Appraisal accounting think-piece

Version 2

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For the Department for Transport

Introduction

This report responds to a call from the Department for Transport which raised the following key questions:

"1. Provide advice to the Department on how best to handle the approach to uplifting appraisal values, given the recent OBR forecast changes and all of the issues discussed in this specification document.
2. Enhance the Department's understanding of what the existing evidence base, including academic literature, has to say on these issues.
3. Provide a robust basis for the Department to develop new Transport Analysis Guidance in February 2021, on the subject of profiling growth in appraisal values over the longer term"

Consistent with our tender document, in this thinkpiece we briefly cover a range of issues that we think are relevant to the brief. In the first section, we consider issues related to growth and the relative price of transport infrastructure. In the second section, we introduce both macroeconomic and project uncertainty into the analysis and consider the implications of these for discount rates and the valuation of transportation projects.

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Summary Points:

1. **Growth**: The growth rate of per capita consumption in the SDR for the Green Book is based on an historical average between 1948 and 1999. It needs to be updated to reflect the current long-term view;

2. **Relative prices, the wealth effect and health discounting**: The value of the Value of Travel Time Saved (VTTS) "uprate" is an empirical question and should be estimated as a separate entity. This circumvents the question of which growth rate to calibrate the uprate around, and whether the uprate perfectly offsets the SDR wealth effect. Health benefits are discounted using pure time preference because they are measured directly in utility terms. This is therefore irrelevant for the VTTS, as are any purely normative arguments;

3. **DDRs**: DDRs do not reflect expected deterministic drops in growth, but rather uncertainty about future interest rates or growth. The Green Book term structure is not based on uncertainty around the preference parameters of the Ramsey Rule but instead primarily over market interest rates. Without additional rationales, the pure rate of time preference should not be declining with the time horizon. There is a strong argument for rooting the Declining Discount Rate (DDR) term structure in a detailed analysis of growth and its persistency;

4. **Relative prices and infinite values**: Infinite present values of VTTS are not a practical concern since time horizons of analysis are limited. If rapidly increasing present values with project maturity, caused either by uprating effects and / or DDRs, are perceived as a problem, then there are theoretical arguments, such as limiting budget shares, that could be drawn upon to cap these effects. Otherwise, high present values may accurately reflect the high expected future price of transportation benefits which have correctly been included in CBA;

5. **Levelling up**: Addressing the levelling up agenda in the SDR means evaluating social welfare separately for different regions. If levelling up through public investment (rather than through public transfers), projects which are financed in high growth areas and pay-off in low-growth areas should have a lower social discount rate. In theory the best levelling up approach is to invest in the highest return projects and use inter-regional transfers to reduce inequality, but this ignores institutional constraints and political economy issues.

6. **Project and Systematic risk and DDRs:** If the project benefits are uncertain and correlated with consumption growth, a systematic risk premium in the SDR may be necessary: a positive premium for a positive correlation and a negative one for a negative correlation. The Green Book argues that this systematic risk premium is small, but transport regulators apply an economically significant risk premium in practice. DDRs in the Green Book are based on risk free projects yet systematic risk premiums may also have a term structure which is increasing in the time horizon for positive correlations. The net effect on the term structure is project and parameter specific. In addition, uncertainty affects expected project benefits, altering the numerator as well as the denominator of the NPV equation.

7. **Catastrophic risk:** The Green Book discount rate incorporates an element, L, that is "an allowance for unpredictable risks not normally included in appraisal, known as 'catastrophic' and 'systemic' risk" (paragraph A6.9). But this combines a range of very different risks, from the possibility that society will no longer exist to enjoy a project's benefits, to catastrophic declines in consumption, to project failure. The potential for low probability catastrophic risk reduces the risk-free Social Discount Rate but may increase the SDR for highly pro-cyclical projects. Alternatively, some argue that investments which protect against catastrophe, such as climate change mitigation projects, should have very high present values because of the insurance they provide to society. It would be more consistent to treat each risks category separately rather than in a single term, L.

Section 1: Growth and Relative Prices

The Social Discount rate in the UK Green Book is based on the Ramsey Rule:

$$r = \rho + L + \gamma g \qquad (1).$$

This has 4 components which reflect the way in which social welfare is measured over time: the sum of discounted utilities for a representative (mean) agent: $W_0 = \sum_t exp(-(\rho + L)t)U(c_t)$. When utility takes standard power/logarithmic form, the four components are: 1) the pure rate of time preference, ρ; 2) a risk adjustment term, L; 3) the elasticity of marginal utility, γ; and 4) the growth rate of per capita consumption (c_t), g.

In this section, we focus on growth and how it should be estimated in the context of social discounting, and on whether specific categories of benefits should be affected in equal measure by the wealth effect in the SDR term , γg. The issue of declining discount rates and how that relates to relative prices (uprating) will also be touched upon.

1.1. Estimating growth for the Social Discount Rate

The Green Book uses an explicit social welfare function to inform intertemporal decision making. Welfare is dependent on consumption, and the benefits and costs of the projects being appraised are measured in units of consumption terms.[2] The growth rate in the Ramsey Rule should, in principle, reflect growth in consumption not income. As mentioned in the positioning note, ONS data show that consumption per capita has grown faster than GDP per capita since 1987, but only by 0.2-0.3%. This difference may therefore be of only limited relevance within a policy context, and would not be expected to persist on average in the long-run.

There are some circumstances in which using income growth instead of consumption growth in the SDR could be problematic; e.g. when growing (possibly non-marginal) damages from climate change drive an ever larger wedge between output and consumption at the aggregate level, thereby depressing consumption growth (Kelleher and Wagner, 2018). Another situation in which the data used are likely to be of consequence for operationalising the discount rate is when we are interested in the term structure of the discount rate. Estimating this requires an understanding of the persistence of the series, which may differ between consumption and income (GDP).

In the γg term, the discount rate captures the anticipated future welfare effects by predicting the economic "state of the world" for society in the future. The growth rate that captures this should be a long-run prediction for this reason. For policy purposes it ought not to change frequently, but it should be updated if it no longer accurately reflects the expected future state of the world.[3]

Predictions of growth may be *forward looking* using an econometric modelling approach or based on assessments by experts, or a combination of the two. The OBR forecast of GDP takes this forward-looking approach, first predicting the output gap, then making an assessment of how quickly the gap will close. An alternative approach is to rely on historical data under the assumption that the past is a good predictor of the future. The Green Book currently uses an historical rate to calibrate the SDR: 2% reflecting per capita consumption growth between 1948 and 1999. Table 1 shows that the historical growth rate is sensitive to the period of analyses, using per capita consumption growth data over different periods and ONS or Bank of England data

[2] It is frequently argued that the cost side should be "uprated" to reflect the costs of raising public funds, as is recommended in Ireland (See also Spackman, 2020).

[3] See e.g. Sunstein, 2014 on institutional inertia and political capital.

One final question, which is discussed further below in relation to the levelling-up agenda, is whose income is growing? Emmerling et al. (2017) discuss how per capita consumption growth can be driven by many different types of distributional effects. For example, per capita growth could be driven entirely by growth in the upper tail of the income distribution, leaving the median household no better off. Such inequality increasing growth has occurred in the US and the UK over the past 20 years. Alternatively, growth in per capita consumption could be accompanied by reductions in inequality, such as in France and Ireland. Emmerling et al. (2017) show that if there is inequality aversion in society, the appropriate growth rate should be adjusted to reflect these effects by the term: $g_{pc} - \gamma(g_{pc} - g_{med})$, where g_{pc} is per capita consumption growth and g_{med} is the growth of the median household income and γ is a measure of aversion to consumption/income inequality. Turk et al. (2020) expand the analysis to growth policy and we return to this discussion in subsection 1.5.

Table 1: Historical Per Capita Consumption Growth

Period	Per capita consumption growth
1991 - 2019 (ONS)	1.8%
2000 - 2019 (ONS)	1.3%
2009 - 2019 (ONS)	0.65%
1948 - 1999 (Groom and Maddison, 2019)	2.2%
1830 - 2009 (Groom and Maddison, 2019)	1%
2020 (OBR, Long-run forecast, labour productivity)	1.5%

Note: Groom and Maddison (2019) use data from the Bank of England.

Recommendations:

- **Growth:** There is a strong argument for updating the consumption per capita growth rate for the SDR in the Green Book. The current rate refers to a pre-financial crisis, pre-Brexit, pre-Covid 19 world, and so the information contained in the historical rates of growth from 1948-1999 most likely does not reflect the future trajectory of the economy, and hence the state of the economy in which future project costs and benefits will accrue. If purely historical rates are to be used then it is possible to extract information about long-term trends from shorter-time horizons (Muller and Watson, 2016), but a more up-to-date period of analysis is probably due. Otherwise, using the OBR rate, or that of some other independent body, might be another appropriate method. The rate estimated should be a long-run rate and subject to revision infrequently but periodically. In our opinion, the long-run 1.5% estimate of labour productivity growth seems like a good starting point for the revision.
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- **Social Discount Rate (SDR):** Coupled with an updated growth rate, the SDR should also update the estimate of γ. Groom and Maddison (2019) illustrate the latest evidence for the UK that γ = 1.5. Coupled with an updated growth rate of 1.5%, this would mean no change to the medium term SDR: it would remain at 3.5% other things equal.

