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# Occupational Choice and Inequality Traps*

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**Abstract**

The paper presents a model where individuals decide to become workers or entrepreneurs in the presence of capital constraints and where individuals differ in wealth levels. The model shows that the higher the initial level of inequality in wealth is, the lower the long run aggregate wealth of the economy and the higher the long run inequality will be.

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# 1 Introduction

Why countries that start poor and with high levels of inequality tend to stay that way over time? We study this question by exploring the conditions under which initial wealth inequalities determine the long-run wealth distribution and how. For this purpose an occupational choice model is developed which considers the effects of imperfect credit markets.

Our economies consist of individuals who live for two periods. At the beginning of the first period each individual faces an occupational choice problem. Individuals decide whether to become workers or entrepreneurs. Entrepreneurs optimise the size of their firm according to their initial wealth and the credit they can get. Following the dynasty of each individual over time, the model shows that the occupation-choice of each offspring depends on the initial distribution of wealth, and the structure of occupational choice in turns determines people's rents and savings giving rise to a new distribution of wealth. The findings from the model are that an initial wealth distribution can converge to a wide range of distributions over time. Given the presence of multiple equilibria and non-linear dynamics, rather than providing a rigorous analytical description of the equilibria, the type of distributions the model predicts, and the conditions that could lead to these distributions are described. These depend on the wealth distribution of the credit and non-credit constrained individuals. The larger and the poorer the bottom of the distribution is, the lower the long-run equilibrium salary will be. The wealthier and the more egalitarian the top of the distribution is the higher the equilibrium salary will be in the long-run.

Overall the parameters in the model show that low levels of wealth are associated with a high cost of setting up a business, a high proportion of credit constrained people, and few and small entrepreneurs. Equilibrium salaries depend on the productivity of labour, but also on the proportion of credit constrained individuals (supply of labour) and how wealthy the proportion of the non-credit constrained individuals is. The wealthier and more numerous the entrepreneurs are, the more likely that they will expand their scales of production and raise salaries.

This model differ in various aspects to previous models that have addressed this topic within an occupational choice framework. An important contribution within this view, is the theoretical model of Banerjee and Newman (1993) further simplified by Gathak and Jiang (2002). In the Banerjee and Newman model two types of equilibria can be reached, depending on the initial distribution of wealth. If there is a large ratio of very poor to very rich the demand for labour remains low over time, and so does the equilibrium salary. This is because entrepreneurs cannot increase their scale of production over time, due to the assumed technological constraints[1] and that credit-constrained people cannot set up new firms. In contrast, if there is a large ratio of very rich to very poor the demand for labour is high, and the economy achieves a high employment and salary equilibrium. The ratio of credit to non-credit constrained individuals then determines the equilibrium returns to occupations and hence the long run wealth distribution.

Although the Banerjee and Newman model links the long run wealth distribution to the original distribution, this link depends on the condition that all entrepreneurs set up firms of exactly the same scale, irrespective of differences in their initial wealth levels[2]. Entrepreneurs cannot expand their scales of production, and therefore cannot raise salaries either. As a result growth stops and so does the process of wealth trickling down from entrepreneurs to workers. In contrast in this paper by allowing entrepreneurs to optimise their size of production a wider type of wealth distributions can be reached. In particular when there are low levels of inequality among the non-credit constrained, it is possible to trickle down wealth to poorer individuals. This is achieved by entrepreneurs expanding their scales of production over time thereby raising salaries. A contrasting scenario can be achieved in the presence of high levels of inequality among the non-credit constrained people. In par-

[1]Entrepreneurs need to acquire a monitoring technology that will enable them to obtain the maximum worker effort. It is assumed that entrepreneurs will all set up the same scale of production, since technology does not allow increases in monitoring capacity.

[2]Banerjee and Newman (1993) assumed that there are self-employed individuals that set up businesses at a smaller scale than entrepreneurs. However, there are no differences in scales of production among entrepreneurs nor among the self-employed.

ticular if initially there were high levels of inequality among non-credit constrained people such that only a few but big entrepreneurs manage to expand their businesses by raising salaries, they would drive out of business small entrepreneurs which are unable to stay afloat with the rising salaries. In this case, inequality remains high in the long-run as the remaining big entrepreneurs have no more incentives to keep raising salaries over time.

The issues introduced are analysed in three sections. Section 2 presents a model where entrepreneurs set up the size of their firms according to their wealth level. Section 3 presents the concluding remarks.

## 2 Occupational Choice Model

Consider an economy there is a constant population, $N$, of two-period lived individuals belonging to generations of altruistic families. At the beginning of the first period each individual is given an initial wealth $w_i$ and a unit of labour which he can use in one of the three occupations available in the economy: worker, entrepreneur and unemployed[3]. During the second period of life individuals consume, $x$. The proportion consumed from their wealth, $(1-\beta)$, is assumed to lie between 0 and 1. The utility preferences over consumption and bequest $b$, $U(x,b)$, are expressed in the following equation.

$$U(x,b) = (1-\beta)\log x + \beta \log b \tag{1}$$

Individuals choose the occupation that maximises their utility subject to their wealth constraint. It is assumed that neither the unemployed nor the workers need any starting wealth in their occupation. The unemployed can produce some fixed amount $\gamma$ with their labour. A worker earns a salary $s$, which is endogenously determined in the labour market. To be profitable to become a worker the salary $s$ has to be greater than or equal to $\gamma$. It is assumed that both workers and the unemployed save

[3] A similar simplification of the Banerjee and Newman (1993) model to the one presented here was developed by Ray (1998) and by Gathak and Jiang (2002). Nonetheless, the model presented here is developed using different credit market and technology specifications.

their initial wealth in the first period and in the second period receive the returns on savings $(1+r)w_i$ in addition to the payment for their labour.

