



# HMRC's CGE model documentation

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## Contents

<b>Overview of Computable General Equilibrium modelling</b>	<b>1</b>
What is a CGE model?	1
Why use a CGE model?	2
How does CGE differ from other modelling?	3
How HMRC's CGE model is used?	4
Limitations of HMRC's CGE model	5
<b>HMRC's CGE model</b>	<b>6</b>
Model overview	6
Social Accounting Matrix	7
Households	9
Firms	11
Government	13
Foreign Sector	15
Model dynamics	15
<b>Technical Annex</b>	<b>17</b>
Functional Forms	17
Investment Dynamics	20

## Overview of Computable General Equilibrium modelling

- 1.1. Computable General Equilibrium (CGE) models have been used since the early 1970s<sup>1</sup> to analyse the economic effects of changes in taxation, trade and environmental policies. Governments and institutions such as the World Bank, OECD, WTO and IMF use CGE models in the policy development process. HMRC currently uses and maintains a CGE model with a detailed representation of the UK tax system.
- 1.2. HMRC's CGE model has mainly been developed in conjunction with the University of Nottingham and Professor Adam Blake and undergoes continual development internally and using external consultants. The current version of the model has recently been peer reviewed by Professor Thomas Rutherford from the University of Wisconsin and Professor Christoph Böhringer from the University of Oldenburg. The peer review finds the model's "implementation meets the requirements of state-of-the-art CGE-based tax policy analysis."

### What is a CGE model?

- 1.3. A CGE model is a large-scale numerical model that simulates the core economic interactions in the economy. It uses data on the structure of the economy along with a set of equations based on economic theory to estimate the effects of fiscal policies on the economy.
- 1.4. HMRC's CGE model uses National Accounts data published by the Office for National Statistics (ONS), UK survey data on households and firms, and HMRC's administrative data to capture the complex transactions in the economy. This data provides a snapshot of the economy in a single year, which is used as a starting point for a baseline to compare policy simulations against.
- 1.5. The underlying data is linked together through a set of equations that governs how the economy evolves over time in response to a policy change. These equations, which are based on the economic theory of general equilibrium, ensure supply and demand for goods, services and factors of production in the economy are balanced, and determine how firms and households respond in response to changes in incentives.

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<sup>1</sup> Shoven, J.B., and J. Whalley. 1972. "A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S." *Journal of Public Economics* 1:281-322.

- 1.6. Economic relationships in the model are based on theory and empirical evidence from the academic literature, but in some cases there is no consensus on the relevant parameters. In such cases it is important to be clear about the assumptions being made and recognise that there are differences of opinion.

### Why use a CGE model?

- 1.7. CGE models capture the inter-dependencies between the different product markets, factor markets, and public and private sectors in the economy, enabling analysis of how a policy change targeted in one part of the economy will affect the rest of the economy. Hence, this is a useful tool for analysis as policies can often have indirect effects that are difficult to quantify.

#### Box 1: A summary of the wider economic impacts that CGE analysis captures

**Inter-sectoral linkages:** firms across sectors are interlinked as they use intermediate inputs produced by each other and compete for factors of production (labour and capital). A tax on a given sector or product will cascade to other sectors and products via the supply chain. These inter-sectoral linkages are extensively captured in CGE models in a more sophisticated way than the traditional input-output multiplier approach.

**Inter-institutional linkages:** CGE models capture the linkages between the main institutions in the economy: households, firms, investors, exporters, importers, and government. They all compete for goods and services produced domestically or imported. The household sector also sells factor inputs (labour and capital) to firms. Both firms and households pay taxes to the government which in turn subsidises firms and provides for the welfare state and public goods.

**Inter-temporal dynamics:** CGE models capture inter-temporal decision making of economic agents. It can be used to evaluate how the effects of current policies propagate over time and the immediate effect of pre-announced policies.

## How does CGE differ from other modelling?

- 1.8. Some classes of models that attempt to estimate the economic impact of tax changes are partial equilibrium models which look closely at one section of the economy. These models are useful for understanding the detailed mechanisms of the market they are examining, but assume the rest of the economy is unaffected. Unlike CGE models, they do not capture the linkages discussed in Box 1 above.
- 1.9. Other models that do attempt to estimate wider economic effects of policy changes are based on a combination of historical trends and economic theory. These classes of models, called macro-econometric forecasting models, are estimated statistically and are mainly used for forecasting economic variables. However, a key criticism of this class of models is that the estimated parameters are not policy-invariant (Lucas, 1976); estimating the effects of new policies by relying on past relationships may not make sense if the policy changes itself affects the relationships. They are also predominantly demand-led ignoring the supply side of the economy. On the other hand, unlike CGE models, they are statistical models that can provide a confidence interval around estimates making them suitable for forecasting.
- 1.10. Although CGE models can simulate future effects of policy changes, they are not a forecasting tool. Policies are evaluated by comparing the economy between two states of the world. The pre-policy baseline is generated from the base year data and the impact of a policy is estimated by measuring deviations from the baseline following the policy change.
- 1.11. CGE models are similar to Dynamic Stochastic General Equilibrium (DSGE) models in that both model classes are based on microeconomic foundations rather than historical relationships. The main difference between the two types of model is DSGE models attempts to capture fluctuations in business cycles whereas CGE models tend to focus more on medium-run and long-run macroeconomic analysis. Standard DSGE models also tend to have less detailed representation of firms and households than CGE models. On the other hand, DSGE models allow for random variations to account for uncertainty whereas CGE models are deterministic, with agents facing no uncertainty about the future.