On the question of which growth rate to use for uprating versus discounting, the next section makes clear that the rate at which uprating occurs is an empirical question, which may or may not be best related to growth. For instance, an uprating for the Value of Transport Time Saved could arise in the absence of growth because of changes in time-saving technology. Rooting the change in VTTS over time in income or consumption growth may not or may not be appropriate.

1.2. Relative pricing and "Uprating" in CBA

Uprating refers to taking into account the relative prices associated with particular benefit or cost components of the project under appraisal. The potential to uprate the value of transportation benefits over time is reminiscent of similar practices proposed for the environment and health. Yet, as noted in the concept note, the motivations in each case are rather different. This subsection uses the environment as an example, the principles from which can be applied to health, VTTS, and other sectors.

For the environment/environmental quality the arguments are organised around three key issues:

> i) **Non-marketed good**: The non-marketed nature of environmental goods and services, hence the need for a careful and separate analysis of the shadow price and how it evolves over time;
>
> ii) **Environmental scarcity**: There is a structural reason why the shadow price may vary over time due to environmental degradation and associated physical scarcity of natural resources;
>
> iii) **Non-substitutability**: Environmental resources may not have close substitutes. As Krutilla (1967) puts it: "While the supply of fabricated goods and commercial services may be capable of continuous expansion for a given resource base by reason of scientific discovery and mastery of technique, the supply of natural phenomena is virtually inelastic."

These issues are primarily matters of fact which lend themselves to an empirical approach to find out how physical changes are taking place and the preferences over these changes (e.g. Venmans and Groom, 2019; Baumgartner et al. 2015; Drupp 2016).

These rationales for uprating should be seen as distinct from that found in the guidelines for the valuation of health in the Green Book which we return to below, although both approaches can be put into the same formal framework.

Relative prices and uprating: Brief formal analysis

If we are concerned about a particular category of benefits, say, environment, then formally that can be separated out in the welfare function and an expression for the shadow prices of both consumption, C, and environment, E, can be analysed. Hoel and Sterner (2007) show that when we treat C and E separately in the utility function: $U(C_t, E_t)$ the shadow price of a marginal unit of good *i = C, E* at time *t* becomes:[4]

$$P_i(t,0) = \frac{U_i(C_t, E_t)}{U_i(C_0, E_0)}\, exp(-\rho t) \quad for \quad i = C, E$$

The rate of change of this shadow price for consumption, the numeraire in CBA, is the SDR. The rate of change for the shadow price of the environmental commodity *E* (not typically the numeraire in CBA) is also a discount rate. The rates of change of the respective shadow prices, i.e. SDRs, are given by:

$$SDR_C(t) = \rho + \gamma_{CC} g_C + \gamma_{EC} g_E \qquad (2)$$

$$SDR_E(t) = \rho + \gamma_{EE} g_E + \gamma_{CE} g_C \qquad (3)$$

Where $\gamma_{ij} = -j\frac{U_{ji}}{U_i}$ for $i = C, E$; that is these are the equivalent elasticities and cross-elasticities of marginal utility that were simply captured by γ term in the single good SDR framework: $SDR = \rho + \gamma g$. Each SDR measures the rate of change in the shadow price of the respective commodity.

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[4] Where Ui is the derivative with respect to argument *i.*

Importantly, when values are placed in terms of the common numeraire for CBA, consumption C, the marginal willingness to pay for a unit of the environment E (its price) is $\frac{U_E}{U_C}$. Denoting the rate of change of the price of E compared to C as $\frac{\left(\frac{d\,U_E}{dtU_C}\right)}{\frac{U_E}{U_C}}$, simple algebra shows that this relative price change, $\Delta RP_{EC}(t)$, is given by:

$$\Delta RP_{EC}(t) = (\gamma_{CC} g_C + \gamma_{EC} g_E) - (\gamma_{EE} g_E + \gamma_{CE} g_C) \qquad (4)$$

But this is just the difference between the two discount rates: (2) – (3) (Weikard and Zhu, 2005). Dual discounting and relative price adjustments are equivalent. It should be recognised, though, that the theoretical approach is essentially treating the “environment” heroically either as a single composite good, in the same way that consumption is treated as a composite of many goods, or as a single element of the environment. This is a strong stylisation that serves only to make the theoretical point.

Expression (4) justifies an uprating, and the relative price changes reflect the practical characteristics (described above) which motivate the focus on relative price changes and uprating in the first place. For environment these were: (i) non-marketability; (ii) increasing scarcity: reflected by g_E; and, (iii) substitutability: reflected by the social preference parameters γ_{ij}.

Expression (4) is complicated so consider two simple examples:

Example 1: Cross elasticities are zero. In this case the utility function is additively separable $U(C_t, E_t) = U(C_t) + V(E_t)$, hence the marginal utility of C does not depend on E or vice versa. Changing relative prices are now given by:

$$\Delta RP_{EC}(t) = \gamma_{CC} g_C - \gamma_{EE} g_E \qquad (5)$$

which is the difference between two “wealth effects” and depends on the relative growth of each commodity and how growth affects marginal utility. For environmental goods we would want to remove this rate of change from the SDR for consumption, leaving $SDR_E(t) = \rho + \gamma_{EE} g_E$ as the effective discount rate for E. Calibration of this discount rate requires estimating the growth of E and γ_{EE}; e.g., as in Venmans and Groom (2020).

Example 2: CES Utility. If $U(C_t, E_t) = \frac{1}{1-\gamma}\left[\alpha C^{1-\frac{1}{\varphi}} + (1-\alpha)E^{1-\frac{1}{\varphi}}\right]^{\frac{\varphi(1-\gamma)}{\varphi-1}}$ then the change in relative prices becomes the following:

$$\Delta RP_{EC}(t) = \frac{1}{\varphi}(g_C - g_E) \qquad (6)$$

Where φ is the elasticity of substitution between C and E measuring how easy it is to compensate a loss in E with a gain in C. This illustrates clearly the importance of substitutability. If φ is small (large) then, for a given difference in growth rates, relative prices of E will diverge quickly (slowly) reflecting rapidly (moderately) increasing scarcity values. If E is perfectly substitutable then there will be no relative price effect since E is not really economically scarce.

This brief theoretical overview allows us to think theoretically about a number of questions concerning uprating of components in a CBA.

How to view the Green Book Health Discount Rate

The recommendation in the Green Book is that health should be discounted solely at the rate of pure time preference, ρ plus the term L which takes into account various dimensions of catastrophic and other hazards, that is, ignoring the wealth effect int eh Ramsey SDR. In terms of the above framework, labelling E as health, this recommendation can be understood in several ways: 1) As purely an outcome of social preferences over health and consumption; 2) in normative or ethical terms; 3) In terms of the units of measurement for Health E.

1) Social preferences for health

One justification for using only the pure rate of time preference for health might be that it is a result of the form of social preferences. For instance:

- Utility: $U(C_t, E_t) = U(C_t) + \theta E_t$ is linear in Health (e.g. sick days);[5]
- SDR for consumption: $SDR_C(t) = \rho + \gamma_{CC} g_C$;
- SDR for health: $SDR_E(t) = \rho$;
- Change in relative *prices* ("uprating") for health (measured in consumption) is the difference between the good specific discount rates: $\Delta RP_{EC}(t) = \gamma_{CC} g_C$;
- So the net consumption discount rate for health is: $SDR_C(t) - \Delta RP_{EC}(t) = \rho$

With this interpretation, the substitution of health for consumption is not perfect, the marginal utility of consumption decreases with consumption, and the relative price increases and perfectly offsets the wealth effect. Whether or not the relative price growth term is in reality equal to $\gamma_{CC} g_C$ is an empirical question, not a normative one. The empirical approach would be to estimate the relative growth of marginal willingness to pay for health changes over time, , and use this as the uprating for health benefits. If it turns out that this rate perfectly offsets the wealth effect, then that would merely be an empirical coincidence. See Gollier and Hammitt (2014) for a discussion of relative prices in a health context.

2) A Normative position on health?

Another approach is to simply argue that health ought not to be treated differently across different income groups. Looked at via a social welfare function, the normative stance is to posit a welfare function of the form $U(C_t, E_t) = U(C_t) + \theta E_t$, as above, which is linear in health. On reflection this seems like a peculiar normative position since it suggests that in relation to health there is no inequality aversion: we do not prefer to give a unit of health to someone with low health compared to someone with high health. Not only is it a peculiar normative stance, it is also refuted by empirical evidence (Dolan and Tsuchiya 2011). One issue that also arises here is that intra- and inter-temporal inequality aversion are not necessarily the same. High inequality aversion intra-temporally leads to a high SDR, yet there is evidence that people feel differently about inequality in these different dimensions (Venmans and Groom 2020; Emmerling et al. 2017)

3) Units of Health (QALYs): The Green Book position on Health Discounting?