Entrepreneurs can set up firms of different scales depending on their wealth. If entrepreneurs use exclusively their wealth, they will be able to set up a scale of production $\widehat{\alpha}_i$. The technology available in the economy is a Leontief production function. Entrepreneurs invest in machinery at a fixed cost $\sigma$ and incur a variable cost $\widehat{\alpha}_i\sigma$, which depends on the scale of production of the firm $\widehat{\alpha}_i$. A number of $\widehat{\alpha}_i$ workers are hired in the first period. The machinery depreciates, such that the net value of the machinery at time $t+1$ is $c(\sigma+\widehat{\alpha}_i\sigma)(1+r)$.

The project yields a return proportional to the scale of production $\widehat{\alpha}_i\rho$ and a salary bill $\widehat{\alpha}_i s$ in the second period.

Therefore, an entrepreneur using exclusively his initial wealth equal to $w_i$ can set up a scale of production $\widehat{\alpha}_i$ if he covers the start up cost,

$$w_i = \sigma + \widehat{\alpha}_i\sigma \tag{2}$$

Given the scale of production $\widehat{\alpha}_i$, the net return $\widehat{\eta}_i$ of the project at time $t+1$ is the return $\widehat{\alpha}_i\rho$ plus the value of the depreciated machinery $c(\sigma+\widehat{\alpha}_i\sigma)(1+r)$, less the salary bill $\widehat{\alpha}_i s$.

$$\widehat{\eta}_i = \widehat{\alpha}_i\rho + c(\sigma+\widehat{\alpha}_i\sigma)(1+r) - \widehat{\alpha}_i s \tag{3}$$

The scale of production $\widehat{\alpha}_i$ can be increased further if the entrepreneur gets a loan.

## 2.1 Credit Market

Banks offer a loan contract conditional on borrowers providing collateral. This collateral is equal to the net value of the machinery, which depends on the scale of production, $\widehat{\alpha}_i$.

An entrepreneur will honour the loan if the cost of the loan is less than or equal to the net value of the collateral at time $t+1$.

$$(1+r)L_i \leq c(\sigma+\widehat{\alpha}_i\sigma)(1+r) \tag{4}$$

From eq.(2) $\sigma + \widehat{\alpha}_i\sigma$ is equal to $w_i$, hence the loan $L_i$ is proportional to the wealth level of the entrepreneur,

$$L_i = cw_i \tag{5}$$

An entrepreneur with initial wealth, $w_i$ and loan $L_i$ will run the project at scale $\alpha_i$,which is greater than $\widehat{\alpha}_i$ the scale of the project the entrepreneur would have run without the loan.

$$w_i + L_i = \sigma + \alpha_i\sigma \tag{6}$$

Solving for the scale $\alpha_i$, the optimum scale of the project is given by:

$$\alpha_i = \frac{w_i + L_i - \sigma}{\sigma} \tag{7}$$

Substituting $L_i$, the optimal scale is,

$$\alpha_i = \frac{w_i(1+c) - \sigma}{\sigma} \tag{8}$$

The net returns $\eta_i$ of entrepreneurs with loan $L_i$ are determined by the net returns of the scale of production $\alpha_i\rho$, plus the value of the depreciated machinery $c(\sigma + \alpha_i\sigma)(1+r)$, minus the salary bill $\alpha_i s$ and minus the repayment of the loan $(1+r)L_i$.

$$\eta_i = \alpha_i\rho + c(\sigma + \alpha_i\sigma)(1+r) - \alpha_i s - (1+r)L_i \tag{9}$$

Entrepreneurs require loans only if the project yields higher returns than the net return earned from being a worker.

$$\eta_i \geq w_i(1+r) + s \tag{10}$$

Substituting $\eta_i$ eq.(9), $\alpha_i$ eq.(8) and $L_i$ eq.(5) into eq.(10), the initial wealth $w_i$, has to be equal to or greater than a wealth threshold $w^A$ (see eq.(11)) in order for entrepreneurs to enjoy returns $\eta_i$ higher than the workers' return,

$$w_i \geq \frac{\rho}{\frac{1+c}{\sigma}\rho + (1+r)c^2 - (1+r) - (\frac{1+c}{\sigma})s} = w^A \tag{11}$$

Another interpretation of eq.(10) can be given by solving for the maximum salary that the entrepreneur $i$ can afford could still earn pay to have a higher return than workers.

$$s_i \leq \frac{w_i\{\frac{1+c}{\sigma}\rho + (1+r)c^2 - (1+r)\} - \rho}{w_i(\frac{1+c}{\sigma})} = \overline{s}_i \tag{12}$$

Note that the wealth threshold $w^A$ varies with the level of the equilibrium salary, while the maximum salary that entrepreneurs are willing to pay $\overline{s}_i$ depends on the individuals' wealth.

