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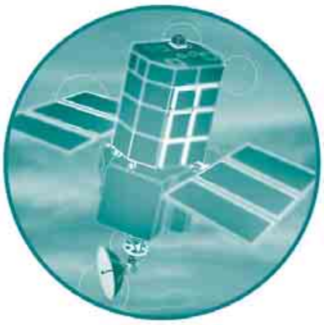  

Benchmarking Hydraulic River Modelling Software Packages

Results – Test E (Ippen Wave)

R&D Technical Report: W5-105/TR2E

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Environment Agency

TECHNICAL REPORT W5-105/TR2E

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BENCHMARKING HYDRAULIC RIVER MODELLING SOFTWARE PACKAGES

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Statement of use
This document provides the results and findings from undertaking the Environment Agency's Benchmarking Test E (Ippen Wave) for hydraulic river modelling software. The results only relate to the ISIS, MIKE 11 and HEC-RAS software packages and inference to the likely performance to other software packages should not be made.

The findings are intended to be a supplementary resource for Defra and Agency staff, research contractors and consultants, academics and students for assessing the applicability of any one of these software packages for their own modelling requirements. This report should not be considered in isolation and should be read in conjunction with the other tests reports produced as part of this R&D project.

Keywords
Hydraulic Modelling, River Modelling, Benchmarking, Test Specifications, Flood Wave, Ippen Wave

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EXECUTIVE SUMMARY

Each software package has modelled the Ippen wave to a reasonable degree of accuracy with all the results showing a good degree of similarity to one another and the analytical solution.

All calculated water levels prior to any superimposition of incident and reflected waves closely follow the analytical values. However, as soon as the two waves meet, the calculated water levels noticeably diverge from the analytical values.

To a reasonable degree of accuracy all the models can predict the times at which the reflected wave falls into phase with the incident wave.

Each software package over-estimates the peak water level at the closed boundary, which in part may be a result of the wave length being slightly longer than the channel length (by 800m).

Sensitivity analysis on the model time-step has shown that all three software packages can suffer from numerical diffusion (damping), however, the degree of sensitivity has shown to be software dependant.

Inspection of the results would suggest that MIKE 11 may be the most appropriate software package for modelling this specific problem, as it is the least influenced/effected by time-step.

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1 INTRODUCTION

1.1 Background

This report presents the results and findings from Test E (Ippen Wave) of the Environment Agency of England and Wales (EA), Benchmarking and Scoping Study (2004). The study, which encompasses a series of tests, is intended to be an independent research investigation into the accuracy, capability and suitability of the following one-dimensional hydraulic river modelling software packages:

Software	Version		Developer
ISIS	User Interface:	2.0 (13/01/01)	Halcrow /
	Flow Engine:	5.0.1 (27/06/01)	Wallingford Software
MIKE11	User Interface:	Build 5-052 (2001b)	DHI Water and Environment
	Flow Engine:	5.0.5.5	
HEC-RAS	User Interface:	3.1.0 (Beta) (03/02)	US Corps of Engineers
	Pre-processor:	3.1.0 (Beta) (03/02)	
	Steady Flow Engine:	3.1.0 (Beta) (03/02)	
	Unsteady Flow Engine:	3.1.0 (Beta) (03/02)	
	Post-processor:	3.1.0 (Beta) (03/02)	

Each of the above software packages was tested in the previously undertaken benchmarking study (Crowder *et al*, 1997). They are currently on the EA's BIS-A list of software packages for one-dimensional hydraulic river modelling.

The test has been undertaken on behalf of the EA by the following team in accordance with the Benchmarking Test Specification - Test E (Ippen Wave), (Crowder *et al,* 2004):

	Role	Affiliation
Mr Andrew Pepper	EA Project Manager	ATPEC River Engineering
Dr Richard Crowder	Study Project Manager	Bullen Consultants Ltd
Dr Nigel Wright	Advisor	University of Nottingham
Dr Chris Whitlow	Advisor	Eden Vale Modelling Services
Dr Andrew Sleigh	Advisor	University of Leeds
Dr Chris Tomlin	Advisor	Environment Agency
Mr David Cross	Tester	University of Leeds

1.2 Aim of Test

The aim of the test is to:

- compare the results generated by the software packages with an analytical solution based on the hydrodynamic theory of tidal wave propagation in a horizontal channel of uniform cross-section and finite length, as presented by Ippen (1966); and

- present the particulars for developing and undertaking the test (Model Build) with each of the software packages and the associated results so that others can repeat the test with their own software.

1.3 The Ippen Flood Wave

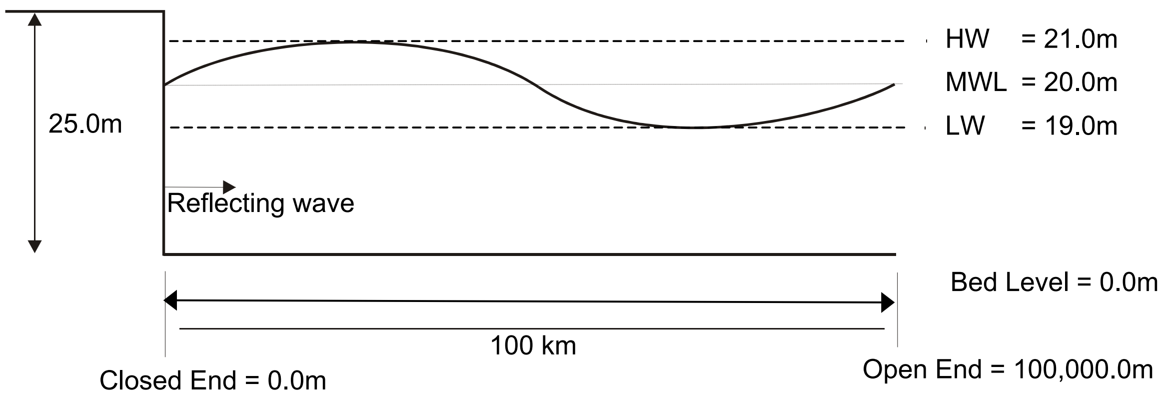
The analytical solution for the Ippen Flood Wave, which is for a channel of finite length, is based upon two crucial assumptions:

- bed friction may be linearised; and
- the non-linear advective acceleration term may be neglected.

These assumptions are acceptable when the ratio of wave amplitude to the mean water depth is relatively small.

The analytical solution calculates the water elevation in a channel of finite length with a closed end by superimposing the incident wave with the reflecting wave, as illustrated in Figure 1.1.

Figure 1.1: Schematic diagram of tidal wave in the idealised estuarial channel.



The analytical solution (which follows that of Ippen) is used as a baseline when assessing the numerical solutions is outlined in Appendix A. In addition, an Excel spreadsheet is available with the data set, which can be downloaded from the Environment Agency's web site (http://www.environment-agency.gov.uk/floodresearch).

For the test the analytical solution has been based upon a tidal wave period T of 2hrs, a mean depth of water H of 20mAD, and an initial wave amplitude a0 of 1m. Given that the celerity of the flow at mean water level is 14m/s, and the wave period is 7200s, this gives rise to a wavelength of 100,800m, as determined by the relationship T=L0/c. Hence, the length of the channel has been set at 100,000m so as to approximately coincide with the tidal wavelength.

