DRAFT

Department for Education

Mathematics

GCE AS and A level subject content

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Contents

The content for mathematics AS and A levels

Introduction

1. AS and A level subject content sets out the knowledge, understanding and skills common to all AS and A level specifications in mathematics.

Purpose

2. A level mathematics provides the framework within which a large number of young people continue the subject beyond GCSE level. It supports their mathematical needs across a broad range of other subjects at this level and provides a basis for subsequent quantitative work in a very wide range of higher education courses and in employment. It also supports the study of AS and A level further mathematics.

3. A level mathematics builds from GCSE level mathematics and introduces calculus and its applications. It emphasises how mathematical ideas are interconnected and how mathematics can be applied to help make sense of data, to understand the physical world and to solve problems in a variety of contexts. It prepares students for further study and employment in a wide range of disciplines involving the use of mathematics.

4. AS mathematics, which can be co-taught with the A level as a separate qualification, is a very useful qualification in its own right. It is valuable preparation for less mathematical disciplines, such as social sciences or business, which nonetheless require serious use of quantitative analysis and calculus.

Aims and objectives

5. AS and A level specifications in mathematics must encourage students to:

- understand mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general

- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- take increasing responsibility for their own learning and the evaluation of their own mathematical development
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate.

Subject content

Background knowledge

6. AS and A level mathematics specifications must build on the skills, knowledge and understanding set out in the 2015 GCSE subject content for mathematics.

Overarching themes

7. A level specifications in mathematics must require students to demonstrate the following knowledge and skills. These must be applied, along with associated mathematical thinking and understanding, across the whole content of the A level set out below.

8. The knowledge and skills required for AS mathematics are shown in bold text within square brackets.

OT1 Mathematical argument, language and proof

	Knowledge/skill
OT1.1	**[Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable]**
OT1.2	**[Understand and use mathematical language and syntax including 'equals', 'identically equals', 'therefore', 'because', 'implies', 'is implied by', 'necessary', 'sufficient',** $\therefore$**,** $=$**,** $\equiv$**,** $\neq$**,** $\Rightarrow$**,** $\Leftarrow$ **and** $\Leftrightarrow$**]**
OT1.3	**[Understand and use Venn diagrams, language and symbols associated with set theory including 'complement',** $\varnothing$**,** $\cap$**,** $\cup$**,** $\in$**,** $\notin$**,** $\mathscr{E}$**]** **[Apply to probability and solutions of inequalities]**
OT1.4	**[Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion]** and proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs) **[Disproof by counter example]**
OT1.5	Understand and use the definition of a function; domain and range of functions

OT1.6	Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics

OT2 Problem solving

	Knowledge/skill
OT2.1	**[Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved]**
OT2.2	**[Construct extended arguments to solve problems presented in an unstructured form, including problems in context]**
OT2.3	**[Interpret and communicate solutions in the context of the original problem]**
OT2.4	Understand that many mathematical problems cannot be solved analytically, but numerical methods permit solution to a required level of accuracy
OT2.5	**[Evaluate, including by making reasoned estimates, the accuracy or limitations of solutions]**, including those obtained using numerical methods
OT2.6	**[Understand the concept of a problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to collect further information, etc.]**
OT2.7	**[Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics]**

OT3 Mathematical modelling

	Knowledge/skill
OT3.1	**[Translate a situation in context into a mathematical model, making simplifying assumptions]**
OT3.2	**[Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student)]**
OT3.3	**[Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student)]**
OT3.4	**[Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate]**

OT3.5	**[Understand and use modelling assumptions]**

Use of technology

9. The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the AS/A level mathematics specifications.

10. Students must not have access to technology with a computer algebra system (CAS) function in examinations.

Use of data in statistics

11. AS and A level mathematics specifications must require students to:

- become familiar with one or more specific large data set(s) in advance of the final assessment (these data must be real and sufficiently rich to enable the statistical concepts in the specification to be explored)
- have used technology such as spreadsheets or specialist statistical packages to explore the data set(s) in advance of the final assessment
- interpret real data presented in summary or graphical form
- use data to investigate questions arising in real contexts.

Detailed content statement

12. A level specifications in mathematics must include the following content. This represents 100% of the content.

13. Content required for AS mathematics is shown in bold text within square brackets. This represents 100% of the AS content.