At very low levels of wealth, although individuals may be able to set up a small business, individuals cannot afford to pay positive salaries. Both the wealth threshold $w^A$ and $s_i$ increase with respect to wealth. However, the wealth threshold $w^A$ reaches an upper limit above which further salary increases will make the returns to entrepreneurial activities negative. This can be seen in eq. (11). When the salary is higher than or equal to the expression below, the wealth threshold $w^A$ becomes negative and entrepreneurs make negative profits.

$$s \geq \rho + \frac{(1+r)(c^2-1)\sigma}{(1+c)} \tag{13}$$

Hence, there is a maximum level of salaries in the economy, above which further salary increases will make profits negative even for the wealthiest entrepreneur.

## 2.2 Labour Market

Note that every entrepreneur is willing to pay different salaries, and this depends on the scale of production $\alpha_i$ and hence on their level of wealth. The higher the scale of production, the higher the salaries individuals are willing to pay. This is because returns are proportional to the scale of production. Hence, the larger the scale, the higher the returns and the higher the salary bill that entrepreneurs can afford to pay. The equilibrium salary in period $t$ is obtained from labour demand, $D_t$ and labour supply $Z_t$.

In order to find the equilibrium salary, define $w^p$ as the wealth level of the individual that is willing to pay the salary $s_i^*$ which clears the labour market, where $s_i^* \geq \gamma$. This equilibrium salary $s_i^*$ also determines the wealth threshold $w^A$.

The demand for labour $D_t$ is given by the number of entrepreneurs that have initial wealth $w_i \geq w^A$.

$$D_t = \left\{ \begin{array}{ll} 0 & \text{if } s_t > \overline{s}_j \\ \int_{w^A}^{\overline{w}} \alpha_i(w) W_t^{'}(w) dw & \text{if } \ s_t \leq s_i^* \end{array} \right\} \tag{14}$$

where $\overline{s}_j$ indicates the maximum salary that the wealthiest entrepreneur is willing to pay.

The supply of labour $Z_t$ is given by the number of people with initial wealth $w_i < w^A$, if the salary $s_i^* \leq s < \overline{s}_j$. If the salary is $s > \overline{s}_j$ all the $N$ individuals of the economy will offer their labour.

$$Z_t = \left\{ \begin{array}{ll} 0 & \text{if } s_t < \gamma \\ \int_{\underline{w}}^{w^A} W_t^{'}(w) dw & \text{if } s_t \geq s_i^* \\ N & \text{if } s_t \geq \overline{s}_j \end{array} \right\} \tag{15}$$

Every individual with wealth $w_i > \frac{\sigma}{(1+c)}$ will be able to set up a business with a positive scale of production, independently of the salaries offered in the labour market. However, depending on the equilibrium salary, individuals will decide whether to become workers or entrepreneurs.

If at the lowest possible salary $\gamma$, there is an excess supply of labour, then all the individuals that are able to pay a salary greater than or equal to $\gamma$ will set up a business.

$$s_t = \gamma \quad if \int_{w^A}^{\overline{w}} \alpha_i(w) W_t^{'}(w) dw < \int_{\underline{w}}^{w^A} W_t^{'}(w), (w^A | s_t = \gamma) \tag{16}$$

If at the lowest possible salary $\gamma$, there is an excess demand for labour, then the salary will rise to the level that it will clear the labour market. This will be given by the salary $s_i^*$ that makes the individual with wealth

$w^p$ indifferent between becoming a worker and an entrepreneur. $s_t = s_i^* > \gamma$.

$$s_t = s_i^* \quad if \int_{w^A}^{\overline{w}} \alpha_i(w) W_t^{'}(w) dw > \int_{\underline{w}}^{w^A} W_t^{'}(w), (w^A | s_t = \gamma) \qquad (17)$$

The equilibrium salary will be equal to $\overline{s}_j$ (the salary that the wealthiest individuals in the economy are willing to pay) if the labour market clears, or if there is an excess demand for labour at $\overline{s}_j$.

**Assumption 1** *The returns of unemployed individuals $\gamma$ are strictly smaller than the maximum salary entrepreneurs are willing to offer.* In this model, then $\gamma < \overline{s}_j$, where $\overline{s}_j$ is the salary that the wealthiest individuals in the economy are willing to pay.

Note that if only the wealthiest person in the economy is willing to set up a business, the equilibrium salary will be given by the salary that the second wealthiest person in the economy would be willing to pay. This is because the labour market is competitive, and in order to clear the labour market entrepreneurs will raise salaries up to the level that leaves the person with wealth equal to $w_i^p$ indifferent between becoming an entrepreneur and a worker.

$$s_t = \overline{s}_j \quad if \int_{w^A}^{\overline{w}} \alpha_i(w) W_t^{'}(w) dw \geq \int_{\underline{w}}^{w^A} W_t^{'}(w), (w^A | s_t = \overline{s}_j) \qquad (18)$$

**Assumption 2** *If the demand for and supply of labour are equal the salaries offered will make individuals indifferent between becoming workers and entrepreneurs.*

However in this model the equilibrium salary can take any value between $[\gamma, \overline{s}_j]$ depending on the wealth distribution. The reason for this is that the demand for labour is not inelastic to salaries as in the first model. If there is excess demand for labour, the only way to clear this excess is by offering a higher salary that will make some individuals indifferent between becoming workers and entrepreneurs, such that the labour market clears. Although changes in salaries do not change the scale of production people can set up, the profits that entrepreneurs will make change. Hence, as long as the salary does not exceed the individual limit $\overline{s}_i$ people will remain in business.