2 MODEL BUILD

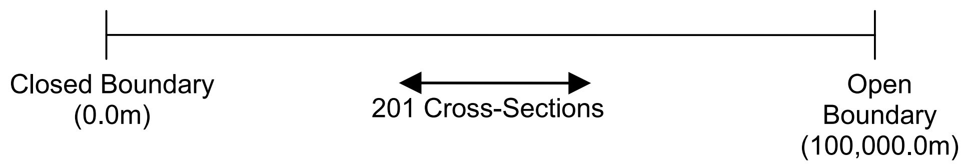
2.1 Test Configuration

The test has been undertaken in accordance with the Benchmarking Specification – Test E.

In order to build the channel in each of the software packages, the model was set up using 201 cross-sections with equal spacing of 500m. The rectangular channel width was set at 1000m, with side walls of 25m, and a constant Manning's 'n' roughness value of 0.025 along the whole length of the channel. The channel is of uniform elevation along its length and there is no bed slope.

The test configuration is illustrated schematically in Figure 2.1.

Figure 2.1: Schematic Illustration of Test Configuration



The closed boundary was specified at the upstream end of the channel and was defined in all three packages as a flow/time boundary of zero inflow for the duration of the simulation. The open boundary was specified at the downstream end of the channel. A head/time boundary, simulating a sinusoidal wave profile of time period 2hrs and amplitude 1m, was specified in all three packages at the downstream end.

2.2 Building the Model in ISIS, MIKE 11 and HEC-RAS

The model build with ISIS, MIKE 11 and HEC-RAS was undertaken in accordance with the test specification as defined by the dataset.

2.3 Initial Conditions

As no flow is specified in the channel at the upstream end, initial steady and quasi-steady flow simulations could not be run from which to generate initial conditions for the unsteady simulation. For each software package it was necessary to specify the initial mean water level (MWL) of 20m at all points along the channel. The unsteady simulation was then run with the sine wave of the head/time boundary starting at this mean water level value and then rising to the high tide of 21mAD.

3 RUNNING THE MODEL

3.1 Running the Model in ISIS, HEC-RAS and MIKE 11

Performing the unsteady run in each of the packages was straightforward. A time-step of 60s was used for each simulation, and the results recorded at 60s intervals.

Given that the length of the channel was specified so that it approximately coincided with the length of the propagating wave, the incident wave should take approximately 2hrs to reach the closed boundary.

Since the test is designed to ensure that each software package correctly superimposes the reflected wave with the incident wave, as described by Ippen's analytical solution, the results have only been analysed between 2hrs and 4hrs of the simulation time. Hence, the simulation was run for 4hrs.

To assess the effect of Manning's n value on the results, additional runs have been undertaken with ISIS and HEC-RAS with a range of n values. The results for HEC-RAS and ISIS showed expected trends/changes and hence it was not considered necessary to also test MIKE 11.

Although not part of the test specification, investigations have been made with all three software packages to assess the impact of model time step on the results.

When running the model the default calculation settings were used for each of the software packages.

None of the software packages reported errors or warnings when undertaking the simulations.

4 RESULTS

4.1 Introduction

As the analytical solutions for this test give water surface elevations and the velocity of flow at any given time and location, it is necessary to compare the calculated water surface levels and flow velocities over the length of the channel at chosen fixed simulation times, and for the duration of the simulation at fixed locations. For this reason, calculated water levels and velocities are compared against the analytical values over the duration of the simulation at the closed boundary and at locations of 25km, 50km and 75km downstream of the closed boundary.

It is unnecessary to analyse the water levels and corresponding velocities at the open boundary, 100km downstream of the closed boundary, as this is the location at which the head/time boundary condition is applied.

Calculated longitudinal water level and velocity profiles will also be compared with the analytical solutions over the length of the channel at the fixed simulation times of 2.0hrs, 2.5hrs, 3.0hrs, 3.5hrs and 4.0hrs.

It was decided to compare the calculated values at 2.0hrs in order to assess how well each of the packages generates the effective initial values of the test.

4.2 Analysis of Results – Water Level Histories

It can be seen from Graphs 1 to 4, Appendix B, that the variation in calculated water levels is very similar for each of the three packages and comparable to the analytical solutions at each of the four channel locations. However, even though all the calculated water levels are generally very close to the analytical values, and thus follow the expected trend, none follow the analytical values exactly at any location for the duration of the test.

From Graph 1, Appendix B, it can be seen that each of the three packages generates higher than expected water levels at the start of the simulation until approximately 2.6hrs.

These higher water levels may be due to the fact that the actual length of the wave is slightly longer than the length of the channel (by 800m). Therefore, as the wave is supposed to travel beyond this point within the first 2hrs, it would be expected that the water levels would be slightly higher even without any reflection. However, as the wave will have reached the closed boundary slightly before 2hrs of the simulation time, some reflection will have already occurred despite the fact that none should have, and the resultant superimposition will have forced the water levels to rise even further.

The water levels calculated by all three packages at chainage –25km (Graph 2, Appendix B) follow the analytical values almost perfectly between 2.0hrs and 2.3hrs, after which time the levels calculated by ISIS and HEC-RAS break away at 2.4hrs. MIKE 11 however, does not break from the analytical values at this chainage until 2.5hrs, the time at which the two propagating waves become superimposed.

All three packages demonstrate slightly varied responses to the effect of the superimposing waves. It can be seen from Graph 2, Appendix B, that the water levels generated by HEC-RAS are almost completely unaffected at this time of superimposition, while both ISIS and MIKE 11, which record lower water levels at this point, quickly rise to slightly greater values than those produced by HEC-RAS, MIKE 11 in particular demonstrating a more erratic change in levels. All three packages then calculate essentially the same variation in water levels for the remainder of the simulation, wavering above and below the analytical values.

From Graphs 3 and 4, Appendix B, which show the variation in water levels at –50km and –75km, it can be seen that the two propagating waves become superimposed at 3.0hrs and 3.5hrs respectively. In addition, these graphs show that all calculated water levels prior to any superimposition closely follow the analytical values. However, as soon as the two waves meet, the calculated water levels noticeably diverge from the analytical values.

The same response recorded by each of the three packages at chainage –25km to the superimposition of the propagating waves is again demonstrated clearly in Graph 4 at chainage –75km. Essentially, the water levels generated by HEC-RAS rise sooner than those generated by the other two packages, passing through the incident seemingly unaffected. In contrast, MIKE 11 only breaks from the analytical values at 3.5hrs, and rises very quickly above those produced by HEC-RAS and ISIS.

4.3 Analysis of Results – Water Level Profiles

It is stated by Ippen (1966), that the result of a tidal wave entering a channel closed at one end is

"…a standing wave of maximum amplitude 2a at the closed end and at multiple distances of L/2. Nodal points of zero amplitude and maximum velocities at L/4 and odd multiples thereof."

The standing wave that Ippen refers to in the above quote, is that which would result from the total superimposition of the reflected wave with the incident wave after one time period of the reflected wave. However, it can also be seen from Ippen's mathematical solutions, that the harmonic function for the wave amplitude applied at the open end of the channel follows the following cosine relationship:

$$\eta = 2a \cos \sigma t \cos kx, \qquad \text{(Equation 1)}$$

where η represents the wave amplitude, $\sigma = 2\pi/T$, and $k = 2\pi/L$.

In comparison, the harmonic function applied at the downstream end of the model followed a sine curve relationship, as confirmed by the shape of the curve representing the water levels along the length of the channel at only 2hrs.

For this reason, the generated wave formation is out of phase with Ippen's theory for tidal waves entering channels of finite length (Ippen, 1966).

