An Empirical Analysis of School Choice under Uncertainty

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Preliminary Draft. Comments welcome.

Abstract

Uncertainty about admission chances and constraints on the number of application choices are two integral aspects of most coordinated school assignment systems. And yet, there is sparse empirical evidence on the behavioral and welfare consequences of these features. This paper builds and estimates a model of optimal portfolio choice to quantify the effects of constrained choice and uncertainty. Using the Marginal Improvement Algorithm proposed by Chade and Smith (2006), our strategy allows us to handle cases with large numbers of students and schools and to recover underlying structural parameters. An application based on administrative data from Ghana finds that total welfare is a concave function of the number of ranked choices – expanding the number of choices from 1 to 2 increases total welfare by 26% and reduces the number of unassigned students by approximately 18%. Allowing students to submit an unrestricted number of choices generates three times as much total welfare as allowing for a single choice.

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1 Introduction

A large number of public school systems assign students to schools based on performance on a standardized examination and a submitted ranking of available options. Two common features of these systems are that students are uncertain about their admission chances and have constraints on the number of choices they can submit (Haeringer and Klijn, 2009). Truthful reporting of preferences is no longer a dominant strategy, which complicates the analysis of student welfare in the absence a behavioral model of school choice.

This paper proposes a novel strategy to identify students’ preferences in mechanisms that are vulnerable to manipulation. We take students’ reported ranking of schools as the outcome of an optimal portfolio choice problem (Weitzman, 1979). Our approach is the empirical counterpart to Chade and Smith (2006), who theoretically examine this complex multi-armed bandit problem. Our strategy allows us to handle cases with large numbers of both students and schools by explicitly modeling the initial portfolio choice process. Existing studies either consider a small number of available choices (Agarwal and Somaini, 2014; Walters, 2014) or abstract away from the portfolio choice problem and identify preferences using rank orderings within a submitted portfolio (Abdulkadiroglu et al., 2015).

Most school choice reforms are motivated by the fact that constraining the number of choices and forcing students to strategize is costly. The traditional approach in comparing the efficiency of various assignment mechanisms consists of running contextual experiments, with potential issues of external validity (Chen and Sonmez, 2006; Calsamiglia et al., 2010; Troyan, 2012). Since our model does not only consider students’ ranking of selected choices but also the fundamental decision-making process that gives rise to such decisions, we can use our model as a true laboratory, simulate individual behavior under a range of scenarios and estimate the welfare effects of alternative policy options. We conduct two classes of policy experiments. The first consists of relaxing constraints on the number of ranked choices. The second consists of reducing uncertainty.

The paper proceeds as follows. Section 2 describes our context and the data generated by the secondary school admission system in Ghana. Section 3 introduces our model of the optimal portfolio choice problem and describes our solution concept. Section 4 outlines our parametric assumptions and estimation strategy. Section 5 presents our results and policy simulations. Finally,
2 Secondary School Choice in Ghana

2.1 Background

The national school system in Ghana consists of six years of primary school, three years of junior high school, and three years of senior high school. Students completing junior high school apply for admission to senior high school through a centralized application system. Students apply to specific academic programs within a school and can submit a ranked list of up to six programs. Available programs include agriculture, business, general arts, general science, home economics, technical studies, visual arts, and several occupational programs offered by technical or vocational institutes. After submitting their ranked lists of choices, students take a standardized Basic Education Certification Exam (BECE). The application system then allocates students to schools based on their BECE scores and an assignment algorithm we describe in more detail when we discuss our policy experiments in Section 5.

As in many other coordinated school choice systems, students in Ghana are uncertain about their admission chances and constrained in the number of choices they can list. Students apply before taking the BECE so they do not know their exam scores when they submit their ranked lists. Moreover, schools do not specify the required scores for admission but instead only report the number of vacancies available in each academic program they offer. Admission cutoffs are endogenously determined by the distribution of vacancies and application choices in a given year.

2.2 Data

We use administrative data on the universe of senior high school applicants in Ghana’s centralized school choice system for our empirical analysis. The data include students’ exam scores, their ranked list of chosen programs, their admission outcomes, and basic demographics (gender, age, and the junior high school they attended). Our primary estimation sample focuses on the 2008 cohort of senior high school admits. Students could rank up to six choices from the 2,655 programs available in 618 schools that year. We have information on 130,060 students who submitted a
complete list of six programs ranked in order of preference.

