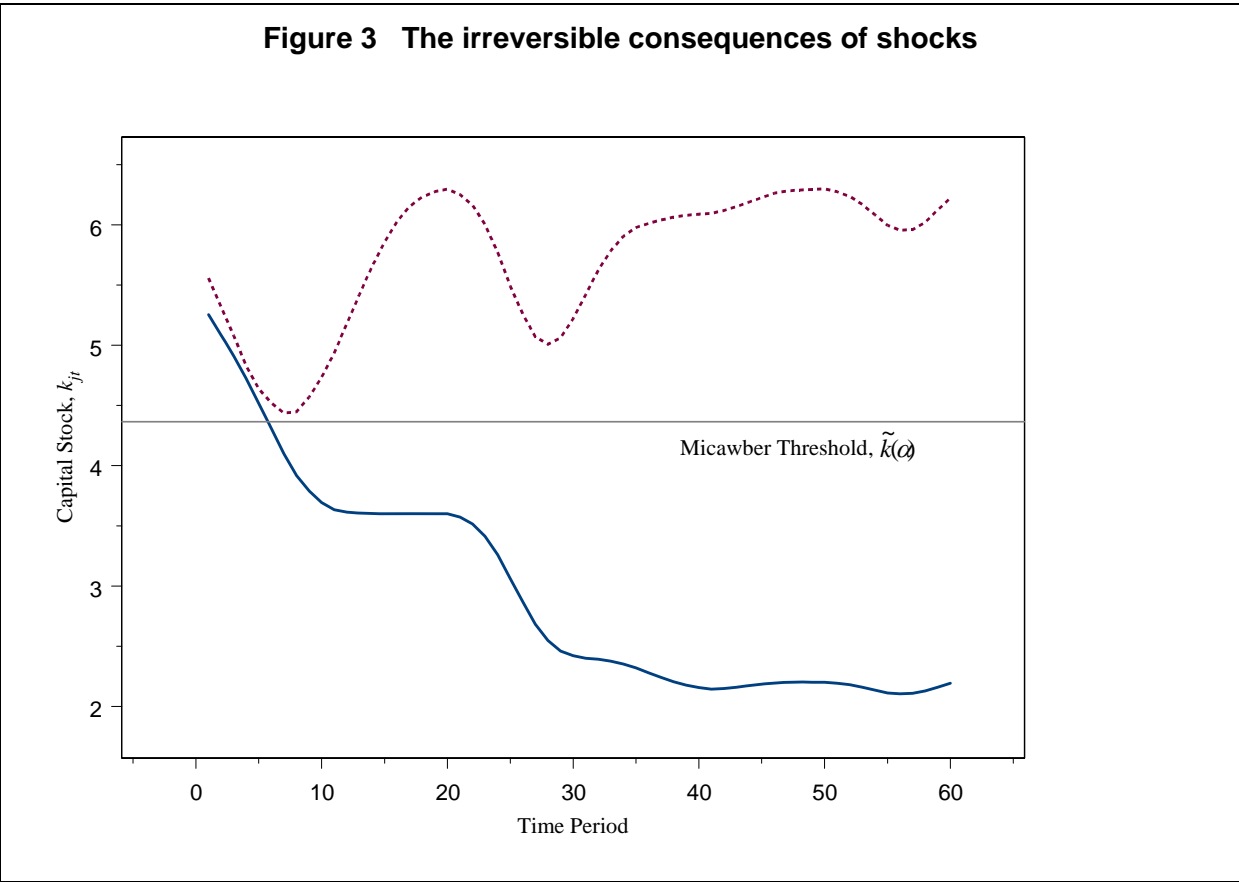


Figure 4 Vulnerability shifts out the Micawber frontier

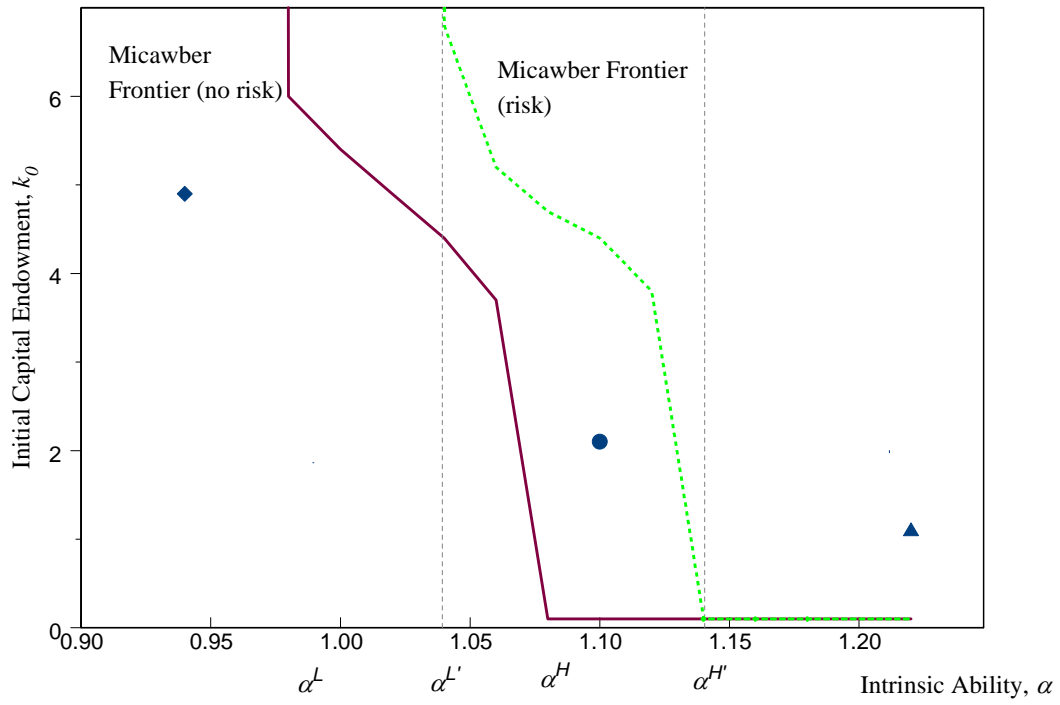
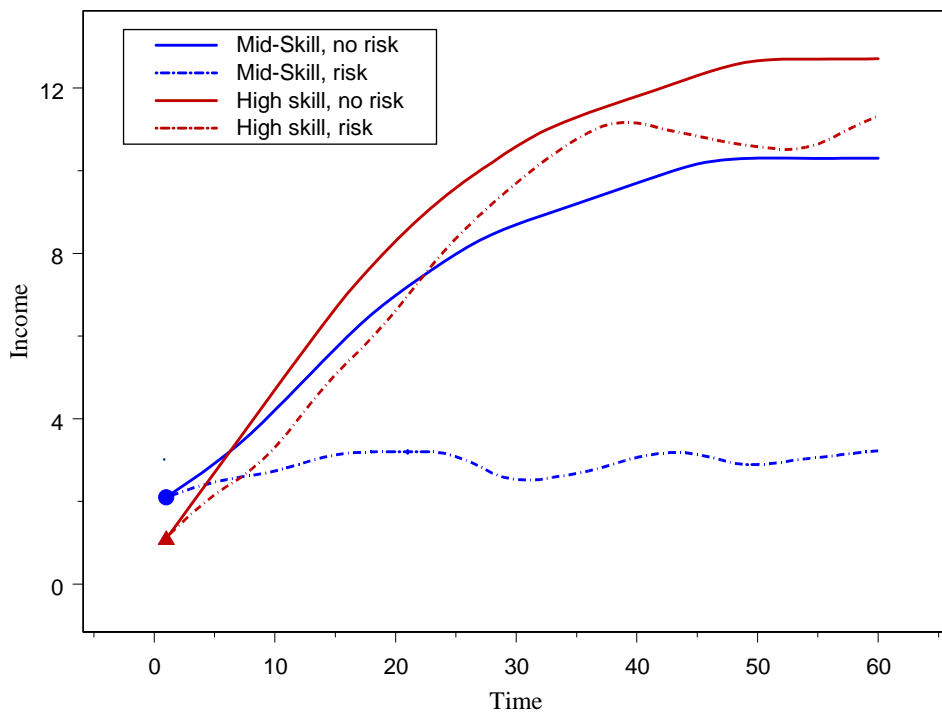


Figure 5 Vulnerability hurts 'average' individuals most



Simulation of the BCI model provides additional insight into the impact of shocks and risk on dynamic behaviour and chronic poverty. Consider the following three simulations:

- i) A *Non-stochastic simulation* in which repeated application of the accumulation rule, $i^*(k_t | \alpha)$, can be used to define the sequence of optimal capital stocks for any individual i with initial endowments k_{i0} and α_i . At each point in time, the implied income and consumption of the individual can also be calculated.
- ii) A *risk without shocks simulation* in which repeated application of the risk-adjusted optimal accumulation rule, $i_r^*(k_t | \alpha, \Omega)$, is used to define a sequence of capital stocks. To isolate the pure *ex ante* effect of risk, no shocks are actually realized in this simulation (despite the fact that individuals fearfully behave as if shocks are expected to occur).
- iii) A *full stochastic simulation* in which the risk-adjusted optimal accumulation rule, $i_r^*(k_t | \alpha, \Omega)$, is applied, but after each application, the individual receives a random shock generated in accordance with the probability structure Ω . This simulation permits us to isolate the full effect of random events (both *ex ante* and *ex post*) on the time path of capital, income and consumption for an individual.

Figure 5 illustrates the generated timepaths when the non-stochastic and the full stochastic simulations are applied to the mid-skill and high skill individuals whose initial endowment positions are shown in Figure 4 by the circle and triangle, respectively. The simulation was run for 60 time periods. As can be seen, in the non-stochastic simulation, both individuals move smoothly toward the high equilibrium.⁹ Autarchic accumulation provides a pathway from poverty for both individuals. Both become non-poor around year 10 of the simulation as their achieved capital stocks exceed the asset poverty line.

In the stochastic simulation, the high-skill individual is occasionally buffeted about by shocks and must continually rebuild his/her assets in order to retain the desired steady state capital stock.¹⁰ In sharp contrast, the middle skill agent undertakes a fundamental shift in strategy in the presence of risk. As seen in Figure 4, this individual has fallen below the Micawber frontier once risk is taken into account. While this individual suffers fluctuations akin to those suffered by the high-skill individual, the more fundamental effect results from his/her retreat from trying to reach the high equilibrium (i.e. a pathway from poverty is no longer attainable nor sustainable).