As the applied water levels at the open end of the channel follow a sine wave relationship as opposed to a cosine relationship, the resultant phase difference is a 90-degree time angle, equivalent to a time lag of 30 minutes. Therefore, in order to achieve the final standing wave that Ippen refers to, it would be necessary to run the simulation for 4.5hrs as opposed to 4.0hrs. Total superimposition of the reflected wave with the incident wave would still be achieved after only 4.0hrs. However, no greater effect of further reflection and superimposition should occur at anytime thereafter as the reflected wave simply travels out of the open boundary after 4.0hrs.

Consideration was given to continuing the simulation for the further 30 minutes necessary to achieve Ippen's standing wave. However, because the propagating sine wave is ahead of the related cosine wave by 90 degrees, superimposition of the two propagating waves will have occurred over the first 25km of the channel downstream of the closed boundary, by the time the initial phase difference is caught up. Furthermore, the two propagating waves fall into phase with one another at 2.5hrs, and every 180 degree time angle thereafter, equating to 3.5hrs, 4.5hrs, and every hour thereafter, for as long as the simulation is allowed to run.

By relating these patterns of interference to Ippen's observations regarding the locations of maximum and minimum amplitude, and theoretical maximum velocity, it becomes clear that it is unnecessary to continue the simulation beyond 4.0hrs as wave interference will have already occurred in the channel giving rise to the same desired observations at 2.5hrs and 3.5hrs. Therefore, it can be seen from Graphs 6 and 8, Appendix B, displaying the generated water levels at the times the reflected wave falls into phase with the incident wave, that maximum amplitudes are achieved at the closed boundary and at multiples of L/2 chainage in the downstream direction.

As the two waves are only superimposed up to chainage –25km at 2.5hrs, and –75km at 3.5hrs, the maximum amplitudes of 2a are only achieved at the closed boundary in Graphs 6 and 8, Appendix B, and at the mid-channel chainage also in Graph 8. However, it can be seen from these graphs, that the maximum amplitudes demonstrated by the software packages and the analytical values are less than 2m variation from the mean water level. This will be due to the fact that the wave is damped as a result of the bed friction. Despite the effect of damping, it is clear to see that superimposition of the propagating waves has occurred in a positive manner at these channel locations, as the resultant water levels produced by all three packages are greater than the wave amplitude 'a' of 1m.

Further to Ippen's observations, Graphs 6 and 8, Appendix B, clearly demonstrate zero, or approximately zero amplitude at channel chainages of –25km and –75km, equivalent to chainages of L/4 and 3L/4 downstream of the origin of the reflected wave.

Graphs 7 and 8, Appendix B, provide further evidence of the correct superimposition of the reflected wave with the incident wave. At 3.0hrs the reflected wave will have travelled half the length of the channel, but would be out of phase with the incident wave by 180 degrees. As such, it would be expected that the two waves would cancel each other out up to channel chainage –50km, whilst the water levels between –50km and the open boundary would be representative of the remainder of the incident wave. These observations are confirmed in Graph 7. The water levels generated by each of the software packages are approximately horizontal up to chainage –50km, showing complete cancellation of the two waves. However, due to the damping effect of the channel bed, all calculated water levels, with the exception of ISIS, fall below the mean water level in the same manner as the analytical values. After chainage –50km, each of the three packages then returns to the sine wave formation of the incident wave as expected.

It can be seen in Graph 7, Appendix B, that the same interference is demonstrated by each of the three software packages in Graph 9, Appendix B, showing the calculated water levels at 4.0hrs. As the reflected wave will have returned to the open end of the channel at this time, though again be out of phase with the incident wave by 180 degrees, it would be expected that both waves would cancel each other out over the entire length of the channel. It is clear to see from the graph that this has happened to a certain extent, the slight remaining water levels being due again to the damping effect of the channel bed. Essentially, the amplitude of the returning wave would be less than that of the incident wave as it has travelled twice the length of the channel by the time it reaches the open boundary.

4.4 Analysis of Results – Velocity Histories

All velocities calculated at the closed boundary in Graph 10, Appendix B, are zero for the duration of the test as expected. This is not a very significant result, but does show that each of the three packages are correctly treating the specified flow time boundary in the intended way and hence forcing complete reflection of the incident wave.

The calculated velocities produced by each of the three packages at all other chainages, -25km, -50km and -75km, follow the analytical solutions fairly closely. HEC-RAS and ISIS produce almost identical values at all times and locations, while MIKE 11 shows the greatest variation, especially at the instance of superimposition at all chainages. The rate of change of this effect is masked when the rate of change of velocity is greatest, as can be seen after 2.5hrs at chainage –25km, and 3.5hrs at chainage –75km. When the rate of change of velocity is small, the effect of superimposition recorded by MIKE 11 is more pronounced as can be seen on Graph 12, Appendix B, at 3.0hrs.

4.5 Analysis of Results – Velocity Profiles

As highlighted earlier, Ippen3 describes a ‘Standing wave...’ with ‘…nodal points of zero amplitude of and maximum velocity at a distance of L/4 and at odd multiples thereof.’ Because the wave that Ippen describes is the result of an initial cosine wave, as opposed to the sine wave applied to this model, it would be expected that the points of maximum velocity, at distances of L/4 and 3L/4, would occur at the simulation times of 2.5 and 3.5 hours. Furthermore, this would be consistent with the observations made earlier that there was no amplitude of the water surface about the mean water level at these corresponding times and locations. However, inspection of Graphs 14 through to 18, Appendix B, show that maximum positive or negative velocities are achieved at these channel chainages, but not at the same times zero amplitudes are recorded at these locations. Instead maximum velocities are generated by the three packages at the approximate locations after 2.0, 3.0, and 4.0hrs. Zero velocities are also recorded at these times at the closed boundary and at multiples of L/2 chainage in the downstream direction, which correspond with the locations of zero amplitude.

All calculated and analytical velocities, at any given time and location, can be observed to switch between positive and negative values. It must be noted that the nature of the direction of the velocities will coincide with the direction each software package calculates positive flow. It is important not to confuse this with the way in which the software determines channel chainage. By default, each package calculates flow positively from the upstream to the downstream direction. Therefore, positive velocities refer to flow travelling towards the open boundary, and negative velocities towards the closed boundary. It can therefore be deduced from Graphs 14 to 18, Appendix B, that positive values of velocity correspond with low tide water levels, negative velocities with high tide levels, and zero velocity when the water level returns to mean water level.

The most significant observation about the calculated velocities at the given times is the relative variation from the analytical values, before and after superimposition of the two waves. No superimposition occurs at 2.0hrs, but thereafter up to –25km at 2.5hrs, -50km at 3.0hrs, -75km at 3.5hrs, and over the full length of the channel at 4hrs. It can be seen from Graphs 15, 16 and 17, Appendix B, that prior to any superimposition, the calculated water levels generated by all three packages follow the analytical values very closely and, on occasions, almost exactly. However, the calculated velocities within the region of the channel subject to the two waves simultaneously, deviate from the analytical velocities in the same manner observed for the water levels.

This deviation might again be due to the marginal difference in the length of the wave and the length of the channel. As there is some premature reflection at the closed boundary, forcing the water levels slightly above the expected values at the start of the simulation, the resultant imbalance of water within the system could give rise to slightly varied velocities.

4.6 Analysis of Results – Varying Manning's Values

To assess the effect of Manning's n value on the results, additional runs were undertaken with ISIS and HEC-RAS with a second value of n of 0.035. These solutions are plotted against the original level history plots (with n = 0.025) in Graphs 19 to 22.

