Complementarity with incomplete preferences*

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Abstract

This paper extends the formulation of complementarity in Milgrom and Shannon (1994) to the case of incomplete but acyclic preferences. It is shown that the problem can be reformulated as one with complete but intransitive preferences. In this case, quasi-supermodularity and single-crossing on their own do not guarantee either monotone comparative statics or equilibrium existence in pure strategies: an additional condition, monotone closure, is required. The results obtained here relax the requirement of convexity in Shafer and Sonnenschein (1975)'s existence result with incomplete preferences. In an application, it is shown that pure strategy equilibria exist in incomplete information games with Knightian uncertainty.

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1 Introduction

This paper extends the formulation of complementarity in Milgrom and Shannon (1994) to the case of incomplete[1] but acyclic preferences. It is shown that the problem can be reformulated as one with complete but intransitive preferences. However, quasi-supermodularity and single-crossing on their own no longer suffice to guarantee either monotone comparative statics or existence: an additional condition, monotone closure, is required. Taken together, quasi-supermodularity, single-crossing and monotone closure guarantee both monotone comparative statics and equilibrium existence (via Tarski's (1955) theorem) thus extending the existence results contained in Vives (1990), Milgrom and Shannon (1994), Topkis (1998).

The results obtained here relax the requirement of convexity[2] in Shafer and Sonnenschein (1975)'s existence result.

As an application, it is shown that pure strategy equilibria exist in incomplete information games with Knightian uncertainty where the formulation of Knightian uncertainty used is follows Bewley (2002)[3].

In what follows, section 2 studies monotone comparative statics and existence while section 3 applies the results obtained in section 2 to incomplete information games with Knightian uncertainty.

2 Comparative statics and existence with intransitive preferences

In this section, I study comparative statics and existence of pure strategy equilibria with intransitive preferences.

2.1 A decision problem with intransitive preferences

To begin with, consider a single decision maker who has to choose an action from a set $A \subset \Re^k$. The preferences of the decision-maker is described

[1]There is a large and growing literature on games and markets where agents have incomplete preferences. A selective list of references (not referred to elsewhere in this paper) includes Shapley (1959), Aumann (1962), Gale and MasCollel (1975) and Shafer and Sonnenschein (1975). Incomplete preferences arise in an essential way in models of coalition (and party) formation as in Ray and Vohra (1997), Roemer (1999) and Levy (2004).

[2]The convexity assumption used by Shafer and Sonnenschein (1975) rules out scenarios with indivisibilities, with increasing returns, with the fundamental non-convexity of feasible sets in the presence of externalities identified by Starret (1972) and the non-convexity of reference-dependent preferences with loss aversion studied by Khaneman and Tversky (1979).

[3]Current applications of Knightian uncertainty (Knight (1921)) such as Rigotti and Shannon (2005) and Lopomo, Rigotti and Shannon (2006) all use Bewley's approach to Knightian uncertainty.

by a map, $\succ: P \rightarrow A \times A$, where P (indexed by p) is a set of preference parameters, $P \subset \Re^n$, and for each $p \in P$, $\succ_p$ describes a preference relation over A. The expression $(a, a') \in \succ_p$ is written as $a \succ_p a'$ and is to be read as "a is preferred to a' by the decision-maker when the utility parameter is p". Define the sets $\succ_p (a) = \{a' \in A : a' \succ_p a\}$ (the upper section of $\succ_p$), $\succ_p^{-1} (a) = \{a' \in A : a \succ_p a'\}$ (the lower section of $\succ_p$). It is assumed that for each $p \in P$, (i) $\succ_p$ is acyclic i.e. there is no finite set $\{a^1, ..., a^T\}$ such that $a^{t-1} \succ_p a^t$, $t = 2, ..., T$, and $a^T \succ_p a^1$, and (ii) $\succ_p^{-1} (a)$ is open relative to A i.e. $\succ_p$ has an open lower section[4]. Write $a' \notin \succ_p (a)$ as $a \not\succ_p a'$.

Define a map $\Psi : P \rightarrow A$, where $\Psi(p) = \{a' \in A : \succ_p (a') = \emptyset\}$: for each $p \in P$, $\Psi(p)$ is the set of maximal elements of the preference relation $\succ_p$.