Table 1: Student Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>0.577</td>
<td>1.000</td>
<td>0.494</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Age</td>
<td>16.610</td>
<td>16.000</td>
<td>1.686</td>
<td>9</td>
<td>54</td>
<td>130060</td>
</tr>
<tr>
<td>BECE exam score</td>
<td>292.770</td>
<td>285.000</td>
<td>50.434</td>
<td>185</td>
<td>469</td>
<td>130060</td>
</tr>
<tr>
<td>Mean BECE in JHS</td>
<td>292.270</td>
<td>281.800</td>
<td>40.164</td>
<td>199</td>
<td>432</td>
<td>130060</td>
</tr>
<tr>
<td>Attended public JHS</td>
<td>0.757</td>
<td>1.000</td>
<td>0.429</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Admitted to first choice</td>
<td>0.267</td>
<td>0.000</td>
<td>0.442</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Admitted to second choice</td>
<td>0.204</td>
<td>0.000</td>
<td>0.403</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Admitted to third choice</td>
<td>0.187</td>
<td>0.000</td>
<td>0.390</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Admitted to fourth choice</td>
<td>0.165</td>
<td>0.000</td>
<td>0.372</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Admitted to fifth choice</td>
<td>0.026</td>
<td>0.000</td>
<td>0.158</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Admitted to sixth choice</td>
<td>0.019</td>
<td>0.000</td>
<td>0.137</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
<tr>
<td>Administratively assigned</td>
<td>0.131</td>
<td>0.000</td>
<td>0.338</td>
<td>0</td>
<td>1</td>
<td>130060</td>
</tr>
</tbody>
</table>

Table 1 summarizes student demographics and admission outcomes. The majority of students are male and the median age is 16. Student performance on the BECE exam ranges from 185 to 469 points out of a possible 600, so students have very different chances of gaining admission to any given program. We do not have information on family background, so we use junior high school characteristics as our proxy of students’ socio-economic status. Junior high schools vary considerably in their average performance and 76 percent of students attended a public junior high school. Almost 27 percent of students were admitted to their first choice program, while less than 2 percent were admitted to their sixth choice. A sizable 13 percent of students were rejected from all six of their chosen schools and administratively assigned to an undersubscribed program at the end of the assignment process.

Table 2 summarizes school and program characteristics. There is substantial variation across programs. The average exam score of students admitted to each program in the previous year ranged from 180 to 466 points, with a mean of 266 and standard deviation of 58 points. The average program admitted 47 students but this ranged from 0 to 358. Each school offered an average of 5 programs, with some offering as many as 14 different programs. 90 percent of programs were offered by public schools, 55 percent of them were in schools with boarding facilities, and 94 percent were in coeducational schools. Only 7 percent of programs were offered by schools established by the British colonial administration before Ghana gained independence in 1957, and 22 percent of
Table 2: Program Characteristics

<table>
<thead>
<tr>
<th>Program Characteristics</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean BECE of admits</td>
<td>265.994</td>
<td>253.286</td>
<td>58.414</td>
<td>180</td>
<td>446</td>
<td>2655</td>
</tr>
<tr>
<td>Number of admits</td>
<td>47.210</td>
<td>40.000</td>
<td>48.097</td>
<td>0</td>
<td>358</td>
<td>2655</td>
</tr>
<tr>
<td>Programs offered</td>
<td>4.940</td>
<td>5.000</td>
<td>1.917</td>
<td>1</td>
<td>14</td>
<td>2655</td>
</tr>
<tr>
<td>Public</td>
<td>0.896</td>
<td>1.000</td>
<td>0.306</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Boarding facilities</td>
<td>0.552</td>
<td>1.000</td>
<td>0.497</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Coeducational (mixed sex)</td>
<td>0.942</td>
<td>1.000</td>
<td>0.233</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Pre-independence</td>
<td>0.066</td>
<td>0.000</td>
<td>0.248</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Religiously affiliated</td>
<td>0.217</td>
<td>0.000</td>
<td>0.412</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Technical/vocational inst.</td>
<td>0.106</td>
<td>0.000</td>
<td>0.308</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.123</td>
<td>0.000</td>
<td>0.328</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Business</td>
<td>0.165</td>
<td>0.000</td>
<td>0.371</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>General Arts</td>
<td>0.195</td>
<td>0.000</td>
<td>0.396</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>General Science</td>
<td>0.102</td>
<td>0.000</td>
<td>0.302</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Home Economics</td>
<td>0.150</td>
<td>0.000</td>
<td>0.357</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Technical Studies</td>
<td>0.056</td>
<td>0.000</td>
<td>0.229</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
<tr>
<td>Visual Arts</td>
<td>0.102</td>
<td>0.000</td>
<td>0.303</td>
<td>0</td>
<td>1</td>
<td>2655</td>
</tr>
</tbody>
</table>

1 programs were in schools with a religious affiliation.

