A Multidimensional Approach: Poverty Measurement & Beyond

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Introduction to the Special Issue

Poverty has probably always been understood as a multidimensional problem, yet traditionally it has been measured with one dimension: income.[1] The assumption was that the income level could capture fairly well whether people were able to achieve certain minimum thresholds in a variety of dimensions such as nutrition, clothing and housing.[2]

In recent years there has been a growing consensus regarding the insufficiency of income poverty measures (Sen 1992). In the first place, some important needs are not satisfied in the market, or markets function very imperfectly. In those cases, non-market goods or institutions are required to provide for those needs. One example of this is access to clean water and education, which are sometimes provided by the state or NGOs. Second, each household has a different capacity to convert income into functionings. At one level, the problem is converting income to resources: households with disabled people, households in rural areas far away from markets and public services, and households with very low educational levels or with high interest loans may not be able to access the basket of goods and services that in theory they should be able to access with the income they earn. Even if equal resources are available, they may generate different levels of capabilities and functionings across diverse people: for example the same caloric bundle will generate different nutritional outcomes across a pregnant woman, a labourer with a high metabolism, a sedentary office worker, or a very elderly person. Third, participatory exercises reveal that poor people describe their state of deprivation with a wide range of dimensions, from health, nutrition, lack of adequate sanitation and water, social exclusion, low education, bad housing conditions, violence, shame and disempowerment, to name a few. Fourth, income is merely a means to ends. It is the ends which are valuable, not the means.

The recognition of these limitations has led to the development of methodologies to measure poverty in a multidimensional way and to an increasing demand from governments to design official poverty measures of this kind which can complement the income poverty measures. This trend has been fostered by the recent availability of household survey data that enables the implementation of multidimensional measures.

Several methodologies for multidimensional poverty measurement have been proposed, which can be broadly grouped into axiomatic and information theory approaches, fuzzy set theories, and latent variable methods (AF, 2011a, p.476).[3] The methodology proposed by

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[1] Here we refer to income in a general way. It may actually be income, or consumption, or expenditure.

[2] Interestingly, however, the dominant method to compute the income poverty line estimates the cost of a food basket which provides with the minimum amount of calorie intake for a representative adult and incorporates the non-food items by expanding this cost by the inverse of the Engel coefficient, estimated for the group of people whose income is just above the cost of the basic food basket. The Engel coefficient is given by the ratio of expenditure in food items to total expenditure. Thus there is not an estimation of the resources needed item-per-item.

[3] On axiomatic approaches see Chakravarty, Mukherjee and Renade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Chakravarty and Silber (2008), Bossert, Chakravarty and D'Ambrosio (2009), and Alkire and Foster (2011). On information theory, see Maasoumi and Lugo (2008). On fuzzy sets, see Lemmi and Betti (2006) and Chiappero-Martinetti and Roche (2009) and on latent variable see Kakwani and Silber (2008) and

Alkire and Foster (2007, 2011a) (AF hereafter) which belongs to the axiomatic approach, is the one which has been empirically implemented to the largest scale through the Multidimensional Poverty Index (Alkire and Santos, 2010; UNDP, 2010; Alkire et al., 2011). It is also the one which has been used in national multidimensional poverty measures developed by governments of Colombia and Bhutan, among others.

This special issue comprises a set of nine papers that utilise the AF methodology. Six papers present multidimensional poverty estimates in different developing regions of the world, with three of them focusing on certain unprivileged groups: poverty among women in Sub-Saharan Africa (14 countries), poverty among children in Afghanistan and in Bangladesh; and three providing poverty estimates in China, Bhutan, and Latin America (6 countries). Some of these papers analyze the evolution of poverty in these countries over time, or scrutinize poverty levels by population subgroups. Two other papers illustrate the use of the AF methodology for targeting purposes and compare it against other currently-used methods. One studies the methodology used to identify and target the Below the Poverty Line households in India, and the other explores the targeting method for urban beneficiaries in the Oportunidades Program in Mexico. Finally, one paper uses the AF methodology to construct a governance index for African countries and compares the results with the Mo Ibrahim Index, upon whose indicators it draws.

In this Introduction we set out the AF methodology used throughout this issue, define terms that are common across papers, and highlight the advantages and limitations of this method. We also present other multidimensional poverty measures to which the AF measures are compared in some papers, namely, the Unsatisfied Basic Needs (UBN) Index and the family of Bourguignon and Chakravarty (2003) measures. We close with a succinct overview of each paper in this issue.

1. The Alkire and Foster Methodology[4]

Since Sen (1976), the measurement of poverty has been conceptualised as following two main steps: identification of who is poor, and aggregation of information about poverty across society. In the unidimensional income space, the identification of who is poor is relatively straightforward. An income poverty line – the level of income necessary to purchase a basic basket of goods and services – dichotomises the population into the poor and the non-poor. Greater consideration is given to the properties that should be satisfied by the poverty index aggregating poor individuals' data into an overall indicator. The indices most frequently used are those of the Foster, Greer and Thorbecke (1984) family (FGT henceforth).

In the multidimensional context, the identification of the poor is more complex. The AF methodology combines a method for identifying the poor based on counting the number of (weighted) deprivations, and a method for aggregation, based on an extension of the unidimensional FGT family of measures to the multidimensional case. We present each in turn following the notation introduced in Alkire and Foster (2011).

Asselin (2009). Atkinson (2003) offers an excellent analysis of counting approaches vs. social welfare approaches.

[4] For a detailed explanation of the methodology, see AF (2011a). See AF (2011b) for a more pedagogical presentation.

1.1 Identification

When multiple dimensions are considered, distributional data are presented in the form of a $n \times d$ matrix $X^{n,d}$, in which the typical element x_{ij} corresponds to the achievement of individual i in indicator j, with $i = 1,....,n$ and $j = 1,....,d$. For clarity, we refer to each column of the matrix as containing the distribution of achievements in one 'indicator' rather than one 'dimension', as in empirical applications several indicators may be associated with a conceptual dimension which does not enter the matrix but may affect weights.

The selection of dimensions and indicators is evidently a crucial step in designing a multidimensional measure which demands careful justification. A number of methods are typically followed for this purpose (see Alkire, 2008; Robeyns, 2005). Each of the papers contained in this special volume has made an effort to appropriately justify the particular selection used.

In terms of identification, the AF methodology (as well as most of the other measures proposed within the axiomatic framework) follows a *counting approach*. Counting approaches have been used in measurement exercises prior to the AF proposal as early as with Townsend (1979)'s study of poverty in Britain, with the Unsatisfied Basic Needs Method used since the early 1980s by national governments in Latin America as well as in European measures (Atkinson 2003), to mention just a few.

Counting approaches require defining for each selected indicator a deprivation cutoff which expresses a normative minimum level of achievement considered necessary to be non-deprived.[5] As with the selection of dimensions and indicators, choosing the deprivation cutoffs also demands strong justification based for example on national or international consensus, empirical evidence or human rights, to mention a few. The *deprivation cutoffs* are collected in vector $z = (z_1,...., z_d)$. A person is defined to be deprived in each indicator if her achievement is strictly less than the corresponding cutoff (i.e. whether $x_{ij} < z_j$ or not). However, that is not sufficient for identifying who is poor. One needs to determine what number or share of deprivations a poor person should experience in order to be considered multidimensionally poor.