The solutions from both packages are as to be expected – the main features of the solutions remain the same with the peaks and troughs (particularly at 0km, Graph 19, and –50km, Graph 21) being reduced in size. The position of these peaks and troughs, with respect to time, appear not to have been affected by the change in roughness.

4.7 Analysis of Results – Varying Time-Step

Investigations were made with all three software packages to assess the impact of model time step on the results. In theory, the accuracy of a numerical solution is dependent on the CFL number. The closer this is to unity the more accurate will be the solution. (For explicit numerical methods, the CFL number must be below 1.0 to obtain a stable solution. However, the packages tested here all use implicit techniques so that solutions may be obtained with CFL numbers much greater than 1.0.) The CFL number is defined as:

$$CFL = \frac{\Delta t}{\Delta x}(v + c) \qquad \text{(Equation 2)}$$

where Δt = Time step,
Δx = Space step, v = local velocity and

$c = \sqrt{gd}$ = local wave speed.

In practice, as all other variables are fixed, the CFL number defines the time step to be used.

Representative values for the velocity and wave speed were taken. Given the initial depth of 20m the wave speed is 14.01m/s. As this is much greater than the velocities of flow obtained in the solutions (a maximum of 1.0m/s was obtained), and since it was not necessary to find a strict time step via a precise CFL number, a value of zero was used for the velocity (same as the initial velocity). Using the parameters specified in Section 2.0, this results in a time step of 35.7 seconds for a CFL of 1.

The test was performed with 5 CFL numbers/time-steps as given in Table 3.1, below:

Table 3.1: CFL number and time step relationship

Δt seconds	CFL
3.6	0.1
6.0	0.17
35.7	1.0
60.0	1.68
600.0	16.81

Results for ISIS are shown in Graphs 23 - 26, for HEC-RAS in Graphs 27 - 30, and for MIKE 11 in Graphs 31 - 43, Appendix B.

For ISIS it can be seen that three distinct solutions have been obtained. The solutions with the same order of magnitude for CFL number are almost identical: the CFL 1.0 and 1.68 solutions, and the CFL 0.1 and 0.17 solutions. There is a notable phase shift between these two sets of results with little or no damping of the solution with increased time-step. However, with the highest time-step (600s) considerable damping is observed.

For HEC-RAS only the 6s, 60s and 600s time-steps could be tested as the software package limits the modeller to select from a predefined list of time-steps.

The results for the three time-steps tested with HEC-RAS show three distinct solutions, as were observed for ISIS. However, unlike the ISIS results there is no noticeable phase shift in the results instead there is an ever-increasing level of damping with an increasing time-step.

With the exception of the 600s time-step result the MIKE 11 results consistently show close agreement to one another. Using a 600s time-step the results are damped although not to the same degree as exhibited by ISIS and HEC-RAS. Coincidentally, the effect of this damping with the 600s time-step produces a result that is closer to the analytical solution.

When using a CFL of 0.17 and 0.1 with all the software packages there appears to be some inflexion that is generated at the boundary which then propagates through the solution, as illustrated in Graphs 26, 30, and 34, Appendix B.

5 DISCUSSION AND CONCLUSIONS

All calculated water levels prior to any superimposition closely follow the analytical values. However, as soon as the two waves meet, the calculated water levels show a varying degree of divergence from the analytical values. Nevertheless, to a reasonable degree of accuracy all the models predict the times at which the reflected wave falls into phase with the incident wave.

Each software package over-estimates the peak water level at the closed boundary, which in part may be a result of the wave length being slightly longer than the channel length (800m).

When using a 60s time-step the maximum water level at the closed boundary, which is achieved by the standing wave, is approximately the same for ISIS and MIKE 11, however, for HEC-RAS the result is noticeably lower by comparison. Although if the time-step is reduced to 6s, there is a much closer agreement.

For this test the results have been shown to be insensitive to Manning's n.

Sensitivity analysis on the model time-step has shown for all three software packages that the results can be appreciably affected. Model time-step can significantly produce numerical diffusion (damping) and can also lead to a phase change.

The sensitivity results have shown that level of damping is software dependant. A consequence of the damping is an underestimation of peak water levels along the length of the channel. For engineering design purposes this may have significant implications.

Inspection of the results would suggest that MIKE 11 may be the most appropriate software package for modelling this specific problem with default calculation parameters and tolerances as it would appear to be the least influenced/affected by time-step. However, if calculation parameters (i.e. implicit weighting) and tolerances (i.e. water elevation) were consistent across the software packages then it could be expected that results from the software packages would be in close agreement.

The developers of HEC-RAS may wish to consider providing more flexibility with respect to model time-step as having to select a model time-step from a predefined list of model time-steps was found to be limiting in this test.

5.1 Numerical Damping Parameter/Weighting Parameter

The numerical damping parameter or weighting parameter in both the Preissmann Box Scheme (used in ISIS and HEC-RAS) and the Abbott Scheme (used in MIKE 11) reflect the forwards or backwards time weighting of a function or derivative evaluated using the current or previous time-step values. If this parameter is zero, the function is fully weighted to the current or new time-step. If it is 0.5, the function is equally weighted towards the previous or current time-step and if it is 1.0, the function is fully weighted towards the previous time-step. Only values greater than or equal to 0.5 and less than or equal to unity are stable.

A value of 0.5 is formally second order accurate but is only marginally stable which explains why oscillations in a solution can be obtained in both ISIS and MIKE 11 if the weighting parameter is set to 0.5. In practice, from an accuracy perspective, it is best to set the weighting parameter as close to 0.5 as possible without compromising stability.

The default value in ISIS is 0.7, which the user is advised to reduce to 0.55 for tidal problems as the numerical dissipation can compromise the peak level and flow results. The default value in MIKE 11 is 0.5, which should be more accurate but can generate instability as demonstrated in the Culvert Test case. The default value in HEC-RAS is unity (1.0), which introduces a considerable amount of mathematical damping and associated error but should guarantee stability. However, the HEC-RAS user manual (Chapter 8) does clearly state that once a model is up and running, the theta value should be reduced towards 0.6 so as to improve the accuracy of the solution scheme, so long as the model will remain stable.

Given the above comments it should be noted that the use of the default damping/weighting parameters may affect the outcome of the results obtained for this test. It has been beyond the scope of this study to investigate this and as such it is recommended that this be investigated as part of further study.

6 RECOMMENDATIONS

From undertaking the test it is clear that the model time step can have a significant impact on the numerical solution produced by each of the software packages. Since there is no accepted method of assessing the appropriate selection of time step within the software packages it is recommended that some inbuilt procedures, methods or checks be incorporated to the software so as to assist and guide the modeller.

The test could be repeated with the channel length increased by 800m so that the channel length is exactly the same as the wave length.

Additional analysis could be undertaken with the use of interpolated cross-sections, at various intervals, so as to assess the extent of any numerical damping that may occur. This could also include the assessment of various methods of cross section interpolation.

Consideration should be given to undertaking the test with a small (nominal) inflow at the closed boundary. This may well enable the software packages to be run in steady or quasi-steady modes and thus remove the need for user defined initial conditions.

The test specification should include the time-stepping sensitivity analysis as undertaken and reported upon within this document.

HEC-RAS limits the modeller to a predefined list of model time-steps. Although this should not pose a problem for most modelling problems, the developers of HEC-RAS may wish to consider providing greater flexibility in the selection of the model time-step.

The affect of the damping/weighting parameters on the results obtained for this test should be investigated with each of the software packages.