Figures 1 and 2 represent descriptive statistics on students’ ranked program choices. We begin by examining the distance between a student’s junior high school and selected senior high school in Figure 1. We do not have exact coordinates for school locations so we measure the distance between centroids of the 110 administrative districts. Ghana’s school choice system is truly national and some students apply to schools as far as 450 miles away (roughly the distance from Boston to Washington, DC). Preferences for distance are convex. Students’ first choice programs are on average 35.1 miles away from their junior high schools and their second choice programs are 1.3 miles closer to them. Their third and fourth ranked choices are 31.4 and 27.1 miles away, but their last two choices are further away at a distance of 31.1 and 31.7 miles on average.

In contrast to preferences for distance, peer quality in ranked programs decreases monotonically. The average exam score of admits to a students’ first choice program is 343 and this falls to 273 for the lowest ranked choice. This is a difference of 1.2 standard deviations in the peer quality distribution. Considering preferences for distance together with preferences for academic quality, it

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1 We do not observe religious affiliation directly, so we classify schools as being Christian if they include any of the following words in their name: “apostle”, “adventist”, “anglican”, “bishop”, “catholic”, “church”, “Christ”, “faith”, “holy”, “King David”, “methodist”, “notre dame”, “our lady”, “pentecost”, “pope”, “presbytarian”, “prince of peace”, “queen of peace”, “reverend”, “sacred”, “salvation”, “seminary”, “seventh day adventist”, “sina”, “St.”, “wesley”, or “zion”; and as being Islamic if their school name contains the word “Ahmad”, “Islam”, or “Muslim”.
Figure 1: Distribution of continuous variables

Notes: Each panel plots the distribution of a given characteristic of programs listed for each choice. The horizontal lines represent the median, first and third quartile of each choice; the whiskers extend from each box edge to the nearest value within $1.5 \times$ the interquartile range; and the black dots indicate outliers.
appears that students are willing to travel for the opportunity to attend a high quality program but less willing to travel for their lower ranked, lower quality choices.

Program size also decreases monotonically for students’ lower ranked choices, although the gradient is not as steep as that for peer quality. The average first choice program admits 105 students and the typical sixth choice program admits 86. The average number of programs offered by schools is relatively constant across choices and ranges from 5.2 to 4.8 programs.

Figure 2 examines discrete program characteristics and reveals more patterns in aggregate preferences. Students prefer programs in boarding schools and have decreasing preferences for single sex schools and schools established before Ghana gained independence – 88 percent of students choose a program in a boarding school as their first choice and only 59 percent select one as their lowest ranked choice; 83 percent of students choose programs in mixed sex schools as a first choice but 98 percent do for their sixth choice; 27 percent of students choose programs in schools constructed before independence as a first choice but only 2 percent do for a sixth choice. Preferences for religious schools are more stable across choices, with roughly a quarter of students choosing programs in religious schools.

Finally, we illustrate students’ preferences over academic program tracks in the second panel of Figure 2. General arts is the most popular program track, with 40 percent of students choosing this program as their first choice and 43 percent choosing a general arts program as their sixth choice. General science has the steepest gradient in choices. 14 percent of students choose a general science program as their first choice and only 7 percent choose one as their sixth choice. Preferences for agriculture programs show the reverse pattern, with 6 percent of students choosing one as their first choice and 9 percent choosing one as their sixth choice. The remaining programs are relatively equally represented across choices with an average of 20 percent of students choosing business programs, 10 percent choosing home economics programs, 7 percent choosing visual arts programs, and 4 percent choosing technical programs.
Figure 2: Distribution of discrete variables

School characteristics

Variable
- Boarding
- Mixed
- Old
- Religious

Program choice

Variable
- Agric.
- Business
- Gen. Arts
- Gen. Sci.
- Home Ec.
- Technical
- Vis. Arts
3 Theory

3.1 Model

We define the school choice problem as a finite set of students $I = \{1, 2, \ldots, I\}$ who must choose to apply to a finite set of schools $J = \{1, 2, \ldots, J\}$, with the understanding that each student has to be assigned to one school. Each student is assumed to have observed attributes $X_i$ and a test score $\psi_i$, which is unknown when they submit their choices. Schools have an observable set of characteristics given by $Z_j$, and a fixed capacity denoted by $C_j$. The utility for an individual $i$ with characteristics $X_i$ matched with a school with attributes $Z_j$ is given by $U(X_i, Z_j)$. Students who do not apply to any school $j$ obtain a normalized utility $U_0$. We denote by $q_{ij}$ the admission chance of an individual with test score $\psi_i$ to a school with capacity $C_j$.