A natural starting point is to consider poor all those who fall short in at least one indicator, the so called *union criterion*.. Yet, more demanding criteria can be used, even to the extreme of requiring deprivation in all the considered indicators, the so called *intersection criterion*. As noted by AF (2011a, p. 478), as the number of considered indicators increases, the proportion of the population identified as poor normally increases. In fact it can be very large, depending upon the considered indicators, and may include people with deprivations that do not reflect poverty, but rather measurement error or personal preference or an idiosyncratic state. Thus, the union criterion may not be helpful for distinguishing and targeting the extensively deprived. At the other extreme, the intersection criterion is likely to overlook people who are experiencing extensive but not universal deprivation. Thus, AF (2007, 2011a)'s methodology proposes an identification function which includes the union and intersection criteria but also allows intermediate cases. Any identification function other than union or intersection requires a weighting vector (Alkire 2011). The AF methodology allows

[5] Other multidimensional approaches have been proposed which first aggregate the individual *i*'s achievements across indicators and compare this with an overall poverty threshold in the space of the well-being metric, which can be a function of the dimension specific poverty thresholds. See Maasoumi and Lugo (2008) for an example.

any weighting structure although weights – like the poverty cutoff we will describe shortly – must be both justified and tested for robustness.

Let vector $w=(w_1,....,w_d)$ be the vector of indicators' weights, with w_j being the weight attached to indicator j such that $\sum_{j=1}^{d} w_j = d$. Note that the weights add up to the total number of columns of matrix *X*.[6] To identify the poor one needs to construct the deprivation matrix $g^0 = \left[g_{ij}^0 \right]$ where the typical element g_{ij}^0 is defined as $g_{ij}^0 = w_j$ if $x_{ij} < z_j$ and $g_{ij}^0 = 0$ otherwise. The deprivation matrix provides a snapshot of who is deprived in which indicator and how much weight the deprivation carries (AF, 2011b, p. 296).

From there, a vector of deprivation count $c=[c_i]$ can be constructed, whose typical element is defined as $c_i = \sum_{j=1}^{d} g_{ij}^0$. In words, c_i is the weighted number of deprivations suffered by individual i. The identification function $\rho_k(x_i;z)$ is such that $\rho_k(x_i;z)=1$ if $c_i \geq k$, where k is the number of weighted deprivations required to be considered multidimensionally poor, and $\rho_k(x_i;z)=0$ otherwise. Parameter k is called the *poverty cutoff* and it ranges from the minimum weight assigned to any indicator (which corresponds to the union criterion), to the total number of considered indicators (which corresponds to the intersection criterion): $\min(w_j) \leq k \leq d$. The poverty cutoff k could equivalently be defined as the proportion – rather than the number – of (weighted) deprivations a person needs to experience in order to be identified as poor. If this is the case, person i is poor whenever $(c_i / d) \geq k$ where $(\min(w_j)/d) \leq k \leq 1$. Note that ρ_k depends on both the indicators' cutoffs z_j and the poverty cut-off k, and that is why the AF methodology is said to follow a dual cut-off method of identification.

Identification is a necessary step in poverty measurement in order to be focused on the poor. Such focus is accomplished through a censoring of the data of the non-poor. Thus, once identification has been completed, one can define the *censored* matrix of deprivations $g^0(k) = \left[g_{ij}^0(k) \right]$, where $g_{ij}^0(k) = g_{ij}^0$ if $c_i \geq k$ and $g_{ij}^\alpha(k) = 0$ if $c_i < k$. That is, the deprivations of the multidimensionally poor remain unchanged whereas the deprivations of those who have not been identified as multidimensionally poor are replaced by zero. An alternative way of seeing the censored matrix is that each row of the g^0 matrix is multiplied by the identification function, such that: $g_{ij}^0(k) = g_{ij}^0 \rho_k(x_i;z)$. Additionally, a vector of censored deprivation counts $c(k)=[c_i(k)]$ can be constructed such that $c_i(k)=c_i$ if $c_i \geq k$ and $c_i(k)=0$ otherwise. This will be used in the measure of poverty intensity.

In the case in which all the variables in X have cardinal meaning, one can consider information on the depth of deprivation in each indicator. In particular, a matrix of normalized gaps raised to the power α can be constructed, $g^\alpha = \left[g_{ij}^\alpha \right]$, where $g_{ij}^\alpha = w_j \; (z_j - x_{ij}) / z_j \;^\alpha$ if $x_{ij} < z_j$ and $g_{ij}^\alpha = 0$ otherwise.[7] When $\alpha = 0$, the matrix is the deprivation matrix already described. When $\alpha = 1$, the matrix is the normalised gap matrix,

[6] Weights can equivalently be defined in a relative way, adding up to 1 (or 100%).
[7] Note that the indicator's weight is *not* raised to the power α.

which provides additional information on the depth of deprivation of each person in each indicator, weighted by its relative importance (AF, 2011b, p. 297). When $\alpha = 2$, the matrix is the squared gap matrix, which places more emphasis on larger shortfalls from the deprivation cutoff. In all cases, one needs the deprivation matrix g^0 to identify the poor and to create a censored version of the matrix $g^{\alpha}(k) = \left[g_{ij}^{\alpha}(k) \right]$, such that the normalized gaps of the non-poor are replaced by zeroes ($g_{ij}^{\alpha}(k) = g_{ij}^{\alpha} \rho_k(x_i; z)$).

1.2 Aggregation

Once identification has been completed and the censored matrices have been obtained, the aggregate M_α measure simply requires taking the mean across people and indicators of the corresponding censored matrix $g^{\alpha}(k)$:

$$M_{\alpha}(x;z) = \frac{1}{nd} \sum_{i=1}^{n} \sum_{j=1}^{d} g_{ij}^{\alpha}(k) \qquad \text{with} \quad \alpha \geq 0 \qquad (1)$$

Note that the aggregation function is an extension of the unidimensional FGT measures. It is the average of the censored normalized gaps raised to the power α, weighted by the respective indicators' weights. The difference is that multiple gaps are considered (in each included indicator) rather than only one (typically income). The mean is naturally taken both across all the *d* indicators for all people.[8] Analogous to the unidimensional case, three members of this family are worth mentioning.

First, when $\alpha = 0$, the measure M_0 is the so-called adjusted headcount ratio. This name derives from the fact that M_0 is the headcount ratio of multidimensional poverty *H*, **poverty incidence**, multiplied – i.e. *adjusted* – by **poverty intensity** *A*: $M_0 = HA$. The headcount ratio of multidimensional poverty *H* is defined as the proportion of people who have been identified as multidimensionally poor: $H = q(k)/n$, where $q(k)$ is the number (or headcount) of multidimensionally poor people according to parameter *k* ($q(k) = \sum_{i=1}^{n} \rho_k(x_i, z)$). The intensity of poverty *A*, also sometimes referred as the ***breadth*** of poverty, is defined as the average deprivation share across the poor; in a formula this is given by $A = \sum_{i=1}^{q} c_i(k) / (q(k) * d)$. The intensity of poverty indicates the fraction of the *d* indicators in which the average multidimensionally poor person is deprived. This adjustment is not trivial. For example, countries B and C may both have 40% of their population in multidimensional poverty. However, while in country B the poor are deprived on average in 30% of the (weighted) indicators, in country C they are deprived in 70% of the (weighted) indicators. The 30% of poor people is adjusted by the intensity of poverty yielding an M_0 of 0.12 in country B and of 0.28 in country C. Only in the extreme case in which *all* the poor are deprived in *all* the indicators (such that A=100%) will M_0 be equal to *H*; otherwise M_0 will be smaller than *H*.