## 2.3 Static equilibrium

The next three equations summarize the earnings of the unemployed, $w_u$, the workers $w_w$, and the entrepreneurs, $w_e$, given that $\gamma \leq s < \overline{s}_j$.

$$w_u = w_i(1+r) + \gamma \tag{19}$$

$$w_w = w_i(1+r) + s_t \tag{20}$$

$$w_e = w_i\mu_t - \delta_t \tag{21}$$

where $\mu_t = (\frac{1+c}{\sigma})(\rho - s_t) + (1+r)c^2$ and $\delta_t = \rho - s_t$. The term $\mu_t > 0$ and $\delta_t > 0$, since $s_t < \rho$ given that the salaries are endogenously determined to have a positive return and that $c > 0$.

**Assumption 3** *The returns of individuals as entrepreneurs are strictly higher than the returns individuals would obtain as unemployed* $\delta_t > \gamma$, $\delta_t > 0$ and $\mu_t > (1+r)$.

The equilibrium salary at time $t$ will determine the earnings of individuals in each occupation. There will be no employment if no one in the economy has wealth equal to or greater than the wealth threshold necessary to set up a business, $w_i < w^A$, not even when the salary is at the lowest level, $\gamma$.

## 2.4 Dynamics of Wealth Distribution

The dynamics of wealth distribution depend on the equilibrium salary, and this can take any value between $[\gamma, \overline{s}_j]$, which depends on the distribution of wealth.

To examine the dynamics of wealth for each dynasty, first, the fixed points are obtained and second their stability is analysed.

The future wealth of the child of an unemployed individual is,

$$w_{u,t+1} = \beta\{w_{i,t}(1+r) + \gamma\} \tag{22}$$

The future wealth of the child of a worker is,

$$w_{w,t+1} = \beta\{w_{i,t}(1+r) + s_t\} \tag{23}$$

The future wealth of the child of an entrepreneur with high earnings is,

$$w_{e,t+1} = \beta\{\alpha_{i,t}(\rho - s_t) + c(\sigma + \alpha_{i,t}\sigma)(1+r) - (1+r)L_{i,t}\} \tag{24}$$

Substituting the values of $\alpha_{i,t}$, $L_{i,t}$,

$$w_{e,t+1} = \beta\{w_t\mu_t - \delta_t\} \tag{25}$$

where $\mu_t = (\frac{1+c}{\sigma})(\rho - s_t) + (1+r)c^2$ and $\delta_t = \rho - s_t$. The term $\mu_t > 0$ and $\delta_t > 0$, since $s_t < \rho$ given that the salaries are endogenously determined to have a positive return and that $c > 0$.

The difference equations $w_{u,t+1}$, $w_{w,t+1}$,$w_{e,t+1}$ describe the evolution of wealth over time. If one thinks of the relationship between $w_t$ and $w_{t+1}$ as a function $w_{t+1} = f(w_t)$, then the wealth of today equals the wealth of tomorrow when $w_{t+1}$ and $w_t$ are equal. This common value denoted by $\widehat{w}$ is usually called a fixed point. In addition to find when wealth reaches a fixed point, it is important to analyse its stability. It is known that the conditions for the stability of the dynamics of linear recurrence relations with **constant coefficients** can be described by the following *Theorem 1.*

**Theorem 1** *If $\widehat{w}$ is a fixed point of the first order recurrence equation $w_{t+1} = f(w_t) = Rw_t + a$, then $\widehat{w}$ is a stable fixed point if $-1 < R < 1$ and an unstable fixed point if $R > 1$.*

***Proof.*** *Let $\upsilon_t$ be the difference between $w_t$ and $\widehat{w}$. Then $\upsilon_t = w_t - \widehat{w}$ and $\upsilon_{t+1} = w_{t+1} - \widehat{w} = f(w_t) - \widehat{w} = f(\widehat{w} + \upsilon_t) - \widehat{w}$.*

*By Taylor's theorem it follows that*

$\upsilon_{t+1} \approx f(\widehat{w}) + f'(\widehat{w})\upsilon_t - \widehat{w}.$

*But $\widehat{w}$ is a fixed point so $\widehat{w} = f(\widehat{w})$ and $f'(\widehat{w}) = R$. Thus $\upsilon_{t+1} \approx R\upsilon_t$.*

*Since $R$ is a constant, the error $\upsilon_t$ decays to zero if $-1 < R < 1$. However if $R > 1$ the error $\upsilon_t$ continuously increases.* ■

Nevertheless, Theorem 1 cannot be applied directly to analyse the dynamics of wealth distribution in the difference equations. Note that the coefficients associated to how wealth changes over time, i.e. $(w_{i,t+1} - w_{i,t})$, are not constant. The difference equations for the workers and entrepreneurs depend on the salaries, which depend on the distribution of wealth. Only the difference equation for the unemployed depends on a constant coefficient $\gamma$. However, the existence of unemployment itself depends on the proportion of people that can set up business and this also depends on the distribution of wealth.