A Algebra and functions

	Content
A1	**[Understand and use the laws of indices for all rational exponents]**
A2	**[Use and manipulate surds, including rationalising the denominator]**
A3	**[Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the condition for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown]**
A4	**[Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation]**

A5	**[Solve linear and quadratic inequalities and interpret such inequalities graphically, including inequalities with brackets and simple algebraic fractions such as $\frac{4}{y} > 3$]** **[Express solutions through correct use of 'and' and 'or', or through set notation]** **[Represent quadratic inequalities such as $y > ax^2 + bx + c$ graphically]**
A6	**[Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem]** Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only)
A7	**[Understand and use graphs of functions; sketch curves defined by simple equations including polynomials]**, the modulus of a linear function, **[$y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ (including their vertical and horizontal asymptotes); interpret algebraic solution of equations geometrically; use intersection points of graphs to solve equations]** **[Understand and use proportional relationships and their graphs]**
A8	Understand and use composite functions; inverse functions and their graphs
A9	**[Understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ including sketching associated graphs:** $y = a\,\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$**]**, and combinations of these transformations Link graphical transformations to transformations of the equation of the Normal probability curve for N(μ, σ^2) for fixed σ and changing μ, and for μ = 0 and changing σ
A10	Decompose rational functions into partial fractions (denominators not more complicated than repeated linear terms, numerators constant or linear)
A11	Use of functions in modelling, including consideration of limitations and refinements of the models

B Coordinate geometry in the (x,y) plane

	Content
B1	**[Understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular]** **[Be able to use straight line models in a variety of contexts]**
B2	**[Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x-a)^2 + (y-b)^2 = r^2$; completing the square to find the centre and radius of a circle; use of the standard circle theorems; finding the equation of the tangent to a circle through a given point on the circumference]**
B3	Understand and use the parametric equations of curves and conversion between cartesian and parametric forms
B4	Use parametric equations in modelling in a variety of contexts

C Sequences and series

	Content
C1	**[Work with sequences including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$ for linear and quadratic functions; increasing sequences; decreasing sequences; periodic sequences]**
C2	**[Understand and use sigma notation for sums of series]**
C3	**[Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms]**
C4	**[Work with geometric sequences and series including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation]**
C5	**[Understand and use the binomial expansion of $(a+bx)^n$ for positive integer n; the notations $n!$ and ${}^nC_r$** **Link to binomial probabilities]** Extend to any rational n, including its use for approximation; be aware that the expansion may not be valid for all values of x
C6	**[Use sequences and series in modelling]**

D Trigonometry

	Content
D1	**[Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form** $\frac{1}{2}ab\sin C$ **]** **[Work with radian measure, including use for arc length and area of sector]**
D2	Understand and use the standard small angle approximations of sine, cosine and tangent $\sin\theta \approx \theta,\ \cos\theta \approx 1-\frac{\theta^2}{2}, \tan\theta \approx \theta$ where θ is in radians
D3	**[Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity]** **[Know and use exact values of sin, cos, tan for** $0,\ \frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3},\ \frac{\pi}{2},\ \pi$ **and multiples thereof]**
D4	Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and appropriate restricted domains
D5	**[Understand and use** $\tan\theta = \frac{\sin\theta}{\cos\theta}$ **]** **[Understand, use and be able to derive** $\sin^2\theta + \cos^2\theta = 1$**]**; $\sec^2\theta = 1+\tan^2\theta$ and $\operatorname{cosec}^2\theta = 1+\cot^2\theta$
D6	Understand and use formulae for $\sin(A\pm B),\ \cos(A\pm B)$ and $\tan(A\pm B)$; Understand geometrical proofs of these formulae Understand and use double angle formulae Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta\pm\alpha)$ or $r\sin(\theta\pm\alpha)$
D7	**[Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle]**
D8	Construct proofs involving trigonometric functions
D9	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces

E Exponentials and logarithms

	Content
E1	**[Know and use the function a^x and its graph, where a is positive]** **[Know and use the function e^x and its graph]**
E2	**[Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive]** Know and use the function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x
E3	**[Understand and use the laws of logarithms:** $\log_a x + \log_a y = \log_a (xy)$; $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$; $k \log_a x = \log_a x^k$ **(including, for example, $k = -1$ and $k = -\frac{1}{2}$)]**
E4	**[Solve equations of the form $a^x = b$]**
E5	**[Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y]**
E6	**[Understand and use exponential growth and decay; use in modelling, including the use of e in continuous compound interest, radioactive decay, exponential growth as a model for population growth; consideration of limitations and refinements of the models]**

F Differentiation

	Content
F1	**[Understand and use the derivative of $f(x)$ as a function for the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for powers of x]** and for $\sin x$ and $\cos x$ **[Understand and use the second derivative as the rate of change of gradient]**; connection to convex and concave sections of curves and points of inflexion
F2	**[Differentiate x^n, and related sums and differences]** Differentiate e^x and a^x, $\sin x$, $\cos x$, $\tan x$ and related sums, differences and

	constant multiples Understand and use the derivative of $\ln x$
F3	**[Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points]**, points of inflection **[Identify increasing and decreasing functions]**
F4	Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change
F5	Differentiate simple functions and relations defined implicitly or parametrically; parametric differentiation for first derivative only
F6	Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand)

G Integration

	Content
G1	**[Know and use the Fundamental Theorem of Calculus]**
G2	**[Integrate x^n]** Integrate e^x, $\frac{1}{x}$, $\sin x$, $\cos x$ and related sums, differences and constant multiples
G3	**[Evaluate definite integrals; use a definite integral to find the area under a curve]** and the area between two curves
G4	Understand and use integration as the limit of a sum
G5	Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae)
G6	Integrate using partial fractions that are linear in the denominator
G7	Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions

	(Separation of variables may require factorisation involving a common factor)
G8	Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics

H Numerical methods

	Content
H1	Locate roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x on which $f(x)$ is sufficiently well-behaved Understand how change of sign methods can fail
H2	Solve equations approximately using simple iterative methods, including the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$; be able to draw associated cobweb and staircase diagrams Understand how such methods can fail
H3	Understand and use numerical integration of functions, including finding the approximate area under a curve and limits that it must lie between
H4	Use numerical methods to solve problems in context

I Vectors

	Content
I1	**[Use vectors in two dimensions]** and in three dimensions
I2	**[Evaluate the magnitude and direction of a vector]**
I3	**[Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations]**
I4	**[Understand and use position vectors; evaluate the distance between two points]**
I5	**[Use vectors to solve problems in pure mathematics and in context, including forces]** and kinematics

14. For sections J to N students must be able to use calculator technology in the examinations that will enable them to compute summary statistics and access probabilities from standard statistical distributions.

J Statistical sampling

	Content
J1	**[Understand and use the terms 'population' and 'sample']** **[Use samples to make informal inferences about the population]** **[Understand and use sampling techniques, including simple random sampling and opportunity sampling]** **[Select or evaluate sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population]**

K Data presentation and interpretation

	Content
K1	**[Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency]** **[Connect to probability distributions]**
K2	**[Interpret scatter diagrams for bivariate data, including recognition of scatter diagrams which include distinct sections of the population]**
K3	**[Interpret measures of central tendency and variation, extending to standard deviation]** **[Understand informal interpretation of correlation]** **[Understand that correlation does not imply causation]**
K4	**[Recognise and interpret possible outliers in data sets and statistical diagrams]** **[Select or critique data presentation techniques in the context of a statistical problem]**

L Probability

	Content
L1	**[Understand and use mutually exclusive and independent events when calculating probabilities]**

	[Link to discrete and continuous distributions]
L2	Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables Understand and use the conditional probability formula $\mathbf{P}(A \mid B) = \dfrac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)}$
L3	Apply probability in context, including critiquing assumptions made and the likely effect of more realistic assumptions

M Statistical distributions

	Content
M1	**[Understand and use probability distributions, including the binomial distribution as a model; calculate probabilities using the binomial distribution]** **[Link to the binomial expansion and tree diagrams]**
M2	Understand and use the Normal distribution as a model; find probabilities using the Normal distribution Link to histograms, mean, standard deviation, points of inflection and the binomial distribution
M3	Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate

N Statistical hypothesis testing

	Content
N1	**[Understand and use the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value]**

N2	**[Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context]** **[Understand that a sample is being used to make an inference about the population and appreciate that, for example, with a significance level of 5%, there is a probability of one in twenty of incorrect rejection of the null hypothesis when it is true]**
N3	Conduct a statistical hypothesis test for the mean of a Normal distribution and interpret the results in context
N4	Interpret a correlation coefficient as a test statistic and use it in a hypothesis test

O Quantities and units in mechanics

	Content
O1	**[Understand and use fundamental quantities and units in the S.I. system: length, time, mass]** **[Understand and use derived quantities and units: velocity, acceleration, force, weight, moment]**

P Kinematics

	Content
P1	**[Understand and use the language of kinematics: position; displacement; distance travelled; velocity; speed; acceleration]**
P2	**[Understand, use and interpret graphs in one-dimensional kinematics: displacement against time and interpretation of gradient; velocity against time and interpretation of gradient and area under the graph]**
P3	**[Understand, use and derive the formulae for constant acceleration in 1 dimension]**; extend to 2 dimensions using vectors
P4	**[Use calculus in kinematics in 1 dimension:** $\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}$, $\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}$, $\boldsymbol{r} = \int \boldsymbol{v} \ \ t$, $\boldsymbol{v} = \int \boldsymbol{a}\ \mathrm{d}t$ **]**; extend to 2 dimensions using vectors
P5	Model motion under gravity in 2 dimensions using vectors; projectiles

Q Forces and Newton's laws

	Content
Q1	**[Understand and use Newton's first law and the concept of a force]**
Q2	**[Understand and use Newton's second law in 1 dimension]**; extend to 2 dimensions
Q3	**[Understand and use weight and motion in one dimension under gravity; gravitational acceleration, *g*, and its value in S.I. units to varying degrees of accuracy]** **[(The inverse square law for gravitation is not required and *g* may be assumed to be constant, but students should be aware that *g* is not a universal constant but depends on location in the universe)]**
Q4	**[Understand and use Newton's third law; equilibrium of forces on a particle in 1 dimension; simple cases in 2 dimensions using vectors]**; resolving forces in 2 dimensions; equilibrium of a particle under coplanar forces
Q5	Understand and use addition of forces; resultant forces; dynamics in 2 dimensions
Q6	Understand and use Coulomb's model of friction in 1 and 2 dimensions; coefficient of friction; motion of a body on a rough surface; limiting friction and statics

R Moments

	Content
R1	Understand and use moments in simple static contexts

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