However, the Green Book position on health related discounting is as follows (paragraph A6.16):

> "*The recommended rate for risk to health and life values is 1.5%. This is because the 'wealth effect' or real per capital consumption growth element of the discount rate is excluded. As set out in Annex 2 [sic, actually Annex 1], health and life effects are expressed using welfare or utility values such as Quality Adjusted Life Years (QALYs), as opposed to monetary values.*"

[5] This is one possibility, but there are other ways in which the terms of the SDR for consumption could be perfectly offset by the relative price effect. E.g. if in equation (4) it were the case that $\gamma_{EC} g_E - (\gamma_{EE} g_E + \gamma_{CE} g_C)$ = 0.

This is an entirely different argument to the previous ones, and refers to the units which the relevant health values are measured in. Since QALYs are measured in *utils,* that is utils are the numeraire, the appropriate discount rate is the pure rate of time preference, ρ. Importantly, this is a completely different argument to the uprating argument.

Recommendation: The rationale for discounting health at the pure rate of time preference only is specific to health and the units in which QALYs are measured. The rationale has some theoretical justification: it is correct that utility should be discounted using a utility discount rate, yet the actual application, which involves a monetary valuation of QALYs, remains ad hoc. As a side note, it is also questionable whether the catastrophic risk components are either a) well estimated by a 1% premium; or, b) relevant at all to the health type outcomes (issue of catastrophic risk are discussed below). In short though, VTTS should follow its own theoretical and empirical approach.

1.3. Relative prices and "Uprating" in transport

The uprating approach for VTTS falls into the social preferences category. There is no particular normative argument for treating VTTS differently to other classes of benefit and costs and it is certainly not measured in terms of utility directly. Estimating the uprating of the VTTS is therefore an empirical question. The willingness to pay can be estimated from revealed preference or stated preference studies in order to calculate the uprating, as in Abrantes and Wardman (2011). The rate of change of VTTS over time will depend on similar issues of substitutability, growth and preferences to the environmental example above. The empirical work would usually estimate $\Delta RP_{EC}(t)$ directly by revealed or stated preference estimates of willingness to pay, rather than the separate theoretical component parts (own and cross price elasticities of marginal utility and so forth).

It has been pointed out to us that VTTS as a derived demand, rather than an argument in utility per se, ought to receive a separate treatment in the relative prices framework discussed in section 1.2. DeSerpa (1971) makes the point VTTS is only likely to be valued by the value of leisure itself when the time spent travelling is greater than some minimum amount required to achieve the ends associated with travel, e.g. work. If the minimum time requirement is binding, then time may be valued differently in different uses. For instance, imagine that the minimum time required to go to and return from work, T_t^w, is dependent on the constraint that $T_t^w \geq b_t X_t$, where X_t is the good that is demanded and b_t is a measure of how much time is required to obtain a unit of X_t, if this constraint is binding then effectively utility can be written as: $U(C_t, T_t) = U(C_t, b_t X_t)$, reflecting that the time allocated to X_t, is not *free* time, but rather necessary time.

Applying the logic of relative prices to this framework where the constraint on time is binding, the relative price change for the VTTS then becomes:

$$\Delta RP_{TC}(t) = \frac{\frac{d}{dt} U_T/U_C}{U_T/U_C} = \gamma_{CC} g_C - \gamma_{TT}(g_b + g_X)$$

where the right hand term reflects how the constraint changes over time. Improved time saving technology is reflected by $g_b < 0$ and increasing consumption of the good requiring time is reflected by $g_X > 0$. Notice that in this framework, relative prices only increase exactly in accordance with the wealth effect, $\gamma_{CC} g_C$, if $g_b + g_X = 0$, or $\gamma_{TT} = 0$. Both could be true. It is an empirical question.[6]

The point being, that the relationship between the VTTS and the growth in income or consumption is only a partial motivation for the uprating/relative price correction. More directly important are the specific preference-related and structural attributes of the VTTS. For instance, imagine a study had been undertaken which showed that VTTS was independent of growth in income, but steadfastly increased with time, due to

[6] Note that according to DeSerpa (1971) the value of time saved is composed of two elements, the value of time in other uses, and the value of time in the particular use. The elasticity γ_{TT} should be interpreted as embodying both effects of changes in time allocated. More work is needed on this to see whether the individual components could be easily characterised.

an aging population, or because preferences are changing over time. An analysis of relative prices would still be valid, but it would be independent of growth, however it is measured, and the relative price correction may or may not cancel the wealth effect of the discount rate.

Finally, note that in the convenient case of equation (6): $\Delta RP_{EC}(t) = \varphi^{-1}(g_C - g_E)$, the parameter φ, the elasticity of substitution, can be estimated by the inverse of the elasticity of marginal willingness to pay for the environment or health. Drupp (2017) has many examples in the context of the environment, and shows that the empirical evidence suggests an estimate of φ^{-1} of around 0.5. Using stated preference experiments, Börjesson et al. (2020) suggest that φ is equal to 1 (but also varies with income), so φ^{-1} is also equal to 1. This just leaves the components of growth to estimate: growth in consumption (g_C) and growth in travel time saved ($g_b + g_X$) in this case. One uninformed prior is that both these two components are quite small. This would lead to an increase in relative prices which increases at the rate g_C. This is close to the DfT's current approach.

Recommendation on uprating and growth: There are a number of recommendations that follow:

- **Wealth effect?** There is no reason to simply ignore the wealth effect of the Ramsey discount rate when it comes to VTTS uprating. As discussed for health, this practice stems from very different assumptions concerning the units (utility) in which health benefits are measured. Such a cancellation would only be appropriate by coincidence.

- **Empirical Evidence 1**: To inform the uprating of VTTS, evidence is required on the way in which relative prices for travel time change over time. If it is found that VTTS is increasing at the same rate as the wealth effect, $\gamma_{CC} g_C$, then this will cancel the wealth effect of the Ramsey Rule.

- **Empirical Evidence 2**: There is some evidence that VTTS is increasing with income. Only if this is a relative price change compared to consumption should this fact be used to uprate VTTA for CBA. If growth in income is the best predicter for future relative prices, then growth can be used to uprate prices. For consistency, the growth rate for VTTS should be the same as the growth rate in the Ramsey discount rate. Both are trying to predict the future state of the world and evaluate welfare in that future state.

- **Growth 1**: The OBR growth rate varies with the time horizon, but the growth rate for the Ramsey discount rate does not. For consistency, if growth is the forecast with which future prices will be predicted, then the future scenario for growth that is used for uprating should be the same as for the SDR. This means the consumption growth rate.

- **Growth 2**: There is a strong argument for revisiting the growth rates used for the Ramsey discount rate due to the historical period covered, and the recent structural changes in the economy. A 1.5% growth rate would be a good first step. The growth rate that informs the SDR is based on the post war period until 1999, which seems out of date now.

1.4. Declining discount rates: History, explanation and response to queries

The Green Book term structure of declining discount rates (DDRs) arose from a survey paper (Oxera, 2002) and subsequent academic contributions (Pearce et al., 2003; Groom et al., 2005). The term structure in the Green Book was based on empirical work by Newell and Pizer (2003) and Groom et al. (2007), which were calibrated by uncertainty in the interest rate in the US (!). In short, the term structure was developed in an approximate manner (rate of decline, and stepped schedule) and was never calibrated clearly to UK data on interest rate or growth uncertainty. Yet the approach has a clear theoretical basis within the overall Ramsey Rule discounting framework of the Green Book, so basing DDRs on a consumption-based approach would appear to be most theoretically consistent.

Why is this important? First, the DDRs in the Green Book are certainly not based on uncertainty in the parameters of the Ramsey Rule: ρ and γ. These parameters reflect the utility of the representative agent and therefore are scalars and not stochastic variables and so the original motivation for the DDRs found in Oxera (2002) did not rely on declining ρ or γ. Years after, Heal and Millner (2014) did describe a motivation for DDRs based on the pure rate of time preference only, based on expert disagreements, but this has never been deployed in the Green Book. The arguments for a DDR probably should not apply to health benefits for this reason.

Second, re: Figure 3 and 4 in the briefing note, DDRs are not determined by an assumed drop in growth at a particular date in the future. Whichever theoretical backdrop is deployed (interest rates or growth) the DDR term structure is based on the fact that there is uncertainty about the state of the world in the future rather than because there is a deterministic drop of growth or interest rates. As mentioned above, there is a model of declining growth which determines DDRs in Gollier (2012, Ch 4) but this assumes above trend incomes now. An increasing term structure of discount rates can arise when income is below trend.

Overall, it is true that the DDRs could be more empirically based. For instance, Groom and Maddison (2019) estimated a Ramsey type DDR model from Gollier (2012) on UK data on consumption growth. The persistence of this time series determines the DDR schedule. They found that the persistence in growth was limited and only warranted a decline of 0.5% in the long run (200+ years), rather than 2.5% as in the Green Book. Further work is required to properly characterise the DDR schedule in our view. We return to this issue in the context of systematic risk in Section 2.