## How HMRC's CGE model is used?

- 1.12. HMRC's CGE model is used to provide evidence on the macroeconomic effects of fiscal policy options over time. Modelling typically focuses on policies likely to have significant wider macroeconomic effects.
- 1.13. Many different variables can be reported, with percentage change in GDP being the primary result. GDP can be decomposed into individual components: consumption, investment, government spending, exports and imports; to see how these variables change over time. In addition, the model can report the distributional impact on households, changes in sector-level output and changes in tax revenue, to get an in-depth understanding of the underlying mechanisms in the economy.
- 1.14. Results are presented as a range alongside a discussion of the key mechanisms and economic theory underlying them. This serves a dual purpose of demonstrating the degree of uncertainty of the results and highlights which parameters are underpinning the results.
- 1.15. There are several steps to conducting policy analysis using the CGE model. Firstly, the policy has to be coded into a simulation file using specialist computer software. The core model code may have to be adapted in order to run certain simulations. Additional data may also be needed to implement the policy, for instance outputs from other models. Secondly, the results have to be checked to ensure the simulation is correctly specified; sensitivity analysis is conducted to understand key the mechanisms underpinning the results. Thirdly, results are transferred from the specialist software into mainstream packages for data visualisation. Numerical results are presented to policy customers alongside a written narrative and sense-checked against existing evidence. These steps can be very time consuming with robust analysis of a policy often taking a small team more than one month to deliver.
- 1.16. It should be noted that the model's results are not forecasts. They are indicative results showing where the key effects are likely to happen and their relative magnitude. As such CGE model results are not directly comparable with results from forecasting models.

## Limitations of HMRC's CGE model

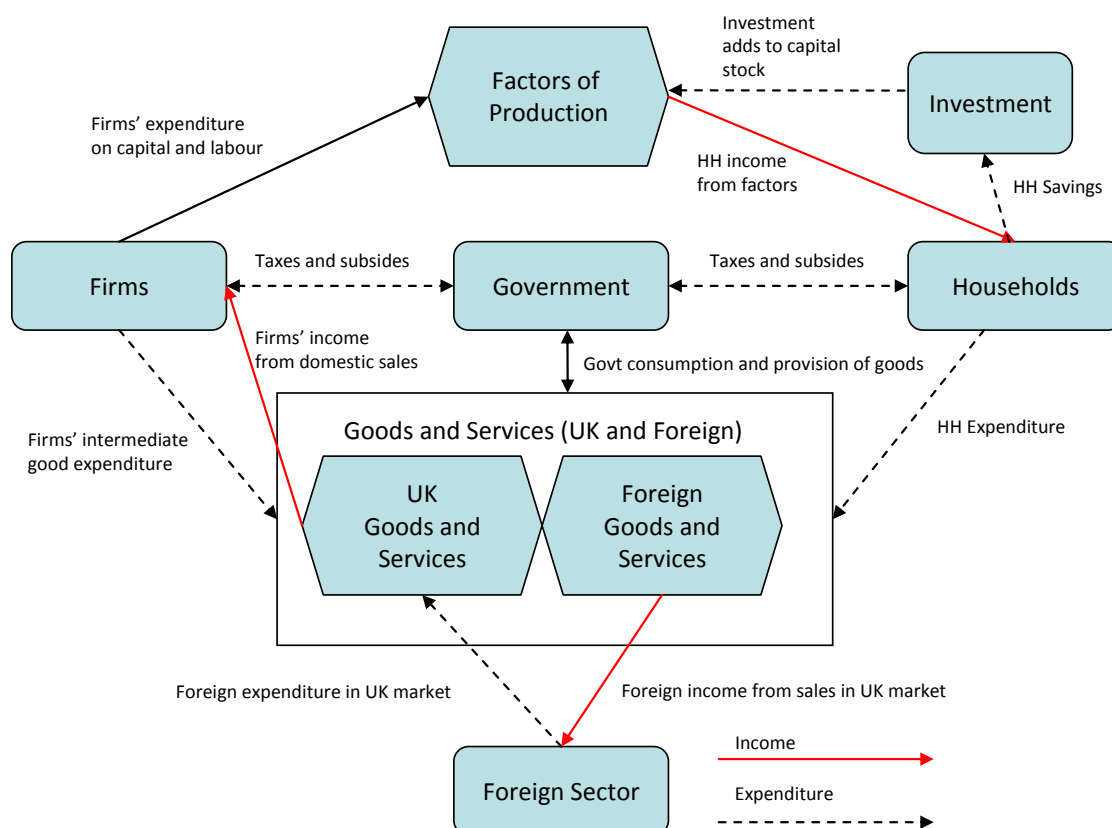
- 1.17. **Externalities:** The model captures the value of market transactions in the economy as it is based on National Accounts data. However, costs or benefits caused by non-marketable transactions, such as externalities, are not captured. An example of an externality would be the effect of tobacco duties on health costs, or the effect of fuel duties on pollution. These externalities can be estimated off-model by applying a factor to a change in demand but are not part of the optimisation processes within the model.
- 1.18. **No excess capacity:** The model proxies unemployment as voluntary in that some households will find it optimal not to participate in the labour market, given current prices and wages in the economy. This assumption enables the model to capture the change in labour costs to firms and labour income to households following a policy change. However, it does not distinguish between the change in voluntary and involuntary employment. Similarly, there is no spare capacity of capital utilisation. A measure of capacity utilisation is important for economic forecasting models, but is less important for CGE models that are often looking at long term effects of a policy change. Future model developments may consider how to improve the functionality of the labour market.
- 1.19. **No monetary side:** Monetary effects are not captured in the model as there is no market for money. This means the model does not have a detailed treatment of the financial sector. Consequently the model is unable to capture the effects of taxes levied on savings or assets such as capital gains tax, stamp duty, or inheritance tax. This limitation may be addressed in future model developments.
- 1.20. **Simplified foreign sector:** The model does not have a detailed treatment of foreign trade and investment. All investment is currently assumed to be domestically sourced. The foreign sector does not have any production activity but merely absorbs UK exports depending on the level of supply, and supplies UK imports depending on the level of demand.