Ignoring for a moment how the proportion of people in unemployment changes and how salaries change over time, the dynamics of wealth for the three types of occupation can be analysed if the fixed points for each occupation are obtained and further assumptions are introduced. Aplying Theorem 1 the fixed points for each of the three ocupations are given by $\widehat{w}_u$ for the unemployed, $\widehat{w}_w$ for workers and $\widehat{w}_e$ for entrepreneurs.

$$\widehat{w}_u = \frac{\beta\gamma}{1 - \beta(1+r)} \tag{26}$$

$$\widehat{w}_w = \frac{\beta s}{1 - \beta(1+r)} \tag{27}$$

$$\widehat{w}_e = \frac{-\beta\delta_t}{1 - \beta\mu_t} \tag{28}$$

The stability of these fixed points can be analysed if further assumptions are made.

**Assumption 4** *A dynasty cannot become rich over time just by saving a fraction of its wealth $\beta(1+r)w_{i,t}$. Therefore it will be assumed that $\beta(1+r) < 1$.*

**Assumption 5** *A dynasty of unemployed people cannot accumulate wealth over time sufficient to set up business. Therefore, it is assumed that $\frac{\beta\gamma}{1-\beta(1+r)} < w^A$.*

**Assumption 6** *The fixed point for workers' earnings lies below the wealth threshold to set up a business* $w^E > \widehat{w}_w$, *if salaries remain low over time* $s_t = \gamma$.

**Assumption 7** *The fixed point for entrepreneurs' earnings is equal to or greater than the wealth threshold to set up business,* $w^A \leq \widehat{w}_e$.

Thefixed point for entrepreneurs can be negative for certain parameter values. To guarantee a positive wealth level for entrepreneurs the following proposition is made.

**Proposition 1.1** *The long run earnings of workers and the unemployed are positive and stable if* $\beta(1+r) < 1$.The long run earnings of the entrepreneurs are not guaranteed to be positive and stable if $\beta(1+r) < 1$.

**Proposition 1.2** *The long run earnings of entrepreneurs have a positive fixed point if* $\beta\mu_t > 1$.

This is because the long run earnings of entrepreneurs are given by eq.(9). The numerator of this equation is negative since $-\beta\delta_t < 0$ , $\beta > 0$ and $\delta_t > 0$. To secure a positive fixed point the denominator must also be negative, which implies $\beta\mu_t > 1$.

If $\beta(1 + r) < 1$ and $\beta\mu_t > 1$ and assuming that salaries are constant over time at the lowest possible value, $\gamma$, the dynamics of wealth will be as depicted in figure (1).

Individuals with wealth below $w^A$ will become workers or unemployed, while individuals with wealth above $w^A$ will become entrepreneurs. Entrepreneurs with wealth below $\widehat{w}_e$ will "decapitalise" over time even if the salaries are kept constant at $\gamma$. For entrepreneurs with wealth above $\widehat{w}_e$ their wealth will expand towards $\infty$. Nonetheless, if some entrepreneurs keep expanding their firms, and others keep dropping out of business it is unlikely that salaries will remain constant.

In the case where there is a larger proportion of entrepreneurs with expanding rather than contracting wealth, salaries will increase. Figure (2) shows how an increase in salaries changes the dynamics of wealth for individuals in each occupation. The fixed wealth point for workers will increase as a result of having higher salaries. In other words the line that denotes the dynamics of wealth for workers will shift upwards from line $WB$ towards the line $WB'$. Since labour is now more expensive this will