Table 1, which presents statistics related to all three simulations, further substantiates this latter point. The table includes results for the low-skill individual whose initial asset position is indicated by the diamond in Figure 4. For each of the three individuals, the table displays the discounted stream of utility, which is obtained under each simulation. In addition, the discounted value of income produced by each individual over the simulation is also listed. In contrast to the mid-skill individual, the 'risk without shocks' simulation barely perturbs the time paths and outcomes of both the low-skill and high-skill agents as neither of these agents shifts strategy in the face of risk.¹¹ When shocks are actually realised (the full stochastic simulation), then these agents suffer more substantial losses, especially in terms of utility.¹²

⁹ Note that the high-skill individual has a higher level of steady state capital stock because his/her skill level boosts the marginal productivity of capital.

¹⁰ Note that the desired steady state level of capital is reduced by the presence of risk.

¹¹ Their desired steady state values do diminish and hence production and consumption fall modestly, generating the changes shown in the table.

¹² Realised shocks affect utility more strongly than income. For example, for the low-skill agent, the discounted stream of the utility of consumption falls from 2.7 to 1.2, while the discounted stream of

Table 1 Simulations for archetypical individuals

	Non-stochastic	Risk without shocks	Full stochastic
Low skill ($\alpha = 0.94, k_0 = 4.9$)			
Discounted stream of utility	2.7	2.7	1.2
Discounted stream of income	26	26	25
Dynamic asset gap $k^e(\alpha) - k_0$	∞	∞	∞
Calvo-Dercon $P^{fia}(0)$	60	60	60
Calvo-Dercon $P^{fia}(1)$	10.2	10.3	14.3
Middle skill ($\alpha = 1.10, k_0 = 2.1$)			
Discounted stream of utility	4.0	3.8	3.3
Discounted stream of income	37	29	28
Dynamic asset gap $k^e(\alpha) - k_0$	0	2.3	2.3
Calvo-Dercon $P^{fia}(0)$	8	60	60
Calvo-Dercon $P^{fia}(1)$	0.6	2.8	3.9
High skill ($\alpha = 1.22, k_0 = 1.1$)			
Discounted stream of utility	5.5	5.5	4.7
Discounted stream of income	44	43	40
Dynamic asset gap $k^e(\alpha) - k_0$	0	0	0
Calvo-Dercon $P^{fia}(0)$	4	5	6
Calvo-Dercon $P^{fia}(1)$	0.5	0.6	0.6

In contrast, for the mid-skill individual, the ‘risk without shocks’ simulation brings a major drop in production (from 37 to 29) and utility (from 4.0 to 3.8) compared to the non-stochastic simulation. While the full stochastic simulation brings some additional income losses for this individual, they are modest compared to the losses occasioned by the risk-induced strategy shift. Among other things, these simulations show that in the presence of critical asset thresholds, risk takes on particular importance for those individuals subject to multiple equilibria.

2. Forward-looking measures of chronic poverty and vulnerability

The theoretical analysis in the prior section has used dynamic economic theory to elucidate the multiple dimensions of chronic and persistent poverty, and to demonstrate how vulnerability to economic shocks further increases chronic poverty. Building on those ideas and insights, this section puts forward two types of chronic poverty measures. The first generalises a suggestion in Carter and Barrett (2006) and uses information on the Micawber

income only falls from 26 to 25. The proportionately larger drop in utility occurs because individuals will end up spending more of their income on re-accumulating assets destroyed by shocks.

frontier to create an asset-based chronic poverty measure. The second uses the asset dynamics implied by the BCI model to create a forward-looking income stream that can then be used to calculate any of the income-based chronic poverty measures discussed by Calvo and Dercon (2006). Both the asset and income-based measures can be utilised to create explicit chronic poverty vulnerability measures, where vulnerability is understood as the increase in the chronic poverty measure induced by risk and shocks.

The measures put forward in this section rely on structure of the standard, backward-looking Foster-Greer-Thorbecke (FGT) class of poverty measures defined as:

$$P(\gamma) = \frac{1}{M} \sum_{i=1}^N I_i \left(\frac{z - f_i}{z} \right)^\gamma$$

where M is the total population size (poor and non-poor), i indexes individual observations, z is the scalar-valued poverty line, f_i is the flow-based measure of welfare (income or expenditures) as measured retrospectively at the time of the survey, I_i is an indicator variable taking value one if $f_i < z$ and zero otherwise, and γ is a parameter reflecting the weight placed on the severity of poverty. Setting $\gamma = 0$ yields the headcount poverty ratio $P(0)$ (the share of a population falling below the poverty line). The higher order measures, $P(1)$ and $P(2)$, yield the poverty gap measure (the money metric measure of the average financial transfer needed to bring all poor households up to the poverty line) and the squared poverty gap (an indicator of severity poverty that is sensitive to the distribution of wellbeing amongst the poor).

2.1 Chronic poverty measures based on the Micawber threshold

As suggested by Carter and Barrett (2006), information on asset dynamics that permits identification of the Micawber frontier opens the door to a forward-looking poverty measure. The standard, money-metric poverty line is frequently criticised as an arbitrary construct, which has no behavioural foundation. In contrast, the Micawber frontier is an empirical construct whose foundation is observed behaviour. Conceptually, the Micawber frontier can separate households expected to be persistently poor from those for whom time is an ally that promises better standards of living in the future. Poverty measures based on the Micawber frontier thus promise to help identify the long-run health of an economy as judged by its ability to facilitate growth in living standards amongst its least well-off members.