Consider the following extension of $\succ$: for each p, define $\succeq_p$ on A by

$$a \succeq_p a' \Leftrightarrow a' \not\succ_p a.$$

As for each p, $\succ_p$ is acyclic and therefore irreflexive, it follows that $\succeq_p$ is complete.

Let $\hat{\succ}_p$denote the strict preference relation corresponding to $\succeq_p$ i.e. $a \hat{\succ}_p a'$ if and only if $a \succeq_p a'$ but $a' \not\succeq_p a$. For each $p \in P$ and $a, a' \in A$, $a \succ_p a'$ if and only if $a \hat{\succ}_p a'$.

Define a map $\hat{\Psi} : P \rightarrow A$, where $\hat{\Psi}(p) = \{a' \in A : \hat{\succ}_p (a') = \emptyset\}$: for each $p \in P$, $\hat{\Psi}(p)$ is the set of maximal elements of the preference relation $\hat{\succ}_p$.

The following lemma establishes that for each p, $\succ_p$ and $\hat{\succ}_p$ are equivalent and have the same set of maximal elements:

Lemma 1: For each $p \in P$ and $a, a' \in A$, $a \succ_p a'$ if and only if $a \hat{\succ}_p a'$ and therefore, $\Psi(p) = \hat{\Psi}(p)$.

Proof. Fix $p \in P$. Consider a pair $a, a' \in A$ such that $a \succ_p a'$. Then, $a' \not\succ_p a$ and therefore, $a \succeq_p a'$ and as $\succ_p$ is acyclic, $a' \not\succeq_p a$. It follows that $a \hat{\succ}_p a'$. Next, consider a pair $a, a' \in A$ such that $a \hat{\succ}_p a'$. Then, $a \succeq_p a'$ and $a' \not\succeq_p a$. Therefore, $a' \not\succ_p a$ and $a \succ_p a'$. ■

With this result in place the decision problem with incomplete but acyclic preferences is rephrased as a decision problem with complete and acyclic but not necessarily transitive preferences.

Consider the following assumptions:

(A1) A is a compact lattice with the vector ordering[5];

[4]The continuity assumption, that $\succ_p$ has an open lower section, is weaker than the continuity assumption made by Debreu (1959) (who requires that preferences have both open upper and lower sections), which in turn is weaker than the assumption by Shafer and Sonnenschein (1975) (who assume that preferences have open graphs). Note that assuming $\succ_p$ has an open lower section is consistent with $\succ_p$ being a lexicographic preference ordering over A.

[5]A lattice is a partially ordered subset of $\Re^k$ with the vector ordering (the usual component wise ordering: $x \geq y$ if and only if $x_i \geq y_i$ for each $i = 1, .., K$, and $x > y$ if and only if both $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for each $i = 1, .., K$). A lattice that is compact (in the usual topology) is a compact lattice.

(A2) For each p, and a, a', (i) if $a \succeq_p \inf(a, a')$, then $\sup(a, a') \succeq_p a'$ (ii) if $a \succeq_p \sup(a, a')$ then $\inf(a, a') \succeq_p a'$ (quasi-supermodularity);

(A3) For each $a \geq a'$ and $p \geq p'$, (i) if $a \succeq_{p'} a'$, then $a \succeq_p a'$ and (ii) if $a' \succeq_p a$ then $a' \succeq_{p'} a$ (single-crossing property);

(A4) For each p and $a \geq a'$, (i) if $\succ_p (a') = \emptyset$ and $a \succeq_p a'$, then $\succ_p (a) = \emptyset$, and (ii) $\succ_p (a) = \emptyset$ and $a' \succeq_p a$, then $\succ_p (a') = \emptyset$,(monotone closure).

Assumptions (A2)-(A3) are quasi-supermodularity and single-crossing property defined by Milgrom and Shannon (1994). Assumption (A4) is new. It requires that for each p, in any mutually unranked pair of vector ordered actions, either both actions are maximal elements of $\succ_p$or neither action is.