Uprating and DDRs

In the emails that we received prior to the 2003 Green Book, in which the DDRs were first adopted, the issue of infinite values was raised in the case that the uprating factors actually outweigh the discount rate at some point in the future. This would be the situation in which the relative price growth term would be greater than the consumption discount rate. In terms of the CES model above, it would mean:

$$SDR_C < \frac{1}{\sigma}(g_C - g_E) \qquad (7)$$

at some point in time. With the proposed extended time horizon, it has again been perceived as a potential problem for CBA of transport projects. There are several possible responses:

1) DDRs do not introduce this issue, they just make it more likely;
2) The problem of infinite values only exists at the limit: infinite time. It cannot be an issue over the 125 year time horizon that the Green Book recommends for standard discounting-NPV analysis.
3) It is, however unrealistic to expect relative prices to increase greater than growth in incomes ad infinitum. In such cases, budget shares for the good in question would exhaust the budget if there were no change in the relative price trajectory. The fact is there would be a correction to prices or consumption that would halt the price rises in the future in most cases, within the bounds of substitutability;
4) The current approach probably is not that problematic. However, it might be capped to reflect subsistence needs in other areas of household consumption.

The point is that the uprating is supposed to be capturing the social value of a cost or benefit in the future. If that component is growing in value, this should be reflected in the social cost benefit analysis, but cannot become infinite within any realistic practical scenario.

Recommendations:

- **Growth and SDR:** Should the Ramsey discount rate be reduced because of the lower OBR growth predictions for longer horizons? Dietz (2008) seems to suggest that this is what should happen since this is what happens in the Stern Review's Page model. But the Stern Review uses an Integrated Assessment Model which endogenises growth. CBA is marginal analysis and treats growth as exogenous. Nevertheless, the growth rate used in the SDR should be the best prediction. If there is a strong belief that growth will be declining in the future this should be embodied in the SDR since this is the best prediction of future consumption. Gollier (2012, Ch 4) provides a model of time varying discount rates under uncertainty that shows how when the economy is seen as above trend, future discount rates should be declining, and vice versa when below trend. Groom and Maddison (2019, footnote 14) explain this point. So, if there were a strong sense that growth is in decline in the long-run, a declining growth rate would be justified.

- **Growth and the SDR 2:** the DDR term structure should be rooted in a specific analysis of UK growth persistence rather than the rather ad hoc uncertainty motivation that currently exists.

1.5. What is the growth rate for discounting when the government aims to level up?

The discount rate that is recommended in the Green Book has been calculated based on the Simple Ramsey Rule. This, in turn, is founded on the existence of a representative agent whose consumption grows at a rate, g; a value that is incorporated in the Ramsey equation. The existence of a representative agent depends on there being complete insurance markets that allows for all individual risk to be traded away. Under this assumption, all individuals will take full advantage of these markets and experience identical consumption growth rate (but not all will have the same consumption level). This allows for a single rate g to be used within a discounting framework as this is the growth rate that all individuals experience.

More recently, there has been Government interest in the levelling-up agenda. This reflects the fact that, in practice, different regions of the UK not only have different consumption levels but also different growth rates. Allowing for this observation invalidates the representative agent assumption used to derive the Simple Ramsey Rule. Here, we set up a very simple alternative theoretical framework that allows for the modelling of two separated regions with different representative agents in each. This enables us to draw some broad conclusions about how the levelling-up agenda might adjust the discount rate.

Consider a simple economy with two regions, A and B, with one representative agent in each. Without loss of generality, region A consumes at least as much as region B at time zero. The Social Planner places equal weight on each region in the Social Welfare Function. We consider the social discount rate when a project is funded across the two regions, and where the benefits potentially occur over both regions, but where these distributions need not be equal. Following standard Ramsey-style analysis, there is no uncertainty within this economy.

Let consumption in regions A and B at time 0 be given by $c_{0a} \geq c_{0b}$. Let $g_a = ln(c_{1a}/c_{0a})$, with a similar expression for g_b, represent the regional growth rate in consumption between time 0 and 1. $g = ln((c_{1a} + c_{1b})/(c_{0a} + c_{0b}))$ denotes the per-capita growth rate across the two regions combined.
Assume there is a project that requires initial investment, p, which will be paid for by region A paying $\pi_a p$ and region B paying the remainder; $\pi_b p = (1 - \pi_a)p$. The project's monetised social benefits are denoted by d and occur at time 1. These benefits are shared $\delta_a d$ to region A and $\delta_b d = (1 - \delta_a)d$ to region B.
In this case, if the utility function has standard constant relative risk aversion power / log form, with rate of pure time preference ρ and elasticity of marginal utility γ, the adjusted version of the simple Ramsey Rule is:[7]

$$r = \rho + \gamma G$$

[7] This result has been derived as part of our response to this commission and the proof is available on request.

$$G = g - ln\left[\frac{1 + c_{1b}/c_{1a}}{1 + c_{0b}/c_{0a}}\right] + \frac{1}{\gamma} ln\left[\frac{\pi_a + (1 - \pi_a)(c_{0a}/c_{0b})^\gamma}{\delta_a + (1 - \delta_a)(c_{1a}/c_{1b})^\gamma}\right]$$

A sufficient condition for this to collapse to the Simple Ramsey Rule occurs under two assumptions. First, growth is equal in the two regions, $c_{1b}/c_{1a} = c_{0b}/c_{0a}$. Second, the division of benefits between the two regions must be matched by the division of initial spending: $\delta_a = \pi_a$. There is no requirement that either the two regions are equally wealthy ($c_{0a} = c_{0b}$), or that each benefits equally from the project ($\delta_a = 0.5$). This very simple analysis reveals that the discount rate depends on the relative initial consumption levels of the two regions, the growth rate in consumption in the two regions, where the money is derived from to invest in the project, and the region where the benefits accrue. The discount rate will be lowest when taxation is taken from the highest wealth region to fund a transportation project where the benefits accrue to the lowest wealth region. Within this very simple one-period framework, the swings in discount rate can be highly, and unrealistically, dramatic even for relatively small changes in reasonable parameter choices.

This adjustment to the discount rate bears similarities to the sub-national and regional distributional weights adjustment described in paragraph 5.74 and expanded upon in Annex 3 of the Green Book. This provides an alternative mechanism through which benefits to less well-off parts of society can be given greater weight in the social welfare analysis of public projects. The mechanism is, though, different, as here it is the numerator of the NPV equation that is altered (by giving higher benefit weightings to poorer regions) rather than through the discount rate. While qualitatively similar, the quantitative results will not be the same. Section A3.12 of the Green Book says that the impact on society is the change in income for the main beneficiary group multiplied by a weighting factor added to the change in income to the taxpayer group, which is assumed to be the median income group. The weighting factor divides equivalised income for the median household by that for the benefit group and raises this to the power of γ (A3.11). There are a number of key differences to applying this rule rather than adjusting the discount rate, including: (i) It is assumed that the tax payer is the median income household (A3.10) rather than tax falling largely on the wealthier, (ii) the time delay between taxation being raised and the benefit being realised is not factored into the analysis, (iii) $\gamma = 1.3$ in these distributional weights (A3.11) rather than $\gamma = 1$ in the derivation of the Ramsey Rule discount rate.

To illustrate the levelling-up discount rate effect with a more realistic example, suppose that Region A has consumption of c_{0a} =£24,545 and a growth rate of $g_a = 2.25\%$. Region B has initial consumption of c_{0b} =£15,727 and a growth rate of g_b =1.75%. The consumption levels broadly represent household expenditure in London and the North East from Table 2 here. The aggregate growth rate in consumption is then g =2.06%. We set the pure rate of time preference at $\rho = 0.5\%$ and the elasticity of marginal utility of γ =1. The Simple Ramsey Rule discount rate is 2.56%, very similar to the Green Book rate when ignoring the L −effect for project and macroeconomic risks (see Section 2).

We consider a major transport programme, taking 25 years, funded by a payment p in each of the years 0-24. Because of the initial wealth differential, Region A will pay 70% of this investment, with Region B paying the remaining 30%. The benefits of this investment all accrue to region B. The growth rate in consumption in this region will rise from 1.75% per year in year 0 at a linear rate of 0.01% a year until it reaches a maximum growth rate of 2.12% in year 38. The growth rate then linearly declines by 0.01% a year until year 75, when it is back at its initial level of 1.75%.[8] The effect of this major transportation project is to increase consumption in region B in year 75 from £58,432 to £67,005. Region A has consumption in year 75 of £132,689. These consumption paths, before deducting the investment costs, are illustrated by the black and red lines in Figure 1. We assume that no discounting occurs beyond year 75 for simplicity only (rather than because we believe this approach is theoretically correct).

By summing total utility in each region with and without the investment and using a numerical solver to ensure that this social welfare is equal in each case, it is straightforward to numerically demonstrate that the

[8] The consumption path is calculated using these growth rates assuming no investment in the project. The cost of the project is then deducted from this consumption path in years 0-24 before deriving the utility of consumption for each region in each year. Therefore consumption levels in years 25-75 are not affected by the investment costs of the project.

representative agent in region B would pay £1,289 in each year 0-24 to derive this benefit, with region A paying £3,008 a year (based on the 70%:30% split). The internal rate of return to this project, which corresponds to the discount rate (since it is precisely the welfare compensating rate of return), is equal to 1.71%. This is some way below the 2.5% rate of the Green Book excluding the $L-$risk adjustment.