## HMRC's CGE model

### Model overview

- 2.1. CGE models are based on the circular flow of income which illustrates how economic agents receive and spend income. Figure 2.1 shows HMRC's CGE representation of the circular flow of income, where households receive income from owning the factors of production, and firms and the foreign sector receive income from selling goods and services. Government provides public goods, and receives taxes from and distributes transfer payments to households and firms.
- 2.2. HMRC's CGE model is dynamic as it tracks the evolution of the economy over time in response to a policy change. In the baseline, it is assumed the economy follows a steady-state growth path where all economic activities grow at constant rate.

Figure 2.1: Circular flow of income





## Social Accounting Matrix

2.3. The monetary flows shown in figure 2.1 are represented coherently in a social accounting matrix (SAM). The SAM, shown below in table 2.2, is the main data source for HMRC's CGE model and is a compact display of the connections between economic agents in the economy. The SAM embodies the double-entry accounting principle, where one agent's expenditure is another agent's income. Each cell denotes the expenditure of the column agent as income to the row agent. The SAM is balanced which means corresponding column totals and row totals must be equal. For example, household income is equal to household expenditure plus savings. The data that populates the SAM is formed from a number of sub-matrices (see square brackets in SAM table) the details of which are given in table 2.1.

**Table 2.1: Sub-matrices of the SAM**

Matrix	Description	Data Source
DEMAND	Demand for products by households, foreign sector, govt, and investors	Input-Output Supply and Use Tables (Table 2)
SUPPLY	Total supply of each product; including imports, excise duties and VAT	Input-Output Supply and Use Tables (Table 1)
MAKE	Domestic output of each industry split by the products they produce	ONS (not publically available) and internal calculations
USE	Intermediate consumption for each industry by the product consumed	Input-Output Supply and Use Tables (Table 2)
HH INC	Labour income, self-employed income, investment income, private pension, state pension, disability benefit, child benefit, unemployment benefits, tax credits, consumption, savings, Income Tax, employee NICs split by households	Input-Output Supply and Use Tables (Table 2), Family Resource Survey, Households Below Average Income, HMRC administrative data
HH EXP	Households' expenditure by household type and product	Input-Output Supply and Use Tables (Table 3), Living Costs and Food Survey
HH EARN	Households' labour income from each sector by household type, including labour skill type and industry	Input-Output Supply and Use Tables (Table 2), Family Resource Survey
GVA	Compensation to employees (split by labour skill type) and employer NICs by industry. Income to capital (split by debt and equity), corporation tax and capital allowances both split by firm size and industry. Net (tax plus subsidy) production taxes by industry	Input-Output Supply and Use Tables (Table 2), FAME, HMRC administrative data
GFCF	Gross fixed capital formation by industry and product	Input-Output Supply and Use Tables (Table 4)