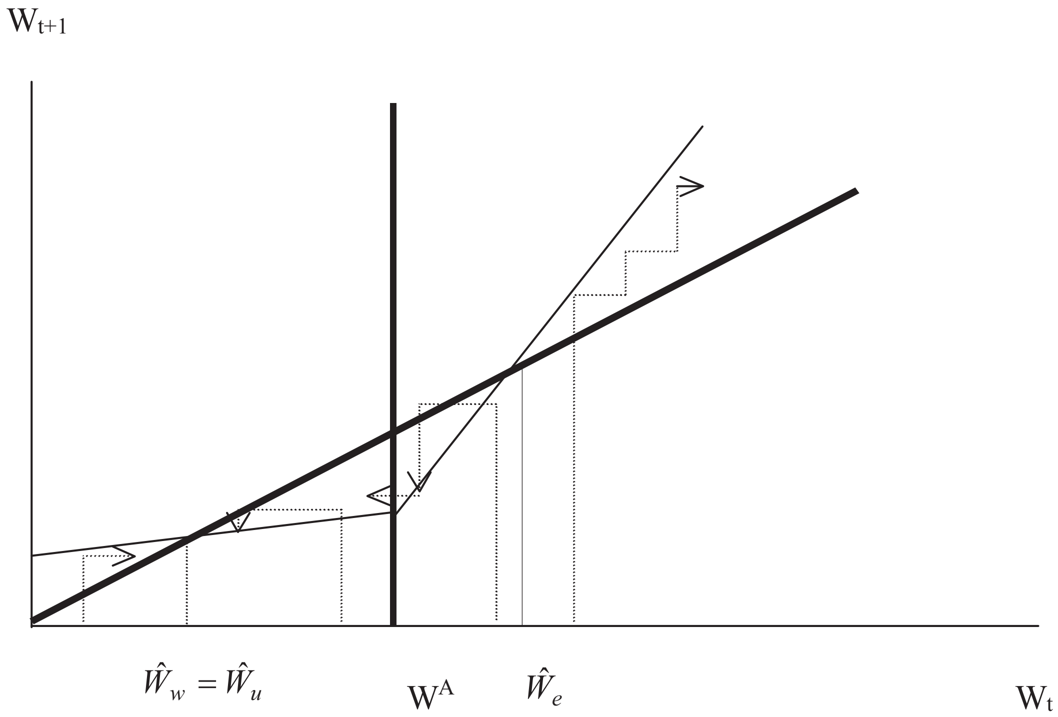


Figure 1: Dynamics of Wealth Distribution (having $s_t = \gamma$ constant)

reduce the rate of accumulation of wealth by entrepreneurs. As a result of the increase in labour payments the line representing the dynamics of wealth for entrepreneurs shifts from $BA$ rightwards towards $B'A'$. The wealth threshold for becoming an entrepreneur will move from $W^A$ to $W^{A}{\prime}$ and only the wealthiest entrepreneurs will remain in business, since they are the only ones that can afford to continue expanding their scale of production after the increase in salaries.

Therefore, the dynamics of wealth will not follow a linear path, but the one presented in figure (3). The wealth of some entrepreneurs will decrease towards $-\infty$. The scale of production of these entrepreneurs' firms is too small to keep financing the cost of replacing the depreciated machinery. Therefore at some point they will be forced to shut down

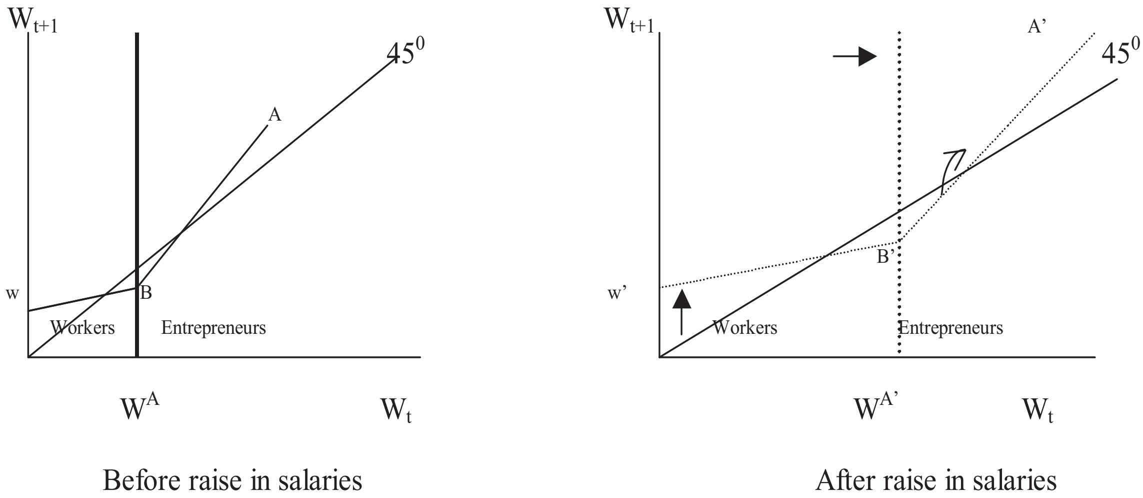


Figure 2: Dynamics of Wealth Distribution when Salaries Increase

their business and become either workers or unemployed. In contrast, the wealth of some entrepreneurs will continuously expand towards $\infty$. However, the growth of their wealth will be constrained by the labour capacity of the economy. Hence, their wealth will tend towards an upper threshold denoted by the dot in the curve that intersects the $45^o$ line.

Since there is an upper limit to wealth expansion, the wealth threshold $w^A$ also tends to stabilize at an upper level defined by $\overline{w}^A$.

Although the wealth of some people keeps increasing, it is not certain what happens to the dynamics of salaries. This depends on the distribution of wealth, how wealthy are the richest in the economy and the inequality among entrepreneurs.

For instance, if at the initial distribution there are already very wealthy entrepreneurs hiring almost the entire population, they will not expand their scale of production much further, nor will they raise salaries significantly. If there is no inequality among the wealthiest entrepreneurs all of them will remain in business since their willingness to pay higher salaries is the same. Nonetheless, if among the entrepreneurs there is one that is much richer, he could raise salaries to keep potential entre-