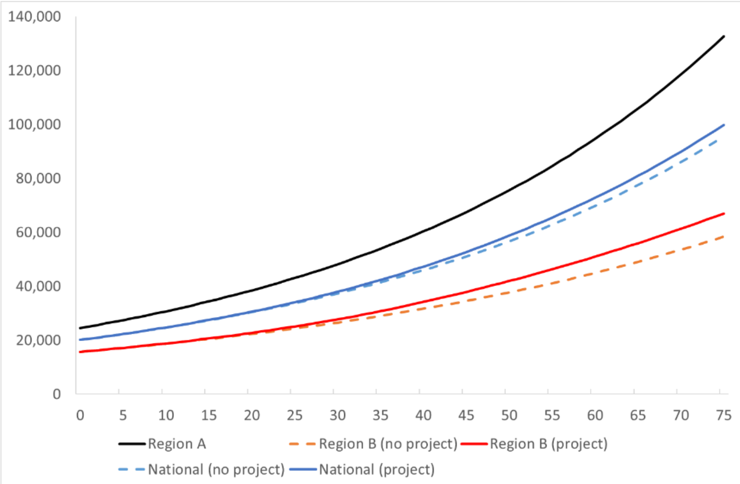

Figure 1. Changes in regional consumption as a consequence of major transport investment.

We can also consider this example within a representative agent framework where we aggregate consumption in each year across regions A and B into a single national representative agent's consumption stream; this is illustrated by the blue lines in Figure 1. In this case, it can be shown that the representative agent would pay £1,431 a year for the project. The project would have an internal rate of return of 2.68%, almost 1% greater than the regional analysis would imply and slightly above the Green Book rate.[9]

The sensitivity of the discount rate to the regional distribution of taxation and benefits reflects the fact that, within this discounting framework, investment policy is the only mechanism that is available to reduce inter-regional inequality, and this is also true of the distributional weights in the Green Book. This is a feature of a number of regional climate change integrated assessment models, such as RICE, where Negishi weights are used to overcome transfers from wealthier to less wealthy regions (see, for example, Stanton 2010). This requires that the weight that the social planner places on the utility of each region is in inverse proportion to the marginal utility of consumption in that region; giving region A greater policy weight than region B in this example. Applying Negishi weights would offset the distributional weight assumptions in the Green Book. Not only do these weights have a number of technical limitations (see, for example, Dennig et al., 2015), politically it would be almost impossible to argue for these wealth-based voting rights; that the North East should be weighted (on a per-capita basis) less than London because it is poorer. We therefore do not consider this possibility further here.

Another potential policy choice has not been incorporated within this simple model, and this also applies to distributional weights in the current Green Book. In principle, investment could be made at a higher rate of return in the wealthier region and then the benefits from this project redistributed to the less wealthy region. As region B would end up wealthier under this scenario than the case above, and as region A would be no less wealthy, this solution, at least theoretically, has greater Pareto efficiency. Therefore discounting within a levelling up agenda (whether through adjustments to the discount rate or by using distributional weightings) naturally raises important questions about whether investment hurdles should concentrate on maximising total benefits through the highest available returns or whether they should also reflect distributional concerns. Clearly, in practice, there are a number of frictions, both economic and political, that

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[9] By altering the change in growth rate for region B to +/-0.000001% a year to make this a marginal project, the IRRs change to 1.49% in the regional case and 2.58% in the single representative agent case. As average growth in per-capita consumption over the 75 years is 2.07%, the Simple Ramsey Rule rate is 2.57%.

would prevent a government from taking all the monetary benefits from a London-based transport project and transferring these in their entirety to the North East.

Section 2: Macroeconomic and project risk

When considering social discounting, there are two distinct types of uncertainty that are relevant. First, within the Ramsey Rule, there is the macroeconomic uncertainty over the consumption growth rate of the representative agent. Allowing for this introduces two effects. First, it reduces the short-term discount rate through a standard precautionary savings motive. This is represented in the Extended Ramsey Rule model by a negative adjustment term, $-0.5\gamma^2\sigma^2$,where σ^2 from here on in represents the volatility of logarithmic consumption growth. As we note in our Report to the Treasury (Freeman, Groom and Spackman 2018, p.10): "In the UK the volatility of consumption growth has been around 2.73%. With $\gamma = 1$ this leads to a prudence correction factor of 0.037%", which is too small a change to be policy relevant and therefore is ignored in Green Book guidance. However, in the long term, persistent shocks to consumption growth lead to a declining term structure of discount rates as we discussed in Section 1. HM Treasury does incorporate this into its guidance and we return to this issue in subsection 2.4. The second type of risk concerns project level, rather than macro-economic risks.

Throughout this section of this report, we consider project as well as macroeconomic level risk. This introduces a theoretical risk premium into the discount rate. HM Treasury's treatment of this is captured in the L −term within its application of the Ramsey Rule. Paragraph A6.10 of the Green Book says: "The risks contained in L could, for example, be disruptions due to unforeseeable and rapid technological advances that lead to obsolescence, or natural disasters that are not directly connected to the appraisal. L also includes a small premium for 'systemic risk' because costs and benefits are usually positively correlated to real income per capita". We believe that Treasury guidance could be usefully enhanced by more explicitly separating out these different risk elements and considering them individually. We raise four issues related to project and macroeconomic risk that we believe are most relevant for the Department for Transport to consider.

2.1 Social vs. regulatory discounting and beta risk

In general, when excluding the possibility of catastrophic outcomes (which we address separately in subsection 2.3.), project risk is theoretically treated in a standard social welfare framework using the Consumption CAPM (CCAPM). As we discussed in our Treasury report, Gollier (2012) shows that the risk premium can be quantified in this case through the term $\gamma\beta\sigma^2$. β here is the consumption beta of the project and captures its systematic risk. It is measured by the covariance of the project's returns with consumption growth divided by the variance of consumption growth. Again taking $\sigma = 2.73\%$ and $\gamma = 1$, this leads to very low estimates of the risk premium for all projects, explaining why the Treasury only incorporates a small premium for systematic risk for all projects within L.

But this social welfare CCAPM approach for dealing with project-specific risk differs markedly from the cost of capital approach taken outside the public sector: see, for example, Armitage (2017) for a detailed account of the differences between social discount rates and private sector discounting. In the private sector, the most commonly used model is the market-based CAPM, where the discount rate is given by $r_f + \beta\Pi$. Here r_f represents the rate of return on a (virtually) risk-free asset; generally the yield on some type of Government bill or bond. Π represents an estimate of the equity premium; the difference in the expected rate of return to a broad stock market index (such as the FTSE100 total returns index) and the risk-free rate. β is again a measure of systematic risk, but now calculated by the covariance of project returns with the returns on the equity market index divided by the variance of the equity market index within this markets-based framework.

Importantly, it is this markets based approach that is frequently used within a regulatory context, including in the transportation sector. For example, the UK Regulators' Network (2020) reports equity premium estimates applied by regulators across different sectors that range from 7.1% to 8.25% and real risk-free rates that range from -2.37% to -1.3%. An earlier version of the UK Regulators' Network (2018) report gives a range of market-based beta estimates for transportation assets.[10] The Office of Rail and Road applied $\beta = 0.37$ to the rail network, and the Civil Aviation Authority (CAA) applied $\beta = 0.50$ and $\beta = 0.56$ respectively for Gatwick and Heathrow. The CAA estimate of beta for Air Traffic Control was 0.51, updated in the 2020 version of the report to 0.52-0.62.

Flint Global (2020, Table 5), commissioned by the CAA to review costs of capital for Heathrow (HAL), estimated the beta to be in the range of 0.50-0.60, against HAL's own estimate of 0.54-0.62 and PwC's of 0.42-0.52. The reported estimates of the real risk-free rate for CMA/HAL/PwC (Table 2) were -2.25% / -1.71% to -1.20% / -1.50% to -1.00%, while the estimates of the equity premium were 7.25% to 8.25% / 7.7% / 6.6%. Sectoral betas for the transport sector in the US are provided by New York University. The quoted average asset beta is 0.79 for the sector in general and 0.74 for railroads.

There are a number of things to note here. First, the quoted market-based real risk-free rates are currently much lower than the Ramsey Rule would imply. There could be a range of reasons for this; the impact of quantitative easing on bond yields at a time when the Bank of England holds over £600bn of Gilts, high demand from pensions funds for risk-free assets, or potentially a high precautionary savings demand caused by the ongoing effects of the 2008 financial crisis and current Covid pandemic. Alternatively, it may just be that normatively constructed SDRs bear no resemblance to positivist market-based yields. Conversely, the quoted equity premium estimates are higher than many current academic estimates. For example, Fernandez et al. (2020) report a median estimate of the equity premium in the UK of 5.8% and a risk-free rate of 0.9% nominal (not real). Broadly, though, if we take the real risk-free rate to be -1%, the equity premium to be 6%, and the average asset beta for a transport project to be 0.6, then the appropriate real risk-adjusted discount rate is 2.6%. Because of negative real bond yields, this is broadly the same as the Green Book risk-free rate of 2.5% excluding the $L-$effect.