By adjusting incidence by intensity, M_0 gains two highly significant advantages over the headcount ratio *H*: it is sensitive to the number of deprivations the multidimensionally poor

[8] If weights were defined adding up to 1 rather than to d, then expression (1) must not contain *d* in the denominator. Analogous modifications apply to the subsequent formulas.

experience. If a poor person becomes less deprived in any indicator but remains poor, M_0 decreases because A decreases, whereas H remains unchanged. Thus M_0 reflects a reduction of deprivations among the poorest of the poor, even if they remain multidimenisonally poor. A second advantage is that, as we shall see shortly, M_0 (and all M_α measures) can be broken down by dimension: a feature that is not possible with the headcount ratio H. In terms of interpretation, M_0 can be interpreted as the proportion of the weighted deprivation counts that are experienced in a society out of all the total potential deprivations that the society (all people, across all indicators) could experience.

This multidimensional poverty measure M_0 can be used in the (very common) case in which one or more considered indicators are categorical, ordinal or dichotomous. Categorical indicators have no cardinal value but may be able to be clustered into groups reflecting deprivation or adequacy – such as water source, sanitation facilities, roof materials, occupational category. Ordinal indicators are such that their rank is known but the values have no cardinal meaning, as in the case of birth order or self rated health. There is no unique way to cardinalize distances between the different categories. By dichotomizing all the variables into deprived and non-deprived categories, the M_0 measure combines the indicators in a robust way.[9]

When $\alpha = 1$, the measure M_1 is the so-called adjusted poverty gap, defined as the weighted average of indicator-specific poverty gaps. It can be shown to be the product of H, A, and the average poverty gap among the poor G: $M_1 = HAG$. G is given by $G = \sum_{i=1}^{n}\sum_{j=1}^{d} g_{ij}^{1}(k) \Big/ \sum_{i=1}^{n}\sum_{j=1}^{d} g_{ij}^{0}(k)$. In words, G is average depth of deprivations among all instances where a poor person is deprived; note that the sum of the (weighted) gaps is divided by all the positive (weighted) entries of the (censored) deprivation matrix. The M_1 measure is not only sensitive to the number of deprivations the poor experience but also to their depth. In other words, if a poor person becomes more deprived in a particular indicator, M_1 increases. Similarly, if the shortfall from the deprivation cutoff in any indicator is reduced, then poverty goes down – even if that person remains poor. This makes the measure more sensitive to changes among the poorest persons.

Finally, when $\alpha = 2$ the measure is the adjusted squared poverty gap, defined as the weighted sum of the dimension-specific squared poverty gaps. M_2 can be shown to be the product of H, A, and the average squared poverty gap among the poor S, namely, the severity of deprivations among the poor: $M_2 = HAS$. S is defined as $S = \sum_{i=1}^{n}\sum_{j=1}^{d} g_{ij}^{2}(k) \Big/ \sum_{i=1}^{n}\sum_{j=1}^{d} g_{ij}^{0}(k)$. In words, S is the average squared gap among all instances where a poor person is deprived; analogous to G, the sum of the (weighted) squared gaps is divided by all the positive weighted entries of the (censored) deprivation matrix. The M_2 measure is sensitive to the number of deprivations the poor experience, the depth of the deprivations and also to the inequality of deprivations among the poor.[10]

1.3 Subgroup Decomposability and Dimensional Break-Down

[9] Robust here refers to the fact that the M_0 measure does not change under any monotonic transformation of the scale of any of the indicators and the corresponding deprivation cutoff.

[10] AF (2011a) offer a detailed presentation of the axioms satisfied by their family of poverty measures.

All members of the M_α family satisfy two properties which are particularly powerful for policy analysis.

First, each measure can be decomposed by subgroups of population. This is because each measure can be expressed as the weighted sum of individual poverty, where each person has a relative weight of $1/n$:

$$M_\alpha(x;z) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{d}\sum_{j=1}^{d} g_{ij}^{\alpha}(k)\right) \qquad (2)$$

Thus, given a population subgroup *I*, its contribution to overall poverty is given by:

$$C_I = \left[\; n_I / n \;\; M_\alpha^I \right] / M_\alpha \qquad (3)$$

where (n_I / n) and M_α^I are the population share and the poverty measure of subgroup I respectively, and M_α is the poverty measure for the overall population. Whenever the contribution to poverty of a region or some other group widely exceeds its population share, this suggests that there is a seriously unequal distribution of poverty in the country, with some regions or groups bearing a disproportionate share of poverty. Clearly, the sum of the contributions of all groups needs to be 100 per cent.

Analogously, once the identification step has been completed, all members of the $M_\alpha(x;z)$ family can be broken down into indicators' subgroups. As mentioned above, this is due to the new factor of poverty intensity, and is not possible using the headcount ratio of multidimensional poverty. Specifically, note that the measures can be expressed as the weighted sum of the censored deprivations by indicator, where each indicator has a relative weight of w_j / d, with w_j being already included in the $g_{ij}^{\alpha}(k)$ gaps defined in Section 1.1

$$M_\alpha(x;z) = \frac{1}{d}\sum_{j=1}^{d}\left(\frac{1}{n}\sum_{i=1}^{n} g_{ij}^{\alpha}(k)\right) \qquad (4)$$

Strictly speaking, this is not decomposability in terms of indicators because it requires the identification step in the first place, and decomposes the censored matrix $g^\alpha(k)$.Yet it is an extremely useful property for policy analysis. Once identification has been applied, and the non-poor rows of g^α have been censored to obtain $g^\alpha(k)$, for each j:

$$C_J = \left(\frac{1}{n}\sum_{i=1}^{n} g_{ij}^{\alpha}(k)\right) / \, dM_\alpha \qquad (5)$$

can be interpreted as the post-identification contribution of indicator j to overall multidimensional poverty. As in the case of the population decomposition, the sum of the contributions of all the indicators needs to be 100 per cent. Whenever the contribution of a particular indicator far exceeds its relative weight, this suggests that there is a relative high deprivation in this indicator. It may also be worth noting that when there are several indicators belonging to the same dimension, the dimensional contribution can be obtained simply adding up the percentage contributions of all the indicators within that dimension.

In the case of the M_0 break-down, the components are called indicators' censored headcount ratios. Given an indicator *j*, its censored headcount ratio CH_j is the proportion of people who are *poor and deprived* in indicator *j*. This is obtained simply by taking the average of the *j* column of the censored deprivation matrix $g^0(k)$ and dividing this average by the indicator's weight. [11]

$$CH_j = \frac{\sum_{i=1}^{n} g_{ij}^0(k)}{w_j n} \qquad (6)$$

When deprivation headcounts are constructed using the original g^0 matrix we call these *raw* headcount ratios.[12]

1.4 Insights and Limitations of the AF method

The AF methodology offers a flexible framework for multidimensional poverty measurement which allows a range of identification criteria to be implemented based on counting the deprivations people experience such that focus can be placed on those exhibiting coupled or simultaneous deprivations, and a parametric family of measures which satisfy a number of desirable properties among which subgroup decomposability and dimensional break down are particularly convenient. Additionally, each measure is composed of intuitive partial indices such as poverty incidence, average intensity, and average poverty gap. Moreover, it is simple to compute.