The role played by assumption (A4) in obtaining the monotone comparative statics with incomplete preferences is clarified by the following example.

Example: P is single valued and A is the four point lattice in $\Re^2$

$$\{(e,e), (f,e), (e,f), (f,f)\}$$

where $f > e$. Suppose that $(f,f) \succ (e,e)$ but no other pair is ranked. Then, Ψ consists of $\{(f,e), (e,f), (f,f)\}$ clearly not a lattice. Note that in this case, preferences satisfy acyclicity and quasi-supermodularity (and trivially, single-crossing property). However, preferences do not satisfy monotone closure: $(f,e) \geq (e,e)$, with $\succ ((f,e)) = \emptyset$ and $(e,e) \succeq (f,e)$, but $\succ ((e,e)) \neq \emptyset$.

The preceding example demonstrates that with intransitive preferences, quasi-supermodularity on its own, is not sufficient to ensure that the set of maximal elements of $\succ$ is a sublattice of A even when $\succ$ is acyclic. The example also demonstrates that $\succ$ can be acyclic without necessarily satisfying monotone closure and therefore, the two are distinct conditions on preferences.

The following result shows that assumptions (A1)-(A4), taken together, are sufficient to ensure monotone comparative statics with incomplete preferences:

Proposition 1: Under assumptions (A1)-(A4), each $p \in P$, $\Psi(p)$ is non-empty and a compact sublattice of A where both the maximal and minimal elements, denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively, are increasing functions on P.

Proof. By assumption, for each p, $\succ_p$ is acyclic, $\succ_p^{-1} (a)$ are open relative to A and A is compact. By Bergstrom (1975), it follows that $\Psi(p)$ is non-empty. As Bergstrom (1975) doesn't contain an explicit proof that $\Psi(p)$ is compact, a proof of this claim follows next. To this end, note that the complement of the set $\Psi(p)$ in A is the set $\Psi^c(p) = \{a' \in A : \succ_p (a') \neq \emptyset\}$. If $\Psi^c(p) = \emptyset$, then $\Psi(p) = A$ is necessarily compact. So suppose $\Psi^c(p) \neq \emptyset$. For each $a' \in \Psi^c(p)$, there is $a'' \in A$ such that $a'' \succ_p a'$. By assumption, $\succ_p^{-1} (a'')$ is open relative to A. By definition of $\Psi(p)$, $\succ_p^{-1} (a'') \subset \Psi^c(p)$. Therefore, $\succ_p^{-1} (a'')$ is a non-empty neighborhood of $a' \in \Psi^c(p)$ and it is clear that $\Psi^c(p)$ is open and therefore, $\Psi(p)$ is closed. As A is compact, $\Psi(p)$ is also compact. Next, I show that for $p \geq p'$ if $a \in \Psi(p)$ and $a' \in \Psi(p')$, then $\sup(a, a') \in \Psi(p)$ and $\inf(a, a') \in \Psi(p')$. Note that as $a' \in \Psi(p')$, $a' \succeq_{p'} \inf(a, a')$. By quasi-supermodularity, $\sup(a, a') \succeq_{p'} a$. By single-crossing, $\sup(a, a') \succeq_p a$. As $a \in \Psi(p)$, $\succ_p (a) = \emptyset$ and therefore, by monotone closure, as $\sup(a, a') \succeq_p a$, $\succ_p (\sup(a, a')) = \emptyset$ and $\sup(a, a') \in \Psi(p)$. Next, note that as $a \in \Psi(p)$, $a \succeq_p \sup(a, a')$. By single-crossing, $a \succeq_{p'} \sup(a, a')$ and by quasi-supermodularity, $\inf(a, a') \succeq_{p'} a'$. As $a' \in \Psi(p')$, $\succ_{p'} (a') = \emptyset$, and therefore, by monotone closure, as $\inf(a, a') \succeq_{p'} a'$, $\succ_{p'} (\inf(a, a')) = \emptyset$ and $\inf(a, a') \in \Psi(p')$. Therefore, (i) $\Psi(p)$ is ordered, (ii) $\Psi(p)$ is a compact sublattice of A and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$, and (iii) both $\bar{a}(p)$ and $\underline{a}(p)$ are increasing functions from P to A. ■

2.2 Normal-form games with incomplete preferences

The set up is as follows. There is a set of I players (indexed by i) and for each player i, a pure action set A_i (indexed by a_i) where $A_i \subset \Re^K$, a finite dimensional Euclidian space. Let $A = \Pi_{i \in I} A_i$ and $A_{-i} = \Pi_{j \neq i} A_j$.