The factors that can be of most significant in tidal (flood wave) modelling and which will improve (i.e. more accurate) numerical solutions include the correct boundary conditions; accurate geometric representation of the river (including roughness coefficients); adequate cross section spacing; computational time step (i.e. Courant stability/condition); theta weighting factor (set as close to 0.5 as possible without introducing numerical instabilities); and numerical solution tolerances (i.e. set low enough such that they do not effect the accuracy of the results). Alternative test specifications should be developed so as to assess these factors more fully for each of the software packages. However, the context in which they are applied needs to be considered, as these factors are often dataset and site specific.

7 REFERENCES

Crowder, R.A., Chen, Y., Falconer, R.A., (1997) Benchmarking and Scoping of Hydraulic River Models, Environment Agency Research and Technical Report, W88, 1997

Crowder, R.A., Pepper, A.T., Whitlow, C., Wright, N., Sleigh, A., Tomlinson, C., (2004), Benchmarking and Scoping of 1D Hydraulic River Models, Environment Agency Research and Technical Report, W5-105/TR1, 2003

Ippen, A.T. (Ed.), (1966) Estuary and Coastline Hydrodynamics, Engineering Society Monographs, New York, MacGraw-Hill Book Co., Chapter 10

APPENDIX A ANALYTICAL SOLUTION

Analytical Solution

Nomenclature

$\eta =$ Tidal Elevation

$u =$ velocity

$x =$ Distance from closed end (negative, i.e. closed end x = 0, open end x = - 100000)

$a_o =$ input wave amplitude

$\mu =$ characteristic of channel, involving friction

$\sigma =$ frequency, $2\pi / T$

$\kappa =$ wave number characteristic of channel, involving friction

$t =$ time

$T =$ wave period

Equation for tidal elevation at any point measured in the negative direction from the closed end, at any time t. i.e. at the closed end x = 0m, at the open end x = - 10000m.

$$\eta = a_o\left(e^{-\mu x}\cos(\sigma t + \kappa x) + e^{\mu x}\cos(\sigma t + \kappa x)\right) \qquad \text{(Equation A.1)}$$

[Ippen's equation 10.41]

Equation for velocity at any point, at anytime:

$$u = \frac{a}{h}\frac{\sigma}{\sqrt{\mu^2 + \kappa^2}}\left(e^{-\mu x}\cos(\sigma t - \kappa x + \alpha) - e^{\mu x}\cos(\sigma t + \kappa x + \alpha)\right) \qquad \text{(Equation A.2)}$$

[Which is very similar to Ippen's equation 10.48, (with the substitution for $\kappa_o = \sigma / C_o$ from equation 10.34), except that the second cos replaces a sin in Ippen's equation.]

The solution of these two equations gives us the analytic solutions which we compare with the model results. A spreadsheet has been prepared for this analytical solution, which accompanies the dataset for the test.

Below are the details of the solution, following Ippen.

We have the wave frequency,

$$\sigma = \frac{2\pi}{T} \qquad \text{(Equation A.3)}$$

where T is the period of the incoming wave, in seconds.

We must convert the Manning's n to a Chezy C by the formula,

$$C = \frac{h^{1/6}}{n} \qquad \text{(Equation A.4)}$$

where h is the mean depth of water in metres.

The friction factor, f,

$$f = \frac{8g}{C^2} \qquad \text{(Equation A.5)}$$

[after Ippens equation 10.24],

Ippen's linear friction coefficient, M, is given from his equation 10.29,

$$M = \frac{f}{3\pi} \frac{u_{\max}}{gh} \qquad \text{(Equation A.6)}$$

where, u_{max}, is some representative velocity in the channel.

Ippen derives the following equations that express the channel geometry and wave characteristics using two variables, κ, and μ:

$$\kappa_o^2 = \kappa - \mu^2 \qquad \text{(Equation A.7)}$$

[Ippen's equation 10.34]

$$\kappa_o = \frac{\sigma}{C_o} = \frac{\alpha}{\sqrt{gh}} \qquad \text{(Equation A.8)}$$

$$\frac{g}{\sigma} M = 2 \frac{\mu}{\kappa} \frac{1}{1 - (\mu / \kappa)^2} \qquad \text{(Equation A.9)}$$

[Ippen's equation 10.35a]

$$\tan\alpha = \frac{\mu}{\kappa} \qquad \text{(Equation A.10)}$$

We need to solve equations A.7 to A.9 for κ , and μ . Equation A.9 can be rearranged to

$$\frac{g}{\sigma} M = 2\mu \frac{\kappa}{\kappa_o^2} \qquad \text{(Equation A.11)}$$

Combining this with equation 7 to eliminate μ, gives

$$\kappa^4 - \kappa_o^2 \kappa - \frac{\kappa_o^2}{4} \left(\frac{gM}{\sigma} \right)^2 = 0 \qquad \text{(Equation A.12)}$$

This can be solved for κ^2 using the formula for quadratic equations [for $ax^2 + bx + c = 0$ then $x = \left(-b \pm \sqrt{b^2 - 4ac}\right) / 2a$] to give

$$\kappa = \left[0.5 \left(\kappa_o^2 + \sqrt{\kappa_o^4 + \kappa_o^4 (gM / \sigma)^2} \right) \right]^{1/2} \qquad \text{(Equation A.13)}$$

and from equation A.11

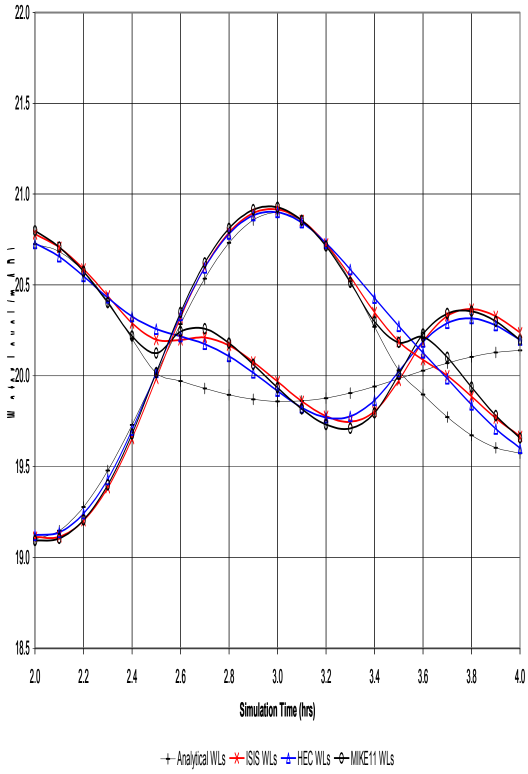
$$\mu = \frac{gM}{\sigma}\frac{\kappa_o^2}{2\kappa} \qquad \text{(Equation A.14)}$$

and

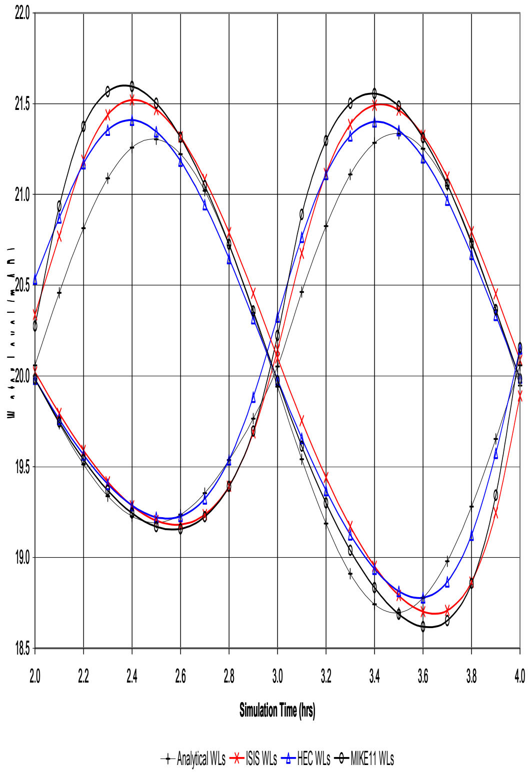
$$\alpha = \tan^{-1}(\mu / \kappa) \qquad \text{(Equation A.15)}$$