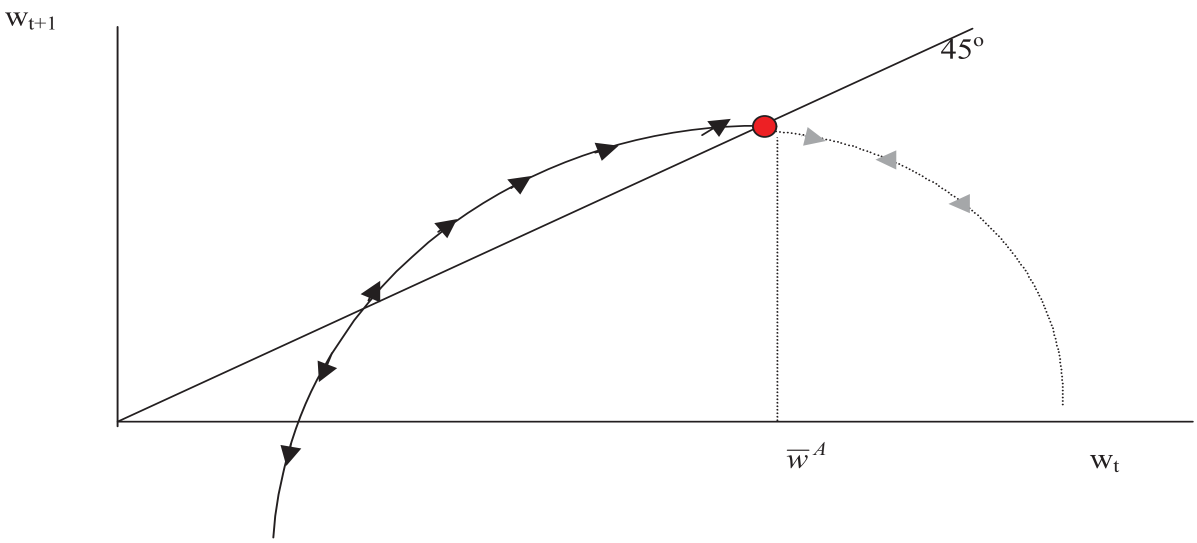


Figure 3: Long Run Wealth Threshold $w^A$ and $\overline{w}^A$

preneurs out of business, in order to secure the expansion of his scale of production.

**Lemma 3** *The equilibrium salary can be characterised by either of the following 4 types of dynamics:*

*Type 1: Salaries remain constant over time, at level $\gamma$. This will happen if at the initial distribution of wealth the labour market clears at the salary $\gamma$, either because there is only one entrepreneur hiring all the population, or because there are various entrepreneurs that hire the entire population and no entrepreneur can expand further his scale of production by increasing salaries. Under these two circumstances salaries will not increase over time, given that the demand for labour cannot expand any further.*

*Type 2: Salaries present fluctuation and then stabilize at any point in $[\gamma, \overline{s}_j]$. This will happen if at any equilibrium salary there is a proportion of people that drops out of business next period, while others keep expanding their wealth. The changes in salaries therefore are either positive or negative depending on whether the proportion that is expanding* *business is greater than, equal to or smaller than the proportion of people dropping out of business. In the long run, when the remaining entrepreneurs reach full employment, the labour demand will remain constant at a point in* $[\gamma, \overline{s}_j]$.

*Type 3: Salaries increase over time to then stabilize at any point in* $(\gamma, \overline{s}_j]$. *This will happen if the proportion of entrepreneurs expanding business is always greater than the proportion of people dropping out of business. In the long run, when entrepreneurs reach full employment, the labour demand will remain stable at a point in* $(\gamma, \overline{s}_j]$.

*Type 4: Salaries increase to reach a peak and then decrease over time to* $\gamma$. *This will happen if there are various entrepreneurs expanding business over time, but there is only one entrepreneur that keeps increasing salaries such that he hires all the population. Notice that although the richest entrepreneur behaves competitively, the equilibrium salaries are equal to the highest salary that the second wealthiest person would be willing to pay. Since the second wealthiest person is a worker, his wealth is declining over time to the fixed point* $\widehat{w}_w$. *Therefore, the salary that the second wealthiest individual can pay keeps falling over time, and so do the equilibrium salaries over time.*

*The aggregate wealth level and the distribution of wealth in the long run depend on the dynamics of salaries. The type of long run wealth distribution that can be achieved is described in proposition 2.*

**Proposition 2** *Given any initial wealth distribution, there exists a unique stationary wealth distribution to which it converges.*

**Proposition 3** *The initial distribution of wealth converges to either of three types of stationary distributions.*

*Stationary Distribution Type 1: The long run distribution converges to two fixed and stable points. For individuals with initial wealth below the wealth threshold required for setting up business at the equilibrium salary* $w_{i,t} < \overline{w}^A$ *the fixed points converge to* $\widehat{w}_u = \widehat{w}_w = \frac{\beta s}{1-\beta(1+r)}$. *For individuals with initial wealth equal to or greater than the wealth threshold to set up business* $w_{i,t} \geq \overline{w}^A$ *their wealth converges to* $\widehat{w}_e = \frac{\beta \psi_t}{1-\beta(1+r)}$, at which there is full employment and the scales of production cannot be expanded further even if salaries increase.

*Stationary Distribution Type 2: The long run distribution converges to one fixed and stable point* $\widehat{w} = \frac{\beta \overline{s}_j}{1-\beta(1+r)}$, *where both entrepreneurs and workers get the same wealth. In this case there will be full employment and the equilibrium salary will equalise the returns of entrepreneurs and workers.*

*Stationary Distribution Type 3: The long run distribution converges to one fixed and stable point* $\widehat{w}_u = \frac{\beta\gamma}{1-\beta(1+r)}$ $\forall i$. *This will happen if the initial wealth of all individuals is below the wealth threshold necessary for setting up business* $w_{i,t} < \overline{w}^A$ *at the lowest possible salary* $\gamma$. *Then the only option available is unemployment. Alternatively, this stationary distribution can also be reached, if at the lowest possible salary* $\gamma$, *the scale of production of all the firms is too small to keep financing the cost of replacing machinery. Therefore, in the long run all firms will shut down and unemployment will be the only option available. This will happen when the wealth of all individuals is below the fixed point* $\widehat{w}_e$ *at* $s_t = \gamma$.