Each player i is endowed with a preference relation $\succ_i : A_{-i} \to A_i \times A_i$. The expression $(a_i, a_i') \in \succ_{i,a_{-i}}$ is written as $a_i \succ_{i,a_{-i}} a_i'$ and is to be read as "a_i is preferred to a_i' by the decision-maker when the action profile chosen by other players is a_{-i}". Let $\succeq_{i,a_{-i}}$ (respectively, $\sim_{i,a_{-i}}$) denote the weak preference relation (respectively, indifference preference relation) associated with $\succ_{i,a_{-i}}$.

A pure strategy equilibrium is a profile of actions a^* such that for each $i \in I$, given a^*_{-i}, $\succ_{i,a^*_{-i}} (a^*_i) = \emptyset$.

For each $i \in I$, assume that A_i is a compact lattice and $\succ_{i,a_{-i}}$and $\succeq_{i,a_{-i}}$satisfies assumption (A2)-(A4) made in the preceding subsection.

Proposition 2: Under assumptions (A1)-(A4), a pure strategy equilibrium exists.

Proof. Define a map $\Psi : A \to A$, $\Psi(a) = (\Psi_i(a_{-i}) : i \in I)$ as follows: for each a, $\Psi_i(a_{-i}) = \left\{a_i' \in A_i : \succ_{i,a_{-i}} (a_i') = \emptyset\right\}$. By proposition 1, for each $i \in I$ and $a_{-i} \in A_{-i}$, $\Psi_i(a_{-i})$ is non-empty and compact and for $a_{-i} \geq a'_{-i}$ if $a_i \in \Psi_i(a_{-i})$ and $a_i' \in \Psi_1(a'_{-i})$, then $\sup(a_i, a_i') \in \Psi_i(a_{-i})$ and $\inf(a_i, a_i') \in \Psi_1(a'_{-i})$. Therefore, the map $\bar{a}(a) = (\bar{a}_i(a_{-i}) : i \in I)$ is an increasing function from A to itself and as A is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $\bar{a} = \bar{a}(\bar{a})$ is a fix-point of Ψ. By a symmetric argument, $\underline{a}(a)$ is an increasing function from A to itself and $\underline{a} = \underline{a}(\underline{a})$ is also a fix-point of Ψ. Moreover, $\bar{a}$ and $\underline{a}$ are respectively the largest and smallest fix-points of Ψ. ■

Remarks:

1. A different approach to the existence of equilibrium would be to deduce the existence result for games with incomplete preferences from the

standard existence result for games with complete preferences in special cases as in Bade (2005).

2. Schofield (1984) shows that if action sets are convex or are smooth manifolds with a special topological property, the (global) convexity assumption made by Shafer and Sonnenschein (1975) can be replaced by a "local" convexity restriction, which, in turn, is equivalent to a local version of acyclicity (and which guarantees the existence of a maximal element). However, here, as action sets are not necessarily convex and are allowed to be a collection of discrete points, Schofield's equivalence result does not apply.

3 Existence in a game with Knightian uncertainty

In this section, the pure strategy equilibria are shown to exist in a incomplete information game with Kinghtian uncertainty.

As before, there is a set of I players (indexed by i) and for each player the finite pure action set A_i (indexed by a_i). Let $A = \Pi_{i \in I} A_i$ and $A_{-i} = \Pi_{j \neq i} A_j$. There is a finite set of types for each player T_i (indexed by t_i) with $T = \Pi_{i \in I} T_i$ (indexed by t) denoting the set of type profiles and $T_{-i} = \Pi_{j \neq i} T_j$ (indexed by t_{-i}) the set of types of all other players excluding player i.