APPENDIX B   RESULTS

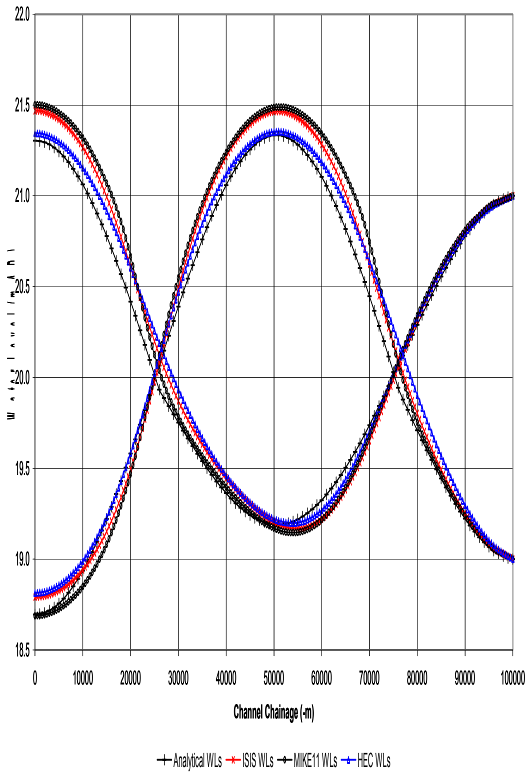
Graph 2 - Test E: Comparison of Calculated Water Level Histories at Chainage -25km



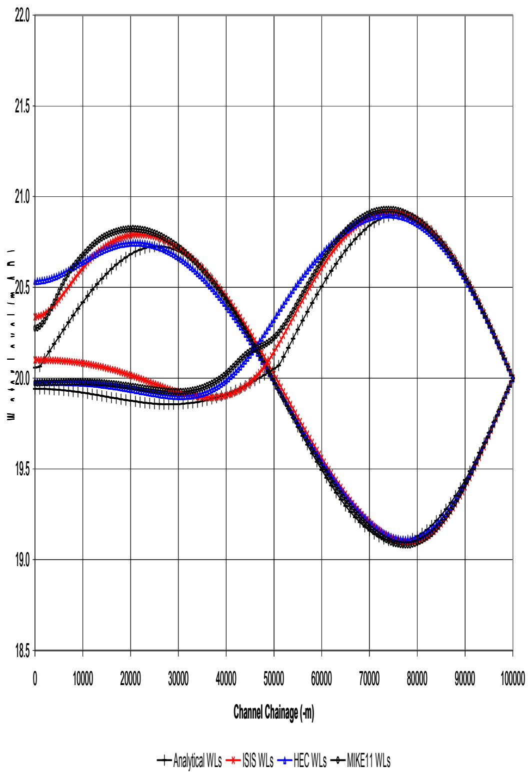
Graph 3 - Test E: Comparison of Calculated Water Level Histories at Chainage -50km



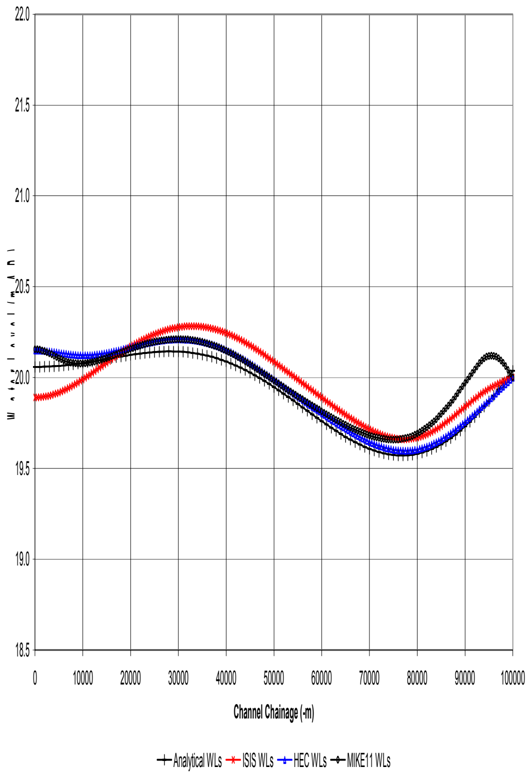
Graph 8 - Test E: Comparison of Calculated Longitudinal Water Level Profiles at 2.5hrs



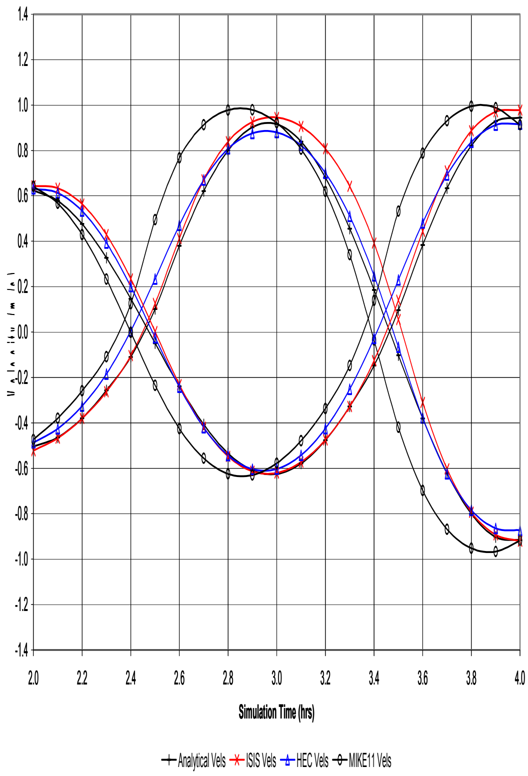
Graph 7 - Test E: Comparison of Calculated Longitudinal Water Level Profiles at 2.0hrs



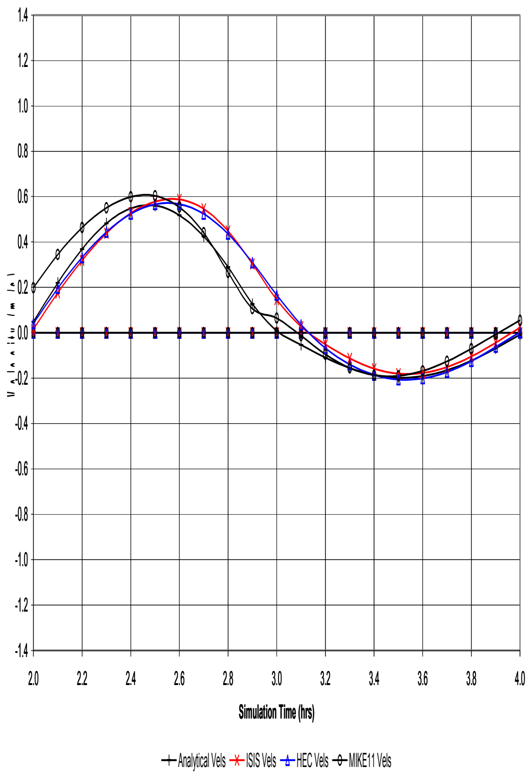
Graph 9 - Test E: Comparison of Calculated Longitudinal Water Level Profiles at 4.0hrs