The largest scale application of the AF methodology to date has been the Multidimensional Poverty Index (MPI) (Alkire and Santos, 2010) developed at the Oxford Poverty and Human Development Initiative in collaboration with the Human Development Report Office. The MPI uses the M_0 measure. It covers three dimensions – health, education and living standards – using ten indicators: nutrition and child mortality, school attendance and years of education, access to drinking water, improved sanitation, electricity, clean cooking fuel, non dirt floor and two small assets or a big one. The MPI was designed to reflect acute multidimensional poverty in a cross-country comparable way. It has been tested for robustness to changes in weights, deprivation cutoffs, poverty cutoffs and sample variability (Alkire and Santos, 2012 and Alkire et al. 2010).[13] Like any poverty measure, particular implementations of a methodology, such as the MPI, are limited by the quality, content, and frequency of data available.

Although the AF methodology provides a general framework, it is inspired in one particular approach to development: Sen's capability approach. The capability approach proposes a fundamental shift in the focus of attention from means of living to the actual freedoms a person has (Sen 2009, p. 253). Capabilities are defined as the various combinations of functionings (beings and doings) that the person can achieve. Capability is "a set of vectors of functionings, reflecting the person's freedom to lead one type of life or another... to choose from possible livings" (Sen 1992, p. 40). Human development in this context is "the process

[11] Also note that censored headcount ratios are not the proportion of the poor deprived in a certain indicator. That is an interesting but different statistic, which can be obtained by dividing the censored headcount ratio by *H*.

[12] In the case of *union* identification – when any deprivation is sufficient to identify a person as poor – the raw and censored headcount ratios coincide.

[13] See Yalonetzky (2011) for the development of standard errors for the AF family of measures.

of expanding the real freedoms that people enjoy" (Sen, 1999, p. 3) and poverty is defined as "deprivation of basic capabilities rather than merely as lowness of incomes" (Sen, 1999, p. 87).

The AF method embodies the capability approach's normative view of poverty. Poverty is measured in a multidimensional way, not only because more than one single indicator is considered, but fundamentally because shortfalls in *each* included indicator enter the measure, and each deprivation is given independent recognition as being 'inherently undesirable' (AF, 2011a, p. 478). Note that this does not necessarily mean that any multidimensional poverty reflects capabilities; if the indicators used reflect resources or subjective states, for example, they may or may not reflect people's achieved functionings. Data constraints affect empirical options for measuring achieved functionings in many contexts. However if the indicators do reflect valuable functionings (directly or via good proxies), then the M_0 measure can be viewed as a measure of 'unfreedom', analogous to Pattanaik and Xu (1990) (AF, 2011a, p. 482).[14]

Many alleged limitations of the AF methodology refer to 'misunderstandings' with which we have tried to clarify (AF 2011b, Alkire, Foster and Santos 2011). These critiques include (1) that it does not add value over marginal data arrays (although through the identification step, which is based on the experience of simultaneous deprivations, it does); (2) that it 'loses' information by creating one overall number (yet the key novelty is that the overall number is linked to an array of consistent partial indices which show the contribution of each indicator, of intensity overall, and of the headcount ratio); (3) that it is too sensitive to weights (but our empirical results of robustness tests for the MPI 2010 do not support such critique, see Alkire and Santos, 2010, 2012 and Alkire *et al.* 2010); and (4) that the requirement that indicators are available for the same household are too demanding (however, the papers in this special issue rightly indicate that such data are often available). The other legitimate limitations have been specified in AF 2011a and particularly regard the fact that while the methodology respects the same principle of non-decreasing poverty under correlation increasing switch as Bourguignon and Chakravarty (2003)'s measures, the methodology is 'neutral' between dimensional contributions when data are cardinal, rather than affixing a given elasticity of substitutability to all indicators. While it would be possible to incorporate that elasticity, this would change other properties like dimensional breakdown which is vital for policy. More to the point, given disagreement about complementarities and substitutabilities across dimensions, this method offers a neutral starting point. This is further explained in Section 3.

2 The Multidimensional Headcount and the Unsatisfied Basic Needs Approach

The oldest antecedent of a multidimensional poverty measure is the unsatisfied basic needs index from the mid seventies and early eighties. This index stemmed from the Basic Needs approach to development (UNEP-UNCTAD, 1974; Dag Hammarskjöld, 1976; Herrera et al. 1976, ILO, 1976; Streeten et al., 1981) which arose as a reaction to the prevailing economic growth-centred approach to development of the time.

"Basic needs may be interpreted in terms of minimum specified quantities of such things as food, clothing, shelter, water and sanitation that are necessary to prevent ill health, undernourishment, and the like" (Streeten et al. 1981, p. 25). The approach led to the

[14] See AF (2011a) for a formal characterization of the M_0 measure as a measure of unfreedom.

construction of an index which would consider whether people were failing to reach the minimum quantities considered to guarantee the satisfaction of these basic needs. Note that the procedure to determine deprivations – unsatisfied basic needs – (UBN hereafter) is the same as in the AF framework: the achievement x_{ij} of person i in a particular indicator j is compared against a normative cutoff z_j, for d number of indicators. There is a conceptual difference however, which is that the basic needs approach focused strongly on access to resources (supposed to guarantee the satisfaction of certain needs), whereas if inspired by the capability approach, implementations of the AF methodology would favour indicators of actual functionings whenever information on these were available, rather than resources or subjective states.

Once it has been determined who has unsatisfied basic needs and which these are, the UBN follows a counting approach for identification, determining people with one, two, and other number of UBNs. For aggregation, the approach used the multidimensional headcount ratio. Following the notation and definitions introduced in the previous section, the UBN index can be expressed as:

$$H(x;z)=\frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{d}g_{ij}(k)\right]^{0}=\frac{q(k)}{n}\text{ with } w_j=1\text{ for all } j \tag{7}$$

Most commonly, the UBN Index is reported as the proportion of people in households with one or more UBN, thus following a union criterion to identifying the poor (*k=1*). However, proportions of people with other number of UBNs are also frequently reported.

The UBN Index has been extensively used in the Latin America and Caribbean (LAC) region to measure poverty both by national governments as well as by the Economic Commission for Latin America and the Caribbean (ECLAC). Each country implemented a different variant of the UBN Index in terms of the indicators and cutoffs. It used the best available data at the moment – census data – and thus remained quite restricted in terms of the indicators that could incorporate. Following a pioneer study in the region performed by INDEC (1984), the UBN Index in LAC most commonly comprised indicators of access to clean water and improved sanitation, type of dwelling, overcrowding, attendance of children to school and some indicator of education of the household head and dependency ratio. Counting-based indices have also been "widely used in applied studies" in Europe (Atkinson 2003 and the references therein).