The type of a player describes her private information and thus, associated with each i and $t_i \in T_i$ is a closed and convex set of probability distributions $\Delta_{i,t_i} \subset \Delta(T_{-i})$ (indexed by π_{i,t_i}) over the type profile of other players. A strategy is a map $\sigma_i : T_i \to A_i$ (with Σ_i the corresponding set). Let $\sigma = (\sigma_i : i \in I)$ (with Σ the corresponding set) denote a profile of strategies for all players and $\sigma_{-i} = (\sigma_j : j \neq i)$ (with Σ_{-i} the corresponding set) denote the strategy profile for all players excluding player i.

For each σ_{-i},i,t_i, it is possible to directly specify some incomplete preference $\succ_{i,t_i,\sigma_{-i}}$ ranking pairs of actions in A_i. Instead, in what follows, I apply Bewley (2002)'s approach to Knightian uncertainty (Knight (1921)). Each player has a utility function $u_i : A \times T \to \Re$. For each σ_{-i}, i, t_i, $a_i \in A_i$ and $\pi_{i,t_i} \in \Delta_{i,t_i}(T_{-i})$, let

$$v_i \left(a_i, \sigma_{-i}, t_{-i}, \pi_{i,t_i} | t_i\right) = \sum_{t_{-i} \in T_{-i}} \pi_{i,t_i} \left(t_{-i}\right) u_i \left(a_i, \sigma_{-i} \left(t_{-i}\right), t_i, t_{-i}\right)$$

For each σ_{-i},i,t_i and two actions $a_i, a_i' \in A_i$,

$$\begin{array}{rcl} a_i & \succ & {}_{i,t_i,\sigma_{-i}} a_i' \Leftrightarrow \\ v_i \left(a_i, \sigma_{-i}, t, \pi_{i,t_i} | t_i\right) & > & v_i \left(a_i, \sigma_{-i}, t, \pi_{i,t_i} | t_i\right) \end{array}$$

for all $\pi_{i,t_i} \in \Delta_{i,t_i}(T_{-i})$. In the standard Bayesian set-up, $\Delta_{i,t_i}(T_{-i})$ is a singleton for each i, t_i and corresponds to the case when there is no Knightian uncertainty. In general, the preference relation $\succ_{i,t_i,\sigma_{-i}}$, defined over $A_i \times A_i$

is incomplete. Following Bewley, the preference relation $\succ_{i,t_i,\sigma_{-i}}$ is complete if $cl.\left\{a, a' \in A : either\ a \succ_{i,t_i,\sigma_{-i}} a'\ or\ a' \succ_{i,t_i,\sigma_{-i}} a\right\} = A$.

A pure-strategy equilibrium with uncertainty is a σ^* such that $\succ_{i,t_i,\sigma^*_{-i}} (\sigma^*_i(t_i)) = \emptyset$ for all $t_i \in T_i$ and $i \in I$.

As $\succ_{i,t_i,\sigma_{-i}}$ is generated by the utility function $u_i : A \times T \to \Re$, it is straightforward to check that $\succ_{i,t_i,\sigma_{-i}}$ is transitive and as long as u_i is continuous, both the upper and lower sections of $\succ_{i,t_i,\sigma_{-i}}$ are open. As before, consider the following complete preferences extension of $\succ_{i,t_i,\sigma_{-i}}$: for each t_i, σ_{-i}, define $\tilde{\succeq}_{i,t_i,\sigma_{-i}}$ on A by

$$a \tilde{\succeq}_{i,t_i,\sigma_{-i}} a' \Leftrightarrow a' \not\succ_{i,t_i,\sigma_{-i}p}\ a.$$

Suppose the following assumptions hold:

(B2) For each $t \in T$, $i \in I$, $a_{-i} \in A_{-i}$ and $a_i, a'_i \in A_i$,

(i) if $u_i(\inf(a_i, a'_i), a_{-i}, t) \leq u_i(a_i, a_{-i}, t)$, then $u_i(a'_i, a_{-i}, t) \leq u_i(\sup(a_i, a'_i), a_{-i}, t)$,