Generalising the Carter and Barrett measure to allow for heterogeneous ability, yields the following expression:

$$P_k(\gamma) = \frac{1}{M} \sum_{i=1}^M I_i^k \left(\frac{k(\mathbf{a}_i) - k_i}{k(\mathbf{a}_i)} \right)^\gamma, \quad (3)$$

where k_i is asset stock of household i and the binary indicator variable $I_i^k = 1$ if $k_i < k(\mathbf{a}_i)$ and reflects whether the household i 's asset stock is below the Micawber frontier. When $\gamma = 1$, we can use the core part of this measure, $I_i^k (k(\mathbf{a}_i) - k)$, to define the 'dynamic asset poverty gap'. Table 1 reports this measure for three prototypical individuals whose initial asset positions are illustrated in Figure 4. The normalised asset poverty gap for the high-skill individual is always zero. For the mid-skill individual, the gap is zero in the absence of risk, but rises to 2.3 units of capital (or about 58 percent of the frontier value of 4) when the discouraging effect of risk shifts out the Micawber frontier. For the low-skill person, the Micawber frontier and the dynamic asset poverty gap are infinite as there is no level of capital stock from which this individual will find it desirable to sustain the high level equilibrium.

The existence of the intrinsically chronically poor individuals (for whom the dynamic asset poverty gap is infinite) renders the poverty measure (3) mathematically problematic, as the portion of the expression in parentheses is undefined for these individuals. We thus modify the measure as follows:

$$\tilde{P}^k(\gamma) = \frac{1}{M} \sum_{i \in \tilde{M}} I_i^k \left(\frac{\tilde{k}(\alpha_i) - k_i}{\tilde{k}(\alpha_i)} \right)^\gamma$$

Where \tilde{M} is the subset of the total population for whom $k(\alpha_i)$ is finite. Setting $\gamma = 0$, this modified measure ($\tilde{P}^k(0)$) gives the headcount ratio of all non-intrinsically poor individuals who are below the Micawber threshold and who would therefore be expected to be chronically poor in the sense of being trapped at the low level equilibrium. Information on the fraction of the population that is intrinsically chronically poor ($\frac{M-\tilde{M}}{M}$) can be used to create a complete chronic poverty headcount measure, $CPHC = \tilde{P}^k(0) + \frac{M-\tilde{M}}{M}$.

When $\gamma = 1$, $\tilde{P}^k(1)$ yields a normalized measure of the asset transfers that would be necessary to place multiple equilibrium chronically poor households in a position from which they can grow and sustain a non-poor standard of living in the future. In the language of Carter and Barrett (2006), this $\tilde{P}^k(1)$ measure would indicate the resources needed for the cargo net transfers required to eliminate multiple equilibrium chronic poverty. Note that there are no asset transfers that will sustainably eliminate the chronic poverty of the intrinsically chronically poor.

To illustrate these ideas, we used the BCI model to conduct a 60-year simulation of poverty and its evolution for an imaginary community of 100 households. We performed two sets of simulations using the procedures described in the prior section. First we performed the non-stochastic simulations (in which households follow the optimal accumulation rule defined by the non-stochastic version of the BCI model). Second we utilised the full stochastic in which each household follows the optimal accumulation rule defined by the stochastic version of the BCI model and each received their own random (idiosyncratic) shock in each time period.

For the simulation, it was assumed that 25 percent of the households had sufficiently low skill endowments that they were in the intrinsically chronically poor category in the presence of risk (that is, $\alpha_i < \alpha^L$). Another 50 percent were in the mid-skill (multiple equilibrium) range, ($\alpha^L < \alpha_i < \alpha^H$), while the final 25 percent were in the high skill (intrinsically upwardly mobile) range ($\alpha_i < \alpha^H$). While these assumptions are arbitrary, they do match the empirical findings of the Santos and Barrett (2006) study of East African pastoral households. Finally, initial asset endowments for each household were randomly distributed (using a uniform distribution) over the range of 0.1 to 10 units of capital.¹³

Included in Table 2 are the standard (backward-looking) Foster-Greer-Thorbecke (FGT) poverty measures for both the initial period ($t = 1$) and final period of the simulation ($t = 60$). Under the scenario that assumes away economic shocks, the standard poverty headcount drops over the period of the simulation from 34 percent to 21 percent of the population. This latter figure exactly equals the period 1 dynamic asset poverty threshold headcounts of the chronically poor (both the intrinsically and multiple equilibrium chronically poor). As this

¹³ In the real world, one would not expect to find 'initial' endowments uncorrelated with skill. However, for illustrative purposes, this egalitarian assumption permits us to more fully see the operation of the model.

simple example shows, the forward-looking threshold based measure captures the dynamics of the system and thus provides a more informative portrayal of the expected long-run evolution of poverty.¹⁴ Combining the two pieces of information would permit us to say (in period 1) that 34 percent of the population is currently poor and that we would expect (under existing dynamics) to see 13 percent of the population to escape poverty, and the other 21 percent to remain chronically poor. The $\bar{P}^*(1)$ measure of the size of the dynamic asset poverty gap shows that on average, the chronically poor have assets that are 10 percent below the Micawber frontier.