The UBN Index has been criticized on several grounds. In the first place, it weights each indicator equally. This means that when more than one indicator is used for a given dimension, some dimensions may be disproportionately weighted over others (Feres and Mancero, 2001). Secondly, as noted by AF (2011a), by aggregating with the multidimensional headcount ratio, if the poor become deprived in one additional indicator, this is not reflected in the UBN Index. Thirdly, and also by relying on the multidimensional headcount ratio, the UBN Index overlooks information over the depth of deprivation, when this is meaningful (when indicators are cardinal). [15] Finally, while it is possible to decompose the headcount in subgroups of population, it is not possible to display the contribution of each

[15] Within his Integrated Method to Measure Poverty, which combines the income approach with the UBN one, Boltvinik (1992) proposed alternative weighting structures for the UBN indicators as well as assigning a 'cardinal' scale to ordinal indicators in order to compute normalised gaps and incorporate the depth of deprivation. Further research is needed on the legitimate use of information from ordinal data which is lost with dichotomization.

dimension to overall poverty, nor can the measure be broken down seamlessly into censored headcounts. As described in Section 2, the AF methodology solves each of these problems. However, it is worth noting that the UBN Index was pioneer in looking at *joint deprivations*. It used micro-level data to determine who was experiencing simultaneous deprivations and this was certainly a novel contribution at the time.

3. Bourguignon and Chakravarty family of poverty indices

One of the early axiomatic papers on multidimensional measurement was that of Bourguignon and Chakravarty (2003) (BC hereafter). We now present their family of multidimensional poverty measures following the same notation previously used. The BC measures are also an extension of the unidimensional FGT measures. For identification, the authors apply the union criterion, such that anyone experiencing at least one deprivation is to be considered multidimensionally poor. Thus, the BC measures can be expressed in terms of the elements of the (uncensored) matrix g^{α}.

Unlike the AF measures, the BC measures can only be implemented with cardinal data; categorical and ordinal data cannot be used. Using that data, a distinctive feature of the BC family of measures is that it considers that indicators may interact with each other as either substitutes or complements. Given the normalized gaps, the substitutability or complementarity interaction applies under a specific type of transfer or rearrangement, called by BC (2003) *correlation increasing switch*.[16] The transformation is as follows: assume two poor individuals B and C, with B having strictly higher achievements than C in some indicators, but strictly lower achievements in others. Assume that there is a transfer between B and C such that one of them – say B – ends up having higher achievements than C in *all* indicators; that is, the correlation (or the association) between indicators has increased as a result of the transfer.

When indicators are thought to be substitutes of one another, poverty should not decrease under the described transfer. The intuition is that if dimensions are substitutes, before the transfer, both B and C were able to compensate their meagre achievements in some indicators with their higher achievements in the others; after the transfer such possibility is reduced for C who now has less of every attribute, and therefore poverty should increase (or at least not decrease). This property is referred by BC (2003) as 'Non-Decreasing Poverty under Correlation Increasing Switch' (NDCIS).

However, when indicators are thought to be complements, poverty should decrease (or at least not increase) under the described transfer. This is because before the transfer both B and C had difficulties in combining the different attributes in order to achieve a certain level of well-being, as both had very low achievement in some dimension; after the transfer, at least B is in a better position to combine achievements and reach a certain level of well-being. This property is referred by the authors as 'Non-Increasing Poverty under Correlation Increasing

[16] This transformation was introduced by Atkinson and Bourguignon (1982) and Boland and Proschnan (1988). It has been considered by Tsui (2002) for multidimensional poverty measures. AF (2007, 2011a) rename the transformation in a more general way as *association increasing rearrangement*, since the term 'correlation' refers only to a specific type of association.

Switch' (NICIS). Clearly, there is also scope for dimensions to be considered independent, in which case, poverty should not change under the described transformation.[17]

The family of multidimensional poverty indices proposed by BC (2003) is given by the following expression:

$$P_\theta^\alpha(x;z) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{d}\sum_{j=1}^{d} g_{ij}^{\alpha}\right]^{\theta/\alpha} \text{ with } \alpha \geq 1 \text{ and } \theta \geq 0 \qquad (8)$$

where g_{ij}^{α} is the normalized (uncensored) gap raised to power α (and weighted) defined in Section 2.1 Note however that in this case $\alpha \geq 1$, as it is the parameter of a constant elasticity of substitution (CES) function with which deprivations for each person are aggregated. The BC family satisfies a set of desirable properties. In particular, for $\theta > 0$ and $\alpha \geq 1$, the indices satisfy monotonicity, and for $\alpha, \theta > 1$, they satisfy the transfer requirement.[18] The indicators are considered substitutes, complements or independent depending on the value of parameter θ relative to that of α. When $\theta > \alpha$, indicators are considered substitutes, and the indices satisfy the NDCIS property, so that an increase in the association between indicators does not decrease poverty (it actually increases it). On the other hand, when $\theta < \alpha$, indicators are considered complements, and the indices satisfy the NICIS property, so that an increase in the association between indicators does not increase poverty (it actually reduces it). Finally, when $\alpha = \theta$ indicators are considered to be independent, and increases in the association between them do not affect the poverty measure, thus both NDCIS and NICIS are weakly satisfied.

Comparing the AF family of measures (expression 1) with the BC family of measures (expression 8) a number of similarities and differences are worth noting. Rather than aggregating achievements and comparing the aggregate achievement with an aggregate cutoff level of wellbeing, both families of measures consider deprivation in each included indicator. Also by being extensions of the unidimensional FGT measures, both families of measures are based on the normalized gaps. However, one important first difference is that BC restrict their analysis to the union criterion to identify the poor. In other words, they consider all deprivations, even of those who only have one deprivation. That is why the elements of the uncensored g^{α} matrix directly enter equation 8. Thus, in terms of identification, the BC measures are a special case of the AF ones. Also note that while the indicators' weights affect both identification and aggregation in the AF methodology, in the BC measures they only affect aggregation (anyone with at least one deprivation, regardless of its weight, is identified as poor). Second, when $\theta=0$ in expression 8, independently of the value of α and of the weights used, the BC measure is reduced to the headcount ratio of multidimensional poverty – or the UBN Index – using a union approach to identification and equal weights $P_\theta^0(x;z) = H(x;z)$. This is an important difference with the AF family of measures, where the first measure is M_0, sensitive to the intensity of poverty, of which H is just one component. Also, when $\alpha = \theta$, the BC measure coincides with the M_0 indices suggested by

[17] It is worth noting that the substitutability, complementarity or independence relationship between indicators is defined in the multidimensional measurement literature in terms of the second cross partial derivative of the poverty measure with respect to any two dimensions being positive, negative or zero, respectively. This corresponds to the Auspitz-Lieben-Edgeworth-Pareto (ALEP) definition, and differs from Hick's definition traditionally used in the demand theory (which relates to the properties of the indifference contours) (Atkinson, 2003, p. 55). See Kannai (1980) for critiques of the ALEP definition.

[18] BC (2003) provide a formal definition of this transfer in the multidimensional context.

AF in the particular case in which equal weights and union identification are used, assuming independence between the considered indicators. Thus, in terms of aggregation, the AF measures might be seen as special cases of the BC ones.