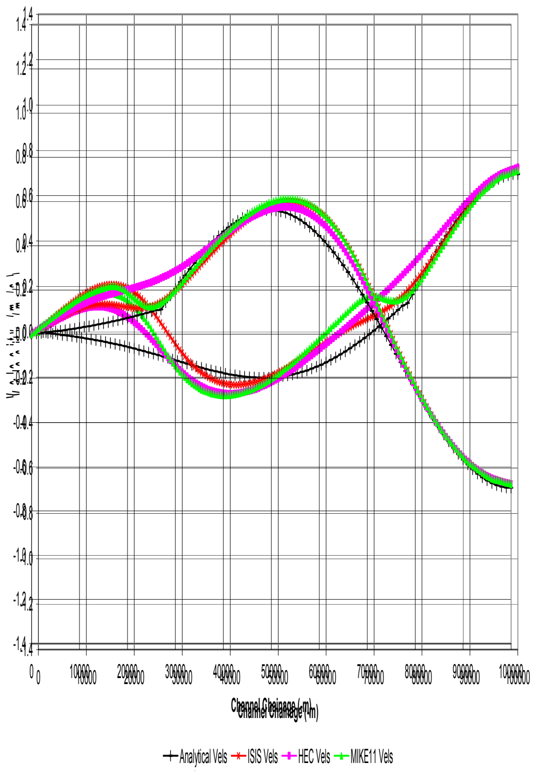
Graph 13 - Test E: Comparison of Calculated Velocity Histories at Chainage -25km



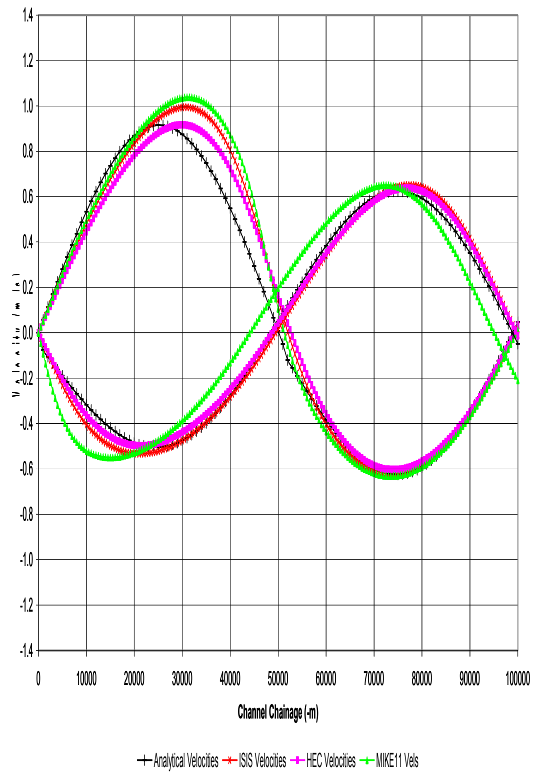
Graph 12 - Test E: Comparison of Calculated Velocity Histories at Chainage -50km



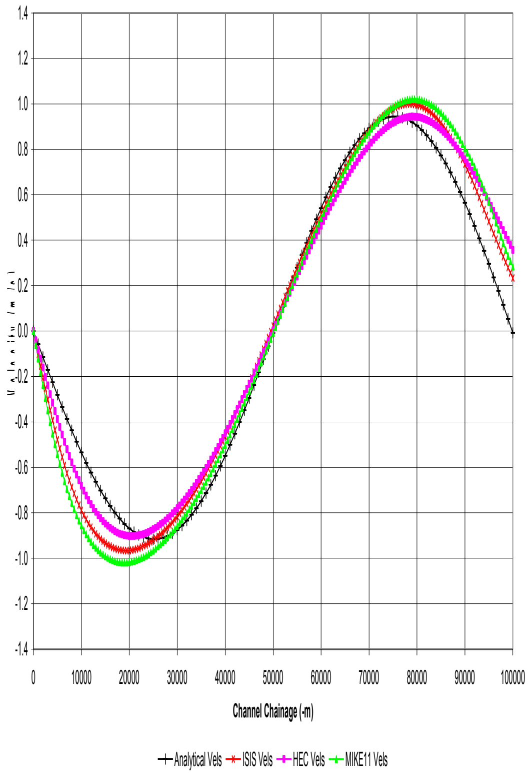
Graph 15 - Test E: Comparison of Calculated Longitudinal Velocity Profiles at 2.5hrs



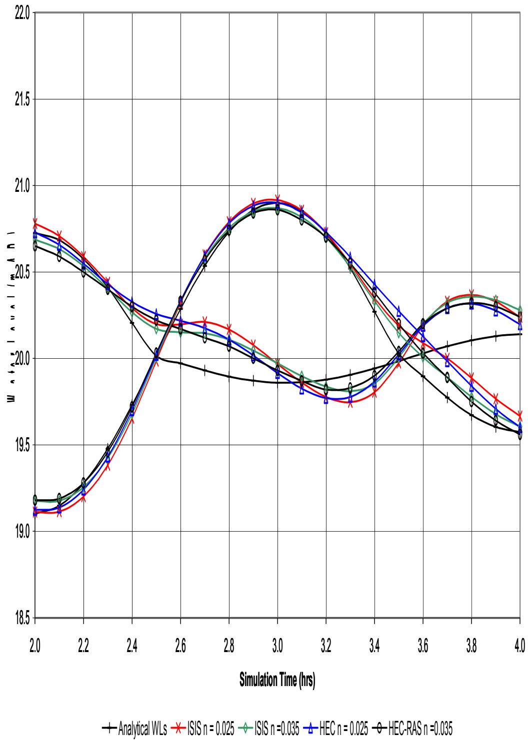
Graph 16 - Test E: Comparison of Calculated Longitudinal Velocity Profiles at 3.0hrs



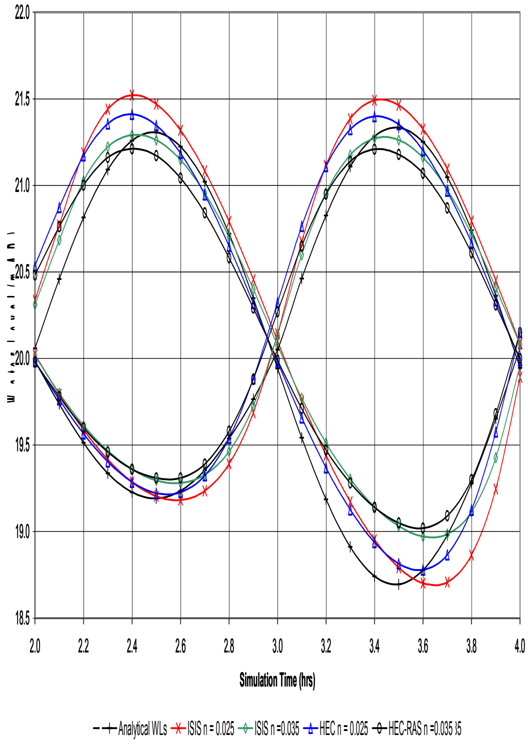
Graph 18 - Test E: Comparison of Calculated Longitudinal Velocities at 4.0hrs