**Table 2.2: HMRC's CGE Social Accounting Matrix**

[NAME] = Name of sub-matrix used by HMRC's CGE model

<i>Expenditure</i> <i>Income</i>	Industries	Goods and Services	Factors of Productions	Households	Foreign Sector	Investment	Government	Total
<b>Industries</b>		Gross Output [MAKE]					Production subsidies [GVA]	<i>Gross Domestic Output</i>
<b>Goods and Services</b>	Intermediate Input [USE]			Household Consumption [HH EXP]	Exports to Foreign Sector [DEMAND]	Purchase goods for investment [GFCF]	Government consumption [DEMAND]	<i>Total Demand</i>
<b>Factors of Productions</b>	Gross income to capital and labour [GVA]							<i>Total factor income</i>
<b>Households</b>			Household earnings from labour and capital [HH INC, HH EARN]				State pensions, disability, child and unemployment benefits [HH INC]	<i>Total Household Income</i>
<b>Foreign Sector</b>		Imports from Foreign Sector [SUPPLY]						<i>Total outflow to Foreign Sector</i>
<b>Investment</b>				Household savings [HH INC]	Foreign savings [GFCF]			<i>Total Savings</i>
<b>Government</b>	Production taxes [GVA]	VAT, Excise Duties [SUPPLY]	Corporation tax, Capital Allowances, Employer NICs [GVA]	Income tax, Employee NICs [HH INC]				<i>Total Government Income</i>
<b>Total</b>	<i>Total Industry Expenditure</i>	<i>Total Supply Expenditure</i>	<i>Total Factor Expenditure</i>	<i>Total Household Expenditure</i>	<i>Total inflows from Foreign Sector</i>	<i>Total Investment Expenditure</i>	<i>Total Government Expenditure</i>	

## Households

### Data

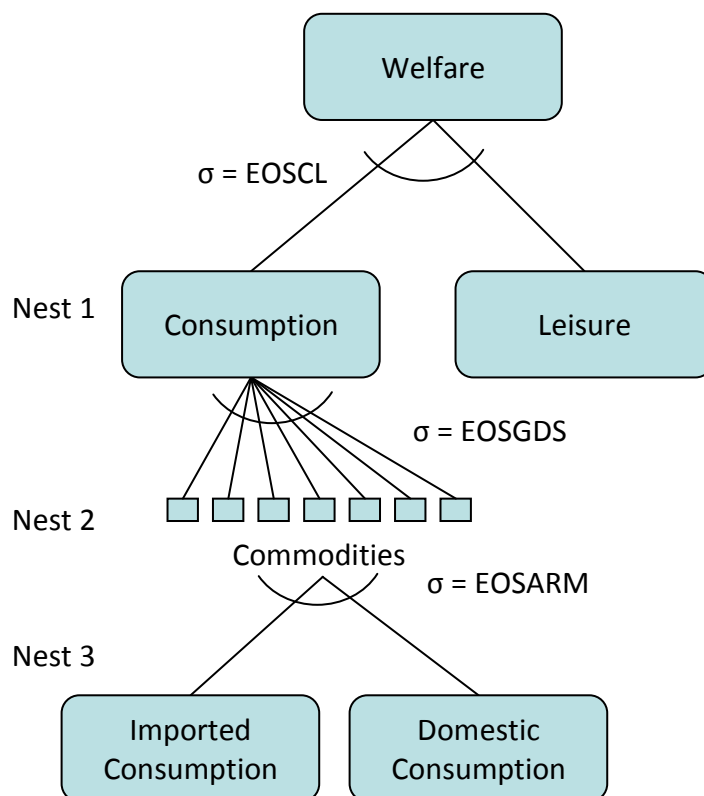
- 2.4. There is a maximum of 50 representative households in the model based on aggregated survey data. Households are sorted into equivalised income deciles and each decile is split into 5 categories: retired, couple with children, couple without children, single with children and single without children. The model has the flexibility to aggregate households by types and/or decile depending on the simulations.
- 2.5. Aggregate data such as total household consumption and income come from the ONS Supply and Use Tables. This data is spread proportionally by households' income sources and expenditure patterns based on the Family Resource Survey (FRS) and Living Cost and Food survey (LCF), respectively.
- 2.6. In the model, household labour income is split into three skill levels: professional, skilled and unskilled, which are derived from the Standard Occupational Classification (SOC) 2007 codes in the FRS. Policies can have different effects on the wage of each skill level affecting households differently depending on the composition of skills level. Household income from work is taxed via income tax and employee NICs. Taxation data is based on HMRC administrative records combined with the FRS data.
- 2.7. Households' income also includes transfers from government which include state pensions, tax credits, child benefits, disability benefits and unemployment related benefits. Benefits can have a positive, negative or neutral impact on labour supply in the model. For instance, benefits conditional on work, like tax credits, can induce households to supply more labour to the market. Unemployment related benefits are likely to have a negative effect on the incentive to work. The composition of benefits differs across household groups, which adds richness to the model. Households also receive income from self-employed activities, private pensions and income from investments.
- 2.8. Household consumption is split by commodity using the LCF survey data and the ONS Supply and Use tables. The spending patterns of households vary across household types and income deciles.
- 2.9. Increasing taxation on consumption has a heterogeneous impact on households. Households that consume more of a taxed good are relatively worse off as their real income declines. This could affect household labour supply, depending on income and substitution effects between work and leisure. Firms supplying the taxed good experience a reduction in demand affecting output and demand for labour and capital, in turn affecting households' labour and capital income. Similarly, changing any tax in the

model has a heterogeneous impact on households as they ultimately own the factors of production as well as consume goods and services.

### Nesting structure

- 2.10. Households maximise an intra-period nested constant elasticity of substitution (CES) utility function across all time periods using an inter-temporal CES utility function subject to a lifetime budget constraint. The lifetime budget constraint ensures that lifetime net income from labour, investments and government transfers is equal to the value of lifetime consumption and leisure. Future consumption is discounted using a time preference coefficient and household inter-period substitution is governed by an inter-temporal elasticity of substitution. The intra-period nested CES utility function for households can be represented in the nesting structure below.