When shocks (and risk) are brought into the model, the results change rather significantly as shown in the second column of Table 2. Over the 60-year period of the simulation, the FGT headcount rises from 34 percent to 55 percent. In this case, the backward-looking FGT measure overstates the long-run health of the economy. In contrast, the year 1 Carter-Barrett asset-based CPHC indicates that 48 percent of the population is chronically poor (with this fraction split evenly between the intrinsically chronically poor and the multiple equilibrium chronically poor). This 48 percent figure is in fact an understatement of the functioning of the economy as it fails to account for multiple equilibrium households that are knocked below the Micawber threshold over the time period of the simulation.¹⁵ The $\bar{P}^*(1)$ measure rises to 15 percent, indicating that the depth of dynamic asset poverty rises for the multiple equilibrium poor. The cargo net transfers needed to lift these individuals over the Micawber frontier has thus increased. As in the non-stochastic case, the combination of the FGT and the dynamic asset poverty measures provide a more comprehensive view of the nature of poverty and its likely future evolution.

Table 2 Simulated chronic poverty measures

	Non-stochastic simulation	Stochastic simulation	Vulnerability
'Backward-looking' measure			
FGT $P(0)$ at time $t = 1$	0.34 [0.07]	0.34 [0.07]	-
FGT $P(0)$ at time $t = 60$	0.21 [0.02]	0.55 [0.08]	-
Forward-looking chronic poverty measure			
Carter-Barrett threshold measures			
Intrinsic headcount, $\left(\frac{M - \bar{P}}{M}\right)$	3%	24%	88%
$\bar{P}^*(0)$ measure at time $t = 1$	18%	24%	25%
Complete headcount, $CPHC$	21%	48%	-
$\bar{P}^*(1)$ measure at time $t = 1$	10%	15%	
Calvo-Dercon income stream measures			
$P_i^{fa}(0)$ (average)	13 [22%]	28.5 [48%]	54%
$P_i^{fa}(1)$ (average)	1.5	3.7	59%

¹⁴ The BCI model assumes that the underlying structural dynamics of the economic do not change over the period of the simulation, a stricture unlikely to be met in the real world.

¹⁵ In principal, the Carter-Barrett measure could be adjusted to account for the likelihood that some individuals will receive shocks that will knock them under the threshold.

2.2 Using asset dynamics to create forward-looking income-based chronic poverty measures

In addition to underwriting chronic poverty measures based on the Carter-Barrett dynamic asset poverty gap, information on asset dynamics can be used to project future asset and income levels. When combined with the income-based chronic poverty measures of Calvo and Dercon (2006), these projections open up another class of forward-looking chronic poverty measures.

Calvo and Dercon suggest a number of ways of consistently analysing a sequence of income levels for a given household over T time periods. While they are primarily thinking of sequences of past incomes, they suggest that their methods can be applied to estimated future income streams. The analysis here follows this suggestion.

For purposes here, we will limit our attention to what Calvo-Dercon call the FTA (focus-transformation-aggregation) chronic poverty measure. Letting z denote the standard income poverty line, and f_{it} denote the income of household i in period t , we can write the FTA measure (P_i^{fta}) as:

$$P_i^{fta}(\gamma) = \sum_{t=1}^T \beta^{T-t} I_{it}^{fta} \left(\frac{z - f_{it}}{z} \right)^\gamma, \quad (4)$$

where I_{it}^{fta} is an indicator variable that takes the value of one if $f_{it} < z$, and β is the discount factor. Note that measure (4) is specific to a particular individual and does not aggregate across individuals. For illustrative purposes here, we set $\beta = 1$, so that all poverty spells are treated identically (see Calvo and Dercon for more discussion on the desirability of this assumption). In the special case when $\gamma = 0$, (4) simply counts the number of poverty spells experienced by the individual.

The various simulations of the BCI model used in the prior section can be used to illustrate our forward-looking use of the Calvo-Dercon measures. Table 1 presents the $P_i^{fta}(0)$ and $P_i^{fta}(1)$ measures for the low-, medium- and high-skill individuals under the various simulation scenarios introduced earlier. The low skill individual is poor all 60 periods under all scenarios, as shown by the degree zero FTA measure. The increase in the degree 1 FTA poverty gap measure under the full stochastic simulation reflects the impact of realised shocks.

The FTA measures for the high-skill agent shows that he/she escapes poverty rather quickly under all scenarios. In contrast, the degree zero FTA measure jumps from 8 to 60 for the mid-skill individual once risk is brought into the picture. As this example illustrates, the forward-looking Calvo-Dercon measure captures the chronic poverty impacts of risk that are overlooked by standard FGT measures. In addition, while not explicitly established to capture threshold effects, the Calvo-Dercon family measures are quite sensitive to their impacts.¹⁶

Table 2 similarly presents FTA measures for the stylised 100 individual economy analysed in the prior section. Results are shown for both the non-stochastic and the full stochastic simulations. While measure (4) is individual specific, Table 2 reports the simple average of the $P_i^{fta}(\gamma)$ measures across the 100 individuals in the simulation. To ease comparability with the other measures, the figures in square brackets divide the $P_i^{fta}(0)$ by the total number of periods and thus yield a measure of the fraction of time that the average individual spends below the poverty line during the course of the 60-period simulation.

¹⁶ This same comment would also apply if the Calvo-Dercon measures were used to look backwards to evaluate the degree of poverty in a past-realised income history.