(ii) if $u_i(\sup(a_i, a'_i), a_{-i}, t) \leq u_i(a_i, a_{-i}, t)$, then $u_i(a'_i, a_{-i}, t) \leq u_i(\inf(a_i, a'_i), a_{-i}, t)$ (quasi-supermodularity);

(B3) For each $t \in T$, $i \in I$, $a_i \geq a'_i$ and $a_{-i} \geq a'_{-i}$,

(i) if $u_i\left(a'_i, a'_{-i}, t\right) \leq u_i\left(a_i, a'_{-i}, t\right)$, then $u_i(a'_i, a_{-i}, t) \leq u_i(a_i, a_{-i}, t)$, and

(ii) if $u_i(a_i, a_{-i}, t) \leq u_i(a'_i, a_{-i}, t)$, then $u_i\left(a_i, a'_{-i}, t\right) \leq u_i\left(a'_i, a'_{-i}, t\right)$ (single-crossing).

Assumptions (B2) and (B3) ensure that for each type profile, the utility function for each individual satisfies quasi-supermodularity and single-crossing over action profiles as in Milgrom and Shannon (1994).

Proposition 4: Assume that for each $i \in I$, A_i is a compact lattice and for each $t \in T$, $i \in I$, $u_i(a_i, a_{-i}, t)$ satisfies (B2) and (B3). Then, a pure strategy equilibrium exists.

Proof. As both quasi-supermodularity and single-crossing are preserved under summation, it follows that for each $\pi_{i,t_i} \in \Delta(T_{-i})$, $v_i(a_i, \sigma_{-i}, t, \pi_{i,t_i}|t_i)$ satisfies the assumptions of quasi-supermodularity and single-crossing and therefore, $\tilde{\succeq}_{i,t_i,\sigma_{-i}}$ satisfies quasi-supermodularity and single-crossing as well. Finally, that $\succ_{i,t_i,\sigma_{-i}}$satisfies monotone closure follows from the fact that for each i, σ_{-i}, t, if $\hat{a}_i \in \arg\max_{a_i \in A_i} v_i(a_i, \sigma_{-i}, t, \pi_{i,t_i}|t_i)$ and $v_i(a'_i, \sigma_{-i}, t, \pi_{i,t_i}) \geq v_i(\hat{a}_i, \sigma_{-i}, t, \pi_{i,t_i}|t_i)$, then necessarily $a'_i \in \arg\max_{a_i \in A_i} v_i(a_i, \sigma_{-i}, t, \pi_{i,t_i}|t_i)$. In the natural ordering defined by Vives (1990), as each A_i is a compact lattice, each Σ_i is a compact lattice as well. Define a map $\Psi : \Sigma \to \Sigma$, $\Psi(\sigma) = (\Psi_i(\sigma_{-i}) : i \in I)$ as follows: for each σ, $\Psi_i(\sigma_{-i}) = \left\{a'_i \in A_i : \succ_{i,t_i,\sigma_{-i}} (a'_i) = \emptyset\right\}$. By proposition 1, for each $i \in I$ and $\sigma_{-i} \in \Sigma_{-i}$, $\Psi_i(\sigma_{-i})$ is non-empty and compact and for $\sigma_{-i} \geq \sigma'_{-i}$ if $a_i \in \Psi_i(\sigma_{-i})$ and $a'_i \in \Psi_1(\sigma'_{-i})$, then $\sup(a_i, a'_i) \in \Psi_i(\sigma_{-i})$ and $\inf(a_i, a'_i) \in \Psi_1(\sigma'_{-i})$. Therefore, the map $\bar{\sigma}(\sigma) = (\bar{\sigma}_i(\sigma_{-i}) : i \in I)$ is an increasing function from Σ to itself and as Σ is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $\bar{\sigma} = \bar{\sigma}(\bar{\sigma})$ is a fix-point of Ψ. By a symmetric argument, $\underline{\sigma}(\sigma)$

is an increasing function from A to itself and $\underline{\sigma} = \underline{\sigma}(\underline{\sigma})$ is also a fix-point of Ψ. Moreover, $\bar{\sigma}$ and $\underline{\sigma}$ are respectively the largest and smallest fix-points of Ψ. ■

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