The optimal portfolio of an individual $i$ is denoted by $S_i$, with $N = \|S\|$. The optimal portfolio of an individual $i$ is denoted by $S_i$, with $N = \|S\|$.

$$\sup_S f(S_i) = \sum_{n=1}^{N} P(q_{in}|q_{i1}, \ldots, q_{i,n-1})U_{in}$$

Utility is assumed to take the additively separable form

$$U_{ij} = \gamma^0 Z_j + \gamma^1 d(l_i, l_j) + \sum_k \Gamma^0_k d(l_i, l_j) X^k_i + \sum_k \Gamma^1_k Z_j X^k$$

where the set of school attributes, $Z_j$, includes quality, size, number of programs, and indicators for boarding facilities, public, mixed sex, religious, old, and program track. These characteristics are summarized in Table 2. The set of individual characteristics, $X_i$, consists of individual test score, gender, age, middle school average test score and an indicator for attending a public middle school. These characteristics are described in Table 1. And $d(\cdot)$ provides the distance between student $i$’s location $l_i$ and school $j$’s location, $l_j$. In addition, we include interaction terms between distance, school characteristics, and individual attributes. Because we have a rich set of observable student characteristics, we can deal with idiosyncratic preferences.\(^2\)

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\(^2\)In our empirical application, we define a choice as a bundle (school,program). We have a total of 618 schools, each offering between 1 and 14 academic programs. This yields a total of 2,655 potential choices.

\(^3\)Allowing for a parametric error term, school unobserved component, or random coefficients complicates tremendously the individual problem because the student optimal portfolio is a highly nonlinear function of these terms. Thus, solving for the probability of submitting a school requires a numerical integration. Since an optimal portfolio of $N$ elements is different from $N$ optimal choices, any numerical integration requires an extremely high number of simulations to avoid extremal probabilities (even with 1,000 draws on a subset of our sample, we could not overcome this issue in a
Finally, to close the model, we describe how students form beliefs about their admission chance at each school. In a fully general equilibrium model, individual admission probabilities would be derived as the Bayesian Nash Equilibrium of the non-cooperative game. Since this equilibrium is unpractical with a large number of individuals, we propose a simplified model for determining admission chances. The aggregate true analog of individual admission probability is the endogenously determined cutoff test score for each school \( \widehat{\Psi}_j \), which results from the school-level capacity. We assume that individuals can have perfect foresight of this cutoff level using historical school admission data given by \( \Psi_j \). However, yearly heterogeneity in the composition of students may create a difference between these two cutoffs, such that the true cutoff is given by:

\[
\widehat{\Psi}_j = \Psi_j + \zeta_j
\]

where \( \zeta_j \) is an error term, which is left unrestricted for now. Similarly, since students submit school applications prior to taking the exam, their test score is measured with error \( \xi_i \), such that the true test score is given by:

\[
\widehat{\psi}_i = \psi_i + \xi_i
\]

Given the former definitions, the admission chance of individual \( i \) into school \( j \) is written as:

\[
P(\widehat{\psi}_i > \widehat{\Psi}_j) = P(\psi_i - \Psi_j > \eta_{ij}) = 1 - F(\psi_i - \Psi_j)
\]

where \( \eta_{ij} = \zeta_j - \xi_i \) with cumulative distribution function \( F(\cdot) \). Since a choice is observed only when an individual \( i \) selects a school \( j \), we can not separately identify the error associated with the measurement of individual test score and cutoffs. As a consequence, we do not take a stand on the respective contribution of individual and school level uncertainty.

Importantly, given this admission probability, the conditional probability of getting admitted to
a school with admission probability $q_{in}$ when the $(n - 1)$ bids have been unsuccessful is given by:

$$P(q_{in} | q_{i1}, \ldots, q_{i,n-1}) = P(\psi_i - \Psi_i > \eta_{in} | \psi_i - \Psi_1 < \eta_{i1}, \ldots, \psi_i - \Psi_{n-1} < \eta_{i,n-1})$$

(6)

$$= F(\psi_i - \Psi_{n-1}) - F(\psi_i - \Psi_{i})$$

(7)

Note that $P(q_{in} | q_{i1}, \ldots, q_{i,n-1}) = P(q_{in} | q_{i,n-1})$.\(^4\)