Figure 2.3: Household nesting structure



- 2.11. There are three nests in the household CES utility function, with each nest representing a trade-off that households face, determined by an elasticity of substitution (EOS) parameter. A high EOS represents higher substitution possibilities in response to relative price changes. It is important to understand that each nest is solved simultaneously, rather than sequentially. For example, a tax affecting imports also affects the ratio of imported to

domestically produced goods, along with aggregate consumption and labour supply.

- 2.12. The first nest determines households' aggregate consumption and leisure demand. The EOS between aggregate consumption and leisure determines the response of the consumption-leisure ratio to changes in prices of goods and wages (price of leisure). This nest also determines labour supply as the amount of time spent enjoying leisure is inversely proportional to employment hours.
- 2.13. The second nest determines how aggregate consumption is split between different goods and services for each household. This is governed by a Stone-Geary utility function, which yields the linear expenditure system of household demand and the EOS between goods. The linear expenditure system imposes a minimum level of consumption for each good to control for heterogeneous price and income elasticities of demand, given constant elasticity of substitution across all goods. The income and price elasticities are derived from the GTAP database<sup>2</sup>.
- 2.14. The final nest determines the ratio of consumption between domestically produced and imported goods. In the model, Armington preferences are used to allocate demand for domestic and imported goods within the same product category. Such preferences mirror real world trading patterns where countries simultaneously import and export goods in the same product category. This is explained in more detail in the next section on production.

## Firms

### Data

- 2.15. Individual firms are aggregated into 110 separate industries using the ONS UK Standard Industrial Classification 2007. The model has the flexibility to aggregate the sectors to any level depending on modelling requirements. Firms' production functions consist of intermediate inputs, labour and capital.
- 2.16. The ONS Supply and Use Tables provide data on what commodities each industry uses as intermediate inputs.
- 2.17. Gross value-added (GVA), which comprises compensation for the use of production factors (physical capital and labour), is the difference between gross value of output and intermediate good use. Data on compensation to employees comes from the ONS Supply and Use tables, and is split out into

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<sup>2</sup> <https://www.gtap.agecon.purdue.edu/resources/download/4184.pdf>

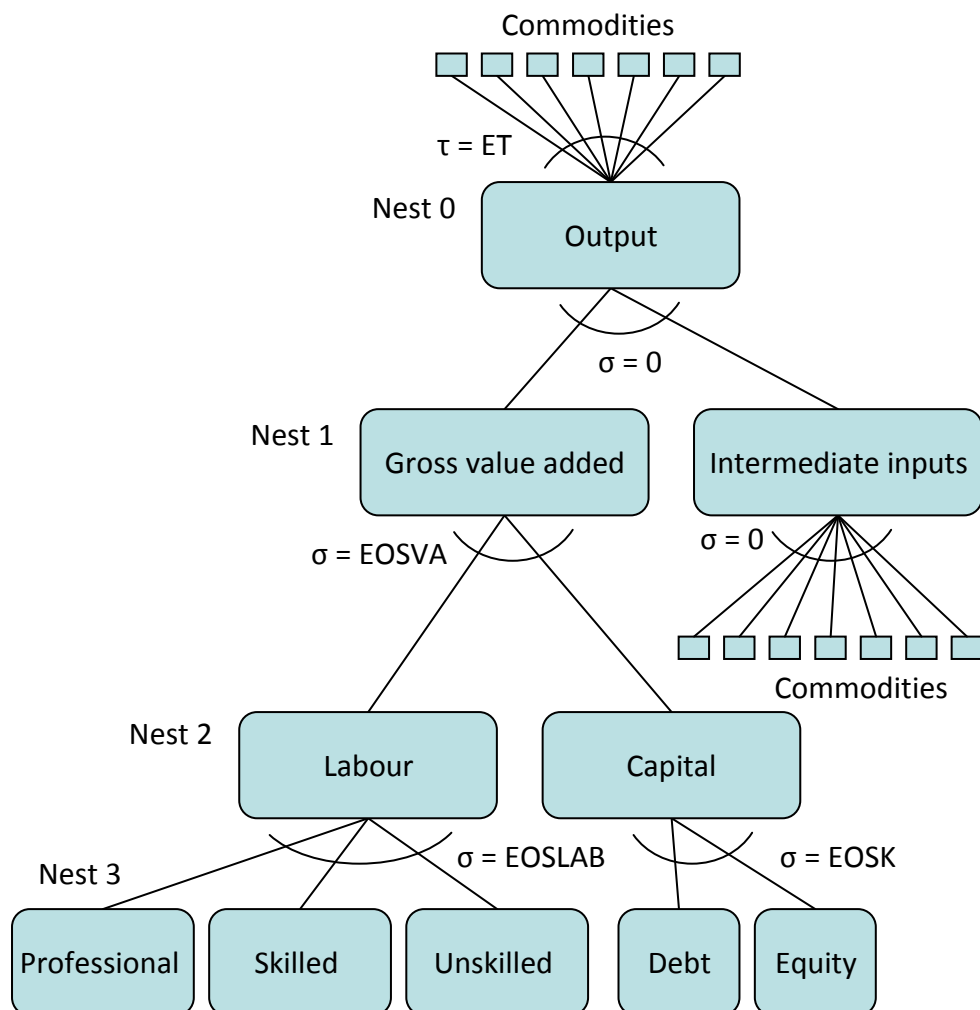
three labour skill types and employer NICs using the FRS and HMRC administrative data respectively.