In practice, some governments use a hybrid CAPM-CCAPM model. The risk-free rate is calculated using either normative, Ramsey Rule type arguments (France) or are based on bond yields (Netherlands). The risk premium is calculated as $\beta\Pi$, where β is calculated based on consumption, but Π reflects a higher rate that is somewhat more in keeping with market-based returns (2% in France, 3.25% in Holland).

2.2. Time-varying betas and the implication for discounting

In the previous section, either within a CCAPM or CAPM framework, we considered how the discount rate varied with the project's beta. But this implicitly assumed that the beta is fixed over time, which is an assumption which may not hold within a transportation context. In more complex situations, we turn to a somewhat old-fashioned method of valuation, which is not heavily applied in practice, but that allows in a simple way for beta to vary over time. This technique uses binomial trees; see, for example Richter (2001).[11] We illustrate a very simple and highly stylised example where each step of the tree represents a 5-year period for a 20-year project that gives one set of benefits at the end of that 20 years. Extending this method for greater complexity is straightforward and is one of its major advantages over closed-form theoretical models. Take Figure 2. In this case, we start at $t = 0$ with the current level of aggregate consumption, here assumed to be £100.[12] At $t = 5$ (one step to the right in the tree), consumption can take one of two levels with equal probability; either £100.57 or £115.01. These are calibrated to match the expected return and standard deviation of consumption growth over the next five years. From the £110.57 node, at the next step in the

[10] Within a market-based CAPM context, it is essential to distinguish between the risk of an underlying project and the risk of the equity claim on that project, which is a levered (and hence riskier) claim on project benefits. All values quoted here are for transportation projects' asset betas.

[11] A binomial tree approach is one of the most common techniques used to value financial derivatives, but is less commonly used in pricing projects. In the former case, discounting is undertaken at a risk-free rate as payoffs from financial options can be perfectly hedged by trading in the underlying asset and the risk-free asset, and the risk-neutral probabilities that are used differ from subjective probabilities. For projects, this hedging cannot be undertaken, and so we use subjective probabilities together with risk-adjusted discount rates.

[12] We use different initial consumption levels in different examples to re-enforce the point that, for power/log utility, it is consumption *growth* and not the consumption *level* itself that drives the discount rate. Re-normalising all consumption levels by a fixed multiplier does not change any of the discount rates presented.

tree ($t = 10$) consumption can either move across horizontally to £115.66 or horizontally and up to £132.27. At each node in the tree, the next move is either directly to the right, or right and up one node, with equal probability. At $t = 20$, there are five possible consumption levels ranging from £102.29 to £174.95 with associated probabilities given by the Binomial Distribution.

To value the project, we start at the final step, t =20 years, and put in the project's expected benefits at each of the five possible consumption levels. In this example, the project is generally pro-cyclical (bottom four nodes) but becomes counter-cyclical in the top node. This, at least intuitively, reflects the idea that some new transportation technology may be developed and implemented within the next 20 years driving strong economic growth, but, should this happen, current transport investment will become partially redundant. This is the "... unforeseeable and rapid technological advances that lead to obsolescence" referred to in Paragraph A6.10 of the Green Book.

		Consumption			Project	Probability
t=0	t=5	t=10	t=15	t=20	Cash Flow	
				174.95	1.3	6.25%
			152.12	152.98	1.5	25.00%
		132.27	133.02	133.78	1.2	37.50%
	115.01	115.66	116.32	116.98	1.1	25.00%
100.00	100.57	101.14	101.71	102.29	1	6.25%

Figure 2. A binomial tree approach to pricing a transport project whose benefits are highly non-linear in expected consumption.

To value the project, we work backwards. The price of the project at a previous node is its expected price five years later, discounted by a discount rate $r = r_f + \beta\Pi$. As normal, β is the covariance of the project's return over the next five years with five-year consumption growth, divided by the variance of five-year consumption growth. Here r_f, Π both reflect the fact that each node of the tree represents five year.
Within this environment, it is straightforward to prove that the price of any node at time t is given by (working from right to left):

$$p_t = \frac{E[p_{t+1}] - Cov[p_{t+1}, g_t]\Pi/Var[g_t]}{1 + r_f}$$

where, again, consumption growth g_t reflects the five-year interval between nodes of the tree as given in Figure 2. Here we set $r_f = 2\%, \Pi = 3\%$ on an annualised basis. Given these assumptions, as shown in the left-hand node in the top part of Figure 3, the initial price of the project is £0.31. As the expected benefit of the project in 20 years is £1.225, the associated average annualised discount rate is 6.93%; a rate that would be associated with a beta of 1.64.

Project values				
t=0	t=5	t=10	t=15	t=20
				1.30
			1.47	1.50
		0.54	0.92	1.20
	0.96	0.86	0.94	1.10
0.31	0.58	0.72	0.85	1.00
Annualised expected returns				
t=0	t=5	t=10	t=15	
			-0.95%	
		15.95%	7.60%	
	-6.28%	1.54%	4.00%	
17.97%	6.38%	4.34%	4.20%	

Figure 3. The value of the project as calculated using the binomial tree approach based on a mixed CCAPM-CAPM model.

However, as shown in the bottom part of Figure 3, the annualised expected rate of return varies materially from node to node. This is particularly noticeable at the top node at the fourth time interval ($t = 15$). Here the value of the project (£1.47) is almost as great as the highest possible final project benefit from this node (£1.50). At this node, the beta is negative (-0.94), reflecting the counter-cyclical payoff over the next five years from this node, and therefore the appropriate discount rate is substantially below the risk-free rate of two percent. From this analysis, it is clear that applying a fixed CCAPM / CAPM type model over long periods may not be appropriate when there are complex relationships between project benefits and aggregate consumption levels.

2.3. Catastrophic risk and the discount rate

A further issue to address with beta risk is that, as it only incorporates covariances and variances, no consideration is given to the higher moments of relevant probability density functions. But, as we argued in our report to HM Treasury, low probability catastrophic risk potentially significantly changes the appropriate discount rate.

There are three reasons why catastrophe risk matters. First, there is the risk of a total failure of society, for example caused by a meteor strike, so that future generations do not exist to benefit in the gains from projects in which we currently invest. This type of catastrophic risk reduces our incentive to save today and so increases the discount rate for all projects. As we noted in our original report to HM Treasury: "The Stern Review used a hazard rate of 0.1% ... The implied probability of survival after 100 years in this case would be 90%" (Freeman, Groom and Spackman 2018 p.13). The second type of catastrophe risk is at the macroeconomic level and refers to Great Depression, or more severe, potential drops in the aggregate consumption level, but where society survives. The risk of such threats raises society's desire to save today ("precautionary saving"), reducing the equilibrium discount rate that should be applied to all projects. The third type of catastrophe risk is project-specific. It relates to how a project's benefits respond to a macroeconomic catastrophe. If the project helps protect against catastrophe (for example, vaccine facilities to help against pandemics) then this further reduces the project-specific discount rate. By contrast, if the project does very poorly in times of macroeconomic shocks, then this will further increase the discount rate. All three of these effects should, at least theoretically, be included within the L term within the Green Book, with clear distinctions made between them and the pure rate of time preference.

To illustrate the impact of the latter two of these three effects, we again use a binomial tree example, but now with only one time interval. We take initial consumption to be £1,000 and set the expected consumption in 10 years at £1,200 with a standard deviation of £300. Consumption can either take an "up" value with probability q or a "down" value with probability $1 - q$. The "up" and "down" values vary with q in order to keep the expected future consumption level, and its standard deviation, fixed. Panel A (top left) of Figure 4 illustrates this. As q gets closer to 1, consumption in the down state drops severely, representing a low-probability catastrophic state.

There are two risky projects, one pro-cyclical and the other counter-cyclical. Both have expected values in 10 years of £2, with standard deviation of £1. The projects pay off either a "high" value or a "low" value. For the pro- (counter-) cyclical project, the "high" payoff comes when consumption is "up" ("down") and the "low" payoff comes when consumption is "down" ("up"). Again, the value of these payoffs change with q in order to keep the expectation and standard deviation of the payoff fixed. These benefits are illustrated in panels B (top right) and C (bottom left) of Figure 4. There is also a risk-free asset that pays off £1 in each state.

To derive the present value of these projects, we apply the standard Euler equation (equation 1.2 in Cochrane 2001[13]), $p = exp(-\rho)E[d(c_1/c_0)^{-\gamma}]$ with $\rho = 1\%, \gamma = 2$. Panel D of Figure 4 (bottom right) compares the price of the risk-free asset with that derived from the simple Ramsey Rule. For most values of q, the

[13] While framed in an asset pricing context, Part 1 of Cochrane (2001) provides an excellent formal introduction to a range of issues that are relevant for social discounting, particularly around project risk, which are discussed in this thinkpiece and elsewhere.

difference, reflecting the precautionary savings motive, is small. This explains why HM Treasury does not explicitly allow for it in the Green Book. However, for large q, reflecting the small probability of catastrophic risk, the price of the risk-free asset rises substantially compared to the Ramsey Rule value due to a strong precautionary savings demand, and this leads directly to a lower discount rate. This observation has been used by Rietz (1988), Barro (2006, 2009) and Gabaix (2012), amongst others, to explain bond yield puzzles in financial markets.