As can be seen in Table 2, the average value of $P_i^{fta}(0)$ when there are no economic shocks is 13, indicating that the average household was below the income poverty line 13 out of the 60 total time periods, or 22 percent of the time. Interestingly, this figure corresponds closely to the period 60 FGT measure, as well as to the dynamic asset poverty measure. Similarly, for the stochastic simulation (in which households anticipate and are subject to economic shocks), the $P_i^{fta}(0)$ averages 28.5 poverty spells across the 100 households, indicating that households are poor roughly 48 percent of the time. However, it should be stressed that the equivalence of the FTA figure to the Carter-Barrett CPHC measure is somewhat coincidental. The former reflects the fact that the intrinsically upwardly mobile may have poverty spells as they accumulate assets and/or recover from shocks. Similarly, the long-term chronically poor may have spells of non-poor income if they fortuitously begin life with an ample (but unsustainable) asset endowment.¹⁷ But despite these differences with the threshold based measure, the Calvo-Dercon FTA measures capture the intrinsic dynamics of the system and provide a more informative, forward-looking picture than does the standard FGT family of measures. Again it should be stressed that these forward-looking measures are in principle estimable in time 1,¹⁸ though their accuracy depends on the stability of the underlying dynamics in the economy.

2.3 Vulnerability as increased chronic poverty

While there is debate over how best to conceptualise and measure vulnerability (compare Calvo and Dercon 2005 with Ligon and Schechter 2003), one natural approach would be to define vulnerability as the increase in chronic poverty that results when individuals are exposed to shocks. Linking vulnerability to increases in chronic poverty captures the sense of drastic and irreversible harm that Calvo and Dercon (2005) identify as the common thread that unites various concepts of vulnerability. In addition, the ability to define vulnerability in terms of increased chronic poverty provides a very compelling policy focus, indicating the fraction of chronic poverty that can be remediated through social protection programmes.

The far right column in Table 2 defines vulnerability using both the Carter-Barrett and the Calvo-Dercon chronic poverty measures. In both cases, vulnerability is defined as the fraction of total chronic poverty revealed by the full stochastic simulation that is created by risk and shocks. That is vulnerability is the difference between chronic poverty between the stochastic and the non-stochastic simulations, normalised by the chronic poverty in the stochastic simulation.

As can be seen in Table 2, nearly 60 percent of total chronic poverty in the simulation analysis is the result of vulnerability under both the Carter-Barrett and the Calvo-Dercon measures. Social protection policies would have an enormous impact on chronic poverty in this case. This large increment in chronic poverty created by vulnerability results from the three forces discussed earlier. First, realised shocks sometimes push individuals below the income poverty line.¹⁹ Second, increased chronic poverty also results when realised negative shocks knock individuals below the Micawber frontier, rendering infeasible a pathway from

¹⁷ Variants on the Calvo-Dercon measures that more heavily weigh final outcomes, would, however, present information that is closer in spirit to the dynamic asset poverty measures.

¹⁸ These are estimable if the accumulation rule can be estimated as well as the error distribution that generates deviations between expected and actual accumulation. With those two pieces of information, a set of forward-looking projections could be generated using either stochastic or non-stochastic simulations.

¹⁹ Note that unlike the Ligon and Schechter vulnerability measure that increases with any fluctuation in income, the vulnerability measure based on the Calvo-Dercon FTA measure has a poverty focus and only increases for fluctuations that drive individuals below the poverty line. Note that the Carter-Barrett measure will not increase for individuals pushed below the income poverty line, but who remain above the Micawber frontier.

poverty, and indeed spelling ruin in the language of Calvo and Dercon (2005) cited above. Third and finally, the prospect that ruin can occur has a discouraging effect on accumulation strategies, shifting the Micawber frontier beyond the reach of some individuals, driving yet additional increases in the measured (multiple equilibrium) chronic poverty.

Calculation of the vulnerability measures in Table 2 is feasible because the BCI model allows us to straightforwardly simulate how individuals would counterfactually behave in the absence of risk. However, the real world does not offer data on how individuals would (counterfactually) behave in the absence of risk. For example, we do not have data that could be used to directly identify what the Micawber frontier would be in the absence of risk as we do not observe individuals behaving in the counterfactual, risk free world. Empirical implementation of this type of vulnerability measure would therefore be far from straightforward.²⁰ Nonetheless, it would be possible in principal to obtain estimates of the parameters that shape behaviour and then simulate what behaviour would counterfactually be in the absence of risk. It might also be possible to take advantage of naturally occurring variation of risk (as Rosenzweig and Binswanger 1993 do) in order to gain insight into how the Micawber frontier shifts with risk. Significant future work will be required to empirically implement the type of vulnerability measures shown in Table 2.

3. A first application to South Africa

The prior sections of this paper have laid out an ambitious agenda, showing how economic theory can be used to underwrite a suite of theoretically grounded, forward-looking chronic poverty measures. This section uses data from South Africa to illustrate the use of these measures, employing the KwaZulu-Natal Income Dynamics Study (KIDS) data that cover the KwaZulu-Natal province that is home to roughly 25 percent of South Africa's population (see Agüero, Carter and May, (forthcoming) for thorough discussion of the KIDS data).