### 3.2 Solution Concept

In standard combinatory analysis, choosing 6 schools out of 2,655 requires computing a total of

$$\frac{2655!}{(2655 - 6)!} = 348,282,630,921,832,064$$

alternative portfolios for each individual, which is impracticable.\(^5\) Instead of computing the full set of potential portfolios, we use the Marginal Improvement Algorithm (MIA) proposed by Chade and Smith (2006). The idea is to construct the global optimum by sequentially iterating on a local optimum. Starting from a first local optimal based on maximizing expected utility, additional optima are sequentially selected using the choice that yields the highest marginal improvement from an initial portfolio. Consequently, the multiplicative nature of the combinatory problem is converted into a situation with an additive number of steps. As such, the MIA requires only 15,915 operations for our sample of 2,655 schools.\(^6\)

The algorithm proceeds as follows.

**Step 1:** In the first step, we select the first ranked choice as the school that yields the maximum expected utility. Formally, we solve the following problem:

$$S_1 = \arg \max_j q_j U_j$$

Then we store the first optimal portfolio as the school that yields the highest utility $U_1 = (U_1^*) = U_{j*}$, and the corresponding admission probability $Q_1 = (q_1^*) = q_j$.

\(^4\)As a consequence, we use this abuse of notation.

\(^5\)2,655 × 2,654 × 2,653 × 2,652 × 2,651 × 2,650

\(^6\)2,655 + 2,654 + 2,653 + 2,652 + 2,651 + 2,650
Step 2: In step 2, the following problem is solved:

\[
S_2 = \arg\max_{j \notin S_t} \left\{ \begin{array}{ll}
q^*_1 U^*_1 + \mathcal{P}(q_j|q^*_1)U_j & \text{if } U^*_1 \geq U_j \\
q^*_j U_j + \mathcal{P}(q^*_1|q_j)U^*_1 & \text{if } U^*_1 > U^*_j
\end{array} \right.
\]

where \( \mathcal{P}(q_j|q^*_1) = P(\hat{\psi} > \Psi_j|\hat{\psi} < \Psi_1) \) is the probability of being accepted in school \( j \) conditional on not being accepted in the first round choice. Given the outcome of this optimization problem, the optimal portfolio is updated \( U_2 = (U^*_1, U^*_j) = (U^*_1, U^*_2) \) with \( U^*_1 > U^*_2 \). Note that the portfolio is arranged in descending order of utilities. The corresponding set of admission chances is given by: \( Q_2 = (q^*_1, q^*_2) \)

Step 3: In the third step, we maximize over all schools that have not been selected in the previous two steps.

\[
S_3 = \arg\max_{j \notin S_2} \left\{ \begin{array}{ll}
q^*_1 U^*_1 + \mathcal{P}(q^*_2|q^*_1)U^*_2 + \mathcal{P}(q_j|q^*_2)U_j & \text{if } U^*_2 \geq U_j \\
q^*_1 U^*_1 + \mathcal{P}(q^*_j|q^*_1)U^*_j + \mathcal{P}(q^*_2|q^*_j)U^*_2 & \text{if } U^*_1 > U_j > U^*_2 \\
q^*_j U_j + \mathcal{P}(q^*_1|q_j)U^*_1 + \mathcal{P}(q^*_2|q^*_1)U^*_2 & \text{if } U_j > U^*_1
\end{array} \right.
\]

Then store \( U_3 = (U^*_1, U^*_2, U^*_j) = (U^*_1, U^*_2, U^*_3) \) with \( U^*_1 > U^*_2 > U^*_3 \).

Step 4: In the fourth step, we select the school that yields the highest utility among all remaining schools.

\[
S_4 = \arg\max_{j \notin S_3} \left\{ \begin{array}{ll}
q^*_1 U^*_1 + \mathcal{P}(q^*_2|q^*_1)U^*_2 + \mathcal{P}(q^*_3|q^*_2)U^*_3 + \mathcal{P}(q_j|q^*_3)U_j & \text{if } U^*_3 \geq U_j \\
q^*_1 U^*_1 + \mathcal{P}(q^*_3|q^*_1)U^*_2 + \mathcal{P}(q^*_j|q^*_3)U^*_j + \mathcal{P}(q^*_2|q^*_j)U^*_3 & \text{if } U^*_2 > U_j > U^*_3 \\
q^*_1 U^*_1 + \mathcal{P}(q^*_j|q^*_1)U^*_j + \mathcal{P}(q^*_2|q^*_j)U^*_2 + \mathcal{P}(q^*_3|q^*_2)U^*_3 & \text{if } U^*_1 > U_j > U^*_2 \\
q^*_j U_j + \mathcal{P}(q^*_1|q_j)U^*_1 + \mathcal{P}(q^*_2|q^*_1)U^*_2 + \mathcal{P}(q^*_3|q^*_2)U^*_3 & \text{if } U_j > U^*_1
\end{array} \right.
\]

Then store \( U_4^* = (U^*_1, U^*_2, U^*_3, U^*_j) = (U^*_1, U^*_2, U^*_3, U^*_4) \) with \( U^*_1 > U^*_2 > U^*_3 > U^*_4 \).