- 2.18. Gross Operating Surplus (compensation to capital) by sectors is also obtained from the ONS Supply and Use Tables. Gross operating surplus is split into retained earnings and interest payments to reflect the level of debt gearing across sectors using FAME (Financial Analysis Made Easy) company accounts. Retained earnings are further split into earnings net of tax, corporation tax and reductions in corporation tax due to capital allowances. To reflect the structure of corporation tax, retained earnings are further decomposed into large, medium and small firms using the FAME dataset.
- 2.19. The GVA table also includes data on production taxes which are modelled as a composite tax that includes business rates net of subsidies from ONS Supply and Use Tables.

### Nesting structure

- 2.20. Production is determined by the profit maximising behaviour of a representative firm in each sector. Firms minimise the costs of production arising from the use of factors, intermediate goods and taxes for a given level of output. Figure 2.4 shows the CES nesting structure depicting the production function of each representative firm. Note the stages are determined simultaneously, and the nesting structure allows for different elasticities of substitution among different inputs.
- 2.21. At the very top nest, firms turn their composite output into commodities. Firms in the model have the ability to produce more than one commodity type. If the prices of commodities change, firms are able to change their output mix to maximise profits. This is captured by the constant elasticity of transformation (CET) production function, with the elasticity of transformation determining the ease with which sectors can change their output mix in response to changes in relative commodity prices.
- 2.22. In the first nest, firms combine aggregate intermediate inputs with Gross Value-Added (GVA). Typically the EOS between these inputs are zero, implying the production function exhibits fixed proportions among its inputs, also known as the Leontief production function.
- 2.23. In the second nest, firms combine aggregate capital and aggregate labour according to a CES production function to form the valued added composite. The EOS in this nest determines the ease with which firms can change their capital-labour ratio in response to changes in relative prices and governs the elasticity of demand for labour and capital. The choice of intermediate goods a firm uses is governed by a Leontief production function.

Figure 2.4: Production nesting structure



2.24. The third nest shows firms use a combination of different labour and capital types as inputs, with associated EOS parameters determining the substitutability between types. This is important as a change in corporation tax, for example, will affect the relative price of equity and debt (affecting the cost of capital), which may lead firms to adjust their gearing ratio.

## Government

### Data

2.25. In the model, government collects revenue from taxes net of subsidies and spends this on the provision of public services, capital infrastructure and benefits payments to households. The model also includes the budget deficit as the difference between tax revenue and spending.

- 2.26. The provision of public services includes expenditure on health, education, public administration and defence. Data is obtained from the ONS Supply and Use Tables. Similarly, data on government investment in infrastructure and other assets is obtained from the ONS Gross Fixed Capital Formation (GFCF) matrix. Data on transfer payments comes from the FRS combined with ONS Public Sector Finance statistics. Tax revenue data is a mixture of ONS public sector finance statistics and HMRC administrative data.

### The role of government

- 2.27. Within the CGE framework, the provision of public services is modelled as representative firms using factors of production to supply education, health, public administration and defence. Firms use the same production structure as defined in section 2.2. The government buys most of the output of these sectors.
- 2.28. The way the government reacts to changes in its budget position (due to changes in tax revenue and spending) in each period is determined by the choice of government 'closure rules'. There are three main closure rules: spending, transfers and debt, which are used interchangeably depending on the policy simulation.
- 2.29. The spending closure rule specifies that any changes in the budgetary position are financed through government spending. This implies changes in the demand for education, health, public administration and defence. The transfer closure rule implies that the budget deficit/surplus is financed through transfer payments to households.
- 2.30. The debt closure rule assumes that changes in the budget position are not offset by other policies, but affect the level of national debt. Change in the debt level is principally a transfer to/from the corporate sector who buy/sell gilt-edged bonds to finance the debt. In a separate version of this closure rule the level of National Debt may be assumed to affect the risk premium faced by investors in the economy. The higher the level of national debt, the greater is the risk premium resulting in lower returns to investors. The sensitivity of the risk premium to debt levels can be adjusted in the model.
- 2.31. The closure rules have different distributional impacts. For example, if a budget surplus is offset by increasing transfer payments, then recipients of welfare income (households in the lower income brackets) benefit. If the debt closure is used, the corporate sector will sell their gilt-edged bonds and investors benefit from higher returns on capital due to the lower risk premium.