The top half of Table 3 displays standard FGT poverty measures for the first two rounds of the KIDS data (1993 and 1998) as reported in Carter and May (2001). As can be seen, this period was characterised by substantial downward mobility as the headcount measure of poverty rose from 27 percent to 43 percent, while the FGT poverty gap measure ($P(1)$) held steady at 33 percent.²¹

These same data can be used to recover the underlying asset dynamics. In a recent paper, Adato, Carter and May (2006) use this KIDS data to estimate the pattern of asset dynamics under the assumption that $\bar{k}(\alpha) = \bar{k} \forall \alpha$. In other words, Adato, Carter and May assume that the Micawber frontier is the same for all agents, irrespective of the individual's skill level. In terms of Figure 2, the Micawber frontier would appear as a horizontal line under the assumptions used by Adato, Carter and May.

²⁰ Note, however, that the partial impact of vulnerability can be recovered rather straightforwardly by doing an empirical analysis that is akin to the middle column of Table 2 (risk without shocks). Using some of the methods of Schechter, it should be possible to simulate the impact that shocks have on individuals, holding the Micawber frontier fixed. Such information could be quite useful from the perspective of designing a social safety net.

²¹ The Carter and May (2001) FGT measures are based on poverty line estimates using the household subsistence line (HSL). The HSL became unavailable after 1998 and subsequent analysis (such as that reported in Agüero, Carter and May 2007) has relied on the poverty line standard suggested by Hoozeveen and Özler (2005). Using this latter poverty line, the poverty headcount in the KIDS data rose from 52 percent to 57 percent over the 1993 to 1998 period.

Table 3 Backward- and forward-looking poverty measures for South Africa

FGT measures	
$P(0)$ in 1993	27%
$P(0)$ in 1998	43%
Carter-Barrett threshold measures	
$\hat{P}^k(0)$ in 1998	59%
$\hat{P}^k(1)$ in 1998	11%

As detailed in that paper, Adato, Carter and May first estimate an asset index for each individual, and then use non-parametric regression techniques to recover the pattern of asset dynamics.²² Interestingly, they estimate the Micawber frontier to be at a level of assets expected to generate a living standard almost twice the poverty line. Individuals below that estimated frontier would be predicted to slide backwards over time towards a sub-poverty line standard of living, while those above it would be predicted to achieve a living standard well above the poverty line. Note also that not everyone who is observed to be poor by standard consumption measures will be predicted to be poor in the longer term. In particular, households that have assets in excess of the frontier and are ‘stochastically poor’ (in the language of Carter and May 2001), would not be predicted to be poor in the long term.

While the Adato, Carter and May analysis rests on several strong assumptions, it does permit us to illustrate the use of asset threshold-based poverty measures. As shown in the bottom half of Table 3, fully 58 percent of KIDS households were below the estimated Micawber threshold in 1998, and are therefore expected to be chronically poor. Because of the assumption that the threshold is the same for all households, it is not possible to partition this group into the intrinsically chronically poor and the multiple equilibrium chronically poor. Nonetheless, the fact that this measure is above the 1998 backward-looking poverty headcount indicates that the underlying asset dynamics predict future increases in poverty. Put differently, the $\hat{P}^k(0)$ measure indicates that the South African economy was not offering a favourable environment for asset accumulation and income growth for the less well-off.

The quality of such a prediction depends on the stability of the underlying asset dynamics, as well as on the quality of the actual estimation. The KIDS 2004 round of data indicate a decline in the standard poverty headcount, rather than further increases as would be expected based on the asset threshold based measure (see Agüero, Carter and May (forthcoming) for results from the 2004 data). The predictive failure of the asset-based measure may reflect an underlying change in asset dynamics (that is, the prospects for accumulation and income growth improved dramatically between 1998 and 2004).²³ It may also reflect the simplifying assumption used by Adato, Carter and May, that the Micawber frontier is the same for all households. The fact that the asset poverty gap measure ($\hat{P}^k(1)$) was a modest 11 percent in 1998 indicates that the typically asset poor household was not too far below the estimated frontier. Either a modest improvement in asset dynamics (or a modest overestimation of the threshold for mid-skill agents) may have lead to the predictive

²² The asset index includes human capital variables as well as tangible physical assets such as land and business equipment. Related methodological approaches to recovering a critical asset threshold can be found in Barrett et al. (2006), Carter et al. (2007) and Lybbert et al. (2004).

²³ The BCI model used in the theoretical analysis here assumes that the income generation process does not change over time. In principle, the model could be modified to reflect growth in productivity and wages (or cycles of macroeconomic boom and bust). The impact on behaviour would depend on how individuals anticipated these changes.

failure of the threshold-based chronic poverty measures. Future efforts are clearly needed to help the empirical measures catch up with the sophistication of the theoretically derived measures discussed earlier.

4. Chronic poverty measurement and policy

This paper began with the challenge of understanding how much poverty is chronic in the sense that it would be expected to persist into the future. The microeconomic theory of asset accumulation and poverty traps suggests a way of approaching this problem and estimating future asset accumulation and income growth. This information can in turn be used as the basis for two families of theoretically grounded, forward-looking chronic poverty measures.

While there is still much to be done to improve the chronic poverty measures put forward here, they are ultimately intended to complement, not replace, conventional, 'backward-looking' poverty measures. While the latter are meant to give us a sense of the current (or at least recent) economic status of people at the bottom of the income distribution, the former use information on patterns of asset accumulation to project forward who is likely to remain poor in the future. Together, the two classes of measures provide a more complete description of the groups for whom the economy is not well functioning.