Step 5: The fifth step carries on using the same strategy as before.
\[ S_5 = \arg\max_{j \in \{1, \ldots, 5\} \setminus S_4} \left\{ q_1 U_1 + \mathcal{P}(q_2 | q_1) U_2 + \mathcal{P}(q_3 | q_2) U_3 + \mathcal{P}(q_4 | q_3) U_4 + \mathcal{P}(q_5 | q_4) U_5 \right\} \text{ if } U_4 > U_5 \]

Then store \( U_5 = (U_1^*, U_2^*, U_3^*, U_4^*, U_5^*) \) with \( U_1^* > U_2^* > U_3^* > U_4^* > U_5^* \).

**Step 6** : Finally, we maximize over the remaining schools to obtain a sixth choice.

\[ S_6 = \arg\max_{j \in \{1, \ldots, 6\} \setminus S_5} \left\{ q_1 U_1 + \mathcal{P}(q_2 | q_1) U_2 + \mathcal{P}(q_3 | q_2) U_3 + \mathcal{P}(q_4 | q_3) U_4 + \mathcal{P}(q_5 | q_4) U_5 + \mathcal{P}(q_6 | q_5) U_6 \right\} \text{ if } U_5 > U_6 \]

Then store \( U_6 = (U_1^*, U_2^*, U_3^*, U_4^*, U_5^*, U_6^*) \) with \( U_1^* > U_2^* > U_3^* > U_4^* > U_5^* > U_6^* \).

The optimization algorithm is extremely easy to implement. Each step constructs the utility associated with each school using simple operations.

### 4 Estimation

We estimate the model by Simulated Method of Moments.\(^7\) That is, we match the empirical characteristics of student ranked choices to their theoretical counterparts generated by the model. Formally, let us denote by \( \theta \) the set of parameters to be estimated. The criterion function is given by:

\[
\mathcal{L}(\theta) = -\frac{1}{2} (\hat{\theta} - \theta)^T \hat{\Sigma} \theta
\]

\(^7\)In a previous version, we implemented a Minimum Distance Estimator. However, this criterion function defined as a sum of 0 and 1 turned out to be highly convex, making the minimization extremely challenging.
where $\hat{m}$ is a set of empirical moments, and $\hat{W}$ is the weighting matrix.

Before computing the empirical moments, we need to empirically define the distribution of uncertainty shocks. Specifically, we assume that $\eta_{ij} \sim \Phi(0, \sigma_i)$. We parametrize the standard deviation of the error terms as follows:

$$\sigma_i = \exp(\sigma_0 + \sigma_1 z_i + \sigma_2 z_m)$$

where $z_i$ is the individual test score, and $z_m$ the quality of the middle school as measured by the average test score of students.

Under this parametrization, we hope to account for the fact that high achieving students may face less uncertainty regarding their performance on the final exam. Similarly, students from high quality middle schools are more likely to have teachers or parents who help them decide which schools to apply to.

Using admission chance probabilities and the marginal improvement algorithm, we can recover individual optimal portfolios and derive a ranked set of optimal schools for each student. We have a total of 84 parameters and 120 moments. We scale each of our continuous variables to range from 0 to 1 to smooth the criterion function.

The identification of our model is based on discrete choice theory. Our preference parameters are identified from heterogeneous individuals who make different choices. In contrast to the standard discrete choice theory, we can identify the admission chance probabilities using the nonlinearity of preferences in the utility function.

## 5 Results

The presentation of results is as follows. In the first part, we analyze the capacity of our model to fit the data. The second part is devoted to the implied utilities and admission chances generated by our model. The third part investigates the welfare consequences of relaxing constraints on the number of ranked choices, and reducing uncertainty.
5.1 Model Fit

Figures 3 and 4 present evidence on how well our model fits the data. In Figure 3, we use our model to generate the distribution of distance and quality for each portfolio and compare these with the real data. Our model fits the distribution of distance for each chosen school reasonably well. Both the data and model suggest that the least preferred school is the furthest away. The fit for the quality distribution of first ranked schools is quite good. However, the median quality of remaining choices is overstated, more seriously for lower ranked choices. Nonetheless, the model captures the sharp decline in school quality over ranked choices.