Thirdly, both AF and BC measures are decomposable in population subgroups. However, while the AF measures can be broken down into indicators' contributions, the BC cannot unless $\theta = \alpha$, that is, unless indicators are considered independent. Thus, the possibility of accounting for association between indicators comes at the cost of losing the possibility of breaking down the measure into its (post-identification) components. Even if one was willing to resign such property, it must be emphasized that accounting for associations requires cardinality of the data. Moreover, it may require a sense or ideally some evidence with respect of the type of association to be assumed, whether substitutability or complementarity. There is too little analysis on these lines. Fourthly, the BC measures assume the same type and degree of association for all considered indicators, but it is likely to be the case that different pairs of indicators behave differently. And finally, as mentioned above, the BC measures are limited to cardinal data which restricts their applicability in practice. In sum, given that quite often indicators considered in multidimensional analysis are ordinal or categorical, and that we still know little about the actual associations between indicators, accounting for these associations has in practice a limited scope as yet.

4. This special issue at glance

This issue contains nine papers which have applied the AF methodology to different contexts. The papers share not only the methodology but also the motivation of Sen's capability approach. Thus each has made an effort to consider indicators of functionings when these were available and has tried to provide good justifications of the selection of dimensions, indicators, weights and cutoffs. Some of them also offer robustness checks of these choices.

The three papers by Santos, Battiston, Cruces, Lopez-Calva, Lugo and Santos, and Yu each analyze the evolution of poverty over time in different countries, namely, China, Bhutan and six Latin American countries.

Santos' paper on Bhutan evaluates multidimensional poverty between two points in time: 2003 and 2007 using the Bhutan Living Standard Survey. The M_0 measure is estimated considering consumption expenditure alongside other six indicators which have directly or indirectly been identified as sources of happiness in the 2007 Gross National Happiness Survey. These are: health, education, access to electricity, safe water, improved sanitation, and enough room per person in dwelling. Two additional indicators are included in the case of rural areas: access to roads and land ownership. The paper performs extensive robustness in terms of the number of indicators, the indicators' cutoffs, the poverty cutoffs and weights, computing confidence intervals of the poverty estimates using the bootstrap technique. Interestingly the paper finds that Bhutan has accomplished a significant reduction in multidimensional poverty, reducing mainly the proportion of poor people albeit not the intensity of poverty among the most intensively poor. The paper also compares multidimensional poverty with income poverty and finds that if income alone is used to target the poor, inclusion errors are marginal, but exclusion errors are sizeable.

Battiston, Cruces, Lopez-Calva, Lugo and Santos evaluate multidimensional poverty in Argentina, Brazil, Chile, El Salvador, Mexico and Uruguay for the period 1992–2006.

The estimated measures overcome the limitations of the two traditional methods of poverty analysis in Latin America – income-based and unmet basic needs – by combining indicators from both methods and using better aggregation methods, namely the AF family of measures as well as that of BC. Two alternative weighting structures are used, one of which is derived from a participatory study performed in Mexico, where the income and children in school indicators receive the highest weights. It is found that over the study period, El Salvador, Brazil, Mexico and Chile experienced significant reductions in multidimensional poverty. In contrast, in urban Uruguay there was a small reduction in multidimensional poverty, while in urban Argentina the estimates did not change significantly. These results hold regardless of the measure used as well as of sample variability (estimates are bootstrapped). It is also found that there are huge disparities within countries, such that the rural areas of Chile can be grouped together with El Salvador, Mexico and Brazil in terms of their poverty incidence and intensity, while the urban areas of Chile have poverty levels similar to those of urban Argentina and Uruguay. Also, in El Salvador, Mexico and Brazil, higher poverty and more coupled disadvantages are found in the rural areas as compared to the urban ones. In all countries, access to proper sanitation and education of the household head are the highest contributors to overall multidimensional poverty.

The paper by Yu estimates the M_0 measure for nine provinces of China at four points in time: 2000, 2004, 2006 and 2009 using the panel data of the China Health and Nutrition Survey. The author considers eight indicators related to five dimensions: (1) per capita household income, (2) access to water, improved sanitation, electricity and clean cooking fuel as measures of living standard, (3) body mass index, as a measure of health, (4) having completed primary education and (5) access to medical insurance as an indicator of social security. Results suggest that China's rapid economic growth has resulted not only in a reduction in income poverty but also in a large reduction in multidimensional poverty. Yet, being China such a populous country, in absolute numbers there is still a huge number of people in poverty. There are also wide disparities across the nine considered provinces and between urban and rural areas, with poverty being 1.5 times higher in rural areas than in urban ones in 2009. Interestingly, the author finds that between 2000 and 2006 the largest contributor to aggregate poverty was deprivation in social security, but this was no longer the case in 2009, likely due to the efforts in re-building the health insurance system in both rural and urban areas after 2003. The author also finds an apparent increase in the deprivation in education which he relates to the migration of educated household members to urban areas over the time span of the panel.

The paper by Batana focuses on women in fourteen Sub-Saharan African countries including seven francophone countries (Benin, Burkina Faso, Guinea, Madagascar, Mali, Niger, and Senegal), six English-speaking countries (Ghana, Kenya, Malawi, Nigeria, Tanzania, Uganda) and Cameroon which is a bilingual (French and English) country. Batana considers four dimensions. Three are traditional: assets, health, schooling. By focusing on women and using the Demographic and Health Survey (DHS) data he is able to incorporate a non-traditional fourth dimension: empowerment. He estimates the M_0 and M_1 measures. His findings suggest three groups of countries according to their incidence of multidimensional poverty. The poorest group is composed of all francophone countries where more than 50% of women are poor; the second group contains countries where between 30 and 50% of women are poor, and the third group consists of countries where less than 30% of women are poor. When comparing with standard poverty measures such as income poverty and asset poverty he finds that there are significant differences in the country rankings with respect to both multidimensional poverty measures, indicating the further dimensions do add new information to the relative performances of countries. As in other papers in this issue, rural areas are significantly poorer than urban ones. Also, interestingly, deprivation in schooling among women is the highest contributor to poverty, followed, in general, by deprivation in empowerment. An important contribution of this paper is the statistical tests of dominance relationships between pairs of countries across the full range of poverty cutoffs k. Batana finds that 61% of the country dominance relations in the M_0 measure hold across the different k values. The author also performs sensitivity analysis using two alternative sets of indicators' deprivation cutoffs.

The papers by Roche and by Trani, Biggeri and Mauro, both focus on another vulnerable group: children. Roche focuses on child poverty in Bangladesh, a country that has made significant progress in reducing poverty in the last fifteen years. The author uses four DHS rounds of data between 1997 and 2007 to analyze in which ways has poverty been reduced in this country. He estimates the M_0 measure using the dimensions and indicators of the measure adopted by UNICEF in the Global Study on Child Poverty and Disparities guide in order to compare this approach with the AF. The author demonstrates the benefits of adjusting the headcount ratio by the intensity of poverty. A novel contribution of this paper is that it implements a Shapley decomposition to study the different paths followed in poverty reduction, both nationally as well as across the geographical regions of the country, and to explain changes in poverty in terms of demographic changes and incidence vs. poverty intensity, as well as to show how changes in individual indicators contributed. Robustness tests are also performed to evaluate the stability of the results.