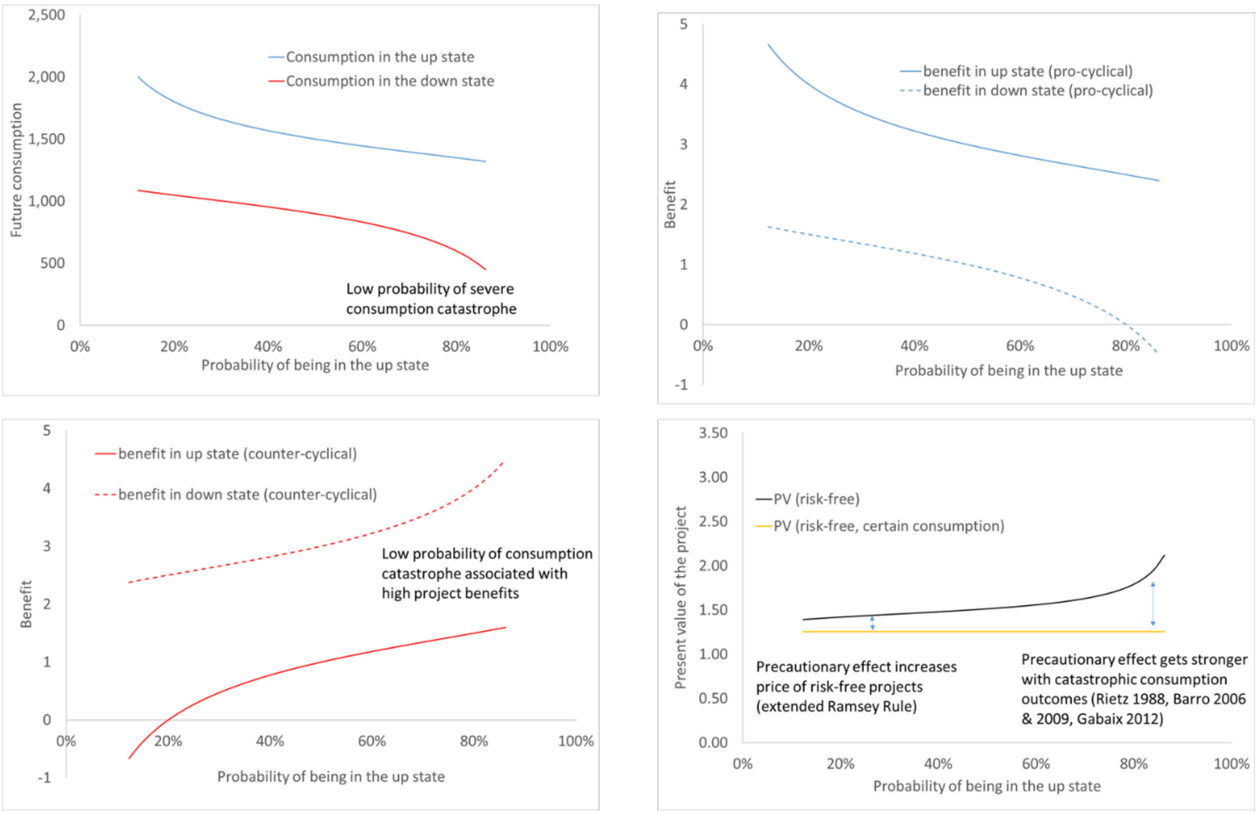

Figure 4. Panel A (top left) reflects consumption levels in the two states. Panels B (top right) and C (bottom left) reflect the payoffs from a pro- and counter-cyclical projects respectively. Panel D (bottom right) shows the price of a risk-free project as the probability of being in the up-state changes, illustrating the sensitivity of the precautionary savings motive to the probability of catastrophic risk.

Figure 5 shows the price of the pro-cyclical project (Panel A on the left) and counter-cyclical project (Panel B on the right) compared to the price of the risk-free asset. In the former case, there are two offsetting effects. The precautionary motive pulls down the price of the asset, but the risk premium pushes it up. The magnitude of the risk premium gets greater as q increases, as does the sensitivity of the valuation to the precise estimate of q (see, for example, Martin 2013 and Gollier 2016).

For the counter-cyclical project, where the risk-premium is negative, the two effects work in the same direction, so the magnitude of the effect is even greater. This argument has been used forcefully in the climate change literature to justify heavy investment in mitigation projects (Weitzman 2007, 2009; Dietz 2011; Pindyck 2013 as example) in order to avoid climate catastrophes.

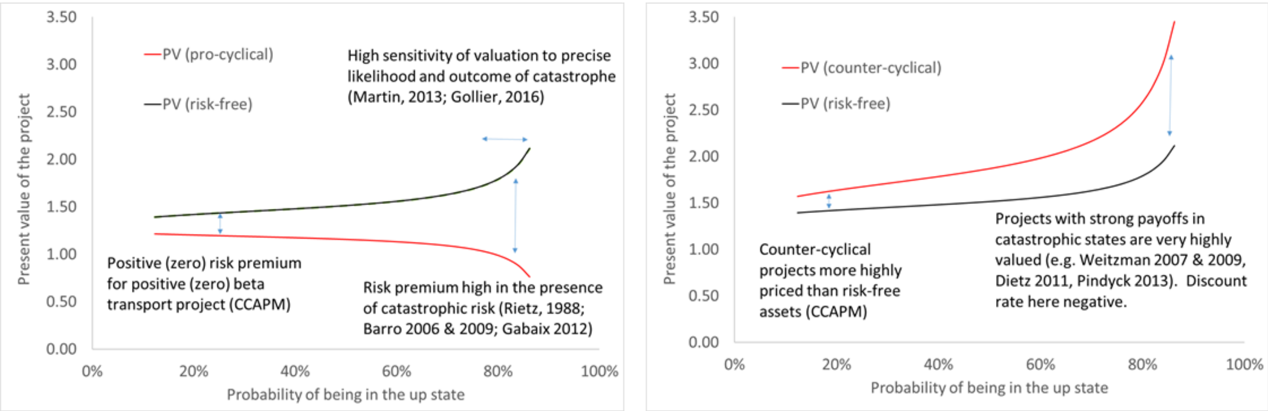

Figure 5. Panel A (left) and B (right) show the price of the pro-cyclical and counter-cyclical projects respectively compared to the price of the risk-free project. This illustrates that the risk premium is highly sensitive to the probability of catastrophic risk.

However, the magnitude of these effects depend heavily on the persistence of the catastrophe. Consider the following example. Again, consumption today is £1,000 and is expected to be £1,200 in 10 years time. We keep $\rho = 1\%, \gamma = 2$. In this case, the rate of return on an asset that pays an expected £1 in 10 years is 4.7% annualised under a no-uncertainty Euler equation analysis, again very close to the standard Simple Ramsey Rule rate. However, if consumption is £1,240 with $q = 95\%$ and £440 otherwise (keeping the expected consumption level at £1,200), and the asset pays £1.10 in the first state and -£0.90 in the second state (keeping the expected benefit at £1), then the project discount rate is 9.1%, well above the risk-free rate.

But now introduce a second period after 20 years, with expected consumption of £1,440 and expected project payoff of £2 in the second period. In the no uncertainty case, the discount rate reduces very slightly to 4.4% using a numerical solver. In the risky case, consumption goes to £1,460 following a good outcome in period 1 and £1,060 otherwise. That is, following a catastrophic outcome in year 10, there is partial recovery over the next decade. Similarly the project payoff goes to £2.05 in year 20 following a good outcome in year 10 and £1.05 following a catastrophe. Again the values match expected consumption and expected payoff with the no uncertainty case and reflect partial recovery of the project. A numerical solver now shows that the discount rate is 5.8%. This remains notably above the no uncertainty rate of 4.4%, but is much lower than the discount rate from the 10-year analysis in the previous paragraph of 9.1%. Therefore the impact of catastrophe risk on transportation project valuation depends both on its severity and its persistency.

2.4 Scenarios and obsolescence risk

We understand that the Department for Transport may be considering a scenario-based approach to NPV analysis. Within this, sub-NPVs would be constructed for a range of different potential scenarios alongside the "core NPV "as part of sensitivity analysis. No probabilities would be assigned to the different scenarios and so it would not be possible to construct a "meta NPV" across the different scenarios' sub-NPVs. This would also preclude binomial tree type analysis as described above as these require probabilistic assessments of the different outcomes.

We are unsure how this will work. In particular, the numerator of the "core NPV" is the expected benefit in each period, where the mathematical expectation is calculated as the sum of each possible outcome multiplied by its associated probability; see A5.15 in the Green Book.[14] Specifically this means that it is ***not*** the modal benefit (the forecast benefit in the most likely scenario). Therefore, unless probabilities are associated with all potential scenarios that the DfT thinks are relevant, we are unsure how the numerator of the "core NPV" equation is constructed. The "core NPV" ***is*** the "meta NPV".[15] Similarly, the consumption

[14] This requires that the scenarios are complete and non-overlapping (every possible outcome is included as part of one and only one scenario). We are unsure if this condition holds for the scenarios that the DfT is considering.
[15] Assuming that the same discount rate is applied within each sub-NPV; see below.

growth rates, when multiplied by their associated probabilities, must for internal consistency reasons equal that used in the derivation of the Green Book discount rate (2.0%).