Figure 3: Actual and Predicted Distributions of Distance and Quality

Figure 4 presents more evidence on the model fit using additional program characteristics. The actual and predicted school profiles (boarding, mixed, old, and religious) are extremely close. We also predict the academic program choice patterns moderately well. Business and general arts are the most popular academic programs in both cases.
Figure 4: Actual and Predicted School Characteristics

School characteristics

Program choice

Program choice

Type
- Data
- --- Model

Variable
- Boarding
- Mixed
- Old
- Religious

Variable
- Agric.
- Business
- Home Ec.

Variable
- Gen. Arts
- Gen. Sci.
- Vis. Arts
5.2 Truthtelling Model

Our model of optimal portfolio choice assumes that students are strategic and rank schools partly based on their expected admission chances. An alternative model of choice behavior is one in which students are completely truthful and simply list the six programs that would give them the highest utility. In this model, students do not consider their admission chances but instead ignore the possibility of being unassigned after exhausting their six choices. To assess how well this truthtelling model fits the data, we estimate another set of parameters using the same Simulated Method of Moments approach described in Section 4 but assuming that students list the six programs with highest utility. [To be completed]

5.3 Implied Utilities and Admission Chances

Given how well the optimal portfolio choice model fits the data, we now move on to discuss its implications. As a starting point, we use our estimated parameters to infer the utilities and admission chances students assign to their chosen programs. Figure 5 shows that our model predicts students apply to schools that give them decreasing utility and increasing expected admission chances. On average, the median utility from first choice schools is approximately 32 compared to 22 for the lowest ranked choice. Meanwhile, the expected admission chance at a first ranked choice ranges from 0 to 15 percent in most cases, and the expected admission chance at a lowest ranked choice is almost 1.

Another way to examine the validity of our model is to compare its predictions for high and low ability students. One would expect that low ability students would be more affected by constraints and uncertainty because they have a higher possibility of being unassigned. We split our sample into students with exam scores in the top and bottom quartiles of the distribution and then separately analyze the choices of students in these two groups. High ability students apply to a more diverse set of schools, which offer higher utility on average. In contrast, there is substantially less variation in the utility of schools selected by low ability students and our parameters imply that these choices provide lower average utilities. Unsurprisingly, high ability students have higher admission chances at their top-ranked schools. Low ability students, however, apply to first choice schools where their expected admission chance is close to zero. Across the board, students have very high expected
Figure 5: Implied Utilities and Admission Chances

Notes: (i) High ability and low ability refer to students with respectively the highest and lowest 25 percent of test scores.
chances of admission to their fifth and sixth ranked choices.

5.4 Policy Experiments

Having examined the basic implications of our model, we turn to the task of simulating the welfare effects of two policies: i) expanding the number of choices students can submit, and ii) reducing students’ uncertainty about their admission chances. To assess the welfare impacts of each policy, we predict student choices and then simulate the assignment process to determine students’ admission outcomes. We then calculate the utility each student would derive from their assigned school and estimate total welfare under each regime. This section proceeds as follows. We first outline the assignment mechanism and specify our welfare function. We then discuss our simulation of the effects of increasing the number of choices and finally discuss our simulation of changing the degree of uncertainty about admission chances.

5.4.1 Assignment Mechanism and Welfare

Ghana’s school choice system uses a student-proposing deferred acceptance algorithm to assign students to schools in the spirit of the matching procedure derived by Gale and Shapley (1962). The algorithm proceeds as follows:

- Step 1: Each student $i$ applies to the first school in her ordered portfolio of choices. Each school $s$ tentatively assigns its seats to applicants one at a time in order of students’ exam scores, and rejects any remaining applicants once all of its seats are tentatively assigned.

- Step $k$: Each student who was rejected in round $k - 1$ applies to the next school in her ordered portfolio of choices. Each school compares the set of students it has been holding to the set of new applicants. It tentatively assigns its seats to these students one at a time in order of students’ exam scores and rejects remaining applicants once all of its seats are tentatively assigned.

- The algorithm terminates when no spaces remain in any of the choices selected by rejected students. Each student is then assigned to her final tentative assignment.
In practice, students who are unassigned at the end of the algorithm are administratively assigned to a program with remaining vacancies. In our simulations, we assign these students the outside option.