Trani, Biggeri and Mauro analyze the case of Afghanistan, a country afflicted by years of conflict, severe droughts, political insecurity and bad governance, all of which has led to high levels of poverty. The authors offer a thorough literature review on measures of child poverty. They perform their estimates using a novel dataset carried out by Handicap International. The survey collected information on dimensions of children's wellbeing identified as relevant by a participatory process (prior to the survey). Such dimensions are typically missing in standard surveys. The authors consider eight dimensions in their measure for children of 5 to 7 years of age, namely: health, material deprivation, food security, care and love, social inclusion, access to schooling, freedom from economic exploitation, and shelter and environment. They include two additional dimensions – autonomy and mobility – in a measure for children between 8 and 12 years of age. As many of the indicators used are either ordinal or dichotomous, the M_0 measure is estimated. The authors find that virtually all children in the country are deprived in at least one dimension. They also find rural areas to be much poorer than urban ones both in terms of incidence and intensity. They also find girls to be poorer than boys; this is linked to a lower access to educational opportunities and healthcare as well as to traditional restrictions on mobility and autonomy. Additionally, they find that both poverty incidence and intensity are higher among disabled children than among non-disabled ones. The authors discuss potential improvements to two critical policies – an educational and a health one – that could contribute to reduce child poverty in the country.

The papers by Alkire and Seth and by Azevedo and Robles illustrate how the identification method of the AF methodology can be used for targeting purposes. Alkire and Seth emphasize that the identification of "who is poor" is a central question in both targeting and measurement exercises. The paper argues that there could be powerful benefits from having linked identification methodologies for the two purposes, such as policy coherence, monitoring and evaluation synergies, and the ability to update the targeting methodology and the targeting census instrument consistently across time. Taking the MPI as a benchmark poverty measure, they compare three alternative targeting methods that have been proposed in the case of India (the Saxena method, the scoring method and the Socio Economic Caste Census method) with a ten-item binary scoring method (in line with AF's identification strategy), in terms of how well they perform in identifying the set of poor. They find a significant better fit of the counting method over the other three. Thus the paper illustrates how a particular targeting method, based on the AF methodology, can be developed and justified, and how it might be linked with a national multidimensional poverty measure based on survey data.

Azevedo and Robles argue that although targeting mechanisms used by conditional cash transfer programmes have been generally successful in identifying the income poor, they have not fared equally well in identifying households that under-invest in human capital. Thus, they propose a multidimensional targeting approach to identifying beneficiaries that explicitly takes into consideration the multiple objectives of CCTs and the multiple deprivations of the poor household. They implement the approach to the case of the Oportunidades Program using data from the nationally representative Mexican household survey and compare it with the current targeting method and with an alternative income proxy-means test. The authors find that the multidimensional model is as good as the current one in identifying the income poor, however, it is significantly better at identifying households with deprivations that matter for the program objectives. Specifically, they find that beneficiaries chosen with the multidimensional method have higher rates of school non-attendance and higher levels of child labor. Moreover, an ex-ante evaluation shows that program transfers can have a greater impact on school attendance of potential beneficiaries selected by the multidimensional model relative to that of alternative targeting models.

Finally, Mitra's paper adapts the AF methodology to propose a governance (deprivation) index. The index draws from the 57 indicators corresponding to five dimensions used in the Mo Ibrahim Index of African governance, which seem to enjoy certain degree of consensus. The five considered dimensions are safety and security, rule of law, transparency and corruption, participation and human rights, sustainable economic opportunity, and human development. The proposed index sets cutoffs for each indicator and determines whether a country is deprived in each of them. In terms of identification, all deprivations count (in other words, a union approach is used). A *hybrid* approach is used for aggregation. In particular, for each country, deprivations are aggregated within each dimension, with a weighted 'count' whenever the indicators in that dimension are of ordinal nature (three out of the total five dimensions). In the case of dimensions which have mostly cardinal indicators (the remaining two dimensions), deprivations are aggregated with the weighted sum of the gaps. In a last stage the dimensional indices are averaged. The index is argued to be more methodologically sound than the Mo Ibrahim index as it does not introduce an arbitrary cardinalisation of ordinal variables. The proposed index also allows the results to be viewed in the form of a simple report card. This report card shows how well or how poorly governments are doing in an absolute sense (i.e. with respect to a normative cut-off) rather than merely relative to others. This facilitates transparency and enables the citizenship to monitor progress in their countries.

In sum, this special issue offers a variety of applications of the AF methodology exhibiting its flexibility of adaptation to poverty measurement in national as well as in cross-country comparable contexts, to monitoring poverty evolution over time, to targeting the poor and to governance evaluation, with comparisons of each of these applications to other available methodologies. Each paper also uses distinct sets of indicators, cutoffs and weights, which demonstrates the flexibility in choice of parameters. In this way there is a double value added in this issue. In the first place, each study introduces empirical findings which are of direct interest, and also demonstrate the distinctive insights that multidimensional poverty measures can shed, even when they are developed from datasets such as Oportunidades, the China Health and Nutrition Survey, or the Latin American household surveys which have been widely analysed using unidimensional analyses. Furthermore, the papers raise some considerations about appropriate indicators, weights and cutoffs that might be relevant in different applications, such as those to measure child poverty, or targeting exercises. In the second place, methodologically, most papers add value by applying different techniques such as dominance tests and robustness analysis with respect to weights and cutoffs. We hope that this issue illustrates the distinctive contribution that multidimensional poverty measures can make to the measurement of poverty and other multidimensional phenomena such as governance, and also motivates further research in this area.

References

Alkire, S. (2008) Choosing dimensions: The capability approach and multidimensional poverty, in N. Kakwani and J. Silber (eds) *The Many Dimensions of Poverty*, Palgrave Macmillan, Basingstoke.

Alkire, S. (2011) Multidimensional Poverty and its Discontents, in *Measure for Measure: How Well Do We Measure Development?* Proceedings of the 8th AFD-EUDN Conference. 2011.

Alkire, S. and J. E. Foster, (2007) "Counting and multidimensional poverty measurement", OPHI Working Paper No 7.

Alkire, S. and J. E. Foster, (2011a) "Counting and multidimensional poverty measurement", Journal of Public Economics, 95 (7-8): 476–487.

Alkire, S. and Foster, J. (2011b). "Understandings and misunderstandings of multidimensional poverty measurement": Journal of Economic Inequality 9, 289-314.

Alkire, S. and M. E. Santos, (2010) "Acute multidimensional poverty: a new index for developing countries". OPHI Working Paper Series, 38. http://www.ophi.org.uk/acute-multidimensional-poverty-a-new-index-for-developing-countries/

Alkire, S. and M. E. Santos, (2012) Acute multidimensional poverty: a new index for developing countries. Revised Version. OPHI Working Paper Series, forthcoming.

Alkire, S., Santos, M. E., Seth, S. and Yalonetzky, G., 2010. Is the Multidimensional Poverty Index robust to different weights?, OPHI Research in Progress. No 22a.

Alkire, S., Roche, J. M., Santos, M. E. And Seth, S. (2011) Multidimensional Poverty Index 2011: Brief Methodological Note ", Oxford Poverty and Human Development Initiative (OPHI). . Available at: www.ophi.org.uk/multidimensional-poverty-index/.

Alkire, S., Foster, J. E., and Santos, M. E. (2011), "Where did identification go?", Journal of Economic Inequality 9, 501-505.