## Foreign Sector

- 2.32. The foreign sector purchases exports from domestic firms, and domestic households and firms purchase imports from the foreign sector. Foreign exchange is used to purchase imports, and its value is determined by the value of exports and imports in the economy. For example, if the value of exports increases, as a result of domestic firms becoming more competitive after a tax cut, then this increases the value of foreign exchange causing the price of imports to fall. Data for the value of imports and exports of goods and services by product come from ONS Supply and Use Tables.
- 2.33. In each period the value of trade must balance, meaning the value of imports must equal the value of exports after accounting for the exchange rate. The foreign sector has fixed income and savings so any changes in exports and imports arise through changes in domestic competitiveness affecting the exchange rate. In addition, the UK is modelled as a small open economy so domestic activity cannot affect the world price of goods, which is fixed in the model.
- 2.34. Foreign investment flows in or out of the UK are not explicitly modelled. All investment is assumed to be domestically sourced. Similarly, repatriation of profits is not captured as all return to investment is allocated to the domestic corporate sector. A planned future model extension will include a more detailed treatment of the foreign sector.

## Model dynamics

- 2.35. The HMRC CGE model is dynamic as it tracks the evolution of the economy over time. The model is not a forecasting model, but it can show how a given forecast (defined in the baseline) may change following a tax change. Dynamics in the model are determined by capital accumulation and households' intertemporal utility optimisation.
- 2.36. Every year some of the existing capital stock depreciates and firms must replace obsolete capital with new capital in order to maintain output. New capital comes about via investment: what is not spent on consumption can be saved and invested, leading to additional capital in the future. The model accounts for the cost of installing new capital over and above the price of capital. This can be thought of as the cost of installing new equipment or training workers to operate new machinery. Initial investment flows in the model are calibrated to data from the GFCF matrix. More details on investment dynamics are given in appendix A.2.

2.37. Households that own capital earn a rate of return by renting capital to firms. Each year, households trade-off the cost of foregone consumption today against future consumption they can gain by earning a return on their investments, taking into account that future consumption is discounted. Households optimise utility across all time periods to maximise lifetime welfare subject to a lifetime budget constraint. Households exhibit 'perfect foresight' which means they can see changes in future prices and adjust their behaviour accordingly. The extent to which households value future consumption as opposed to current consumption, after accounting for discounting, is governed by the intertemporal elasticity of substitution. A higher intertemporal elasticity of substitution means households are more willing to substitute between current and future consumption. This means for pre-announced tax changes, a lower intertemporal elasticity of substitution would mean households start to adjust their behaviour earlier in order to smooth consumption across periods.

## Technical Annex

### A.1 Functional Forms

#### *Production Block*

**Nest-0:** Transformation of a sector's output into commodities is given by the *Constant Elasticity of Transformation (CET)* function.

$$q_i = \alpha_i \left( \sum_j \varepsilon_{j,i} q_{j,i}^{\left(\frac{\tau-1}{\tau}\right)} \right)^{\left(\frac{\tau}{\tau-1}\right)}$$

where  $\varepsilon$  is the commodity ( $j$ ) share in output of sector  $i$  ( $q_{ij}$ ) and  $\sum_j \varepsilon_{j,i} = 1$ .  $\tau$  is the elasticity of transformation between commodities.

**Nest-1:** Composite output of each sector determined by combining gross value-added and aggregated intermediate goods in a *Leontief function*.

$$q_i = \min[\lambda_i \cdot GVA_i, (1 - \lambda_i) \cdot INT_i]$$

where  $GVA$  is gross value added,  $INT$  is composite intermediate goods and  $\lambda$  is the share of  $GVA$  and  $INT$  in output.

**Nest-2 (GVA):** Gross value added is a combination of composite labour and composite capital given by a *Constant Elasticity of Substitution* function.

$$GVA_i = \varphi_i \left( \beta_i L_i^{\frac{(\sigma-1)}{\sigma}} + (1 - \beta_i) K_i^{\frac{(\sigma-1)}{\sigma}} \right)^{\frac{\sigma}{(\sigma-1)}}$$

where  $\sigma$  is the elasticity of substitution between labour and capital,  $L$  is composite labour and  $K$  is composite capital used for each sector.

**Nest-2 (INT):** Intermediate goods aggregate ( $INT$ ) for each sector is given as a *Leontief function* of all intermediate inputs.

$$INT_i = \min(\lambda_{1,i} \cdot qd_{j=1}, \lambda_{2,i} \cdot qd_{j=2}, \dots, \lambda_{n,i} \cdot qd_{j=n})$$

where  $qd$  is composite commodity input  $j$  and  $\lambda$  is the share of each input.  $qd_i$  is a CES aggregation of domestically produced and imported commodity  $j$ .

**Nest-3 (Capital):** Capital composite is an aggregate of debt and equity financing governed by a *Constant Elasticity of Substitution function*.

$$K_i = \left( \beta_i kd_i^{\frac{(\sigma-1)}{\sigma}} + (1-\beta_i) ke_i^{\frac{(\sigma-1)}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where  $\sigma$  is the elasticity of substitution between labour and capital,  $kd$  is capital from debt financing,  $ke$  is capital from equity financing and  $\beta$  is the share of debt financing.