We propose a simple utilitarian welfare function $\mathcal{W}$ to aggregate individual utilities:

$$
\mathcal{W} = \sum_{i} U_i^*
$$

where $U_i^*$ is the utility individual $i$ derives from the school he was assigned. Total welfare is therefore the sum of individual utilities.

### 5.4.2 Number of Ranked Choices

To evaluate the effects of constraints on choices, we expand the number of choices students can submit incrementally from 1 to 7 and then allow students to submit an unlimited number of choices. Table 3 displays the change in utilities and Figure 6 plots the distribution of utilities under each regime. Total welfare is a concave function of the number of choices – expanding the number of choices from 1 to 2 increases total welfare by 26 percent and reduces the number of unassigned students by approximately 18 percent. Allowing students to submit an unrestricted number of choices generates three times as much total welfare as allowing for a single choice.

<table>
<thead>
<tr>
<th>Choices</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Unlimited</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number unassigned</td>
<td>77625</td>
<td>65666</td>
<td>57300</td>
<td>50436</td>
<td>45366</td>
<td>41237</td>
<td>37649</td>
<td></td>
</tr>
<tr>
<td>1st quartile</td>
<td>7.38</td>
<td>7.52</td>
<td>7.54</td>
<td>7.58</td>
<td>7.56</td>
<td>7.54</td>
<td>7.52</td>
<td>8.04</td>
</tr>
<tr>
<td>Median</td>
<td>10.2</td>
<td>10.4</td>
<td>10.6</td>
<td>10.7</td>
<td>10.7</td>
<td>10.7</td>
<td>10.8</td>
<td>11.1</td>
</tr>
<tr>
<td>Average</td>
<td>10.8</td>
<td>11.1</td>
<td>11.3</td>
<td>11.5</td>
<td>11.6</td>
<td>11.7</td>
<td>11.8</td>
<td>12.5</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>4.83</td>
<td>5.04</td>
<td>5.23</td>
<td>5.39</td>
<td>5.5</td>
<td>5.61</td>
<td>5.7</td>
<td>5.77</td>
</tr>
<tr>
<td>3rd quartile</td>
<td>13.4</td>
<td>14</td>
<td>14.2</td>
<td>14.5</td>
<td>14.7</td>
<td>14.8</td>
<td>15</td>
<td>15.7</td>
</tr>
<tr>
<td>Maximum</td>
<td>51.3</td>
<td>51.3</td>
<td>51.3</td>
<td>51.3</td>
<td>51.3</td>
<td>51.3</td>
<td>51.3</td>
<td>51.3</td>
</tr>
<tr>
<td>Total welfare</td>
<td>564687</td>
<td>713610</td>
<td>821222</td>
<td>912509</td>
<td>982298</td>
<td>1039532</td>
<td>1093358</td>
<td>1624961</td>
</tr>
</tbody>
</table>
Figure 6: Distribution of utilities

The figure shows the distribution of utilities for different choice sets. Each graph represents a different choice set with varying density along the y-axis and utility along the x-axis. The choice sets are distinguished by different colors: red for choice 1, blue for choice 2, and green for choice 6. The distributions indicate the likelihood of choosing a particular utility level for each choice set.
5.4.3 Uncertainty

To evaluate the implications of uncertainty about admission chances, we vary the parameters governing the distribution of uncertainty shocks. Recall that we define these shocks as $\eta_{ij} \sim \Phi(0, \sigma_i)$ and we allow the standard deviation of this distribution to vary by individual test scores and middle school quality. In our simulation, we first predict choices under the assumption of homogeneous uncertainty, assigning all students either the minimum, average, or maximum value estimated for $\sigma$. We then simulate the impact of increasing or reducing individual uncertainty by 1 standard deviation of our initially estimated parameters. [To be completed]

6 Conclusion

In this paper, we conduct two sets of policy experiments that simulate the effects of relaxing constraints on choices and reducing uncertainty. We find that there are substantial welfare improvements from allowing students to rank an unlimited number of choices. Given the large advances in computational capacity over recent decades, implementing this kind of policy is not particularly costly in practice.

Although we focus our analysis on two common features of coordinated school assignment systems, our empirical approach has broad applications. We estimate a model of optimal portfolio choice that allows us to conduct other experiments related to changes in mechanism design (such as restricting the categories of schools students can select from, using a lottery instead of merit-based procedure, or modifying the assignment algorithm) as well as experiments related to institutional reforms (such as increasing the capacity of high-performing schools or constructing new schools in remote areas). We leave this for future work.
References