Alkire, S. and Seth, S. (2013). Selecting a Targeting Method to Identify BPL Households in India. Social Indicators Research, this issue.

Asselin, L. M. (2009). *Analysis of Multidimensional Poverty. Theory and Case Studies*. IDRC. Springer Dordrecht Heidelberg London New York.

Atkinson, A. B. (2003). Multidimensional deprivation: contrasting social welfare and counting approaches. Journal of Economic Inequality 1: 51-65.

Atkinson, A.B. and Bourguignon, F., (1982). The comparison of multi-dimensional distribution of economic status. The Review of Economic Studies 49: 183–201.

Azevedo, V. And Robles, M. (2013), Multidimensional Targeting: Identifying Beneficiaries of Conditional Cash Transfer Programs. Social Indicators Research, this issue.

Batana, Y. (2013). Multidimensional Measurement of Poverty in Sub-Sahara Africa, Social Indicators Research, this issue.

Battiston, D., Cruces, G., Lopez-Calva, L. F., Lugo, M. A. and Santos, M. E. (2013) Income and Beyond: Multidimensional Poverty in Six Latin American Countries. Social Indicators Research, this issue.

Boland, P., Proschan, F., (1988). Multivariate arrangement increasing functions with application in probability and statistics. Journal of Multivariate Analysis 25: 286–298.

Boltvinik, J. (1992), El método de medición integrada de la pobreza. Una propuesta para su desarrollo. Comercio Exterior 42: 354-365.

Bossert, W., Chakravarty, S. R. and D'Ambrosio, C. (2009). Multidimensional Poverty and Material Deprivation. ECINEQ Working Paper 129.

Bourguignon, F. and Chakravarty, S. R.R. (2003). The measurement of multidimensional poverty, Journal of Economic Inequality 1: 25–19.

Chakravarty, S. R. and D'Ambrosio, C. (2006), The Measurement of Social Exclusion. Review of Income and Wealth 52: 377-398.

Chakravarty, S. R. and Silber, J. (2008). Measuring Multidimensional Poverty: The Axiomatic Approach in Kakwani, N. and Silber, J. (eds), *Quantitative Approaches to Multidimensional Poverty Measurement*. UNDP. Palgrave Macmillan. New York.

Chakravarty, S. R. Mukherjee, D. and Ranade, R. R. (1998). On the Family of Subgroup and Factor Decomposability Measures of Multidimensional Poverty, in Slottje, D. J. (ed.), Research on Economic Inequality 8: 175-1794. JAI Press, Stanford, CT and London.

Chiappero-Martinetti, E. and Roche, J.M. (2009) "Operationalization of the capability approach, from theory to practice: a review of techniques and empirical applications" in Chiappero-Martinetti, E. (Ed.) Debating Global Society: Reach and Limits of the Capability Approach. Milan: Fondazione Giangiacomo Feltrinelli.

Dag Hammarskjöld Foundation (1976) *What Now- Another Development*. The 1975 Report on Development and International Cooperation. Motala Grafiska, Sweden.

Feres, J. C. & Mancero, X. (2001). El método de las necesidades básicas insatisfechas (NBI) y sus aplicaciones a América Latina, Series Estudios Estadísticos y Prospectivos. Santiago, Chile: CEPAL.Foster, J.E., Greer, J., and Thorbecke, E. (1984)."A Class of Decomposable Poverty Indice", Econometrica 52: 761-766.

Gordon, D., S. Nandy, C. Pantazis, S. Pemberto and P. Townsend (2003). *Child poverty in the developing world*, Bristol, The Policy Press.

Herrera, A. O., H. D. Scolnik, G. Chichilnisky, G. C. Gallopin, J. E. Hardoy, D. Masovich, E. Oteiza, G. L. de Romero Brest, C. E. Suarez, and L. Talavera, (1976) *Catastrophe or New Society? A Latin America World Model*, IDRC-064e. Ottawa.

Instituto Nacional de Estadísticas y Censos (INDEC) (1984). *La Pobreza en la Argentina, Indicadoresde Necesidades Básicas Insatisfechas a partir de los datos del censo nacional de Población y Vivienda 1980*. Presidencia de la Nación. Secretaría de planificación.

International Labour Organization (ILO) (1976) *Employment, Growth and Basic Needs: a One World Problem.* Geneva.

Kakwani, N., Silber, J. (2008). *Quantitative Approaches to Multidimensional Poverty Measurement*. Palgrave Macmillan, Basingstoke.

Kannai, Y. (1980). The ALEP definition of complementarity and least concave utility functions. *Journal of Economic Theory* 22: 115–117.

Lemmi, A., Betti, G. (2006). Fuzzy Set Approach to Multidimensional Poverty Measurement. *Economic Studies in Inequality, Social Exclusion and Well-being*. Springer, New York.

Maasoumi, E. and Lugo, M. A. (2008). The Information Basis of Multivariate Poverty Assessments, in Kakwani, N. and Silber, J. (eds), *Quantitative Approaches to Multidimensional Poverty Measurement*. UNDP. Palgrave Macmillan. New York.

Mitra, S. (2013), Towards a Multidimensional Measure of Governance. Social Indicators Research, this issue.

Pattanaik, P., Xu, Y. (1990). On ranking opportunity sets in terms of freedom of choice. *Recherches Economiques de Louvain* 56: 383–390.

Robeyns, I. (2005) Selecting capabilities for quality of life measurement. Social Indicators Research, 74: 191–215.

Roche, J. M. (2013), Child Poverty Measurement: An assessment of methods and an application to Bangladesh. Social Indicators Research, this issue.

Santos, M. E. (2013) Tracking poverty reduction in Bhutan: Income deprivation alongside deprivation in other sources of happiness. Social Indicators Research, this issue.

Sen, A. K. (1992) *Inequality Reexamined*, Harvard University Press, New York, Cambridge.

Sen, A. K. (1999) *Development as Freedom*, Oxford: Oxford University Press.

Sen, A. K. (2009) *The Idea of Justice*, Allen Lane, London.

Sen, A. K. (1976): "Poverty: An Ordinal Approach to Measurement", Econometrica 44: 219-231.

Streeten, P., J. S. Burki, M. ul Haq, N. Hicks, and F. Stewart, (1981) *First Things First: Meeting Basic Human Needs in Developing Countries*. Oxford University Press, New York.

Townsend, P (1979), *Poverty in the United Kingdom*, Harmondsworth, Penguin.

Trani, J-F., Biggeri, M. and Mauro, V. (2013), The multidimensionality of child poverty: an empirical investigation of children of Afghanistan. Social Indicators Research, this issue.

Tsui, K. (2002). Multidimensional poverty indices, *Social Choice and Welfare* 19: 69–93.

UNDP (2010). *Human Development Report 2010. The Real Wealth of Nations: Pathways to Human Development.* Macmillan, New York.

UNEP/UNCTAD (1974) *The Cocoyoc Declaration*. *International Organization*, 29(3) International Responses to Technology (1975): 893-901.

Yalonetzky, G. (2011), A note on the standard errors of the members of the Alkire Foster family and its components. OPHI Research in Progress 25a. http://www.ophi.org.uk/wp-content/uploads/OPHI-RP-25a.pdf

Yu, J. (2013), Multidimensional Poverty in China: Findings based on CHNS. Social Indicators Research, this issue.