Graph 20 - Test E: Comparison of Calculated Water Level Histories at Chainage -25km
Varying Mannings n

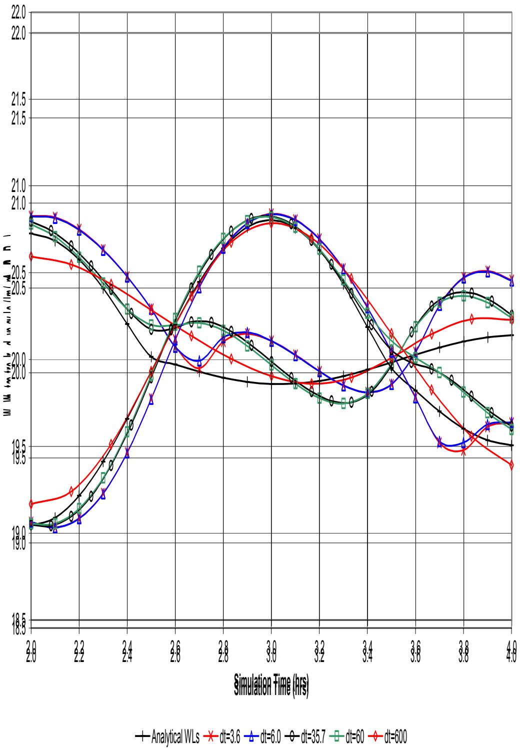


Graph 21 - Test E: Comparison of Calculated Water Level Histories at Chainage 50km
Varying Mannings n



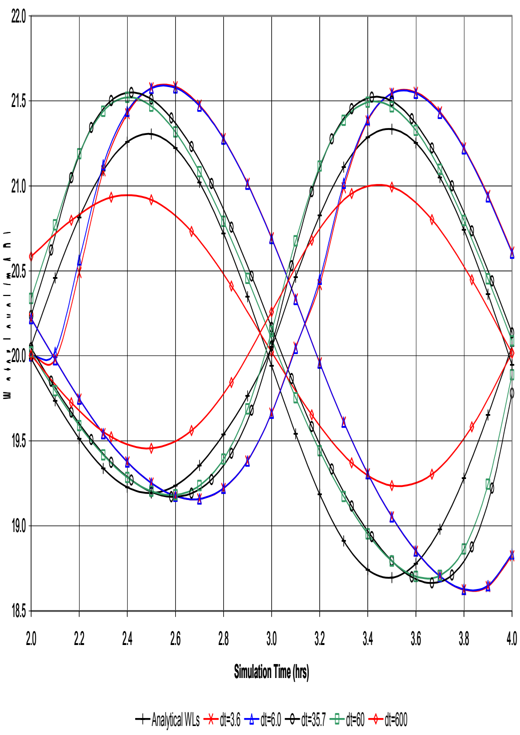
Graph 26 - Test E: Comparison of Calculated Water Level Histories at Chainage -25km

ISIS Results - Varying Time Step



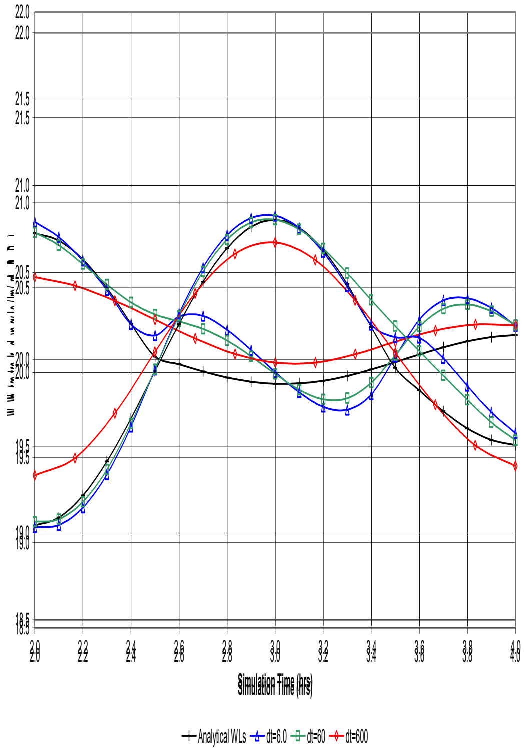
Graph 23 - Test E: Comparison of Calculated Water Level Histories at Chainage -50km

ISIS Results - Varying Time Step



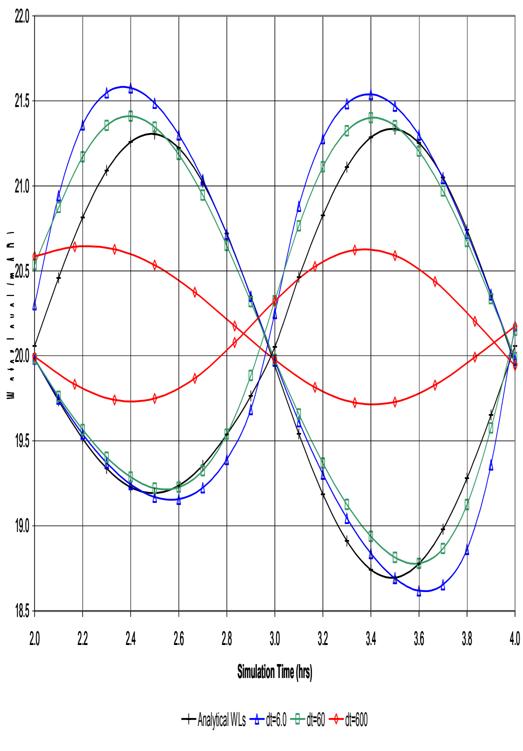
Graph 28 - Test E: Comparison of Calculated Water Level Histories at Chainage -25km

HEC-RAS Results - Varying Time Step



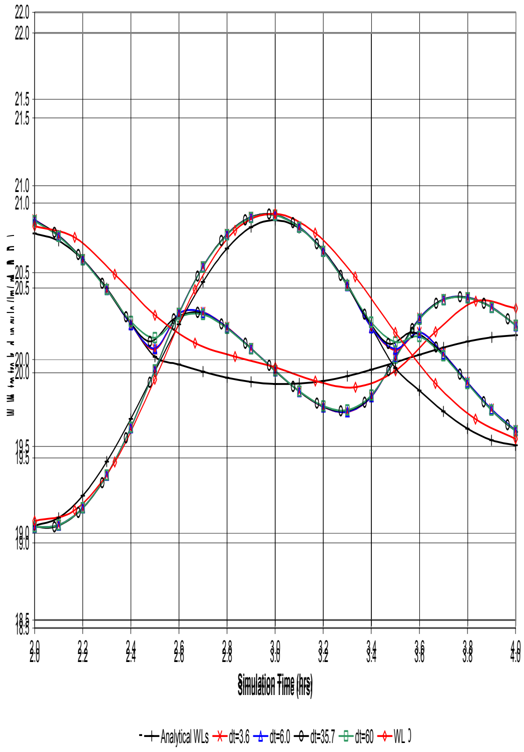
Graph 27 - Test E: Comparison of Calculated Water Level Histories at Chainage -50km

HEC-RAS Results - Varying Time Step



Graph 32 - Test E: Comparison of Calculated Water Level Histories at Chainage -25km

MIKE 11 Results - Varying Time Step



Graph 31 - Test E: Comparison of Calculated Water Level Histories at Chainage -50km

MIKE 11 Results - Varying Time Step