As in any area of economics, looking forward into the future is fraught with difficulties. The information that can be gleaned from the chronic poverty measures suggested here is probably most valuable over a medium-term time horizon when the structure of the economy is relatively stable. But even within these limits, the capacity of the asset-based chronic poverty measures to provide information on the intrinsically chronically poor and the multiple equilibrium chronically poor is potentially quite valuable from a policy perspective. Moreover, while empirical calculation of the vulnerability measures discussed in section 2.3 is likely fraught with difficulty, the theoretical analysis put forward makes clear that vulnerability to economic shocks is potentially an important part of chronic poverty. This is especially true in economies where large numbers of agents find themselves in the multiple equilibrium category, facing a positive but finite Micawber frontier. The theory reviewed here suggests that the provision of social protection measures will lower the Micawber frontier for average individuals, crowding-in private accumulation and rendering feasible new pathways from poverty for at least some. While there is still much to find out about whether social protection can in practice really have these twin effects on reducing chronic poverty, further efforts to more sharply conceptualise and measure chronic poverty will move us in the direction of being able to explore these ideas and pilot new social protection programmes.

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Appendix 1: Details of theoretical model

This section provides additional detail on the formal model used to generate the results discussed in Sections 2 and 3. For additional details, see Barrett, Carter and Ikegami (2007).

Non-stochastic model

Under the technological specification given in Section 1.1, we assume that individuals face the following infinite horizon model as they make the decision about how best to divide income ($f(\alpha, k_t)$) every period t between consumption (c_t) and investment (i_t):

$$\begin{aligned} \max \quad & \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) \\ \text{s.t.} \quad & c_t + i_t \leq f(\alpha, k_t) \\ & f(\alpha, k_t) = \max\{\alpha k^{\gamma_L}, \alpha k^{\gamma_H} - E\} \\ & k_{t+1} = i_t + (1 - \delta)k_t \\ & k_1 \text{ given} \end{aligned}$$

where β is the discount factor, c_t is consumption, $u(\cdot)$ is utility function, i_t is investment and δ is depreciation rate. Note that there is neither saving nor borrowing and that the household is assumed to live forever. Solution of this problem, using the parameters given below, generates the Micawber frontier, $\mathcal{P}(\theta)$, illustrated in Figure 2 in the main body of the text.

Stochastic model

Households face a number of risks. These risks can be classified as (i) asset shocks (ii) income shocks and (iii) marginal utility shocks. While all three types of risk are important, the analysis here focuses on the relatively simple case of asset shocks. In particular we assume that assets evolve according to the following specification:

$$k_{t+1} = \theta_t [i_t + (1 - \delta)k_t]$$

where $\theta_t \in (0, \theta^{\max}]$ is asset shock. Note that this multiplicative specification makes the magnitude of risk increase as accumulated capital stock increases.

We assume that the probability distribution of θ_t is known and that the individual decisionmaker allocates income between consumption and investment in order to solve the following problem:

$$\begin{aligned} \max \quad & E_1 \sum_{t=1}^{\infty} \beta^{t-1} u(c_t) \\ \text{s.t.} \quad & c_t + i_t \leq f(\alpha, k_t) \\ & f(\alpha, k_t) = \max\{\alpha k^{\gamma_L}, \alpha k^{\gamma_H} - E\} \\ & k_{t+1} = \theta_t [i_t + (1 - \delta)k_t] \\ & k_1 \text{ given} \end{aligned}$$

where E_1 is expectation at period 1. Solution of this problem, again using the parameters outlined below, yields the results summarised in Figure 4.

Parameters and other details for numerical simulation

The functional specification for the utility function $u(\cdot)$ is

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

The probability density of θ_t is assumed to be:

$$\text{density of } \theta_t = \begin{cases} 0.90 & \text{if } \theta_t = 1.0 \\ 0.05 & \text{if } \theta_t = 0.9 \\ 0.03 & \text{if } \theta_t = 0.8 \\ 0.02 & \text{if } \theta_t = 0.7 \end{cases}$$

The other structural parameter values are assumed to be as follows:

$$\sigma = 1.5, \delta = 0.08, \beta = 0.95, \gamma_L = 0.3, \gamma_H = 0.45, E = 0.45.$$

We discretise continuous variables k and α as follows: $k = \{0.1, 0.2, \dots, 15.0\}$ and $\alpha = \{0.94, 0.96, \dots, 1.22\}$.

For the simulation of the stylized economy of 100 individuals we draw α from $N(1.08, 0.074^2)$. Parameter values of mean and variance are chosen so that *ex ante* proportion of low, middle, and high type individuals (defined relative to the stochastic Micawber frontier) would be 25 percent, 50 percent, and 25 percent, respectively. We draw k_1 from Uniform[0.1, 10.0] and assume that k_1 and α are statistically independent from each other.

We specify poverty line as follows: Asset level which generates income on the poverty line satisfies the following equation:

$$y_p = f(\alpha, k_p).$$

where y_p is income-based poverty line. Thus, that asset level depends on α and we denote it by $k_p(\alpha)$. We assume that middle type individual would be below income poverty line if he converged in low equilibrium and set poverty line by $k_p(\alpha = 1.12) = 3.4$ and thus $y_p = 1.62$.