**Nest-3 (Labour):** Professional ( $PR$ ), skilled ( $SK$ ) and unskilled ( $US$ ) labour types are aggregated into the labour composite ( $L$ ) using a *Constant Elasticity of Substitution function*.

$$L_i = \left( \beta_i PR_i^{\frac{(\sigma-1)}{\sigma}} + \delta_i SK_i^{\frac{(\sigma-1)}{\sigma}} + (1-\beta_i-\delta_i) US_i^{\frac{(\sigma-1)}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where  $\sigma$  is the elasticity of substitution between the types of labour,  $\beta$  is the share professional and  $\delta$  is the share of labour skilled labour.

### Consumption Block

**Inter-temporal utility function is given as a CES function of current and future utility ( $U_t$ )**

$$U_h = \left[ \sum_{t=1}^T \left( \frac{1}{1+\rho} \right)^t U_{h,t}^{\left( \frac{\lambda-1}{\lambda} \right)} \right]^{\left( \frac{\lambda}{\lambda-1} \right)}$$

where  $\lambda$  is intertemporal elasticity of substitution and  $\rho$  is the discount factor.

**Income balance**

$$\sum_{t=1}^T p_{u,t} U_{h,t} = \sum_{t=1}^T LabInc_{l,h} (1-T_{l,h}) + \sum_{t=1}^T InvInc_h + \sum_{t=1}^T GovTrans_h$$

where  $LabInc$  is income from work,  $T$  is income tax and employee NICs rate,  $InvInc$  is income from investment and  $GovTrans$  includes all transfers income from benefits.

**Nest-1:** Households' utility is a function of leisure and aggregate consumption given in a CES function

$$U_{h,t}(welfare_h) = \left( \alpha_h Leis_h^{\frac{(\sigma-1)}{\sigma}} + (1-\alpha_h)(Con_h)^{\frac{(\sigma-1)}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where  $\sigma$  is the elasticity of substitution between leisure ( $Leis$ ) and consumption ( $Con$ ).

**Nest-2 (Consumption):** Aggregate consumption is represented as a Stone-Geary function given as

$$U_h(Con_h) = \prod_i (qd_i - \pi_i)^{\delta_i}$$

where  $qd$  is a composite (imported and domestically produced) commodity and  $\pi$  represents the minimum requirement for each commodity.

**Linear expenditure function for consumption**

Demand function for each composite commodity.

$$dq_i = \alpha_i + \delta_i p_i^{-1} \left( EXP - \sum_j \alpha_j p_j \right)$$

where  $EXP$  is equal to total expenditure on consumption for each household, and  $dq$  is a composite good, comprised of a domestically produced commodity and a imported commodity.

**Nest-3:** Domestic consumption is a CES aggregate of imports and domestically produced goods given as

$$qd_j = \mu_j \left( \beta_j qa_j^{\frac{(\sigma-1)}{\sigma}} + (1-\beta_j)qm_j^{\frac{(\sigma-1)}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where  $\sigma$  is the Armington elasticity of substitution between imports ( $qm$ ) and domestic supply ( $qa$ ).

**Export Transformation**

Domestic composite commodity transformed into exported and domestic commodity via a CET function.

$$q_j = \eta_j \left( \beta qx_j^{\frac{(\tau-1)}{\tau}} + (1-\beta_j)qa_j^{\frac{(\tau-1)}{\tau}} \right)^{\frac{\tau}{\tau-1}}$$

where  $\tau$  is the elasticity of transformation between domestic consumption and exports,  $q$  is domestic composite commodity,  $qx$  is foreign consumption and  $qa$  is for domestic consumption.

## A.2 Investment Dynamics

Investment in the model is subject to installation costs whereby the cost of investment is related to the current level of capital stock, the magnitude of investment and a 'cost of capital adjustment' parameter. This implies that more rapid capital accumulation becomes increasingly costly:

$$I_{j,t} = J_{j,t} \left( 1 + \phi \frac{J_{j,t}}{2K_{j,t}} \right)$$

where  $\phi$  = adjustment cost of capital,  $I$  is gross investment and  $J$  is *net investment*.

When  $\phi$  is greater than zero, investment incurs additional costs over and above the purchase price of capital. This may include installation costs or learning and training costs.

Investment dynamics are based on the first order conditions from the optimising behaviour of firms and households. Investment is positive if the marginal benefit of new capital is greater than the marginal cost of investment. The ratio of marginal benefit to marginal cost is summed up in Tobin's marginal  $q$

$$q_t = \frac{\mu_t}{\lambda_t}$$

where  $\mu$  is the shadow price of capital, which takes into account the discounted marginal benefit of future consumption, adjusted for depreciation and investment costs.  $\lambda$  is the marginal cost of investment in terms of forgone current consumption. When  $q > 1$ , investment is positive. Hayashi (1982)<sup>3</sup> provides a good summary on the equivalence of Tobin's  $q$  and the neo-classical theory of profit maximisation and capital accumulation.

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<sup>3</sup> Hayashi, Fumio (1982): "Tobin's Marginal Q and Average Q: A Neoclassical Interpretation," *Econometrica*, 50(1), 213–224,