Further, if there are to be project-specific adjustments to the discount rate (which is not currently incorporated into Green Book guidance), then the estimated systematic risk premium for the "core NPV" will depend on the relationship between project benefits and macroeconomic consumption growth both within each of the scenarios, and also across scenarios. This again requires a probabilistic assessment of the likelihood of each scenario occurring. The scenarios do not affect the Green Book, non project-specific, discount rate that should be applied to the "core NPV". The risks of different macroeconomic outcomes are incorporated into the declining discount rate schedule by HM Treasury and there is no reason for the DfT to seek specific adjustments for this.

For each of the scenario-specific sub-NPVs, it is not clear (to us, at least) what discount rate should be applied. Our preference would be for each scenario to reflect one specific ex-post outcome, in which case the appropriate discount rate would be the ex-ante Green Book rate for all scenarios. Alternatively, each scenario could be viewed as the only possible ex-ante outcome within that analysis, in which case the discount rate would vary within each scenario depending on the usual underlying factors. But it is not clear what the output of such an NPV analysis would mean in any economic or policy framework because, ex-ante, many different scenarios are possible. In short, we would wish to have a better understanding of what purpose these sub-NPVs were being used for before recommending a discount rate to apply within them. This is particularly true when the scenarios are potentially overlapping and/or incomplete.

Obsolescence is, under this setting, just one specific scenario. As a consequence, it introduces both numerator and denominator effects to the NPV equation. The "core NPV" can be decomposed into the expected benefit in the absence of obsolescence multiplied by the probability that obsolescence does not occur plus the expected benefit with obsolescence multiplied by the probability of obsolescence. This requires a probabilistic estimate of how likely obsolescence is together with the benefits (or perhaps more likely costs) that arise as a consequence.

The discount rate may, at least theoretically, be affected by the risk of obsolescence depending on whether or not this risk is systematic. If the project collapses at a time of economic decline, then that increases the discount rate, particularly if this is at a time of a Great Depression; see the previous subsection. However if the project becomes obsolete because of new technologies that increase economic growth, then this is a "negative beta" risk and therefore further reduces the theoretical discount rate.

In brief, the numerator adjustment to the NPV depends only on the perceived probabilistic estimate of obsolescence, while the discount rate adjustment, at least theoretically, requires an estimate of the pro-cyclicality, or otherwise, of that risk. This is likely to be of particular policy relevance if obsolescence is likely to occur at a time of severe economic decline. But fundamentally, obsolescence is just another example of a potential future scenario.

2.5. The term structure of discount rates for risky projects

Project risk also impacts upon the shape of the term structure of the discount rate which, within the Green Book, declines as the project maturity increases. Such a declining term structure can be theoretically motivated by introducing uncertainty to the discounting "primatives" in a range of different theoretical environments. For example, DDRs may be motivated by uncertainty in consumption growth, stochasticity in future interest rates, or expert disagreement over the "true" value of the discount rate (Freeman and Groom, 2016).

As discussed in Section 1, within the Green Book, the empirical term structure is derived predominantly within a positivist framework based on the econometric estimation of interest rate processes. But, within its Ramsey Rule setting, the short-term discount rate in the Green Book is based on consumption growth and therefore, at least theoretically, that is the appropriate basis from which to analyse DDRs. Here we extend the macro-level uncertainty that the Green Book incorporates to allow for project-specific risk and consider how this latter type of uncertainty might affect both the term structure of discount rates and the present value of a transportation project.

Imagine a project whose benefits depend on the outcome of future consumption growth. For example, following Gollier (2014), suppose $d_t = ae^{btg_t} + u_t$ where a, b are constants, $g_t = \left(\frac{1}{t}\right)\ln\left(\frac{c_t}{c_0}\right)$ is the annualised growth rate in consumption, and u_t is an idiosyncratic error term which, within a CCAPM environment, does not affect the present value of the project. Through Jensen's inequality, the expected future benefit, $E[d_t]$ increases with uncertainty in g_t.

In this case, using the standard Euler equation with power/log utility, the price of the asset is $p = e^{-\rho t}E[aexp(btg_t)exp(-\gamma t g_t)]$ and the t −period discount rate is $r_t = \rho + (1/t)ln(E[aexp(btg_t)]/E[aexp((b-\gamma)tg_t)])$. Let tg_t be normally distributed with mean $t\mu$ and variance $h_t t\sigma^2$ where h_t captures the persistency in consumption growth shocks. If consumption growth is independently and identically normally distributed over time, then $h_t = 1$ for all t. By contrast, if there is full persistency in consumption growth shocks, then $h_t = t$. Simple algebraic rearrangement shows that the appropriate discount rate is given by:

$$r_t = \rho + \frac{1}{t}ln\left[\frac{aexp(bt\mu + 0.5b^2h_t t\sigma^2)}{aexp((b-\gamma)t\mu + 0.5(b-\gamma)^2h_t t\sigma^2)}\right]$$

and this simplifies to $r_t = \rho + \gamma\mu + (b - 0.5\gamma)\gamma h_t\sigma^2$ (see also equations 4 and 5 in Gollier 2014). This effectively extends the precautionary savings term, $-0.5\gamma^2\sigma^2$, and the CCAPM risk premium term, $\gamma\beta\sigma^2$, both discussed earlier in this section, to a multi-period framework. If consumption growth is independent, so $h_t = 1$, for all t, then the term structure is flat, with the final term representing the standard prudence motive within the Extended Ramsey Rule when the project is risk-free ($b = 0$). However, if there is persistency (positive serial correlation) in growth rates, so that h_t increases with t from its initial value of $h_t = 1$, then the term structure is increasing or declining in time depending on whether $b < 0.5\gamma$ or not as there are two offsetting effects. Uncertainty reduces the risk-free discount rate as reflected in the Green Book, but also increases the project-specific risk premium. See also Gollier (2014, p.535).

The magnitude of the increase or decrease in the term structure depends on how h_t varies with t as well as the magnitude of $(b - 0.5\gamma)\gamma\sigma^2$. 10-year government bond yields tend to be relatively persistent, making this term of policy relevance (e.g., Newell and Pizer 2003 and subsequent work), but consumption growth is much less persistent than real interest rates (e.g. Groom and Maddison, 2019 for the UK) meaning that $h_t \approx 1$ for all t within a Ramsey-type environment, and the shape of the term structure is therefore much flatter. Somewhat depressingly, Freeman and Groom (2016) showed that the rate of decline of the term structure has hypersensitivity to very small changes in assumption, making it difficult to estimate its shape with any precision beyond about 75 years.

To capture this in a "second-best" way within the Green Book's schedule of DDRs would be to decrease the VTTS after 35 years to offset the decline in the discount rate. Assuming that this was to match a flat term structure, the adjustment would be 0.9 at 50 years, 0.8 at 75 years, 0.63 at 100 years and 0.49 at 125 years. However, because these price adjustments occur to cash flows that are most heavily discounted, the impact on any project appraisal may be low. For example, a 125 year annuity discounted at a flat 3.5% has a present value of £28.18, while the same annuity discounted using the Green Book schedule would have a present value of £29.80. This difference is unlikely to be of economic significance to the policy appraisal. Therefore making these adjustments would only be of policy relevance for projects where the benefits are heavily deferred into the future and, in such cases, more work would be required to estimate the Jensen's effect on the overall discount rate. But making a fixed adjustment to the VTTS to account for uncertainty is certainly somewhat ad-hoc and second best.

However, it is not the discount rate per-se that is of policy relevance, but instead the present value of the project and this is given by $p = aexp(-\rho t + (b-\gamma)t\mu + 0.5(b-\gamma)^2 t h_t\sigma^2]$. In this case, increased uncertainty about growth, $th_t\sigma^2$, increases the price if $b \neq \gamma$ and otherwise leaves it unaltered. When $0 < b < 0.5\gamma$, the discount rate is declining if consumption growth is persistent and the expected benefit increases with σ^2, both increasing the price. When $0.5\gamma < b < \gamma$, the expected benefit is increasing, and so is the discount rate, with σ^2, but the former term dominates. At $b = \gamma$, the two effects exactly offset, but as b increases further, the expected benefits effect again dominates the increasing discount rate effect. A related, but slightly different, discussion in the context of climate change discounting is given in Dietz et al. (2018).

These theoretical results, of course, all depend on the assumed functional form of the relationship between d and g_t. We have presented no empirical evidence here that suggests that the benefits from transportation investment takes the form assumed in this section, and a previous sub-section has argued that there may be important non-linearities in the relationship between a project's benefits and consumption outcomes. Nevertheless, there are two important qualitative messages here. First, if the benefits from transportation projects are pro-cyclical, then in a discounting environment which allows for such uncertainty, the term structure will decline less steeply than the Green Book implies and may even become upward sloping. Current French guidance assumes that the risk-adjusted term structure is flat for a $\beta = 1$ project, reflecting a declining risk-free rate and offsetting increasing risk premium. However, in most circumstances, increased consumption uncertainty raises the present value of transportation (and other pro-cyclical) projects because the Jensen's inequality term that drives DDRs also increases the expected benefit in the numerator of the NPV equation.

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