In the stationary distribution type 1 the returns of entrepreneurs will be different to the returns of workers. In this case, the aggregate wealth in the long run will be low and denoted by $\overline{W}_L$. It will be given by the sum of the wealth of workers and the unemployed, a $W(\overline{w}^A)$ proportion of the population and by the wealth of entrepreneurs, a proportion of the population $\int_{\overline{w}^A}^{\overline{w}} W_t'(w)dw$.

$$\overline{W}_L = \widehat{w}_w \int_{\underline{w}}^{\overline{w}^A} W_t'(w)dw + \widehat{w}_e \int_{\overline{w}^A}^{\overline{w}} W_t'(w)dw \tag{29}$$

The long run aggregate wealth achieved by the stationary distribution type 2, is denoted by $\overline{W}_H$. It is achieved when there is an excess demand for labour in the long run, or when the labour market clears at $\overline{s}_j$. In this case, all the population, $N$, enjoys the same level of wealth.

$$\overline{W}_H = \frac{\beta \overline{s}_j}{1-\beta(1+r)} N \tag{30}$$

The economy will achieve the stationary distribution type 3 $\overline{W}_P$, when no one in the economy can afford to set up a business. In this situation, not even the wealthiest person in the economy can set up a business.

$$\overline{W}_P = \frac{\beta\gamma}{1-\beta(1+r)}N \tag{31}$$

In the stationary distribution types 2 and 3 there is no inequality in the long run.

Comparing the long run wealth levels of $\overline{W}_L$,$\overline{W}_H$ and $\overline{W}_P$ it follows that the larger is the proportion of wealthy people, understood as those who can set up a business or remain in business, the higher the salaries and the higher the long run wealth level will be. This is due to by the fact that the wealthiest entrepreneurs will be able to expand their scales of production and raise salaries at the same time. Salaries will increase faster, the less poor the non-entrepreneurs are. This is because the reservation salary is not given only by $\gamma$, but actually by the maximum salary that every individual is willing to pay, $\overline{s}_i$. The higher this is the higher the equilibrium salary will be.

## 2.5 Discussion

The distribution of wealth can converge to three types of "family" distributions, which depend on the initial overall wealth distribution. An egalitarian long run distribution with high levels of wealth will be achieved if many entrepreneurs can keep expanding their firms, such that the equilibrium salary equates the returns of workers and entrepreneurs. An egalitarian long run distribution, but with low levels of wealth, will be achieved if either at the starting point there is no one in the economy that can set up a business, or if all the entrepreneurs in the economy are unable to stay in business even if the salaries remain at the lowest possible level and constant. A less egalitarian distribution will be achieved if entrepreneurs expand their scales of production, hiring all the population but the equilibrium salary does not equate the returns of entrepreneurs and workers. The wider the initial wealth differences between workers and entrepreneurs, the more likely that the salaries will remain low over time.

A one-shot Pigou-Dalton wealth transfer will not permanently increase the wealth level, even if this transfer enabled some people to set up business. The reason for this is that even if salaries remain constant over time, for small entrepreneurs wealth decreases over time, forcing them to shutdown their firm. This is because their scale of production is too small to keep financing the cost of replacing machinery. Even if some workers receive transfers to enable them to set up a business, their dynasties might not afford to keep financing the required investment in machinery. Given that some wealthier entrepreneurs keep expanding their scales of production salaries will rise and hence so will the salary cost for small firms. Therefore wealth transfers will increase the long run wealth only if the transfers are large enough to enable small entrepreneurs to keep increasing their scale of production, despite the increase in salaries. The model points to the relevance of the initial wealth distribution and credit constraints in the analysis of growth and inequality.

## 3 Concluding Remarks

The article presented a model that assessws how credit market imperfections and the overall business-environment affect the dynamics of wealth.

The model allowed entrepreneurs to change their scales of production over time and the equilibria depend not just on the proportion of credit to non-credit constrained individuals, but also on the wealth distribution within these two groups. When there is a large difference in wealth between the rich and the middle-scaled entrepreneurs, the large entrepreneurs can drive out of the market the middle- and smaller-scaled entrepreneurs by increasing salaries in the short run. In the long run, once the middle- and smaller-scaled entrepreneurs are out of the market the large entrepreneurs can end up paying low salaries, given the reduced level of competition in the market. Previous models have not established such a link between the distribution of wealth within the credit and non-credit constrained groups and the long run distribution of wealth.

In the model presented the long run distribution of wealth can converge to a wide range of distributions over time. These depend on the wealth distribution of the credit and non-credit constrained individuals. The larger and the poorer the bottom of the distribution is, the lower the equilibrium salary will be. The wealthier and the more egalitarian the top of the distribution is the higher the equilibrium salary will be.

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